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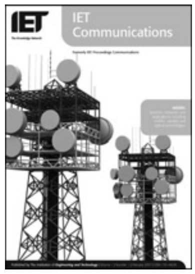

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An efficient joint channel estimation and decoding algorithm for turbo-coded space–time orthogonal frequency division multiplexing receivers

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Abstract: The challenging problem in the design of digital receivers of today's and future high-speed, high data-rate wireless communication systems is to implement the optimal decoding and channel estimation processes jointly in a computationally feasible way. Without realising such a critical function perfectly at receiver, the whole system will not work properly within the desired performance limits. Unfortunately, direct implementation of such optimal algorithms is not possible mainly due to their mathematically intractable and computationally prohibitive nature. A novel algorithm that reaches the performance of the optimal maximum a posteriori (MAP) algorithm with a feasible computational complexity is proposed. The algorithm makes use of a powerful statistical signal processing tool called the expectation–maximisation (EM) technique. It iteratively executes the MAP joint channel estimation and decoding for space–time block-coded orthogonal frequency division multiplexing systems with turbo channel coding in the presence of unknown wireless dispersive channels. The main novelty of the work comes from the facts that the proposed algorithm estimates the channel in a non-data-aided fashion and therefore except a small number of pilot symbols required for initialisation, no training sequence is necessary. Also the approach employs a convenient representation of the discrete multipath fading channel based on the Karhunen–Loeve (KL) orthogonal expansion and finds MAP estimates of the uncorrelated KL series expansion coefficients. Based on such an expansion, no matrix inversion is required in the proposed MAP estimator. Moreover, optimal rank reduction is achieved by exploiting the optimal truncation property of the KL expansion resulting in a smaller computational load on the iterative estimation approach.

1 Introduction

Future wireless communication systems aim to provide various multimedia services, where high-speed data transmission needs to be reliably transmitted. The attainment to succeed high data rates reliably on wireless channels is severely restricted by effects of channel fading. In such a channel, increasing the quality or reducing the effective error rate is extremely difficult.

Space–time block coding (STBC) was introduced in [1] as an effective transmit diversity technique for mitigating the detrimental effects of channel fading and it was later expanded in [2] for an arbitrary number of antennas. Unfortunately, their practical application can present a real challenge to channel estimation algorithms, especially when the signal suffers from frequency-selective multipath channels. One of the solutions alleviating the frequency selectivity is the use of orthogonal frequency division multiplexing (OFDM) together with transmit diversity which combats long channel impulse response by transmitting parallel symbols over many orthogonal subcarriers yielding a unique reduced complexity physical layer capabilities

STBC is not designed to achieve an additional coding gain. Therefore an outer channel code is applied in addition to transmit diversity to further improve the receiver performance. Recent trends in coding favour parallel and/or serially concatenated coding and probabilistic soft-decision iterative (turbo-style) decoding. Such codes are able to exhibit near-Shannon-limit performance with reasonable complexities in many cases and are of significant interest for communications applications that require moderate error rates. We therefore consider the combination of turbo codes with the transmit diversity OFDM systems. Especially we address the design of iterative channel estimation approach for transmit diversity OFDM systems employing an outer channel code.

There have been several studies to estimate channel for transmit diversity OFDM systems. Most of the early studies focus on channel estimation for transmit diversity OFDM systems subjected on uncoded systems. Since the necessity or desirability of using error-correcting codes to protect data is crucial for wireless communication systems, more recent work have addressed coded transmit diversity OFDM systems. Since direct calculation of estimation is computationally prohibitive, expectation–maximisation (EM) algorithm is a good candidate that can iteratively approximate the maximum-likelihood (ML) estimate with practical complexity. Therefore an iterative procedure based on EM applied to channel estimation problem in the context of STBC [3–5] as well as transmit diversity OFDM systems with or without outer channel coding (e.g. convolutional code or turbo code) [6–9]. In [6], maximum a posteriori (MAP) EM-based iterative receivers for STBC-OFDM systems with turbo code are proposed to detect transmitted symbols, directly, assuming that the fading process remains constant across several OFDM words contained in one STBC code word. In [7], an EM approach, proposed for the general estimation from superimposed signals [10], is applied to the channel estimation for transmit diversity OFDM systems with outer channel code (convolutional code) and compared with SAGE version. In [8], a modified version of [7] is suggested for STBC-OFDM and space–frequency block-coding (SFBC)-OFDM systems. Moreover, Kashima *et al.* [9] proposed two new types of MAP receivers for multiple-input-multiple-output and OFDM systems with a channel coding such as the low-density parity-check code. One proposed receiver employs the EM algorithm so as to improve the performance of the approximated MAP detection.

Assuming that the channel varies between every adjacent OFDM symbols, a non-data-aided EM-MAP channel estimation algorithm was proposed [11] for SFBC-OFDM systems unlike the EM approaches used in [6–8]. This receiver structure was also extended to the turbo/convolutionally coded SFBC-OFDM systems [12] assuming that the complex channel gains between adjacent subcarriers are approximately constant. In [12], it was shown that this approach was more effective for time-varying channels. On the other hand, for highly frequency-selective channels, it is obvious that the bit error rate (BER) performances of the turbo/convolutionally coded SFBC-OFDM systems degrade substantially and cause error floors for high SNR values. Therefore in this paper, we extend the work presented in [12] to the turbo receiver structures of the turbo/convolutionally coded STBC-OFDM systems and propose an EM-MAP channel estimation algorithm that reaches the performance of the optimal MAP algorithm with a feasible computational complexity. The algorithm makes use of the EM technique. It iteratively executes the MAP joint channel estimation and decoding for STBC-OFDM systems with turbo channel coding in the presence of unknown wireless dispersive channels. In [12], binary phase shift keying (BPSK) modulation was considered for the channel model having an exponentially decaying power profile [13]. In the current work, both BPSK and high-level modulation schemes are considered in the presence of COST-207 channel model. Moreover, the effects of the correlation mismatch and increasing frame length of the turbo-coded structure are also investigated and important conclusions are drawn for practical considerations.

The rest of the paper is organised as follows. In Section 2, we introduce the signal model for encoded STBC-OFDM systems and corresponding channel model is established. In Section 3, the proposed channel estimation and iterative equalisation and decoding algorithms are presented. Computer simulation results are given with detailed discussion in Section 4, and finally conclusions are drawn in Section 5.

Notation: Vectors (matrices) are denoted by bold italics lower (upper) case letters; all vectors are column vectors; $(\cdot)^*$, $(\cdot)^T$ and $(\cdot)^\dagger$ denote the conjugate, transpose and conjugate transpose, respectively; $\mathbf{I}_L$ denotes the $L \times L$ identity matrix.

2 Received signal model for encoded STBC-OFDM systems

In order to compensate for the reduced data rate of turbo codes, some ST codes having data rates greater than one could be employed. However, it is well known from literature that the Alamouti antenna modulation configuration is the only scheme that retains orthogonality and full rate when the complex-valued data are involved and the low complexity is concerned [1]. As will be seen shortly, orthogonality property is an essential and a required condition for the channel estimation algorithm in our work. Moreover, orthogonality structure of Alamouti allows the decoupling of the channel and reduces the equaliser complexity. Note that the Alamouti's schemes has been adopted in several wireless standards such as WCDMA and CDMA2000. It imposes an orthogonal spatio-temporal structure on the transmitted symbols that guarantees full (i.e. order 2) spatial diversity. In addition to the spatial level, to realise multipath diversity gains over frequency-selective channels, the Alamouti block-coding scheme is implemented at a block level in frequency domain.

As illustrated in Fig. 1, the binary information bits $\boldsymbol{b}$ of random data are encoded by an outer-channel code, resulting in a coded symbol stream $\{\boldsymbol{C}\}$. The coded symbols are then interleaved by a random permutation resulting in a stream of independent symbols. A code-bit interleaver reduces the probability of error bursts and removes correlation in coded symbol stream. The coded and interleaved symbols form a block of independent symbol stream of length $2N_c$, denoted by $\boldsymbol{X} = [X_0, X_1, \ldots, X_{2N_c-1}]^T$. Resorting $\boldsymbol{X}$ into two consecutive disjoint subchannel blocks $\boldsymbol{X}(n)$ and $\boldsymbol{X}(n+1)$ as

$$\boldsymbol{X}(n) = \left[X_0, X_2, \ldots, X_{2N_c-2}\right]^T, \qquad \boldsymbol{X}(n+1) = \left[X_1, X_3, \ldots, X_{2N_c-1}\right]^T \tag{1}$$

the ST encoder maps them to the following $2N_c \times 2$ matrix

$$\text{space} \downarrow \text{antenna\#1} \begin{bmatrix} \boldsymbol{X}(n) & -\boldsymbol{X}^*(n+1) \\ \boldsymbol{X}(n+1) & \boldsymbol{X}^*(n) \end{bmatrix}$$

whose columns are transmitted in successive time intervals with the upper and lower blocks in a given column simultaneously through the first and second transmitted antennas, respectively as shown in Fig. 1.

The frequency-selective wireless channel is assumed to be a quasi-static so that path gains are constant over a frame of length L_{frame} and vary from one frame to another. If the frequency response of the channel between the lth ($l = 1, 2$) transmitter and the receiver is denoted by $H_l(f, t)$, then the discrete channel coefficients $\{H_l(k, n)\}$ at the kth subcarrier frequency and nth time instant is given as

$$H_l(k, n) = H_l\left(\frac{k}{N_c T_s}, n\right), \quad k = 0, 1, \ldots, N_c - 1 \tag{2}$$

where T_s is the sampling period of the OFDM symbol. They are correlated samples, in frequency, of a complex Gaussian process. Moreover, $\boldsymbol{H}_l(n) = [H_l(0, n), \ldots, H_l(N_c - 1, n)]^T$ denotes the channel vector between the lth transmitter and the receiver.

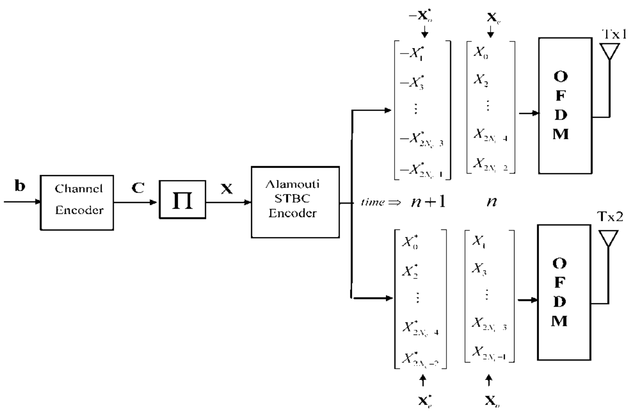

Figure 1 *Transmitter structure for STBC-OFDM with outer-channel coding*

Since the Alamouti's two-branch transmit diversity scheme with one receiver is employed here, each pair of the two consecutive received signal can be expressed in vector form as

$$\begin{aligned} \boldsymbol{R}(n) &= \mathcal{X}(n)\boldsymbol{H}_1(n) + \mathcal{X}(n+1)\boldsymbol{H}_2(n) + \boldsymbol{W}(n) \\ \boldsymbol{R}(n+1) &= -\mathcal{X}^*(n+1))\boldsymbol{H}_1(n+1) + \mathcal{X}^*(n)\boldsymbol{H}_2(n+1) \\ &\quad + \boldsymbol{W}(n+1) \end{aligned} \tag{3}$$

where $\boldsymbol{R}(n) = [R_0, R_2, \ldots, R_{2N_c-2}]^T$, $\boldsymbol{R}(n+1) = [R_1, R_3, \ldots, R_{2N_c-1}]^T$. $\mathcal{X}(n)$ and $\mathcal{X}(n+1)$ are $N_c \times N_c$ diagonal matrices whose elements are $\boldsymbol{X}_e$ and $\boldsymbol{X}_o$, respectively. Finally, $\boldsymbol{W}(n)$ and $\boldsymbol{W}(n+1)$ are $N_c \times 1$ zero-mean, i.i.d. Gaussian vectors that model additive noise in the N_c tones.

Equation (3) shows that the information symbols $\mathcal{X}(n)$ and $\mathcal{X}(n+1)$ are transmitted twice in two consecutive time intervals through two different channels. To simplify the problem, we assume that the complex channel gains remain constant over the duration of one ST-OFDM code word, that is, $\boldsymbol{H}_1(n) \simeq \boldsymbol{H}_1(n+1)$ and $\boldsymbol{H}_2(n) \simeq \boldsymbol{H}_2(n+1)$. In order to estimate the channels and decode $\boldsymbol{X}$ with the embedded diversity gain through the repeated transmission, for each n, we define, $\boldsymbol{R} = [\boldsymbol{R}^T(n) \quad \boldsymbol{R}^T(n+1)]^T$ and write (3) into a matrix form

$$\boldsymbol{R} = \mathcal{X}\boldsymbol{H} + \boldsymbol{W} \tag{4}$$

where $\boldsymbol{H} = [\boldsymbol{H}_1^T \quad \boldsymbol{H}_2^T]^T$, $\boldsymbol{W} = [\boldsymbol{W}^T(n) \quad \boldsymbol{W}^T(n+1)]^T$ and

$$\mathcal{X} = \begin{bmatrix} \mathcal{X}(n) & \mathcal{X}(n+1) \\ -\mathcal{X}^*(n) & \mathcal{X}^*(n+1) \end{bmatrix} \tag{5}$$

Obviously, channel estimation is very essential in digital receivers for decoding of the STBC-OFDM systems with outer-channel encoder. In this paper, a novel channel estimation algorithm is presented by representing the discrete multipath channel based on the Karhunen–Loeve (KL) orthogonal representation and making use of the EM technique.

Modelling the frequency-selective fading channels as random processes, we employ a linear expansion based on the KL series representation involving a complete set of orthogonal deterministic vectors with the corresponding uncorrelated random coefficients. An orthonormal expansion of the vector $\boldsymbol{H}_l(n)$ involves expressing the $\boldsymbol{H}_l(n)$ as a linear combination of the orthonormal basis vectors as $\boldsymbol{H}_l(n) = \boldsymbol{\Psi}\boldsymbol{G}_l(n)$, where $\boldsymbol{\Psi} = [\boldsymbol{\psi}_0, \boldsymbol{\psi}_1, \ldots, \boldsymbol{\psi}_{N_c-1}]$, $\boldsymbol{\psi}_i$'s are the orthonormal basis vectors, $\boldsymbol{G}_l(n) = [G_1(0, n), \ldots, G_l(N_c - 1, n)]^T$, and $G_l(k, n)$ represent the weights of the expansion. The autocorrelation matrices of all the channels between transmitter antennas and receiver are

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same and it can be decomposed as

$$\boldsymbol{C}_H = \boldsymbol{\Psi}\boldsymbol{\Lambda}\boldsymbol{\Psi}^\dagger \tag{6}$$

where $\boldsymbol{\Lambda} = E\{\boldsymbol{G}_l\boldsymbol{G}_l^\dagger\}$. The KL expansion is one where $\boldsymbol{\Lambda}$ of (6) is a diagonal matrix (i.e. the coefficients are uncorrelated). If $\boldsymbol{\Lambda}$ is diagonal, then (6) must be eigendecomposition of $\boldsymbol{C}_H$. The fact that only the eigenvectors diagonalise $\boldsymbol{C}_{H_l}$ leads to the desirable property that the KL coefficients are uncorrelated. Furthermore, in the Gaussian case, the uncorrelatedness of the coefficients renders them independent as well, providing additional simplicity. Thus, the channel estimation problem in this application equivalent to estimating the i.i.d. Gaussian vector $\boldsymbol{G}_l$ KL expansion coefficients.

The channels between transmitter and receiver in this paper are assumed to be doubly selective, where $\boldsymbol{H}_l(n)$'s modelled based on realistic channel model determined by COST-207 project in which typical urban (TU), bad urban (BU), hilly terrain (HT) and rural area (RA) channel model are considered and their channel transfer functions are given.

3 Turbo receiver

Iterative decoding and detection has been a topic of recent interest since the introduction of turbo codes. Iterative receiver structure for coded STBC-OFDM system is illustrated in Fig. 2 which uses three submodules to carry on iterative process: channel estimation, STBC-OFDM decoding and the MAP outer-channel code decoding. We therefore consider an EM-based MAP iterative channel estimation technique in frequency domain for turbo-coded STBC-OFDM systems. Frequency-domain estimator presented in this paper was inspired by the conclusions in [14] in which it has been shown that the time-domain channel estimators based on a discrete Fourier transform approach for non-sample-spaced channels cause aliased spectral leakage and result in an error floor.

3.1 EM-based MAP channel estimation

The MAP criterion is used in the fading channel as seen at the FFT output of the OFDM receiver. Then, the probability density function of the uncorrelated channel vectors $\boldsymbol{G}_l$'s can be expressed as

$$p(\boldsymbol{G}_l) \sim \exp\left(-\boldsymbol{G}_l^\dagger \widetilde{\boldsymbol{\Lambda}}^{-1}\boldsymbol{G}_l\right), \quad l = 1, 2 \tag{7}$$

where $\widetilde{\boldsymbol{\Lambda}} = \mathrm{diag}(\boldsymbol{\Lambda}\ \ \boldsymbol{\Lambda})$. Since we assumed that the channel covariance matrix $\boldsymbol{C}_h$ is known by the receiver, the diagonal matrix $\boldsymbol{\Lambda}$ can be determined easily by the eigendecomposition of $\boldsymbol{C}_h$ in (6). Given the transmitted signals $\boldsymbol{X}$ as coded according to Alamouti's scheme and the discrete channel representations $\boldsymbol{G} = [\boldsymbol{G}_1^{\mathrm{T}}, \boldsymbol{G}_2^{\mathrm{T}}]^{\mathrm{T}}$ and taking into account the independence of the noise components, the conditional probability density function of the received signal $\boldsymbol{R}$ can be expressed as,

$$p(\boldsymbol{R}|\mathcal{X}, \boldsymbol{G}) \sim \exp\left[-(\boldsymbol{R} - \mathcal{X}\widetilde{\boldsymbol{\Psi}}\boldsymbol{G})^\dagger \widetilde{\boldsymbol{\Sigma}}^{-1}(\boldsymbol{R} - \mathcal{X}\widetilde{\boldsymbol{\Psi}}\boldsymbol{G})\right] \tag{8}$$

where $\widetilde{\boldsymbol{\Psi}} = \mathrm{diag}(\boldsymbol{\Psi}\ \ \boldsymbol{\Psi})$ and $\widetilde{\boldsymbol{\Sigma}} = \mathrm{diag}(\boldsymbol{\Sigma}\ \ \boldsymbol{\Sigma})$ and $\boldsymbol{\Sigma}$ is an $N_c \times N_c$ diagonal matrix with $\boldsymbol{\Sigma}[k, k] = \sigma^2$, for $k = 0, 1, \ldots, N_c - 1$.

The MAP estimate $\widehat{\boldsymbol{G}}$ is given by

$$\widehat{\boldsymbol{G}} = \arg\max p(\boldsymbol{G}|\boldsymbol{R}) \tag{9}$$

Directly solving this equation is mathematically intractable. However, the solution can be obtained easily by means of the iterative EM algorithm. This algorithm inductively reestimate $\boldsymbol{G}$ so that a monotonic increase in the a posteriori conditional pdf in (9) is guaranteed. The monotonic increase is realised via the maximisation of the auxiliary function

$$Q(\boldsymbol{G}|\boldsymbol{G}^{(q)}) = \sum_{\boldsymbol{X}} p(\boldsymbol{R}, \mathcal{X}, \boldsymbol{G}) \log p(\boldsymbol{R}, \mathcal{X}, \boldsymbol{G}^{(q)}) \tag{10}$$

where the sum is taken over all possible transmitted data-coded signals and $\boldsymbol{G}^{(q)}$ is the estimation of $\boldsymbol{G}$ at the qth iteration.

Note that the term $\log p(\boldsymbol{R}, \mathcal{X}, \boldsymbol{G})$ in (11) can be expressed as

$$\log p(\boldsymbol{R}, \mathcal{X}, \boldsymbol{G}) = \log p(\mathcal{X}|\boldsymbol{G}) + \log p(\boldsymbol{R}|\mathcal{X}, \boldsymbol{G}) + \log p(\boldsymbol{G}) \tag{11}$$

The first term on the right-hand side of (12) is constant, since the data sequence $\mathcal{X} = \{X_k(n)\}$ and $\boldsymbol{G}$ are independent of each other and $\mathcal{X}$ have equal a priori probability. Therefore (11) can be evaluated by means of (7) and (8). Given the received signal $\boldsymbol{R}$, the EM algorithm starts with an initial value $\boldsymbol{G}^0$ of the unknown channel parameters $\boldsymbol{G}$. The $(q+1)$th estimate of $\boldsymbol{G}$ is obtained by the maximisation step described by

$$\boldsymbol{G}^{(q+1)} = \arg\max_{\boldsymbol{G}} Q(\boldsymbol{G}|\boldsymbol{G}^{(q)})$$

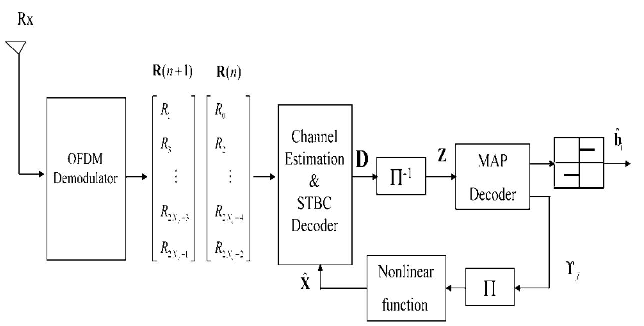

Figure 2 *Turbo-coded STBC-OFDM receiver*

After long algebraic manipulations, the expression of the re-estimate $\hat{\boldsymbol{G}}_l^{(q+1)}(l=1,2)$ can be obtained as follows [11]

$$\hat{\boldsymbol{G}}_1^{(q+1)} = (\boldsymbol{I}+\boldsymbol{\Sigma}\boldsymbol{\Lambda}^{-1})^{-1}\boldsymbol{\Psi}^{\dagger}\left[\hat{\mathcal{X}}_e^{\dagger(q)}\boldsymbol{R}(n) - \hat{\mathcal{X}}_o^{(q)}\boldsymbol{R}(n+1)\right]$$
$$\hat{\boldsymbol{G}}_2^{(q+1)} = (\boldsymbol{I}+\boldsymbol{\Sigma}\boldsymbol{\Lambda}^{-1})^{-1}\boldsymbol{\Psi}^{\dagger}\left[\hat{\mathcal{X}}_o^{\dagger(q)}\boldsymbol{R}(n) + \hat{\mathcal{X}}_e^{(q)}\boldsymbol{R}(n+1)\right] \quad (12)$$

where it can be easily seen that

$$((\boldsymbol{I}+\boldsymbol{\Sigma}\boldsymbol{\Lambda}^{-1})^{-1}) = \mathrm{diag}\left\{\frac{\lambda_0}{\lambda_0+\sigma^2}, \ldots, \frac{\lambda_{N_c-1}}{\lambda_{N_c-1}+\sigma^2}\right\}$$

and $\hat{\mathcal{X}}_e^q$ and $\hat{\mathcal{X}}_o^q$ in (12) is an $N_c \times N_c$-dimensional diagonal matrix representing the a posteriori probabilities of the data symbols at the qth iteration step.

3.1.1 Truncation property: The truncated basis vector $\boldsymbol{G}_{l,r}$ can be formed by selecting r orthonormal basis vectors among all basis vectors that satisfy $\boldsymbol{C}_{\boldsymbol{H}_l}\boldsymbol{\Psi} = \boldsymbol{\Psi}\boldsymbol{\Lambda}$. The optimal solution that yields the smallest average mean-squared truncation error $1/N_c E[\boldsymbol{\epsilon}_r^{\dagger}\boldsymbol{\epsilon}_r]$ is the one expanded with the orthonormal basis vectors associated with the first largest r eigenvalues as given by

$$\frac{1}{N_c - r}E[\boldsymbol{\epsilon}_r^{\dagger}\boldsymbol{\epsilon}_r] = \frac{1}{N_c - r}\sum_{i=r}^{N_c-1}\lambda_i \quad (13)$$

where $\boldsymbol{\epsilon}_r = \boldsymbol{G}_l - \boldsymbol{G}_{l,r}$. For the problem at hand, truncation property of the KL expansion results in a low-rank approximation as well. Thus, a rank-r approximation of $\boldsymbol{\Lambda}$ can be defined as $\boldsymbol{\Lambda}_r = \mathrm{diag}\{\lambda_0, \lambda_1, \ldots, \lambda_{r-1}\}$ by ignoring the trailing $N_c - r$ variances $\{\lambda_l\}_{l=r}^{N_c-1}$, since they are very small compared with the leading r variances $\{\lambda_l\}_{l=0}^{r-1}$. Actually, the pattern of eigenvalues for $\boldsymbol{\Lambda}$ typically splits the eigenvectors into dominant and subdominant sets. Then the choice of r is more or less obvious.

3.1.2 Complexity: Based on the approach presented in [13], the traditional linear minimum mean squared error (LMMSE) estimation for $\boldsymbol{H}_l$ can be easily expressed as

$$\hat{\boldsymbol{H}}_l = \underbrace{\boldsymbol{C}_{\boldsymbol{H}_l}(\boldsymbol{\Sigma}+\boldsymbol{C}_{\boldsymbol{H}_l})^{-1}}_{\text{Precomputed}}\boldsymbol{P}_{\mu}, \quad \mu = 1, 2 \quad (14)$$

where $\boldsymbol{P}_1 = \hat{\mathcal{X}}_e^{\dagger(q)}\boldsymbol{R}(n) - \hat{\mathcal{X}}_o^{(q)}\boldsymbol{R}(n+1)$ and $\mathbf{P}_2 = \hat{\mathcal{X}}_o^{\dagger(q)}\boldsymbol{R}(n) + \hat{\mathcal{X}}_e^{(q)}\boldsymbol{R}(n+1)$. Since $\boldsymbol{C}_{\boldsymbol{H}_l}(\boldsymbol{\Sigma}+\boldsymbol{C}_{\boldsymbol{H}_l})^{-1}$ does not change with data symbols, its inverse can be pre-computed and stored during each OFDM block. Since $\boldsymbol{C}_{\boldsymbol{H}_l}$ and $\boldsymbol{\Sigma}$ are assumed to be known at the receiver, the estimation algorithm in (14) requires N_c^2 complex multiplications after precomputation (Multiplication of $N_c \times N_c$ precomputation matrix with $N_c \times 1P_l$ vector.). However, this direct approach has high computational complexity due to the requirement of large-scale matrix inversion of the precomputation matrix (The computational complexity of an $N_c \times N_c$ matrix inversion, using Gaussian elimination, is $O((N_c)^3)$.). Moreover, the error caused by the small fluctuations in $\boldsymbol{C}_{\boldsymbol{H}_l}$ and $\boldsymbol{\Sigma}$ have an amplified effect on the channel estimation due to the matrix inversion. Furthermore, this effect becomes more severe as the dimension of the matrix, to be inverted, increases [15]. Therefore the KL-based approach is need to avoid matrix inversion. Using (6) and (14), iterative estimate of the $\boldsymbol{H}_l$ with KL expansion can be obtained as

$$\hat{\boldsymbol{H}}_l^{(q+1)} = \boldsymbol{\Psi}((\boldsymbol{I}+\boldsymbol{\Sigma}\boldsymbol{\Lambda}^{-1})^{-1})\boldsymbol{\Psi}^{\dagger}\boldsymbol{P}_1 \quad (15)$$

To reduce the complexity of the estimator further, we proceed with the low-rank approximations by considering only r column vectors of $\boldsymbol{\Psi}$ corresponding to the r largest eigenvalues of $\boldsymbol{\Lambda}$, yielding

$$\hat{\boldsymbol{H}}_l^{(q+1)} = \boldsymbol{\Psi}_r\underbrace{((\boldsymbol{I}+\boldsymbol{\Sigma}_r\boldsymbol{\Lambda}_r^{-1})^{-1})\boldsymbol{\Psi}_r^{\dagger}}_{\text{precomputation}}\mathbf{P}_1 \quad (16)$$

where $((\boldsymbol{I}+\boldsymbol{\Sigma}_r\boldsymbol{\Lambda}_r^{-1})^{-1}) = \mathrm{diag}(\lambda_0/(\lambda_0+\sigma^2), \ldots, \lambda_{r-1}/(\lambda_{r-1}+\sigma^2))$. $\boldsymbol{\Sigma}_r$ in (16) is a $r \times r$ diagonal matrix whose elements are equal to σ^2 and $\boldsymbol{\Psi}_r$ is an $N_c \times r$ matrix which can be formed by omitting the last $N_c - r$ columns of $\boldsymbol{\Psi}$. The low-rank estimator is shown to require $2N_c r$ complex multiplications (First, multiplication of precomputation matrix with P_{μ} has $N_c r$ complex multiplications and then multiplication with $\boldsymbol{\Psi}_r$ has $N_c r$ complex multiplication which totally requires $2N_c r$ complex multiplication.). In comparison with the estimator (traditional), the number of multiplications has been reduced from N_c to $2r$ per tone.

3.2 Iterative equalisation and decoding

We now consider the STBC-OFDM decoding algorithm and the MAP outer-channel code decoding to complete the description of the turbo receiver. Since the channel vectors or equivalently expansion coefficients are estimated through EM-based iterative approach, it is possible to decode $\boldsymbol{R}$ with diversity gains by a simple matrix multiplication. Before dealing with how we resolve decoding, let us first re-express received signal model (3) as

$$\tilde{\boldsymbol{R}} = \mathcal{H}\tilde{\boldsymbol{X}} + \tilde{\boldsymbol{W}} \quad (17)$$

where $\tilde{\boldsymbol{R}} = [\boldsymbol{R}^{\mathrm{T}}(n), \boldsymbol{R}^{\dagger}(n+1)]^{\mathrm{T}}, \tilde{\boldsymbol{X}} = [\boldsymbol{X}_e^{\mathrm{T}}, \boldsymbol{X}_o^{\mathrm{T}}]^{\mathrm{T}}, \tilde{\boldsymbol{W}} = [\boldsymbol{W}^{\mathrm{T}}(n), \boldsymbol{W}^{\dagger}(n+1)]^{\mathrm{T}}$ and

$$\mathcal{H} = \begin{bmatrix} \mathcal{H}_1(n) & \mathcal{H}_2(n) \\ \mathcal{H}_2^{\dagger}(n+1) & -\mathcal{H}_1^{\dagger}(n+1) \end{bmatrix} \quad (18)$$

where $\mathcal{H}_l(n)$ and $\mathcal{H}_l(n+1)$, $l = 1, 2$, are $N_c \times N_c$ diagonal matrices

Depending on complexity against performance tradeoffs, any linear equaliser can be applied to retrieve $\tilde{\boldsymbol{X}}$ from (17). In this paper, we consider linear equaliser where the parameters are updated using the MMSE criterion. We define the linear MMSE estimate $\boldsymbol{D}$ of $\tilde{\boldsymbol{X}}$ given the

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observation $\tilde{\boldsymbol{R}}$ by

$$\boldsymbol{D} = \bar{\tilde{\boldsymbol{X}}} + \boldsymbol{C}_{\tilde{X}}\mathcal{H}^{\dagger}(\mathcal{H}^{\dagger}\boldsymbol{C}_{\tilde{X}}\mathcal{H} + \boldsymbol{C}_{\tilde{W}})^{-1} \times (\tilde{\boldsymbol{R}} - \mathcal{H}\bar{\tilde{\boldsymbol{X}}}) \quad (19)$$

where $\bar{\tilde{\boldsymbol{X}}}$, $\boldsymbol{C}_{\tilde{X}}$ and $\boldsymbol{C}_{\tilde{W}}$ are the mean of $\tilde{\boldsymbol{X}}$, the covariance matrix of $\tilde{\boldsymbol{X}}$ and the covariance matrix of $\tilde{\boldsymbol{W}}$, respectively.

With a scaled unitary matrix $\mathcal{H}$ and approximately constant complex channel gains with $\mathcal{H}_1^2(n) + \mathcal{H}_2^2(n) \simeq 1$ assumptions, we can simplify $\mathcal{H}^{\dagger}\mathcal{H}$ as

$$\mathcal{H}^{\dagger}\mathcal{H} = \boldsymbol{I}_{2N_c \times 2N_c} \quad (20)$$

where $\boldsymbol{I}_{2N_c \times 2N_c}$ is the $2N_c \times 2N_c$ identity matrix. Moreover, following the assumptions used in [16], $\bar{\tilde{\boldsymbol{X}}} = \boldsymbol{0}$, $\boldsymbol{C}_{\tilde{X}} = \boldsymbol{I}$, (19) becomes

$$\boldsymbol{D} = (\boldsymbol{I} + \sigma_n^2\boldsymbol{I})^{-1}\mathcal{H}^{\dagger}\tilde{\boldsymbol{R}} \quad (21)$$

If we set $\boldsymbol{C}_{\tilde{W}} = \boldsymbol{0}$ in (19), further simplified form of linear equaliser called zero forcing equaliser is obtained resulting in

$$\boldsymbol{D} = \mathcal{H}^{\dagger}\tilde{\boldsymbol{R}} = \mathcal{H}^{\dagger}\mathcal{H}\tilde{\boldsymbol{X}} + \boldsymbol{\eta} \quad (22)$$

where $\boldsymbol{\eta} = \mathcal{H}^{\dagger}\tilde{\boldsymbol{W}}$.

We propose a turbo receiver structure for STBC-OFDM systems in this paper, which consists of an iterative MAP-EM channel estimation algorithm, STBC decoder and a soft MAP outer-channel-code decoder. As shown in Fig. 2, first EM-based channel estimator computes channel gains according to pilot symbols. Output of the estimator is used in the STBC demodulator (22). Next, the equalised symbol sequence $\{\boldsymbol{D}\}$ is passed through a channel de-interleaver, resulting in de-interleaved equalised symbols sequence $\{\boldsymbol{Z}\}$. Finally, $\{\boldsymbol{Z}\}$ is applied to a MAP decoder by de-interleaved estimated channel gains. In the MAP decoder submodule, log-likelihood ratios (LLRs) of a posteriori probabilities on the coded and uncoded bits are yielded. In the next iteration, LLRs of coded bits $\{Y_j\}$ are re-interleaved and passed through a nonlinearity [12]. Output of the nonlinearity computes soft value estimation of $\boldsymbol{X}$ as $\hat{\boldsymbol{X}}$ in Fig. 2.

$\hat{\boldsymbol{X}}$ is used as $\hat{\mathcal{X}}_e^{(q)}$ and $\hat{\mathcal{X}}_o^{(q)}$ in (12) for next iteration. Thus, the MAP-EM channel estimator iteratively generates the channel estimates by taking the received signals from receiver antennas and interleaved soft value of LLRs which are computed by the outer-channel code decoder in the previous iteration. Then, STBC-OFDM decoder takes channel estimates together with the received signals and computes equalised symbol sequence for next turbo iteration. Iterative operation is fulfilled between these three submodules. In all simulations, three iterations are employed.

4 Simulations

In this section, BER performances of channel estimators are presented through simulations for turbo-coded STBC-OFDM systems. Moreover, to investigate the sensitivity to channel estimation errors, we also considered convolutionally coded STBC-OFDM systems. In case of the turbo encoder, two identical recursive systematic convolutional component codes with generator $(1, 5_8/7_8)$ were concatenated in parallel via a pseudorandom interleaver form the encoder. For the convolutionally coded system, a $(5_8, 7_8)$ code with rate 1/2 was used. The channels between transmitter and receiver in this paper are assumed to be doubly selective where $\boldsymbol{H}_l(n)$'s were modelled, based on a realistic channel model determined by COST-207 project in which TU, BU, HT and RA channel models are considered.

The scenario for the STBC-OFDM study, with outer channel codes simulation, consists of BPSK and QPSK modulation formats. The system has 1.07 MHz bandwidth (for the raised cosine pulse roll-off factor $\alpha = 0.2$) and is divided into $N_c = 512$ tones with a total period T_s of 328 μs, of which 40 μs constitute the cyclic prefix. The data rate is about 0.8 and 1.6 Mbps for BPSK and QPSK modulation techniques, respectively. In order to choose good initial values for the unknown channel parameters, pilot tones known by the receiver are inserted in OFDM symbol. One pilot tone is inserted for every K data symbol denoted as a pilot insertion rate (PIR = 1:K). The details of the initialisation process are presented in [13, 17]. In all simulations, three iterations are employed.

Fig. 3 compares the BER performances of the EM-MAP channel estimation approach with the EM-ML [7] and widely used LMMSE-PSAM (LMMSE pilot-symbol-assisted modulation) approaches [18], for the turbo-coded STBC-OFDM systems over the BU channels for BPSK-modulated signals. It can be seen from Fig. 3 that the performance of the proposed EM-MAP algorithm outperforms EM-ML as well as PSAM techniques while

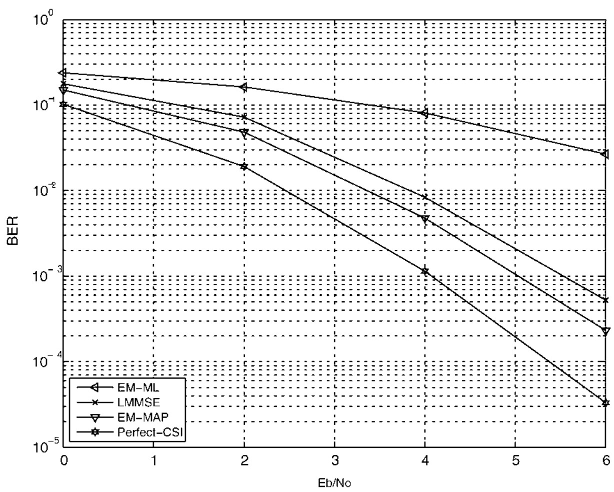

Figure 3 *Comparison of different channel estimation algorithms for BPSK-modulated turbo-coded STBC-OFDM systems over BU channels (f_{Dm} = 100Hz, PIR = 1:8*

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approaching the perfect channel state information (CSI) case for higher SNR values.

In Fig. 4, by using the same bandwidth and total number of subcarriers, QPSK modulation technique is used for BU channels and the overall data rate is increased to 1.6 Mbit/s. Since the turbo-coded data frame length is doubled when compared with BPSK case, it is observed that the performance difference between EM-MAP and LMMSE is decreased, as shown in Fig. 4, and the overall performance is degraded when compared with the BPSK case for PIR = 1:8 case. Therefore it is concluded that the EM-MAP channel estimator precedence could be faded for larger turbo-coded data frames. However, the turbo-coded data frame length is usually limited by the total number of subcarriers for OFDM applications. The coding process over several OFDM symbols is not practical due to the complexity and implementation issues and requires more decoder memory. On the other hand, the superiority of the EM-MAP over the LMMSE algorithm is clearly seen for PIR = 1:16 corresponding to PIR = 1:8, while the overall performance falls.

Computer simulation results with the same simulation parameters for TU channels are presented in Fig. 5. In this figure, it can be observed that the performance difference between the EM-MAP and the perfect CSI scenarios for BU channels is about 1 dB, and in TU channels, about 0.5 dB corresponding to $Pe = 10^{-4}$ for PIR = 1:8. However, the performances of the EM-MAP and the LMMSE are almost similar to each other since estimation accuracy is sufficient for the turbo decoder to work well. On the other hand, the superiority of the EM-MAP over the LMMSE algorithm is clearly seen for PIR = 1:16 corresponding to PIR = 1:8, while overall performance falls. In these figures, it can be concluded that the EM-MAP algorithm performs far better than other methods for the efficient bandwidth usage (i.e. lower PIRs).

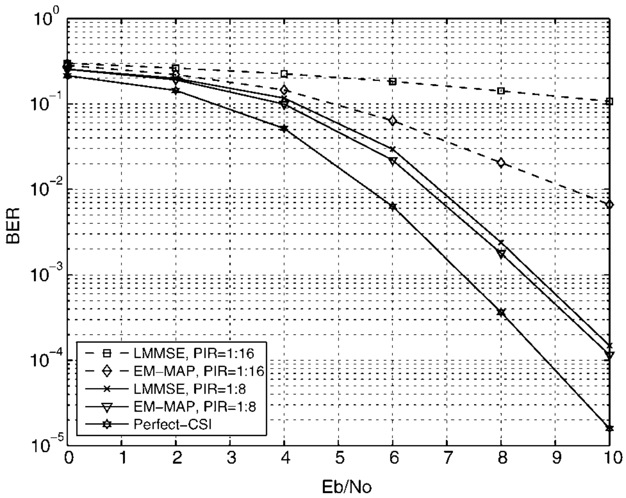

Figure 4 *Comparison of different channel estimation algorithms for QPSK-modulated turbo-coded STBC-OFDM systems over BU channels (f_{Dm} = 100 Hz)*

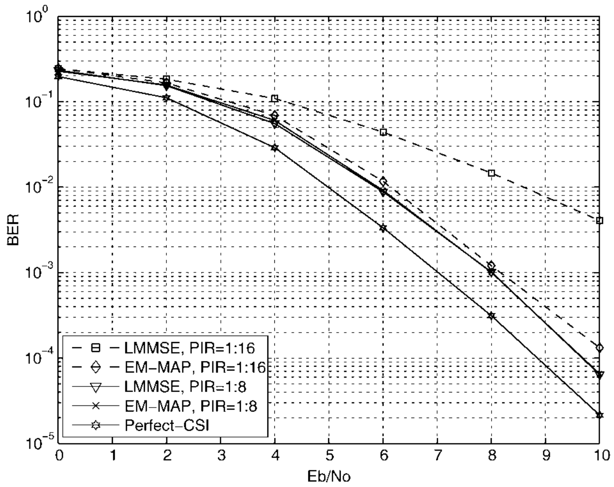

Figure 5 *Comparison of different channel estimation algorithms for QPSK-modulated turbo-coded STBC-OFDM systems over TU channels (f_{Dm} = 100 Hz)*

Moreover, in Fig. 6, the EM-MAP algorithm performance for turbo/convolutionally coded STBC-OFDM systems are investigated as a function of PIRs (1:8, 1:16, 1:32) for the TU channel model. Generally, it is known that a good channel code is more sensitive to channel estimation errors with high dependency among the coded bits that might cause severe error propagation during decoding process. Our simulation results support this argument. We observe that more performance degradation is experienced in the turbo-coded systems when compared with the convolutionally encoded systems in the presence of the TU channels since lower PIRs provide poor initial estimates.

The amount of information required to represent the statistically dependent channel parameters could be minimised by using the optimal truncation property. It is

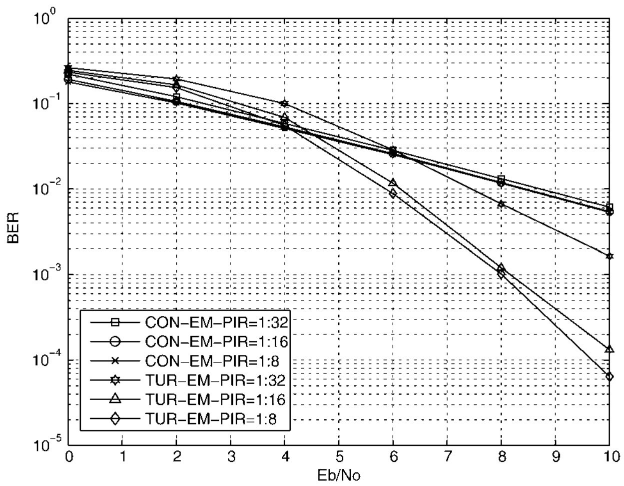

Figure 6 *EM-MAP channel estimator BER performance of the QPSK-modulated turbo/convolutionally coded STBC-OFDM systems as a function of PIRs for TU channels (f_{Dm} = 100 Hz)*

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possible to obtain an excellent approximation, if the dominant eigenvalues are selected in the KL expansion. In this way, more reduction in computational complexity could be achieved on the channel estimation algorithm as explained in Section 3. Since the frequency selectivity of the BU channels is more than the TU channels, low-rank representation of the BU channel requires more KL coefficients than TU channels. In Fig. 7, the BER performance of the EM-MAP algorithm is investigated for SNR = 6dB as a function of the number of KL coefficients employed. It is observed that 15 KL coefficients are sufficient to represent for TU channels, whereas it is about 25 coefficients for the BU channels.

In the preceding simulations, the autocorrelation matrix was assumed to be available as a priori information at the receiver for the COST-207 channel model. However, in practice, the true channel correlation is not known and it is important to analyse the performance degradation due to the mismatch between the approximated and the actual autocorrelation matrices. In this scope, the autocorrelation matrix $\boldsymbol{C} = [c_{m,n}]$ of the TU channel was approximated by

$$c_{m,n} = \begin{cases} 1 & \text{if} \quad m = n \\ \dfrac{1 - e^{-j2\pi L(m-n)/N}}{2\pi j L(m-n)/N} & \text{if} \quad m \neq n \end{cases} \tag{23}$$

where N is the number of subcarriers and L the length of the cyclix prefix. Note that (23) can be obtained from a profile having a uniform power-delay [13]. The simulation results concerning the mismatch analysis are presented in Fig. 8 for turbo-coded STBC-OFDM systems. It is concluded that the using the approximate $\boldsymbol{C}$ does not degrade the BER performance significantly for the TU channels in the case of correct channel length $\boldsymbol{L}$. Moreover, it can be shown that the performance degradation will be visible when $\boldsymbol{L}$ is less or more than the correct channel length.

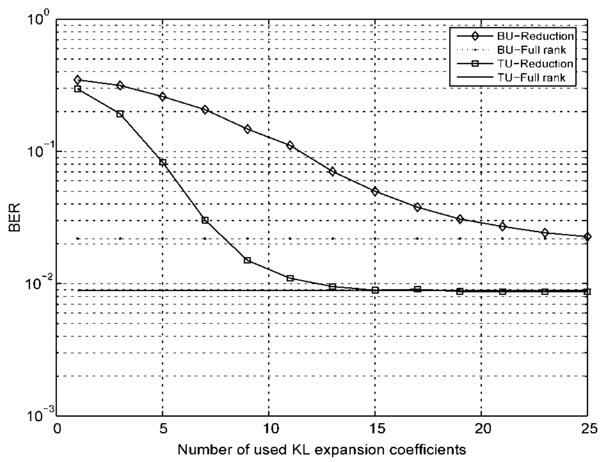

Figure 7 *Optimal truncation property of the KL expansion (PIR = 1:8, f_{Dm} = 100 Hz, QPSK)*

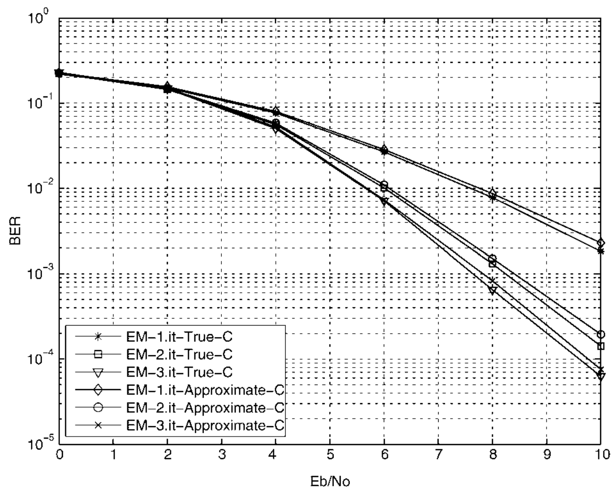

Figure 8 *Mismatch analysis for autocorrelation matrix (PIR = 1:8, f_{Dm} = 10 Hz, QPSK)*

5 Conclusion

In this paper, we proposed the EM-based MAP channel estimation algorithm for turbo/convolutionally coded STBC-OFDM systems, which is crucial for turbo receiver structure. This algorithm iteratively estimates the channel variations according to the MAP criterion, using the EM algorithm based on the orthogonal expansion representation of the channel via KL transform. It is observed that the proposed EM-MAP outperforms the EM-ML as well as PSAM techniques at lower pilot insertion rates. Based on such representation, we show that no matrix inversion is needed in the channel estimation algorithm. Moreover, a simplified approach with rank reduction is also proposed. It has been shown that in comparison with the traditional estimators the number of complex multiplications has been reduced from N_c to $2r$ per tone. Furthermore, sensitivity to channel estimation errors of turbo receivers is investigated. It was concluded that the receiver structures with turbo codes outperform the convolutional coded receiver structures assuming that channel estimation error is sufficient low. Moreover, it is observed that the performance difference between the EM-MAP and LMMSE decreases as the length of turbo-coded frames increases. Therefore it was concluded that the EM-MAP channel estimator precedence could fade out for longer turbo-coded data frames. However, the length of turbo-coded frames is usually limited by the total number of subcarriers for OFDM applications. Finally, we also concluded that the coding process over several OFDM symbols is not practical due to the complexity and implementation issues and requires more decoder memory. Consequently, the EM-MAP receiver structure is very efficient for turbo-coded STBC-OFDM systems.

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