

Data-Aided Autoregressive Sparse Channel Tracking for OFDM Systems

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Abstract—In order to meet future communication system requirements, channel estimation over fast fading and frequency selective channels is crucial. In this paper, Space Alternated Generalized Expectation Maximization Maximum a Posteriori (SAGE-MAP) based channel estimation algorithm is proposed for Orthogonal Frequency Division Multiplexing (OFDM) systems for Autoregressive (AR) modeled time-varying sparse channels. Also, an initialization algorithm has been developed from the widely used sparse approximation algorithm Orthogonal Matching Pursuit (OMP), since the performance of SAGE algorithm strictly depends on initialization. The results show that multipath delay positions can be tracked successfully for every time instant using the proposed SAGE-MAP based approach.

Index Terms—OFDM, Autoregressive model, Fast Time-Varying, SAGE-MAP, OMP, sparse channel estimation.

I. INTRODUCTION

WIRELESS communication systems operate over frequency and time selective channels, which need to be equalized at the receiver side. Also, increasing demand for high data rates in high mobility environments has let the researchers look for more durable wireless communication systems to time variation. Orthogonal Frequency Division Multiplexing (OFDM) has been widely accepted by many communication standards because of its spectral efficiency and capability to mitigate frequency selectivity of the channel. OFDM converts frequency selective channel to be flat for each subchannel, dividing available bandwidth to subchannels. On the contrary to the advantages of OFDM systems, it needs strict time and frequency synchronization for reliable communication. In order to equalize channel effects at the receiver side, many estimation algorithms have been proposed, [1] - [3].

Recent studies showed that wireless communication channels exhibit a sparse structure. In other words, channel impulse response (CIR) consists of a few dominant randomly located impulses. Also with the advances in compressed sensing (CS) techniques [4], [5], it is shown that sparse signals can be reconstructed using less observations than sparse signal dimension under some conditions. As a result, exploited sparse channel structure can help to design more energy efficient and less complex algorithms.

Sparse channel estimation algorithms have been proposed for OFDM systems in the literature. In [6], a sparse channel estimation algorithm is proposed based on Matching Pursuit (MP) algorithm for non-mobile systems. Thus, channel coeffi-

cients are assumed to be constant over a symbol duration and the number of observation samples are greater than the channel length. In [6], MP algorithm performances are compared with Least Squares (LS) channel estimation performances. In [7], a theorem is proposed for the sufficient number of observations to obtain sparse signal. Corresponding to proposed theorem in [7], simulation results are presented. In [1], sparse multipath channel definition is made in the concept of compressed sensing, and LS and Lasso realization of sparse signal reconstruction is given. However, in [1] the channel is assumed to be time-invariant since non-mobile case is investigated as in [6]. On the other hand, [3] assumed that the channel is sparse and fast time-varying, where frequency dispersiveness of the channel is modeled with a few sparse basis expansion coefficients. Using Space Alternating Generalized Expectation Maximization (SAGE) algorithm these coefficients are estimated, where also an MP initialization algorithm has been proposed. In [8], a sparse AR process has been tracked by Kalman Filter and sparsity change has been estimated with CS algorithm. In [8], defining a threshold level for filtering error, channel sparsity pattern changes have been estimated with Dantzig Selector (DS) algorithm. In practical systems DS is too complex to realize.

In this paper, for fast time-varying and frequency selective environments such as high speed trains and aeronautical systems, a channel estimation algorithm exploiting sparsity has been developed. Time variation of multipath coefficients has been modeled as an AR process. Multipath delay positions are tracked for every time instant, which has not been studied in the literature to the best of our knowledge. SAGE algorithm based approaches need good initialization, therefore, OMP based estimation algorithm is adopted to estimate initial channel coefficients and delays. The proposed model is important for tracking fast time-varying sparse channels.

The remainder of the paper is organized as follows. Section II presents the system model of an OFDM based wireless mobile communication system over rapidly varying sparse channels and describes the main parameters. Section III proposes a new channel estimation algorithm based on SAGE-MAP method and an initialization algorithm using OMP. Section IV presents performance results of the proposed SAGE-MAP based channel tracking algorithm. Finally, Section V concludes the paper.

Throughout the paper following notations will be used.

Vectors are represented as boldface lowercase letters, e.g., \mathbf{x} , and matrices as boldface uppercase letters, e.g., \mathbf{X} . For vectors and matrices $(\cdot)^T$, $(\cdot)^*$, $(\cdot)^H$ and $E\{\cdot\}$ denote the transpose, conjugate, Hermitian transpose and expectation, respectively. $\mathbf{1}_{m \times n}$ and $\mathbf{0}_{m \times n}$ represents ones and zeros matrix with m rows and n columns. To take the diagonal of a matrix or diagonalize a vector $\text{diag}\{\cdot\}$ is used. $(\bar{\cdot})$ means that all elements in the set \mathcal{L} except l , where $l \in \mathcal{L}$. $\text{Re}\{x\}$ and $\text{Im}\{x\}$ represent real and imaginary part of x . The symbols \odot and \otimes denote Hadamard and Kronecker product, respectively. l_2 -norm represents $\|\mathbf{x}\| = \sqrt{\mathbf{x}^H \mathbf{x}}$ for complex valued \mathbf{x} column vector.

II. SYSTEM MODEL

OFDM systems with N subcarriers have been considered and all subcarriers are occupied by pilots. Even for the data-aided estimation, there are N -sample observation $r[n]$ and $(N+1)L$ unknowns, which are $\alpha^{(l)}[n]$ and $\eta^{(l)}$, where $n = 0, 1, 2, \dots, N-1$, $l = 0, 1, 2, \dots, L-1$, and L is the number of multipaths. Discrete-time channel coefficient and delay of the l th path are represented with $\alpha^{(l)}[n]$ and $\eta^{(l)}$, respectively. In this paper, it is assumed that l th multipath coefficient at n th time-sample, $\alpha^{(l)}[n]$, changes for each sampling time and the delay sample of the l th multipath, $\eta^{(l)}$, is constant.

An OFDM signal in discrete-time domain can be expressed as

$$s[n] = \frac{1}{N} \sum_{k=-N/2}^{N/2-1} d[k] e^{j\frac{2\pi}{N}nk}, \quad (1)$$

where $d[k]$ is digitally modulated OFDM data symbol which is sent from k th subchannel, n represents each time sample over one symbol duration, $T_{\text{sym}} = NT_s$ and T_s are the symbol duration of one OFDM symbol and the sampling period, respectively. Cyclic prefix (CP) is added in front of the OFDM symbol to avoid inter-symbol-interference (ISI), which occurs due to time dispersiveness of the channel. Received signal is the convolution of OFDM signal and CIR, where the discrete-time channel impulse response can be expressed as

$$h[n, \eta] = \sum_{l=0}^{L-1} \alpha^{(l)}[n] \delta[\eta - \eta^{(l)}]. \quad (2)$$

When wireless communication channels are modeled, it is widely assumed in the literature that normalized path delays are integer values. That is the multipath delays are at the sampling times.

Channel maximum delay spread τ_{max} should be less than cyclic prefix duration (T_{cp}), where multipath delays can be modeled as uniformly distributed. Therefore $\eta^{(l)} \in [0, N_{cp}-1]$, where $T_{cp} = N_{cp}T_s$, and shows the l th normalized path delay, which is in the interval of cyclic prefix length N_{cp} .

Using (1) and (2), CP removed received signal sample at time nT_s can be written as

$$\begin{aligned} r[n] &= \sum_{\eta=0}^{N_{cp}-1} h(n, \eta) s(n - \eta) + w[n] \\ &= \frac{1}{N} \sum_{l=0}^{L-1} \sum_{k=-N/2}^{N/2-1} d[k] e^{j\frac{2\pi}{N}k(n-\eta^{(l)})} \alpha^{(l)}[n] + w[n] \\ &= \mathbf{c}^H [n] \boldsymbol{\alpha}[n] + w[n], \end{aligned} \quad (3)$$

where

$$\begin{aligned} \mathbf{c}^H [n] &= \mathcal{F}^{-1} [n] \mathbf{D} \boldsymbol{\Phi} \\ \mathcal{F}^{-1} [n] &\stackrel{\text{def}}{=} \frac{1}{N} \left[e^{j\frac{2\pi}{N}(-N/2)n}, \dots, e^{j\frac{2\pi}{N}(N/2-1)n} \right] \\ \boldsymbol{\Phi} &\stackrel{\text{def}}{=} \left[\phi^{(0)}, \phi^{(1)}, \dots, \phi^{(N_{cp}-1)} \right] \\ \phi^{(l)} &\stackrel{\text{def}}{=} \left[e^{-j\frac{2\pi}{N}(-N/2)\eta^{(l)}}, \dots, e^{-j\frac{2\pi}{N}(N/2-1)\eta^{(l)}} \right]^T \\ \mathbf{D} &\stackrel{\text{def}}{=} \text{diag} \left\{ d[k] \Big|_{k=-N/2}^{N/2-1} \right\}. \end{aligned} \quad (4)$$

Here $\boldsymbol{\alpha}[n]_{N_{cp} \times 1}$ is the L -sparse channel coefficient vector. $w[n] \sim \mathcal{CN}(0, \sigma_w^2)$ is the additive channel noise. In other words only L elements of $\boldsymbol{\alpha}[n]$ are dominant coefficients, where $L \ll N_{cp}$ implies sparseness of the channel.

Channel coefficient variation over time is modeled as an AR process as

$$\alpha^{(l)}[n] = a \alpha^{(l)}[n-1] + b^{(l)} u^{(l)}[n]. \quad (5)$$

Here $\alpha^{(l)}[n]$ is the l th path channel coefficient at time instant n . $u^{(l)}[n] \sim \mathcal{CN}(0, 1)$ is the driving channel noise. AR process coefficients a and $b^{(l)}$ determine variation of the process and can be found using autocorrelation function of channel coefficients. It is straightforward that each path obeys Jakes' spectrum with the following autocorrelation function

$$\begin{aligned} \rho^{(l)}[n - n'] &= E\{\alpha^{(l)}[n] \alpha^{(l)*}[n']\} \\ &= \Omega^{(l)} J_0(2\pi f_D T_s (n - n')) \end{aligned} \quad (6)$$

with $\eta^{(l)}$ is given l th path channel power

$$\Omega^{(l)} |\eta^{(l)}| = \frac{\lambda}{(1 - e^{-\lambda})L} e^{-\lambda \frac{\eta^{(l)}}{N_{cp}}}, \quad \lambda \geq 0, \quad (7)$$

where λ is multipath power decaying constant and f_D is the maximum Doppler frequency. [2] By applying Yule-Walker equation and with the help of Jakes' autocorrelation function in (6), AR coefficients a and $b^{(l)}$ are determined as follows

$$\begin{aligned} a &= J_0(2\pi f_D T_s), \\ b^{(l)} &= \sqrt{\Omega^{(l)} (1 - J_0^2(2\pi f_D T_s))}. \end{aligned} \quad (8)$$

III. CHANNEL ESTIMATION

In the literature, if path delays are known, channel coefficients can be tracked by Kalman filtering easily. However, for unknown path delay case, Kalman filtering approach can not be applied. In this work, we propose a SAGE based

algorithm to find sparse channel coefficients sequentially as in [2] and [3]. SAGE algorithm, first proposed in [9], has faster convergence rate than Expectation Maximization (EM) algorithm. SAGE algorithm is the generalization of EM algorithm through updating unknown parameters sequentially rather than completely. When updating parameters in one subspace other parameters are held fixed. This process repeats for all subspaces. Unknown parameters' dependency on observation was introduced with admissible hidden data concept, where χ represents admissible hidden data. Also, SAGE algorithm is computationally more feasible for practical systems. SAGE-MAP first proposed in [2] is for highly mobile OFDM systems and simulated for LTE and mobile WIMAX systems.

In [9], it is stressed that choosing subspaces determines algorithm convergence. The current study deals with this problem defining unknown sets and subspaces as follows:

$$\begin{aligned}\theta[n] &= \{\alpha[n], \eta\} \\ \theta^{(l)}[n] &= \{\alpha^{(l)}[n], \eta^{(l)}\}\end{aligned}\quad (9)$$

where $\theta[n]$ represents unknown parameter set and $\theta^{(l)}[n]$ is the l th subspace, which will sequentially be estimated. SAGE-MAP algorithm was applied to OFDM systems in [2]. The main contribution of this paper to literature is that channel coefficients are estimated using AR process information and observation. SAGE-MAP algorithm consists of expectation and maximization steps.

Expectation Step:

$$\begin{aligned}Q(\theta^{(l)}[n] | \theta^{[i_n]}[n]) \\ = E[\log p(\theta^{(l)}[n], \chi_n^{(l)} | \overline{\theta^{(l)}}^{[i_n]}[n]) | \mathbf{r}_n, \theta^{[i_n]}[n]]\end{aligned}\quad (10)$$

Maximization Step:

$$\theta^{(l)[i_n+1]}[n] = \underset{\theta^{(l)}[n]}{\operatorname{argmax}} Q(\theta^{(l)}[n] | \theta^{[i_n]}[n])\quad (11)$$

$$\overline{\theta^{(l)}}^{[i_n+1]}[n] = \overline{\theta^{(l)}}^{[i_n]}[n] \equiv \begin{cases} \alpha^{(\bar{l})[i_n+1]}[n] = \alpha^{(\bar{l})[i_n]}[n] \\ \eta^{(\bar{l})[i_n+1]} = \eta^{(\bar{l})[i_n]} \end{cases}$$

As a result, multipath channel coefficients and delays can be sequentially found with the following expressions

$$\begin{aligned}\alpha^{(l)[i_n+1]}[n] &= \left(\frac{1}{\|b^{(l)}\|^2} + \frac{\|c_{\eta^{(l)}}[n]\|^2}{\sigma_w^2} \right)^{-1} \\ &\quad \left(\frac{c_{\eta^{(l)}}[n] \overline{\chi^{(l)[i_n]}[n]}}{\sigma_w^2} + \frac{a\alpha^{(l)[i_n]}[n-1]}{\|b^{(l)}\|^2} \right)\end{aligned}\quad (12)$$

with

$$\eta^{(l)[i_n+1]} = \underset{\eta^{(l)}}{\operatorname{argmax}} \mathbf{G}^{(l)[i_n]}[\eta^{(l)}]\quad (13)$$

where $\mathbf{G}^{(l)[i_n]}[\eta^{(l)}]$ is the objective function, given as

$$\begin{aligned}\mathbf{G}^{(l)[i_n]}[\eta^{(l)}] &= -\frac{\|\alpha^{(l)[i_n+1]}[n] - a\alpha^{(l)[i_n]}[n-1]\|^2}{\|b^{(l)}\|^2} \\ &\quad + \sum_{m=0}^n \left(2\operatorname{Re} \left\{ \frac{\alpha^{*(l)}[m]^{[i_n+1]} c_{\eta^{(l)}}[m] \overline{\chi^{(l)[i_n]}[m]}}{\sigma_w^2} \right\} \right. \\ &\quad \left. - \frac{\|c_{\eta^{(l)}}^*[m] \alpha^{(l)[i_n+1]}[m]\|^2}{\sigma_w^2} \right) - \log(\|b^{(l)}\|^2).\end{aligned}\quad (14)$$

Expectation of admissible hidden data can be represented as

$$\overline{\chi^{(l)[i_n]}[n]} = r[n] - \sum_{\substack{l'=0 \\ l' \neq l}}^{L-1} c_{\eta^{(l')}[i_n]}[n] \alpha^{(l')}[i_n][n].\quad (15)$$

A. Initialization of the Algorithm

Error performance of SAGE algorithm highly depends on initialization errors. Therefore, we need to apply a suitable algorithm to initialize channel estimation. Since the channel coefficients are sparse, it is appropriate to select the sparse approximation algorithm OMP to find initial estimates. OMP is a widely accepted practical and fast sparse signal recovery algorithm compared to other sparse signal recovery algorithms. In [7], steps of the OMP algorithm are outlined. To employ OMP, every channel coefficient can be represented in terms of initial channel coefficients. Therefore, using AR process the relation between initial and other channel coefficients can be expressed as

$$\alpha^{(l)}[n] = a^{n+1} \alpha^{(l)}[-1] + \sum_{k=0}^n a^k b^{(l)} u[n-k].\quad (16)$$

Received samples can be expressed using (16). It should be noted that, using all observations cause more complexity despite no additional performance contribution. Therefore, using M observations to find initial estimates of channel coefficients and path delays is enough for initial estimation:

$$\begin{aligned}\mathbf{r} &= \mathbf{D}\boldsymbol{\alpha}[-1] + \mathbf{v} \\ \mathbf{D} &= \mathbf{C}^H \odot (\mathbf{a} \otimes \mathbf{1}_{1 \times N_{cp}}) \\ \mathbf{r} &\stackrel{\text{def}}{=} [r[0], r[1], \dots, r[M-1]]^T \\ \mathbf{C}^H &\stackrel{\text{def}}{=} [c[0], c[1], \dots, c[M-1]]^H \\ \mathbf{a} &\stackrel{\text{def}}{=} [a^1, a^2, \dots, a^M]^T,\end{aligned}\quad (17)$$

Here, \mathbf{D} is the dictionary matrix with size $M \times N_{cp}$ and \mathbf{v} represents noise, which includes AWGN and driving noise. Mean and variance statistics of \mathbf{v} are not necessary for the OMP algorithm.

The algorithm steps are outlined in Algorithm 1. When the algorithm is first initialized, it takes first M received signal samples to process OMP and to find initial channel coefficients and delays. In [7], it is assumed that the number of non-zero coefficients in the sparse signal is known, however, in this paper, we used energy threshold as a stopping criterion. Then for every time sample, algorithm will sequentially find

Algorithm 1: SAGE-MAP Based OFDM Sparse Channel Tracking

Data: received signal sample $r[n]$, f_D , SNR

Result: tracked $\alpha^{(l)}[n]$'s and $\eta^{(l)}$'s

- Initialization through OMP;

for $n = 1$ *to* N **do**

- $\alpha^{(l)[i_n=0]}[n] = a^{(l)}\alpha^{(l)[i_n=i_{max}]}[n-1]$;
- $\eta^{(l)[i_n=0]} = \eta^{(l)[i_n=i_{max}]}$;
- Calculate $C_{\eta^{(l)[i_n]}}^H[n]$'s and $\chi^{(l)[i_n]}[n]$'s;

for *SAGE-MAP Step:* $i_n = 1$ *to* i_{max} **do**

for $l = 0$ *to* $L-1$ **do**

for $\eta^{(l)} = 0$ *to* $N_{cp}-1$ **do**

- Find $\Omega^{(l)}$ from (7), $b^{(l)}$ from (8) and $C_{\eta^{(l)}}^H[n]$ from (4)
- Find $\alpha^{(l)[i_n]}[n]$ from (12)
- Find objective function $\mathbf{G}^{(l)[i_n]}[\eta^{(l)}]$ from (14)

end

- Find $\eta^{(l)[i_n+1]}$ through maximizing (14);
- Recalculate $\Omega^{(l)}$, $b^{(l)}$, $C_{\eta^{(l)[i_n]}}^H[n]$
- Find $\alpha^{(l)[i_n+1]}[n]$ at $i_n + 1$ th step in (12)

end

- Update all $\chi^{(l)[i_n]}[n]$'s using (15);

end

the channel coefficients and delays jointly with SAGE and Matching Pursuit (MP) based approach. For each SAGE-MAP iteration all stacked admissible hidden data sets ($\chi^l[n]$) will be updated, and each multipath channel coefficient and delay will be estimated.

IV. SIMULATION RESULTS

Performance of the proposed algorithm will be assessed considering initialization and channel coefficients tracking. Simulation parameters are presented in Table I.

TABLE I
SIMULATION PARAMETERS

Number of subcarriers (N)	1024
Cyclic prefix duration	$128T_s$
Bandwidth	5 kHz
Maximum Doppler Frequency (f_D)	50 – 100 Hz
Number of multipaths (L)	3
Modulation mapping	QPSK
Number of received signal samples for initialization (M)	64
Number of SAGE Iterations	$i_{max} = 3$
Signal to Noise Ratio	SNR= 10, 20 dB

Proposed algorithm takes Doppler frequency and SNR value in addition to received signal as an input to estimate channel coefficients. After AR process coefficients are determined, OMP algorithm is employed. Then proposed SAGE-MAP tracking algorithm finds the channel coefficient and delay of l th path sequentially. The advantage of the proposed algorithm is that channel coefficients and delay positions are tracked real-time, which is different from the proposed algorithms in the literature. In the following, performance results will be provided.

Initialization Part Performance Results: Here, $H[n, k]$ is the channel frequency response (CFR) at the n th time sample and it is defined as

$$H[n, k] = \sum_{l=0}^{L-1} e^{-j\frac{2\pi}{N}k\eta^{(l)}} \alpha^{(l)}[n]. \quad (18)$$

Estimated discrete channel frequency response $\hat{H}[n, k]$ can be found by substituting $\alpha^{(l)}[n] = \alpha^{(l)[i_{max}]}[n]$ and $\eta^{(l)} = \eta^{(l)[i_{max}]}$. Initial estimations of CIR and CFR are plotted in Fig. 1 for $f_D = 50$ Hz and SNR= 20 dB.

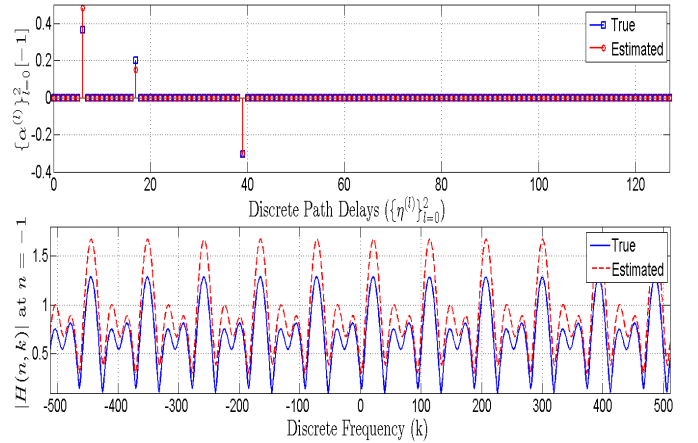


Fig. 1. Initial estimation of CIR and CFR

Increasing number of received signal samples and lowering maximum Doppler frequency values will improve the initialization algorithm performance.

SAGE-MAP Tracking Performance Results: Channel tracking results are presented in Figs. 2 to 5 for different f_D and SNR values. It can be inferred from the figures that the channel variation over time is tracked successfully for instantaneously changing channel coefficients and delays. Note that while the channel tracking is successful for all f_D and SNR values tested, the channel coefficients are estimated with some errors at SNR= 10 dB. This is mainly due to coefficients varying every time instant. It should also be noted that initialization errors are compensated while the channel tracking algorithm is running.

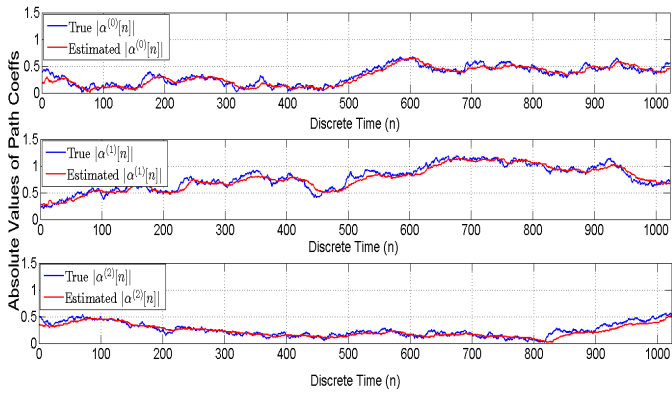


Fig. 2. SAGE-MAP algorithm channel tracking performance for $f_D = 50$ Hz, SNR= 20 dB

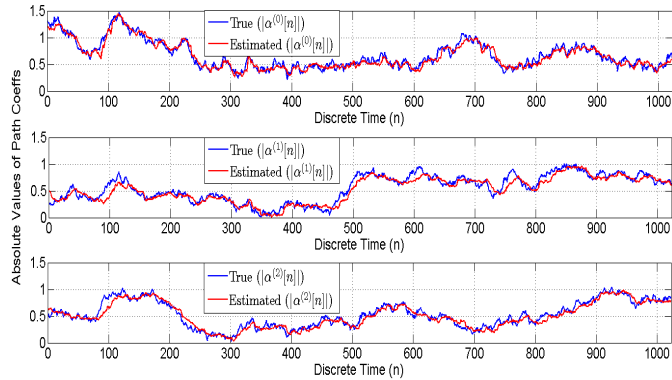


Fig. 3. SAGE-MAP algorithm channel tracking performance for $f_D = 100$ Hz, SNR= 20 dB

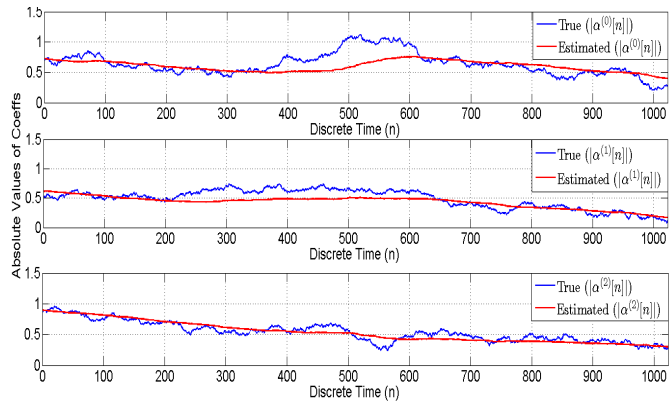


Fig. 4. SAGE-MAP algorithm channel tracking performance for $f_D = 50$ Hz, SNR= 10 dB

V. CONCLUSION

In this paper, a SAGE-MAP based channel estimation algorithm is proposed for tracking channel coefficients and delays at every iteration. The proposed algorithm is a solution for sparse AR process tracking. Since multipath delay positions are also tracked, any change on delays can be estimated in more extended models. This work can be extended to non-integer normalized delay positions and pilot-aided symbol detection and channel estimation case. For future work, mean-

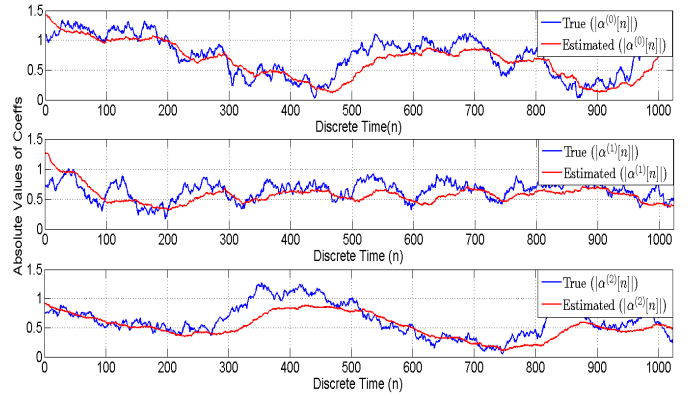


Fig. 5. SAGE-MAP algorithm channel tracking performance for $f_D = 100$ Hz, SNR= 10 dB

square error (MSE) performance results will be analyzed for the proposed tracking algorithm. In addition, complexity analysis will be pursued and complexity reduction will be investigated.

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REFERENCES

- [1] W. U. Bajwa, J. Haupt, G. Raz and R. Nowak. "Compressed Channel Sensing". Proceedings of the IEEE, vol.98(6), pp. 1058-1076, Apr. 2010.
- [2] H. Senol, E. Panayirci, H. V. Poor. "Nondata-Aided Joint Channel Estimation and Equalization for OFDM Systems in Very Rapidly Varying Mobile Channels". IEEE Transactions on Signal Processing, vol.60(8), pp. 4236-4253, July 2012.
- [3] H. Senol. "Joint Channel Estimation and Symbol Detection for OFDM Systems in Rapidly Time Varying Sparse Multipath Channels". Wireless Personal Communications, vol.82(3), 2015, pp. 1161-1178, June 2015. (Published Online: January 14, 2015)
- [4] E. Candes. "Compressive Sampling". Int. Congress of Mathematics, vol.3, pp. 1433-1452, 2006.
- [5] E. Candes and T. Tao. "The Dantzig Selector: statistical estimation when p is much larger than n". The Annals of Statistics, vol.35(6), version-3, pp. 2313-2351, 2007.
- [6] S. F. Cotter, B. D. Rao. "Sparse Channel Estimation via Matching Pursuit With Application to Equalization". IEEE Transactions on Communications, vol. 50(3), pp. 374-377, 2002.
- [7] J. A. Tropp and A. C. Gilbert. "Signal Recovery From Random Measurements Via Orthogonal Matching Pursuit". IEEE Transactions on Information Theory, vol.53(12), pp. 4655-4666, 2007.
- [8] N. Vaswani. "Kalman Filtered Compressed Sensing". 15th IEEE International Conference on Image Processing, 2008, pp. 893-896.
- [9] J.A. Fessler, A.O. Hero. "Space-Alternating Generalized Expectation-Maximization Algorithm". IEEE Transactions on Signal Processing, vol.42(10), pp. 2664-2677, Oct. 1994.
- [10] B. Chen, Q. Cui, F. Yang, J. Xu. "A Novel Channel Estimation Method Based On Kalman Filter Compressed Sensing for Time-Varying OFDM System". Sixth International Conference on Wireless Communications and Signal Processing (WCSP), 2014, pp. 1-5.
- [11] G.L. Stuber. "Principles of Mobile Communications". London: Springer Science Business Media, 2012.
- [12] M. Elad. "Sparse and Redundant Representations From Theory to Applications in Signal and Image Processing". New York: Springer Science Business Media, 2010, pp.35-76.
- [13] S.M. Kay. "Fundamentals of Statistical Signal Processing Volume I: Estimation Theory". New Jersey: Prentice Hall, 1993.