Dual-Hop Amplify-and-Forward Multi-Relay Maximum Ratio Transmission

Eylem Erdogan and Tansal Gucluoglu

Abstract: **In this paper, the performance of dual-hop multi-relay maximum ratio transmission (MRT) over Rayleigh flat fading channels is studied with both conventional (all relays participate the transmission) and opportunistic (best relay is selected to maximize the received signal-to-noise ratio (SNR)) relaying. Performance analysis starts with the derivation of the probability density function, cumulative distribution function and moment generating function of the SNR. Then, both approximate and asymptotic expressions of symbol error rate (SER) and outage probability are derived for arbitrary numbers of antennas and relays. With the help of asymptotic SER and outage probability, diversity and array gains are obtained. In addition, impact of imperfect channel estimations is investigated and optimum power allocation factors for source and relay are calculated. Our analytical findings are validated by numerical examples which indicate that multi-relay MRT can be a low complexity and reliable option in cooperative networks.**

Index Terms: **Channel estimation error, conventional and opportunistic relaying, maximum ratio transmission, multi-relay, power allocation.**

I. INTRODUCTION

WIRELESS channels can experience deep fading leading to unreliable communication, thus, increasing diversity order of the system is highly desirable to reduce symbol error rates and outage probabilities. Similar to well investigated multiple antenna techniques with proper coding such as famous space time block coding (STBC) [1], "cooperative/relay" transmissions [2]–[5] have become popular to obtain spatial diversity. In practice, neighbouring mobile units or fixed relays can help the transmitted signals to be delivered to destination over independent fading channels. For example, with amplify-and-forward (AF) approach, the source signal received at relays can be amplified with a variable gain depending on the channel coefficients and then forwarded to destination. Another relaying method is decode-and-forward (DF) where relays can detect the transmitted symbols and then retransmit to destination, however, this approach has more complexity and may result in significant error propagation due to detection errors at relays and thus reduce the cooperation advantages.

In the last decade, research works on the design and analysis of cooperative/relay communication schemes with multiple relays have been increasing tremendously. In [6]–[8], symbol error rate (SER) and outage probability over Rayleigh fading channels are derived whereas the same performance indicators are obtained in [9]–[10] for Nakagami-*m* fading channels. Like conventional relaying, opportunistic relaying in which the best relay is selected to maximize the received signal-to-noise ratio (SNR), proposed in [11]. In [12], outage probability and SER performance over Nakagami-*m* fading channels are studied whereas the performance of ergodic capacity and SEP are examined for Rayleigh fading channels in [13].

In an attempt to increase degrees of freedom, capacity and diversity gains further, using multiple-antenna techniques in relay/cooperative transmissions can be attractive, although the mathematical analysis can get quite complicated. Reference [14] explores SER and outage probability of a multi-antenna single-relay AF transmission with orthogonal space-time block coding (OSTBC) and maximal ratio combining (MRC). In [15], OSTBC based opportunistic relaying scenario is investigated where SER and outage probability expressions are derived. Recently, employing maximum ratio transmission (MRT), a transmit diversity method, has attracted several interest in the research of cooperative/relay structures since MRT can achieve full available diversity and perform better than the well-known STBCs while requiring low receiver complexity [16]. Although MRT requires feedback of channel state information (CSI) to the transmitter, this may cause negligible overhead when the channel is very slow fading or when the channel is almost reciprocal e.g. indoor wireless mesh networks. In [17], authors investigate a MIMO-MRT network and derives SER and outage probability for Nakagami-*m* fading channels. Besides, employing MRT has been investigated in single-relay dual-hop networks in [18]–[23]. Reference [18] considers a network in which multiple-antennas employ MRT at the source and derives outage probability for Rayleigh fading channels. In [19], DF MRT-based multi-antenna cooperative network is considered and outage probability is derived. Likewise, in [20], MRT both at the source and relay is investigated and SER is derived. Moreover, [21] and [22] consider a network where source and destination employing MRT/MRC and SER and outage probability are derived for Nakagami-*m* and Rayleigh-Rician fading channels. In [23], MRT/MRC scheme is applied at both hops where SER and outage probability in the presence of feedback delay, channel estimation errors and antenna correlation are derived. In addition, partial relay selection schemes employing MRT is investigated in [24]–[26]. In [24]–[25], outage probability and SER are derived over Nakagami-*m* and Rayleigh fading channels respec-

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E. Erdogan is with the Department of Electrical and Electronics Engineering, Kadir Has University, Fatih, 34083, Istanbul, Turkey, email: erdoganeyl@gmail.com.

T. Gucluoglu is with the Department of Electronics and Communications Engineering, Yildiz Technical University, Esenler, 34220, Istanbul, Turkey, email: tansal@yildiz.edu.tr.

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tively whereas [26] considers the impact of feedback delay and channel estimation errors on a similar scenario where ergodic capacity and outage probability are derived.

To the best of our knowledge, there are no previous works which studies multi-relay MRT. In this paper, we investigate a dual-hop AF conventional and opportunistic relay transmissions with MRT technique. We note that this low complexity scheme can be useful in wireless mesh or ad-hoc networks especially with massive number of relays and antennas which prohibits the use of channel coding techniques to obtain high reliability in practice. The main contributions of this paper are outlined as follows:

- A tractable SNR bound is presented and probability density function (PDF), cumulative distribution function (CDF) and moment generating function (MGF) of the received SNR are derived.
- By using CDF and MGF expressions, SER, outage probability and ergodic capacity for both conventional and opportunistic relaying scenarios are derived and compared.
- Diversity and array gains of conventional and opportunistic networks are obtained by using asymptotic behavior of SER and outage probability.
- Impact of imperfect channel estimations which is critical for the performance of MRT, are explored.
- By using asymptotic outage probability, optimal source and relay power allocation factors are obtained.
- To verify the correctness of our analytical study, numerical examples are presented.

The remainder of the paper is organized as follows. In Section II, system model is presented. Section III describes performance analysis for conventional and opportunistic networks. Moreover, impact of imperfect channel estimations are investigated. In Section IV, optimum source and relay powers that minimize asymptotic outage probability is studied. Numerical examples are provided in Section V and finally Section VI concludes the paper.

Notations: Bold letters denote vectors and the following symbols $(\cdot)^T$, $(\cdot)^\dagger$ and $\|\cdot\|$ are used for transpose, conjugate-transpose and Frobenius norm respectively. A complex Gaussian random variable with mean a and variance σ_n^2 is denoted as $\mathcal{CN}(a,\sigma_n^2)$. A $n\times n$ identity matrix is shown as $\boldsymbol{I}_n$. The source-relay and relay-destination paths are shown with $S\to R$ and $R\to D$, respectively. Furthermore, $\Pr[\cdot]$ and $\mathbb{E}[\cdot]$ stand for probability and expectation operations respectively and $Q(\cdot)$ denotes Q-function.

II. SYSTEM MODEL

The block diagram is depicted in Fig. 1. Source node having K antennas transmits to the destination node through R independent relays each having L antennas. We assume each terminal is operating in half-duplex mode and the communication between source to destination takes place in two phases: In conventional relaying, source transmits signal x to all relays by using MRT in the first phase, in the second phase, relays amplify the received signal with an appropriate variable gain and forwards to the destination by using MRT. At the destination, signals coming from R relays are combined by using MRC to obtain maximum diversity gain. Total transmission in conventional relaying is $R+1$ time slots. In opportunistic relaying, best $S\to R\to D$ path is selected to maximize the received SNR at the destination. We assume source, relays and destination know perfect channel state information as needed for optimum MRT. Also, the direct link is assumed to be unavailable due to heavy shadowing.

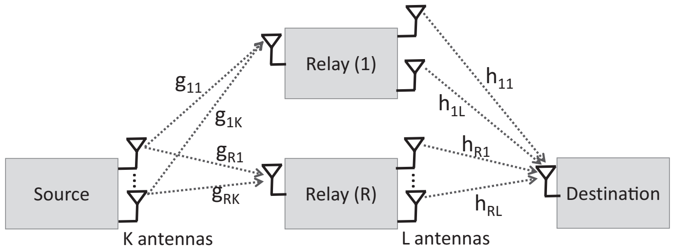

Fig. 1. Block diagram of dual-hop AF multi-relay system with MRT.

For the rth relay $r=\{1,\cdots,R\}$, the channel vectors for $S\to R$ and $R\to D$ paths are given as $\boldsymbol{g}_r=[g_{r_1}\cdots g_{r_K}]$ and $\boldsymbol{h}_r=[h_{r_1}\cdots h_{r_L}]$, respectively. The $\boldsymbol{g}_r$ and $\boldsymbol{h}_r$ row vectors are modeled as $\boldsymbol{g}_r\sim\mathcal{CN}(0,\boldsymbol{I}_K)$ and $\boldsymbol{h}_r\sim\mathcal{CN}(0,\boldsymbol{I}_L)$ respectively. The received signal at the rth relay is written as

$$y_r=\sqrt{\mathcal{P}_s}\boldsymbol{g}_r\boldsymbol{w}_{g_r}x+n_r. \tag{1}$$

As mentioned above, each relay uses AF relaying with a variable gain in order to assist the transmission. Assuming that fading coefficients remain almost constant over each frame, the received signal at the destination from rth relay is given by

$$y_{rd}=\sqrt{\mathcal{P}_r}\beta_r\boldsymbol{h}_r\boldsymbol{w}_{h_r}y_r+n_{rd}. \tag{2}$$

In (2), $\mathcal{P}_s$ and $\mathcal{P}_r$ are denoted as transmit powers at the source and relay respectively. MRT based weight vectors for $S\to R$ and $R\to D$ paths are given as $\boldsymbol{w}_{g_r}=(\boldsymbol{g}_r^\dagger/\|\boldsymbol{g}_r\|)$ and $\boldsymbol{w}_{h_r}=(\boldsymbol{h}_r^\dagger/\|\boldsymbol{h}_r\|)$ respectively. Noise samples (n_r,n_{rd}) are modeled as $n_r,n_{rd}\sim\mathcal{CN}(0,N_0)$ and scaling factor β_r is selected to normalize the power at the relay as shown below

$$\beta_r^2=\frac{1}{\mathcal{P}_s|\boldsymbol{g}_r\boldsymbol{w}_{g_r}|^2}. \tag{3}$$

The noise at the relay is not considered to simplify the scaling factor β_r above. With the help of (1)–(3) and after some manipulations, SNR can be written as follows

$$\gamma_d=\begin{cases}\sum\limits_{r=1}^{R}\left(\frac{\gamma_{g_r}\gamma_{h_r}}{\gamma_{g_r}+\gamma_{h_r}}\right), & \text{Conventional relaying}\\ \max\limits_{0\le r\le R}\left(\frac{\gamma_{g_r}\gamma_{h_r}}{\gamma_{g_r}+\gamma_{h_r}}\right), & \text{Opportunistic relaying}\end{cases} \tag{4}$$

where $\gamma_{g_r}=(\mathcal{P}_s/N_0)\|\boldsymbol{g}_r\|^2$ and $\gamma_{h_r}=(\mathcal{P}_r/N_0)\|\boldsymbol{h}_r\|^2$ represent the received SNRs at $S\to R$ and $R\to D$ transmissions.

III. PERFORMANCE ANALYSIS

In this section, we present the performance analysis of a dual-hop multi-antenna/multi-relay AF MRT transmission scheme. To this end, PDF, CDF and MGF of SNR is obtained, then SER, outage probability and ergodic capacity for both opportunistic and conventional relaying are derived. In addition, diversity and array gains are found by deriving asymptotic expressions of SER and outage probability. Finally, the impact of imperfect channel estimations on the proposed scenario are examined.

A. SNR Statistics

As the analysis of SER and outage probability becomes quite complicated in multi-antenna/multi-relay networks, we resort to compute tight lower bounds on these performance indicators by simplifying the SNR expressions given in (4) similar to [9], [28]–[29] as

$$\sum_{r=1}^{R}\left(\frac{\gamma_{g_r}\gamma_{h_r}}{\gamma_{g_r}+\gamma_{h_r}}\right) \leq \gamma_{\mathrm{up}}^{\mathrm{cv}} = \sum_{r=1}^{R}\min(\gamma_{g_r},\gamma_{h_r}) \tag{5}$$

and

$$\max_{0\leq r\leq R}\left(\frac{\gamma_{g_r}\gamma_{h_r}}{\gamma_{g_r}+\gamma_{h_r}}\right) \leq \gamma_{\mathrm{up}}^{\mathrm{op}} = \max_{0\leq r\leq R}\min(\gamma_{g_r},\gamma_{h_r}) \tag{6}$$

where superscript cv and op denotes conventional and opportunistic schemes. To simplify further, we denote $\rho_r = \min(\gamma_{g_r},\gamma_{h_r})$, then the CDF of ρ_r, can be expressed as

$$\begin{aligned} F_{\rho_r}(\gamma) &= \Pr[\min(\gamma_{g_r},\gamma_{h_r}) < \gamma] \\ &= 1-\Pr[\gamma_{g_r} > \gamma]\Pr[\gamma_{h_r} > \gamma]. \end{aligned} \tag{7}$$

PDF expressions of γ_{gr} and γ_{hr} can be obtained as in [16]. Integrating these PDFs w.r.t. γ gives us the CDFs of γ_{g_r} and γ_{h_r}. By substituting the CDF of γ_{g_r} and γ_{h_r} in (7), $F_{\rho_r}(\gamma)$ can be written as follows

$$F_{\rho_r}(\gamma) = 1-\frac{\Gamma(K,\frac{\gamma}{\Omega_{g_r}})\Gamma\left(L,\frac{\gamma}{\Omega_{h_r}}\right)}{\Gamma(K)\Gamma(L)} \tag{8}$$

where $\Omega_{g_r} = \mathcal{P}_s/N_0$ and $\Omega_{h_r} = \mathcal{P}_r/N_0$ are the average SNRs per antenna, $\Gamma(\cdot)$ is the gamma function as described in [36, eqn. (8.310.1)], $\Gamma(\cdot,\cdot)$ is the upper incomplete gamma function as described in [36, eqn. (8.350.2)]. PDF of ρ_r can be found by taking the derivative of (8) w.r.t. γ

$$\begin{aligned} f_{\rho_r}(\gamma) = \frac{1}{\Gamma(K)\Gamma(L)}&\left(\frac{\gamma^{K-1}}{\Omega_{g_r}^{K}}e^{-\gamma/\Omega_{g_r}}\Gamma\left(L,\frac{\gamma}{\Omega_{h_r}}\right)\right. \\ &\left.+\frac{\gamma^{L-1}}{\Omega_{h_r}^{L}}e^{-\gamma/\Omega_{h_r}}\Gamma\left(K,\frac{\gamma}{\Omega_{g_r}}\right)\right). \end{aligned} \tag{9}$$

MGF of (9) can be obtained by using the definition ($\mathcal{M}_x(s) = \mathbb{E}[e^{-sx}]$) and [36, eqn. (6.455.1)] as shown at the top of the next page. In (10), ${}_2F_1(\cdot,\cdot;\cdot;\cdot)$ denotes Gauss' hypergeometric function which is defined in [36, eqn. (9.100)]. If we assume $K = L = M$, (10) can be simplified as

$$\begin{aligned} \mathcal{M}_{\rho_r}(s) = &\frac{2\Gamma(2M)}{M\Gamma(M)^2\Omega_{\rho_r}^{2M}(s+(2/\Omega_{\rho_r}))^{2M}} \\ &\times {}_2F_1\left(1,2M;M+1;\frac{s\Omega_{\rho_r}+1}{s\Omega_{\rho_r}+2}\right). \end{aligned} \tag{11}$$

B. Symbol Error Rate and Outage Probability

B.1 Conventional Relaying

Having found the MGF of SNR for 1 relay, we can easily extend it to R-relays by using the MGF approach as all channel coefficients between $S \rightarrow R$ and $R \rightarrow D$ path are independent.

$$\mathcal{M}_{\gamma_{\mathrm{up}}^{\mathrm{cv}}}(s) = \prod_{r=1}^{R}\mathcal{M}_{\gamma_{\rho_r}}(s). \tag{12}$$

With the help of (10) and (12), symbol error rate and outage probability for conventional relaying can be obtained. For example, for M-PSK modulation, SER can be obtained as given in [37].

$$P_s^{\mathrm{cv}}(e) = \frac{1}{\pi}\int_0^{\phi}\mathcal{M}_{\gamma_{\mathrm{up}}^{\mathrm{cv}}}\left(\frac{g_{PSK}}{\sin^2(\theta)}\right)d\theta \tag{13}$$

where $\phi = (M-1)\pi/M$, $g_{PSK} = \sin^2(\pi/M)$, i.e., $g_{PSK} = 1$ for BPSK modulation.

Similar to SER, outage probability ($P_{\mathrm{out}}^{\mathrm{cv}}$) is a widely used performance indicator in wireless communication systems. $P_{\mathrm{out}}^{\mathrm{cv}}$ is defined as the probability of SNR falling below a certain threshold γ_{th} and can be computed by taking the inverse Laplace transform of $\mathcal{M}_{\gamma_{\mathrm{up}}^{\mathrm{cv}}}(s)$ at γ_{th} as follows

$$P_{\mathrm{out}}^{\mathrm{cv}} = \left[\mathcal{L}^{-1}\left(\frac{\mathcal{M}_{\gamma_{\mathrm{up}}^{\mathrm{cv}}}(s)}{s}\right)\right]_{s=\gamma_{th}} \tag{14}$$

where $\mathcal{L}^{-1}(\cdot)$ denotes the inverse Laplace transform.

To the best of our knowledge, closed form expressions of SER and outage probability are not available in the literature. However, similar to previous studies in cooperative/relay communication systems, SER can be obtained approximately as shown in [27] and outage probability can be found numerically by using well-known software programs such as MAPLE or MATHEMATICA. For BPSK modulation, approximate SER can be written as shown in [27, eqn. (10)]

$$P_s(e) = \frac{1}{12}\mathcal{M}_{\gamma_{\mathrm{up}}^{\mathrm{cv}}}(1)+\frac{1}{4}\mathcal{M}_{\gamma_{\mathrm{up}}^{\mathrm{cv}}}(1.3)-\frac{1}{12}\mathcal{M}_{\gamma_{\mathrm{up}}^{\mathrm{cv}}}\left(\frac{1}{\sin^2(\theta)}\right). \tag{15}$$

In [27], it is shown that approximate SER expressions are valid and accurate in the whole integral region.

B.2 Opportunistic Relaying

In opportunistic relaying networks, CDF of received SNR ($F_{\gamma_{\mathrm{up}}^{\mathrm{op}}}(\gamma)$) can be written as $F_{\gamma_{\mathrm{up}}^{\mathrm{op}}}(\gamma) = \{F_{\rho_r}(\gamma)\}^R$. With the help of high order statistics [37], equation (8) and [36, eqn. (8.352.7)], $F_{\gamma_{\mathrm{up}}^{\mathrm{op}}}(\gamma)$ can be expressed as

$$\begin{aligned} F_{\gamma_{\mathrm{up}}^{\mathrm{op}}}(\gamma) = &\left\{1-e^{-\frac{\gamma}{\Omega_{g_r}}}\sum_{k=0}^{K-1}\left(\frac{\gamma}{\Omega_{g_r}}\right)^k\frac{1}{k!}\right. \\ &\left.\times e^{-\frac{\gamma}{\Omega_{h_r}}}\sum_{l=0}^{L-1}\left(\frac{\gamma}{\Omega_{h_r}}\right)^l\frac{1}{l!}\right\}^R. \end{aligned} \tag{16}$$

$$\mathcal{M}_{\rho_r}(s) = \frac{\Gamma(K+L)}{\Gamma(K)\Gamma(L)\Omega_{g_r}^K\Omega_{h_r}^L\left(s+(1/\Omega_{g_r})+(1/\Omega_{h_r})\right)^{K+L}} \times \left[(1/K)_2F_1\left(1,K+L;K+1;\frac{s+(1/\Omega_{g_r})}{s+(1/\Omega_{g_r})+(1/\Omega_{h_r})}\right) + (1/L)_2F_1\left(1,K+L;L+1;\frac{s+(1/\Omega_{h_r})}{s+(1/\Omega_{g_r})+(1/\Omega_{h_r})}\right)\right] \quad (10)$$

By applying binomial [36, eqn. (1.111.1)] and multinomial [36, eqn. (0.314)] expansions respectively, $F_{\gamma_{\rm up}^{\rm op}}(\gamma)$ becomes

$$F_{\gamma_{\rm up}^{\rm op}}(\gamma) = \sum_{r=0}^{R}\sum_{k=0}^{r(K-1)}\sum_{l=0}^{r(L-1)}\binom{R}{r}(-1)^r e^{-r\frac{\gamma}{\Omega_{g_r}}}e^{-r\frac{\gamma}{\Omega_{h_r}}} \times \mathcal{X}_k(r)\mathcal{X}_l(r)\gamma^{k+l} \quad (17)$$

where combination operation denotes binomial coefficients and multinomial coefficients can be written as $\mathcal{X}_t(r) = \{1/(tk_0)\}\sum_{\rho=1}^{t}(r\rho-t+\rho)k_\rho\mathcal{X}_{t-\rho}(r), t \geq 1$ [36, eqn. (0.314)], where $k_\rho = (1/\Omega_m)^\rho(1/\rho!)$, $\mathcal{X}_0(r) = k_0^r = 1$, $t \in \{k, l\}$ and $m \in \{g_r, h_r\}$.

As defined above, outage probability is the probability of received SNR falling below a certain threshold and it can be obtained as $P_{\rm out}^{\rm op} = F_{\gamma_{\rm up}^{\rm op}}(\gamma_{th})$. In addition, for the systems whose conditional symbol error rate expression is in the form of $\mathbb{E}[aQ(\sqrt{2b\gamma})]$, SER can be computed by using the CDF of SNR as [36]

$$P_s^{\rm op}(e) = \frac{a\sqrt{b}}{2\sqrt{\pi}}\int_0^\infty \gamma^{-1/2}e^{-b\gamma}F_{\gamma_{\rm up}^{\rm op}}(\gamma)d\gamma \quad (18)$$

where a and b denotes modulation coefficients, i.e., $\{a = 1, b = 0.5\}$ for BFSK modulation, $\{a = 1, b = 1\}$ for BPSK and $\{a = 2(M-1)/M, b = 3/(M^2-1)\}$ for M-PAM. Also, $\{a = 2, b = \sin^2(\pi/M)\}$ for approximate M-PSK. By substituting (17) in (18) with the help of [36, eqn. (3.351.3)], SER can be obtained as

$$P_s^{\rm op}(e) = \frac{a\sqrt{b}}{2\sqrt{\pi}}\sum_{r=0}^{R}\sum_{k=0}^{r(K-1)}\sum_{l=0}^{r(L-1)}\binom{R}{r}(-1)^r\mathcal{X}_k(r) \times \mathcal{X}_l(r)\Gamma\left(k+l-\frac{3}{2}\right)(b+r\Omega_r)^{-k-l-\frac{1}{2}} \quad (19)$$

where $\Omega_r = (\Omega_{g_r}+\Omega_{h_r})/(\Omega_{g_r}\Omega_{h_r})$.

C. Diversity and Array Gains

Here, we examine asymptotic SER and outage probability expressions to obtain diversity (G_d) and array (G_a) gains.

C.1 Conventional Relaying

At high SNR, $F_{\rho_r}(\gamma)$ can be expressed as [30, eqn. (6)]

$$F_{\rho_r}(\gamma) = \frac{\Upsilon\left(K, \frac{\gamma}{\Omega_{g_r}}\right)}{\Gamma(K)} + \frac{\Upsilon\left(L, \frac{\gamma}{\Omega_{h_r}}\right)}{\Gamma(L)} \quad (20)$$

where $\Upsilon(\cdot)$ is lower incomplete Gamma function [36, eqn. (8.350.1)]. By using the asymptotic behavior of lower incomplete Gamma function given in [35, eqn. (45.9.1)], asymptotic $F_{\rho_r}^{\infty}(\gamma)$ can be expressed as

$$F_{\rho_r}^{\infty}(\gamma) = \frac{\gamma^K}{\Gamma(K+1)\Omega_{g_r}^K} + \frac{\gamma^L}{\Gamma(L+1)\Omega_{h_r}^L}. \quad (21)$$

To obtain asymptotic SER and outage probability expressions for conventional relaying, we need to obtain $\mathcal{M}_{\gamma_{\rm up}^{\rm cv,\infty}}(s)$. Therefore, by using the relationship between MGF and CDF i.e., $\mathcal{M}_{\rho_r}^{\infty}(s) = s\int_0^\infty e^{-s\gamma}F_{\rho_r}^{\infty}(\gamma)d\gamma$, with the help of [36, eqn. (3.351.3)] and then substituting $\mathcal{M}_{\rho_r}^{\infty}(s)$ in (12), $\mathcal{M}_{\gamma_{\rm up}^{\rm cv,\infty}}(s)$ can be obtained as

$$\mathcal{M}_{\gamma_{\rm up}^{\rm cv,\infty}}(s) = \prod_{r=1}^{R}\left(\frac{1}{s^K\Omega_{g_r}^K} + \frac{1}{s^L\Omega_{h_r}^L}\right). \quad (22)$$

To obtain the inverse Laplace transform of (22) is highly complicated. For this, we assume both hops are balanced i.e., $K = L = M$ and $\Omega_{g_r} = \Omega_{h_r} = \Omega$. Then for large average SNR, $F_{\gamma_{\rm up}^{\rm cv,\infty}}(\gamma)$ can be expressed as

$$F_{\gamma_{\rm up}^{\rm cv,\infty}}(\gamma) = \mathcal{A}\left(\frac{\gamma}{\Omega}\right)^{MR} \quad (23)$$

where $\mathcal{A} = 2^R/(\Gamma(MR+1))$. As $P_{\rm out}^{\rm cv,\infty} = F_{\gamma_{\rm up}^{\rm cv,\infty}}(\gamma_{th}) = \mathcal{A}(\gamma_{th}/\Omega)^{MR}$ [31], diversity and array gains can be obtained as $G_d = MR$ and $G_a = \left(\frac{2^R\gamma_{th}^{MR}}{\Gamma(MR+1)}\right)^{-1/G_d}$. By substituting (23) in (18) and with the help of [31, prop. (1)], asymptotic SER can be obtained as

$$P_s^{\rm cv,\infty}(e) = \frac{a\mathcal{A}\Gamma(MR+1/2)}{2\sqrt{\pi}(b\Omega)^{MR}} + \text{H.O.T.} \quad (24)$$

where a, b are modulation coefficients as described above.

C.2 Opportunistic Relaying

As mentioned above, in opportunistic networks, $F_{\gamma_{\rm up}^{\rm op,\infty}}(\gamma)$ can be written as $F_{\gamma_{\rm up}^{\rm op,\infty}}(\gamma) = \{F_{\rho_r}^{\infty}(\gamma)\}^R$. By using (21) and replacing γ with $\gamma_{\rm th}$, $P_{\rm out}^{\rm op,\infty}$ can be obtained as

$$P_{\rm out}^{\rm op,\infty} = \left(\frac{\gamma_{th}^K}{\Gamma(K+1)\Omega_{g_r}^K} + \frac{\gamma_{th}^L}{\Gamma(L+1)\Omega_{h_r}^L}\right)^R. \quad (25)$$

By using [31, prop. (5)], $P_{\rm out}^{\rm op,\infty}$ can be expressed as

$$P_{\rm out}^{\rm op,\infty} \approx \mathcal{Z}\left(\frac{\gamma_{th}}{\Omega}\right)^{G_d} + \text{H.O.T.} \quad (26)$$

where $\Omega \in \{\Omega_{g_r}, \Omega_{h_r}\}$, H.O.T denotes high order terms and $\mathcal{Z}$ is

$$\mathcal{Z} = \begin{cases} \prod_{r=1}^{R}\left(\frac{1}{\Gamma(K+1)}\right), & K<L \\ \prod_{r=1}^{R}\left(\frac{1}{\Gamma(K+1)}+\frac{1}{\Gamma(L+1)}\right), & K=L \\ \prod_{r=1}^{R}\left(\frac{1}{\Gamma(L+1)}\right), & K>L. \end{cases} \tag{27}$$

Diversity and array gains can be expressed as

$$\begin{aligned} G_d &= R\min(K,L) \\ G_a &= \mathcal{Z}^{-1/(R\min(K,L))}. \end{aligned} \tag{28}$$

By substituting (26) in (18) and after γ_{th} is replaced with γ, asymptotic SER can be obtained as follows

$$P_s^{\text{op},\infty}(e) = \frac{2^{G_d-1}a\mathcal{Z}\Gamma(G_d+1/2)}{\sqrt{\pi}(2b\Omega)^{G_d}} + \text{H.O.T.}. \tag{29}$$

When the diversity gain obtained from opportunistic is compared with that of conventional one, we infer that conventional scheme has better array gain but equal diversity with opportunistic.

D. Ergodic Capacity

Ergodic capacity can be specified as the maximum mutual information (or expectation of information rate) between source and destination. Ergodic capacity for conventional relaying can be expressed as

$$\begin{aligned} C_{\text{erg}}^{\text{cv}} &= \frac{1}{R+1}\mathbb{E}\left[\log_2(1+\gamma_{\text{up}}^{\text{cv}})\right] \\ &= \frac{1}{R+1}\int_0^\infty \log_2(1+\gamma) f_{\gamma_{\text{up}}^{\text{cv}}}(\gamma)d\gamma \end{aligned} \tag{30}$$

where $f_{\gamma_{\text{up}}^{\text{cv}}}(\gamma)$ can be find by taking the inverse Laplace transform of $\mathcal{M}_{\gamma_{\text{up}}}(s)$ as follows

$$f_{\gamma_{\text{up}}^{\text{cv}}}(\gamma) = \left[\mathcal{L}^{-1}\left(\mathcal{M}_{\gamma_{\text{up}}^{\text{cv}}}(s)\right)\right]_{s=\gamma}. \tag{31}$$

By substituting (31) in (30), an upper bound on $C_{\text{erg}}^{\text{cv}}$ can be computed numerically. As can be seen from (30), ergodic capacity degrades by a factor of $R+1$.

In opportunistic relaying, ergodic capacity can be expressed by using the CDF of SNR as shown in [34]

$$\begin{aligned} C_{\text{erg}}^{\text{op}} &= \frac{1}{2}\mathbb{E}\left[\log_2(1+\gamma_{\text{up}}^{\text{cv}})\right] \\ &= \frac{1}{2}\log_2(e)\int_0^\infty \frac{1}{1+\gamma}F_{\gamma_{\text{up}}^{\text{op}}}(\gamma)d\gamma. \end{aligned} \tag{32}$$

Substituting (17) into (32) with the help of [36, eqn. (3.353.5)], an upper bound on $C_{\text{erg}}^{\text{op}}$ can be found as

$$\begin{aligned} C_{\text{erg}}^{\text{op}} = &\frac{\log_2(e)}{2}\sum_{r=0}^{R}\sum_{k=0}^{r(K-1)}\sum_{l=0}^{r(L-1)}\binom{R}{r}(-1)^r\mathcal{X}_k(r)\mathcal{X}_l(r) \\ &\times\Bigg\{(-1)^{k+l-1}e^{r\Omega_r}\text{Ei}(-\Omega_r) \\ &+\sum_{z=1}^{k+l}(z-1)!(-1)^{k+l-z}(\Omega_r)^{-z}\Bigg\} \end{aligned} \tag{33}$$

where $\text{Ei}(\cdot)$ denotes exponential integral.

E. Impact of Imperfect Channel Estimations

In this section, we investigate the effects of imperfect channel estimations on the proposed scenarios. For this, we assume $S\to R$ and $R\to D$ paths are erroneously estimated as shown below

$$\begin{aligned} \boldsymbol{g}_r &= \tilde{\boldsymbol{g}}_r + \boldsymbol{\xi}_{g_r}, \\ \boldsymbol{h}_r &= \tilde{\boldsymbol{h}}_r + \boldsymbol{\xi}_{h_r} \end{aligned} \tag{34}$$

where channel estimates $\tilde{\boldsymbol{g}}_r$ and $\tilde{\boldsymbol{h}}_r$ are modeled as $\tilde{\boldsymbol{g}}_r \sim \mathcal{CN}(0,\boldsymbol{I}_K\sigma^2_{\tilde{g}_r})$ and $\tilde{\boldsymbol{h}}_r \sim \mathcal{CN}(0,\boldsymbol{I}_L\sigma^2_{\tilde{h}_r})$. Estimation errors ($\boldsymbol{\xi}_{g_r}$ and $\boldsymbol{\xi}_{h_r}$) are given as $\boldsymbol{\xi}_{g_r} \sim \mathcal{CN}(0,\boldsymbol{I}_K\sigma^2_{\xi_{g_r}})$ and $\boldsymbol{\xi}_{g_r} \sim \mathcal{CN}(0,\boldsymbol{I}_L\sigma^2_{\xi_{h_r}})$ [32]-[33]. MRT based weight vectors can be specified as $\boldsymbol{w}_{\tilde{g}_r} = (\tilde{\boldsymbol{g}}_r^\dagger/\|\tilde{\boldsymbol{g}}_r\|)$, $\boldsymbol{w}_{\tilde{h}_r} = (\tilde{\boldsymbol{h}}_r^\dagger/\|\tilde{\boldsymbol{h}}_r\|)$ respectively. The scaling factor becomes

$$\tilde{\beta}_r^2 = \frac{1}{\mathcal{P}_s|\tilde{\boldsymbol{g}}_r\boldsymbol{w}_{\tilde{g}_r}|^2}. \tag{35}$$

By substituting (34), (35) in (1) and (2) and after some manipulations, effective received SNRs can be expressed as

$$\gamma_d^{ef} = \begin{cases} \sum_{r=1}^{R}\left(\frac{\gamma_{g_r}^{ef}\gamma_{h_r}^{ef}}{\mathcal{A}_r\gamma_{g_r}^{ef}+\mathcal{B}_r\gamma_{h_r}^{ef}+\mathcal{C}_r}\right), & \text{Conv. relaying} \\ \max\limits_{0\le r\le R}\left(\frac{\gamma_{g_r}^{ef}\gamma_{h_r}^{ef}}{\mathcal{A}_r\gamma_{g_r}^{ef}+\mathcal{B}_r\gamma_{h_r}^{ef}+\mathcal{C}_r}\right), & \text{Opp. relaying} \end{cases} \tag{36}$$

where $\gamma_{g_r}^{ef} = (\mathcal{P}_s/N_0)\|\tilde{\boldsymbol{g}}_r\|^2$ and $\gamma_{h_r}^{ef} = (\mathcal{P}_r/N_0)\|\tilde{\boldsymbol{h}}_r\|^2$. Also, $\mathcal{A}_r = 1+(\mathcal{P}_r/N_0)\sigma^2_{\xi_{h_r}}$, $\mathcal{B}_r = 1+(\mathcal{P}_s/N_0)\sigma^2_{\xi_{g_r}}$ and $\mathcal{C}_r = (\mathcal{P}_r/N_0)\sigma^2_{\xi_{g_r}} + (\mathcal{P}_s/N_0)(\mathcal{P}_r/N_0)\sigma^2_{\xi_{g_r}}\sigma^2_{\xi_{h_r}}$. After effective SNRs are approximately written as in (5) and (6), $F_{\rho_r}^{ef}(\gamma)$ can be obtained as

$$F_{\rho_r}^{ef}(\gamma) = 1 - \frac{\Gamma(K,\mathcal{B}_r\frac{\gamma}{\Omega_{g_r}})\Gamma\left(L,\mathcal{A}_r\frac{\gamma}{\Omega_{h_r}}\right)}{\Gamma(K)\Gamma(L)}. \tag{37}$$

From (37), it can be observed that the CDF of SNR deteriorates from the negative effects of imperfect channel estimations. By applying the same theoretical steps to (37), SER and outage probability in the presence of channel estimation errors can be obtained for both conventional and opportunistic relay networks.

IV. OPTIMAL POWER ALLOCATION

In this section, we aim to improve the performance of the dual-hop single-relay multi-antenna network by obtaining optimum $\mathcal{P}_s$ and $\mathcal{P}_r$ values to minimize the outage probability under a power fraction α. To this end, by using (25), we rewrite P_{out}^∞ as shown below

$$P_{\text{out}}^\infty = \frac{A}{\mathcal{P}_s^K} + \frac{B}{\mathcal{P}_r^L} \tag{38}$$

where $A = \frac{\left(\gamma_{th}\times N_0\right)^K}{\Gamma(K+1)}$ and $B = \frac{\left(\gamma_{th}\times N_0\right)^L}{\Gamma(L+1)}$. We assume $\mathcal{P}_s = \alpha\mathcal{P}_t$ and $\mathcal{P}_r = (1-\alpha)\mathcal{P}_t$, where $\mathcal{P}_t$ the total transmit power

Table 1. Optimum power values for $\mathcal{P}_t = 10$ and $\gamma_{th} = 7$ dB.

K, L	Optimum α values
$2, 1$	$\alpha = 0.2925$
$1, 2$	$\alpha = 0.7074$
$3, 1$	$\alpha = 0.1711$
$4, 1$	$\alpha = 0.1104$

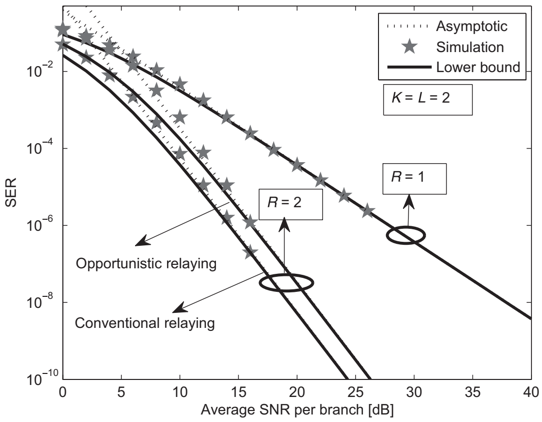

Fig. 2. SER comparison of theoretical bounds with exact simulations.

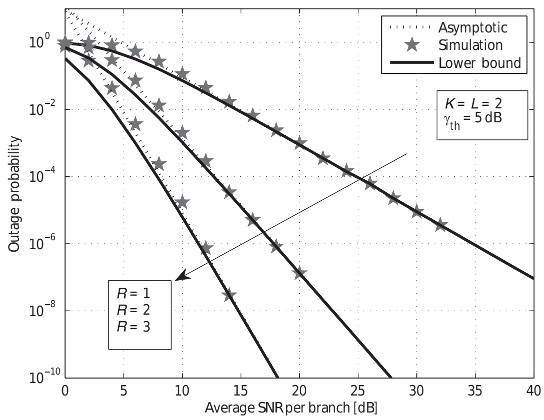

Fig. 3. Outage probability of conventional relaying.

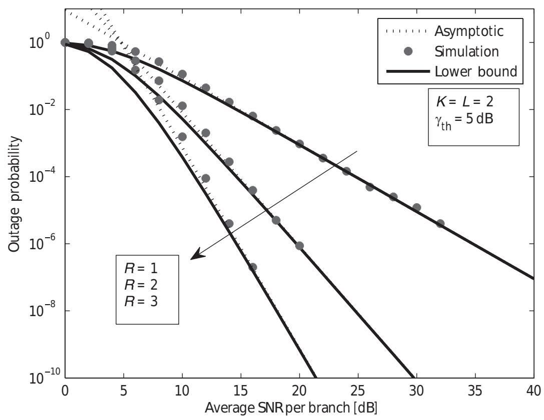

Fig. 4. Outage performance of opportunistic relaying.

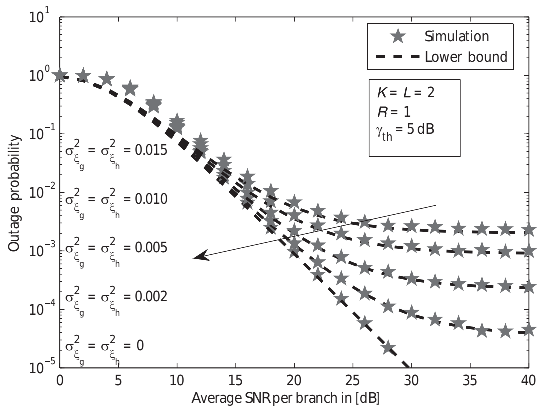

Fig. 5. Impact of imperfect channel estimations on the proposed network when $R = 1$.

available in the network. Hence, the power allocation problem can be formed as follows

$$\min_{\alpha} P_{\text{out}}^{\infty}, \text{ subject to} : 0 < \alpha < 1. \tag{39}$$

By substituting $\mathcal{P}_s = \alpha \mathcal{P}_t$ and $\mathcal{P}_r = (1-\alpha)\mathcal{P}_t$ in (38), then taking the second derivative of P_{out}^{∞} w.r.t α, we recognize that P_{out}^{∞} is a strictly convex function of α. Therefore, taking the first derivative of (38) and equating to zero, we can obtain optimal value of α as follows

$$\begin{aligned} \frac{\alpha^{K+1}}{(1-\alpha)^{L+1}} &= \frac{KA}{LB}\mathcal{P}_t^{L-K}, \text{ when } K \neq L \\ \alpha &= \frac{1}{2}, \qquad\qquad \text{when } K = L. \end{aligned} \tag{40}$$

The closed form solution of (40) is difficult to obtain, but numerical results can be obtained by using root-finding algorithms such as Bisection or Newton. Table 1 gives some examples for $\mathcal{P}_t = 10$ dB. From the table, we understand that when $K > L$, source power decreases and relay power increases, or when $L > K$, source power increases and relay power decreases to minimize outage probability.

V. NUMERICAL EXAMPLES

In this section, several numerical examples are provided to verify and demonstrate our analytical study to gain further insight about the usefulness of the proposed system. SER and outage probabilities are obtained via Monte-Carlo simulations where BPSK signalling and Rayleigh fading channel model are used. For simplicity, we assume that transmit powers between $S \to R$ and $R \to D$ links are equal ($\mathcal{P}_s = \mathcal{P}_r = \mathcal{P}_t/2$) and horizontal axes of all figures represent the average SNR per branch unless otherwise stated.

Fig. 2 depicts the SER of opportunistic and conventional re-

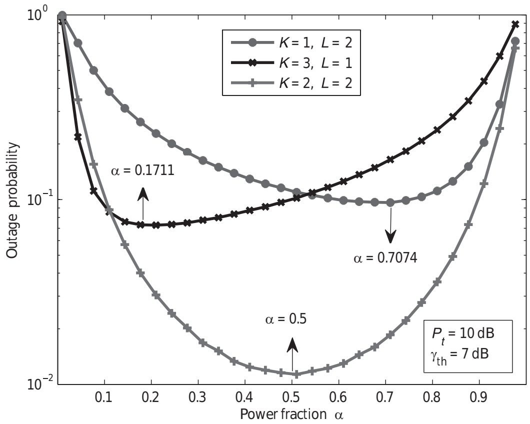

Fig. 6. Outage probability based optimal power allocation under fraction α.

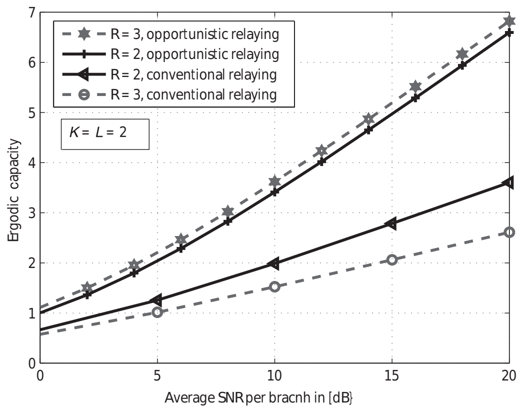

Fig. 7. Ergodic capacity comparison of conventional and opportunistic relaying.

laying schemes for $K = L = 2$ and $R = 1, 2$. Comparing derived lower bound and asymptotic results with the simulation, it can be observed that the theoretical results match almost perfectly with the simulation at especially medium to high SNRs. In addition, we understand that conventional relaying achieves average 2 dB better SER then opportunistic relaying despite the diversity orders are identical, e.g., 2 and 4 for $R = 1, 2$, respectively. Interestingly, due to simple structure of MRT technique, one can satisfy error performance requirements by exploiting few of the available users as relays without the overhead of changing receiver structure and executing channel coding/decoding algorithms.

In Figs. 3 and 4, outage probabilities of conventional and opportunistic relaying is drawn for $R = 1, 2, 3$ and when $K = L = 2$. From both figures, we understand that as the number of relays increase, the performance significantly improves e.g., the difference between $R = 2$ and $R = 3$ is about 9 dB at 10^{-10} P_{out}. Similar to Fig. 2, asymptotic and approximate results of both figures matches perfectly with the simulation at all cases especially at medium to high SNRs. In addition, conventional scheme is complex but average 2–3 dB superior than opportunistic case, despite the diversity orders are exactly the same e.g., $2, 4, 6$ for $R = 1, 2, 3$.

In Fig. 5, the impact of imperfect channel estimations on the outage probability is demonstrated for different values of fixed estimation error variances. From this figure, we can clearly observe error floors due to channel estimation errors when the error variances cannot be improved with increased SNR. After especially 15 dB, error floors results in huge performance loses as no diversity can be obtained. Furthermore, we observe that the lower bound is in an excellent agreement with the simulation results in all cases especially at medium to high SNRs.

Fig. 6 shows the usefulness of power allocation which obtains optimum power fraction values to minimize outage probability. In this figure, total power is set to 10 dB and 3 different cases are drawn. From all cases, we infer that optimum power allocation yields a much better performance then $\alpha = 1/2$. For example, when $K = 3, L = 1$, outage probability is lower than $10^{-1} P_{\text{out}}$ at $\alpha = 0.1711$ or when $K = 1, L = 2$, source power must be increased to 7.074 dB to obtain a much better outage performance. However, when $K = L$, source and relay powers are equal i.e., $\mathcal{P}_s = \mathcal{P}_r = 5$ dB. All these values are obtained numerically as shown in Table 1 can also be verified from Fig. 6.

In Fig. 7, ergodic capacity of conventional and opportunistic relaying schemes, is illustrated. As can be seen, in opportunistic relaying, increasing R increases ergodic capacity. However, as conventional relaying uses $R + 1$ time slots in the transmission, ergodic capacity decreases by a factor of $R + 1$. Therefore, opportunistic relaying is much superior than conventional in terms of ergodic capacity. It should be noted that, to improve the ergodic capacity of conventional scheme, number of antennas at the source and relay can be increased.

VI. CONCLUSIONS

In this paper, multi-antenna/multi-relay AF MRT with both conventional and opportunistic networks are investigated. In conventional relaying, source and all relays employing MRT participate the transmission to obtain considerable diversity gain. In contrast, opportunistic relaying selects the best path to maximize the received SNR at the destination and obtain identical diversity gains with low computational complexity. For both models, PDF, CDF and MGF are derived. Approximate and asymptotic SER and outage probability expressions are obtained, ergodic capacity is derived and diversity and array gains are computed. In addition, optimum source and relay powers are obtained and the theoretical derivations are verified by numerical examples. The proposed multi-relay MRT can be a promising option in practical wireless communication networks as they can provide high diversity gains while requiring low receiver complexity.

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Eylem Erdogan received B.S. and M.S. in Electronics Engineering from Isik University Turkey in 2003 and 2006. He received Ph.D. degree from Kadir Has University, Turkey in 2014. He is now working as a Post-Doc Fellow in Lakehead University, ON, Canada. His current research interests include multi-antenna, multi-relay structures, diversity techniques, cooperative, and two-way networks.

Tansal Gucluoglu received the B.S. degree from Middle East Technical University, Ankara, Turkey in 1997, the M.S. degree from Syracuse University in 2001 and the Ph.D. degree from Arizona State University in 2006, all in Electrical Engineering. He also worked as an RD Engineer in industry between 1997 and 2001 in Ankara and Syracuse. Between 2006-2012, he has been an Assistant Professor at the Department of Electronics Engineering, Kadir Has University, Istanbul. Since 2012, he has been affiliated with Department of Electronics and Communications Engineering, Yildiz Technical University, Istanbul. His research interests include channel coding, decoding and detection algorithms, multiple antenna techniques, and cooperative transmission systems for wireless communications.