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Maintenance optimization for a single wind turbine component under time-varying costs

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ABSTRACT

In this paper, we introduce a new, single-component model for maintenance optimization under time-varying costs, specifically oriented at offshore wind turbine maintenance. We extend the standard age replacement policy (ARP), block replacement policy (BRP) and modified block replacement policy (MBRP) to address time-varying costs. We prove that an optimal maintenance policy under time-varying costs is a time-dependent ARP policy. Via a discretization of time, the optimal time-dependent ARP can be found using a linear programming formulation. We also present mixed integer linear programming models for parameter optimization of BRP and MBRP. We present a business case and apply our policies for maintenance planning of a wind turbine gearbox and show that we can achieve savings up-to 23%.

1. Introduction

One of the most efficient ways to generate sustainable energy is by wind turbines, especially at sea. The United Kingdom, Germany and the Netherlands already generate 14 GW of power in offshore wind farms and plan to triple that by 2030. Nowadays the operation and maintenance costs make up around 25–30% of the total life cycle costs for an offshore wind farm (Röckmann, Lagerveld, & Stavenuiter, 2017). Since larger wind turbines are developed every year to generate more electricity, the operation and maintenance costs are likely to increase. It is therefore important that the maintenance operations are optimized to keep the costs of wind energy at a reasonable level.

To keep wind turbines operating, both preventive and corrective maintenance is done. Preventive maintenance may be time-based or condition-based. Time-based maintenance is much easier to plan than condition-based maintenance as the latter depends on the changes in condition, which are typically unpredictable. Both types of maintenance require preparation and shutdown of the wind turbine, which results in production loss. Yet the level of production loss depends very much on the wind speed, which varies over the year. In winter average wind speeds are much higher than during summer and working conditions are much harsher. According to the (Royal Academy of Engineering, 2021) the energy a wind turbine generates increases with a third power of the windspeed up to a level of about 5 Beaufort (12m/s), after which the energy generated remains constant. Figs. 3 and 4 in Section 6 show these relations in detail. As a result one wants to do maintenance during or around summer. Yet this may conflict with the best moment to do preventive maintenance. Solar energy output also exhibits a cyclic pattern over the year, but in that case one wants to do maintenance in winter. Note that a similar problem setting, maintenance optimization under time-varying downtime cost, is relevant for maintenance problems in other industries, e.g. equipment maintenance in touristic facilities, agriculture industry, and aviation.

Quite some research has been done on maintenance optimization. The classic work by Barlow & Proschan (1965) has been followed by very many papers. Yet few consider time-varying maintenance costs and if they do, they consider opportunity maintenance, where maintenance execution is triggered by a random event. Another related stream considers maintenance execution together with production planning, yet those papers often consider fitting maintenance into a short-term production plan. Finally, there exist some research works directly oriented at wind turbine maintenance. This studies are either technical or consider condition-based maintenance, like (Ciang, Lee, & Bang, 2008), while bypassing the option to coordinate it with the periods with low wind speed. The most relevant papers are by Besnard, Fischer, & Tjernberg (2013),

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who provide a general maintenance cost model for wind turbines but do not present general maintenance optimization results, and by Besnard, Patrikssont, Strombergt, Wojciechowskit, & Bertling (2009), who consider opportunity maintenance for wind turbines.

In this paper, we consider the wind turbine maintenance planning problem for a single component with predictable time-dependent maintenance costs. Maintenance is performed by replacing the component by an as-good-as-new component. We generalize classic, single-component maintenance policies, age-based and block-based replacement, which assume time-invariant costs. Under mild conditions, we prove the structure of the optimal maintenance policy, viz. periodic-age replacement policy. In addition, we suggest periodic-block replacement and modified block replacement policies, which can easily be applied to multi-component systems. We also present mathematical programming models to optimize policy parameters under the conditions where the finiteness of policy parameters are guaranteed. We present a business case for the implementation of our results in practice. Our results show that substantial savings (up to 20%) can be obtained advancing or deferring maintenance actions to the periods with lower cost rates. The modified block replacement policy performs similarly to the age replacement policy as variability in cost rates (over year) increases. In addition, we find that replacement times (over a year) are determined by low cost periods rather than by the increase of component failure rate.

The structure of this paper is as follows. In the next section, we present an overview of the relevant literature. In Section 3, we present the formal model. Section 4 presents three maintenance policies and provides conditions necessary for preventive maintenance to be cost-effective. It also provides mathematical programming formulations to determine the best policy. Numerical results are given in Section 5, while Section 6 presents an application of the maintenance policies to a case study on a wind turbine gearbox.

2. Literature review

The literature focusing on maintenance problems consists of a large number of studies with applications in many industries. In this section, we first review general maintenance optimization models. Next, we summarize models integrating maintenance and production planning decisions and finally we review wind park maintenance.

Literature on maintenance optimization models has been reviewed by Pierskalla & Voelker (1976), Sherif & Smith (1981), Valdez-Flores & Feldman (1989) in the 1980s and by Ding & Kamaruddin (2015) as well as (de Jonge & Scarf, 2020) recently. In maintenance planning, there are two primary decisions: the type and time of maintenance. Most common maintenance policies are the block and the age replacement policies, introduced already by Barlow & Proschan (1965), where preventive maintenance is done if a critical time or age has been reached. One useful result is that for a single component system with constant preventive and corrective maintenance costs, under mild conditions, the age-based maintenance policy is proven to be optimal with respect to average cost criterion (see e.g. Ross, 1970). This result was extended in many directions such as integrating maintenance and production, as executing maintenance often interferes with the latter.

The literature on maintenance planning in production facilities is relevant for our study as, similar to the maintenance of wind turbines, maintaining production equipment leads to downtime cost due to disrupted production. Budai, Dekker, & Nicolai (2008) and Hadidi, Al-Turki, & Rahim (2012) review studies in this part of the maintenance literature. They recognize the relation between job scheduling and opportunity maintenance, where maintenance interferes with production and (seldom and unplanned) maintenance opportunities arrive randomly. Several critical level policies are suggested where an opportunity is used whenever the age or time has exceeded a threshold. Only one level of reduced costs is considered, while for wind turbines there are many levels.

Studies on maintenance policies for wind parks constitute another research stream relevant to this paper. Shafiee & Sørensen (2017) give an overview of the logistical challenges that are encountered in optimizing all aspects of a wind park and present research that has been performed on many different topics. Design, infrastructure, transportation, operations, and maintenance should be jointly optimized for efficient generation of electricity. Lumbreras & Ramos (2013) give an overview of the research that has been performed on the optimization of the design of a wind park. They discuss which optimization models can be used to decide at which sites it is economical to build a wind park and how large the wind park should be. Besnard et al. (2013) suggest an algorithm for the number of maintenance teams required at all times and the number of vessels that should be available for transportation of the maintenance team. Besnard et al. (2013) and Seyr & Muskulus (2019) give an overview of the different maintenance strategies that are used for the maintenance of offshore wind parks. Byon, Ntaimo, & Ding (2010) and Byon & Ding (2010) consider the stochastic nature of weather conditions and solve partially observed Markov decision process models. In our paper, we extend some of these strategies to include time-varying costs.

Many studies on the maintenance optimization of wind turbine components assume the availability of sensor data. In these applications, the deterioration of components can be measured by means of sensors deployed on wind turbines (Ciang et al., 2008). Not all deterioration, however, can be measured and even if measurements are available, sensor readings might be different from the actual deterioration level due to uncertain environmental factors. Another problem with condition-based maintenance is the lack of plannability, which is needed for logistics. That is, we would like to know months ahead whether a failure is impending or not. We, therefore, focus on models that use a deterioration process of which we only know the statistical behavior and where failures occur unexpectedly. Well-known maintenance strategies associated with this problem are the (modified) block-based maintenance policies (or (modified) block replacement policies) and the age-based maintenance policies (or age replacement policies).

In the maintenance literature it is commonly assumed that the cost rates are known (in expectation) and constant over time (Nakagawa, 1984). Also in some studies age-dependent cost rates are considered for the maintenance optimization policies (Boland & Proschan, 1982; Chien, 2010; Sanoubar, Maillart, & Prokopyev, 2021; Sheu & Griffith, 2001). However, these assumptions rarely hold for wind farms where the production rates (and the downtime costs) mainly depend on wind speed and are hence time-varying in a cyclic way and not varying or increasing with age. To address this challenging feature, we generalize the constant cost rate assumption of maintenance policies in Section 3 by allowing costs to change over periods of a cycle, e.g. weeks of a year. To the best of our knowledge, our paper is the first study addressing time-dependent cost rates in single-component maintenance optimization.

3. Model description

We consider a single component in a continuously operating installation. For ease of exposition, we refer to an offshore wind turbine, but the model can also be applied to other installations with predictable time-varying production. We assume that failure of the component reduces production, hence corrective maintenance (CM) has to be performed directly. The system is maintained by replacing the failed component by an as-good-as-new compo-

Table 1
Nomenclature.

N	Number of periods within a year
$\mathbb{N}^+$	Set of positive integers, $\{1, 2, ...\}$
$\bar{\mathbb{N}}$	Set of extended natural numbers, $\{0, 1, 2, ..., \infty\}$
$\mathcal{I}_1$	Set of periods within a year
$\mathcal{I}_2$	Set of component ages
$\mathcal{I}$	State space of the Markov decision process
$\mathcal{I}^b$	Set of states representing a failed component
$c_p(i_1)$	PM cost in period $i_1 \in \mathcal{I}_1$.
$c_f(i_1)$	CM cost in period $i_1 \in \mathcal{I}_1$.
$\bar{c}_p$	Average PM cost
$\bar{c}_f$	Average CM cost
p_x	Failure rate of a component at age x.
X	Random lifetime of a component
$\mathbb{E}(X) = \mu$	Average lifetime of a component
c_X^2	Squared coefficient of variation of X
$g(T)$	Expected cost rate of ARP policy when the replacement age is equal to T.
$g(\infty)$	Cost rate of pure corrective maintenance policy
$\mathcal{A}(i_1, i_2)$	The set of possible actions in Markov decision process in state (i_1, i_2).
$\mathcal{R}_s$	Class of stationary policies.
$h(t)$	Renewal density for age t.
$H(t)$	Renewal function, i.e., the expected number of failures by age t
m	The number of years in a p-BRP cycle.
M	A large number representing the maximum age of the component.

nent. We discretize time into periods (say weeks or months). The lifetime of the component is denoted by X and $\mathbb{P}(X = k) > 0$ for all $k \in \bar{\mathbb{N}} \setminus \{\infty\}$, where $\bar{\mathbb{N}} = \{0, 1, \ldots, \infty\}$ is the set of extended natural numbers. We assume that the cdf of the lifetime is known. Moreover, the component has an increasing failure rate. Let $\mu = \mathbb{E}(X)$ and c_X^2 denote its squared coefficient of variation. Notations used in this paper are given in Table 1.

To prevent failure, the component can be replaced preventively by an as-good-as-new component. This action is called preventive maintenance (PM). We assume a seasonal cycle in which CM and PM execution costs vary periodically, e.g., by weather conditions. Typically we consider a yearly cycle but the model and its results can also be used for any other cycle lengths. The objective is to determine a preventive replacement policy which minimizes the long-run average costs. Such a policy should specify the age and the period (of the cycle) in which a PM is to be performed.

3.1. Markov decision process

We model the life of the component by a discrete-time Markov decision process (MDP) with a partially ordered state space consisting of periods of a year, $\mathcal{I}_1 \subseteq \mathbb{N}^+$, and the component age, $\mathcal{I}_2 \subseteq \bar{\mathbb{N}}$, where $\mathbb{N}^+$ is the set of positive integers. The state space $\mathcal{I}$ is given by $\mathcal{I} = \mathcal{I}_1 \times \mathcal{I}_2$. $\mathcal{I}_1 = \{1, 2, \ldots, N\}$ where N represents the number of periods in a year ($N = 12$ for months or $N = 52$ in case of weeks). The actions in each state, except the ones with the failed state, are either to replace the component ($a = 1$) or to do nothing ($a = 0$). For the states with age 0, i.e. the failed state denoted by $i_2 = 0$, we can only perform a CM. For all other states, we perform a PM. Hence, the state-dependent action space can be expressed as follows:

$$\mathcal{A}(i_1, i_2) = \begin{cases} \{1\} & \text{if } i_2 \in \{0, M\}, \\ \{0, 1\} & \text{otherwise}, \end{cases} \tag{1}$$

where M represents the maximum age. At age M, a PM is performed. Note that M might be equal to ∞, in which case we never perform a PM.

Transitions of the Markov chain are dependent on whether we plan a PM or a failure occurs. Specifically, the system jumps to a state with age 0 and a higher time period in case of failure whereas there is a transition to a state with a higher age and time period if there is no failure. In case of PM, there is an instantaneous jump to age 0, after which the component can reach age 1 at the end of the period, or have a failure and end with age 0.

The probability of failure depends on the age of the component, but is assumed to be independent of the time of the year whereas costs rates of PM and CM are time-dependent. Let $\pi_{(i_1,i_2)(j_1,j_2)}(a)$ be the transition probability from state (i_1, i_2) to state (j_1, j_2) under action $a \in \mathcal{A}(i_1, i_2)$. We have

$$\pi_{(i_1,i_2)(j_1,j_2)}(0) = \begin{cases} 1 - p_{i_2} & \text{for } j_1 = i_1 + 1 \pmod N, \ j_2 = i_2 + 1, \ i_2 \notin \{0, M\} \\ p_{i_2} & \text{for } j_1 = i_1 + 1 \pmod N, \ j_2 = 0, i_2 \notin \{0, M\} \\ 0 & \text{else}, \end{cases} \tag{2a}$$

$$\pi_{(i_1,i_2)(j_1,j_2)}(1) = \begin{cases} 1 - p_1 & \text{for } j_1 = i_1 + 1 \pmod N, \ j_2 = 1, \\ p_1 & \text{for } j_1 = i_1 + 1 \pmod N, \ j_2 = 0, \\ 0 & \text{else}. \end{cases} \tag{2b}$$

where $p_{i_2} = \mathbb{P}(X = i_2 | X \geq i_2)$ indicates the failure probability at age i_2 and mod is the modulo operator. It is easy to verify that the Markov decision chain defined by (1), (2a) and (2b) is unichain.

3.2. Cost parameters

The costs of PM and CM depend on the period i_1 of the year only and are denoted by $c_p(i_1)$ and $c_f(i_1)$ for $i_1 \in \mathcal{I}_1$. The cost of taking action a in state (i_1, i_2) $c_{(i_1,i_2)}(a)$ is

$$c_{(i_1,i_2)}(a) = \begin{cases} 0, & \text{if } a = 0, \\ c_p(i_1), & \text{if } a = 1, i_2 \neq 0, \\ c_f(i_1), & \text{if } a = 1, i_2 = 0. \end{cases} \tag{3}$$

Throughout the rest of this paper we denote the yearly average PM and CM cost rates by

$$\bar{c}_p := \frac{1}{N} \sum_{i_1=1}^{N} c_p(i_1), \qquad \bar{c}_f := \frac{1}{N} \sum_{i_1=1}^{N} c_f(i_1). \tag{4}$$

We will make comparisons with the case where PM and CM costs are constant through the year, and with values $\bar{c}_p$ and $\bar{c}_f$ respectively. We refer to this case as the *constant cost case*.

4. Analysis

4.1. Age replacement policies

In the standard age replacement policy (ARP), PM is performed when the component reaches (or is above) a critical age. In the constant cost case with increasing failure probabilities, ARP is optimal, see e.g. Ross (1970).

When the costs are time-varying, the optimal policy is not necessarily a policy with a constant critical maintenance age over the year. Intuitively, it makes sense to have a lower critical maintenance age in periods where preventive maintenance is relatively cheap. Accordingly, we introduce a period-dependent variant of the ARP policy.

Definition 1 (p-ARP)**.** A p-ARP is a policy in which in each period $i_1 \in \mathcal{I}_1$ of the year, there is a period-dependent critical maintenance age $t(i_1) \in \bar{\mathbb{N}} \setminus \{0\}$ at or above which PM is performed.

Note that critical maintenance age $t(i_1)$ may be ∞ for $i_1 \in \mathcal{I}_1$. Indeed, the CM policy is a special case of p-ARP policy where $t(i_1) = \infty$ for all periods $i_1 \in \mathcal{I}_1$. If a p-ARP policy has $t(i_1) < \infty$ for at least one period $i_1 \in \mathcal{I}_1$, we have a finite p-ARP policy. Similar to (Ross, 1970), we have the following result, for which the proof is in Appendix.

Theorem 1. *There exists an optimal maintenance policy under time-varying costs that is a p-ARP policy.*

This theorem holds due to the following. For $i_1^0 \in \mathcal{I}_1$ and $i_2^* \in \mathcal{I}_2$, if it is optimal to perform a PM in state (i_1^0, i_2^*), then it is optimal to perform a PM in all states (i_1^0, i_2) with $i_2 \geq i_2^*$, which is equivalent to a p-ARP policy.

If the lifetime distribution has infinite support the age of the component can be arbitrarily large and the cut-off to a finite value M may be too restrictive. We, therefore, need a sufficient condition for the optimal critical maintenance age to be finite. In the continuous time case with constant PM and CM costs (denoted by c_p and c_f, respectively), a sufficient condition for the optimal critical maintenance age to be finite is that the failure rate is increasing and $c_f > c_p$, see e.g. Barlow & Proschan (1965). For discrete lifetime distributions with constant costs, Nakagawa (1984) shows that the optimal critical maintenance age is finite if the failure rate is increasing, $c_f > c_p$ and

$$p_\infty = \lim_{k\to\infty} p_k > \frac{1}{\mathbb{E}(X)} \frac{c_f}{c_f - c_p}, \tag{5}$$

In our paper, we extend these results to the time-varying cost case. Lemma 1 is a building block towards the result for time-varying cost case. Its proof is in Appendix.

Lemma 1. *Let $g(T)$ denote the long-run average maintenance cost for a p-ARP policy, where we maintain at age $T \in \bar{\mathbb{N}} \setminus \{0\}$ in all periods $i_1 \in \mathcal{I}_1$. Then, we have $g(T) = \bar{g}(T)$, where $\bar{g}(T)$ is the long-run average cost for the problem where the time-varying maintenance costs are replaced by the averages over the year (i.e. $\bar{c}_p$ and $\bar{c}_f$).*

Lemma 1 implies that the cost $g(\infty)$ under the CM policy is

$$g(\infty) = \frac{\bar{c}_f}{\mathbb{E}(X)}. \tag{6}$$

We can now extend the result of (Nakagawa, 1984) to the time-varying cost case and obtain Theorem 2.

Theorem 2. *If $\bar{c}_f > \bar{c}_p$ and $p_\infty > \frac{1}{\mathbb{E}(X)} \frac{\bar{c}_f}{\bar{c}_f - \bar{c}_p}$, then there exists a finite optimal p-ARP policy.*

Proof. Consider a constant cost case for which condition (5) is satisfied and a time-varying cost case where average PM and CM costs are $\bar{c}_p = c_p$ and $\bar{c}_f = c_f$, respectively. By Lemma 1, the long-run average cost of the time-varying cost case is equal to that of the constant cost case under corrective maintenance policy. For the constant cost case, there exists an optimal finite ARP policy according to Nakagawa (1984). Consider this ARP policy in the time-varying cost case, i.e., consider the same critical maintenance age in all periods. By Lemma 1, this policy has the same long-run average cost as the minimum long-run average cost in the constant cost case and hence less costly than the corrective maintenance policy. Hence, in the time-varying cost case, this policy leads to lower costs than the corrective maintenance policy. □

It is possible to sharpen the condition in Theorem 2, as in fact, the PM cost needs to be low enough in one period only. It is, however, complex to give a simple condition for the time-varying cost case, as there can be a coordination between the period where PM is performed and the periods where failures are likely to occur. Therefore, it may be that if PM is performed in a cheap period, the failures occur in expensive periods. In case failure costs are the same for all periods, we have the following result, for which the proof is provided in Appendix.

Theorem 3. *Suppose $c_f(i_1) = \bar{c}_f$ for all $i_1 \in \mathcal{I}_1$. If there exists $i_1^0 \in \mathcal{I}_1$ satisfying $\bar{c}_f > c_p(i_1^0)$ and $p_\infty > \frac{1}{\mathbb{E}(X)} \frac{\bar{c}_f}{\bar{c}_f - c_p(i_1^0)}$, then there exists a finite optimal p-ARP.*

Notice that, under the existence of a finite optimal p-ARP policy, there exists a finite maximum age the component can attain, hence it is allowed to truncate the state space. That is, if the conditions in Theorem 2 or 3 are met, it is justified to limit the age dimension of the state space to a large number M.

Finding the optimal policy in an MDP can be done in several ways. Tijms (2003) provides a linear programming (LP) formulation. Alternatively, one can use policy improvement or value iteration algorithms for optimization. The LP approach is not only fast and easy-to-implement, but also it allows us to find specific types of policies by manipulating the constraint set and variables. Therefore, we proceed with the this approach.

To optimize the long-run average cost, we need to distinguish the states with a failed component. We denote the set of states representing a failed component by $\mathcal{I}^b = \mathcal{I}_1 \times \{0\}$. The following LP leads to the optimal policy with respect to the long-run average cost criterion (see Tijms (2003)):

$$\text{(p-ARP)} \qquad \min \sum_{i=(i_1,i_2)\in\mathcal{I}\setminus\mathcal{I}^b} c_p(i_1) x_{i,1} + \sum_{i=(i_1,i_2)\in\mathcal{I}^b} c_f(i_1) x_{i,1} \tag{7a}$$

s.t.

$$\sum_{a\in\mathcal{A}(i)} x_{i,a} - \sum_{j\in\mathcal{I}} \sum_{a\in\mathcal{A}(j)} \pi_{ji}(a) x_{j,a} = 0, \;\; \forall i = (i_1, i_2) \in \mathcal{I} \tag{7b}$$

$$\sum_{i_2\in\mathcal{I}_2} \sum_{a\in\mathcal{A}(i_1,i_2)} x_{i_1,i_2,a} = \frac{1}{N}, \;\; \forall i_1 \in \mathcal{I}_1 \tag{7c}$$

$$x_{i,a} \geq 0, \;\; \forall i = (i_1, i_2) \in \mathcal{I}, \; a \in \mathcal{A}(i) \tag{7d}$$

Decision variables $x_{i,a}$ can be interpreted as the long-run probability that the system is in state $i = (i_1, i_2) \in \mathcal{I}$ at the beginning of the period and the decision $a \in \mathcal{A}(i_1, i_2)$ is chosen. The objective function (7a) represents the long-run average cost. Suppose we have an optimal solution $x^*_{i,a}$ to the LP problem. Let the set $\mathcal{I}_{LP} = \{i | x^*_{i,a} > 0 \text{ for some action } a\}$ and define the strategy R^* by $R^*(i) = a$ if $x^*_{i,a} > 0$. For all other states i one chooses an action a such that $p_{ij}(a) > 0$ for some $j \in \mathcal{I}_{LP}$ and add the states i recursively to the set $\mathcal{I}_{LP}$ until no remains. The resulting policy R^* is average optimal. In case the policy R^* does PM for a state (i_1^0, i_2^0) with $i_2^0 \neq M$ then by Theorem 1 the policy which replaces for all states (i_1, i_2) with $i_2 \geq i_2^0$ is optimal as well. Hence, we can construct an optimal p-ARP from the solution to LP (7a)-(7d).

4.2. Block replacement policies

A block replacement policy (BRP) replaces a component preventively after a fixed amount of time since the previous PM. If the component fails before the fixed PM time, an immediate CM is performed. In that case, the next PM is still performed at the predetermined fixed time. In other words, PM times do not depend on the time passed since the preceding CM. Since the costs for wind turbine maintenance vary in time, we allow intervals between the so-called block times to vary as well, yet with a repetitive pattern, e.g. do maintenance after 4 months, then after 6 months and repeat this sequence. Formally, we perform maintenance at periods $T_1, T_2, \ldots, T_n$, where $T_k \in \mathbb{N}^+ \ \forall k = 1, 2, \ldots, n$ and $n \in \mathbb{N}^+$. To limit the scope of possible policies, we restrict ourselves to policies that repeat themselves every m years for some $m \in \mathbb{N}^+$. In this way, we generalize block maintenance policies.

Definition 2 (p–BRP)**.** A p-BRP is a policy with a finite cycle of m years in which PM is performed at times $T_1, T_2, ..., T_n < mN$, for some $n \in \mathbb{N}^+$, since the start of the cycle, regardless of the age of the component at those times.

The intervals between PM may be different within one cycle, but they are the same in subsequent cycles. We chose this way of extending the BRP as it guarantees the execution of maintenance in always the same periods, whereas if we would stick to constant intervals, the time periods in which we maintain may shift every year. We call a p-BRP optimal if it has the lowest long-run average cost compared to all p-BRPs and the CM policy.

The Markov decision process that we discussed in Section 3.1 needs a small adaptation to incorporate p-BRP. Instead of limiting i_1 to values in $\{1, 2, ..., N\}$, we now allow them to take values in $\mathcal{I}_1 = \{1, 2, ..., mN\}$ in order to allow varying block periods in a cycle of multiple years. The set $\mathcal{I}$ is extended accordingly. For the constant cost case, the condition for the optimal maintenance block time T to be finite is given by Nakagawa (1984) and in Lemma 2. We give the proof in order to show the origin of the condition. This condition is stricter than the condition for p-ARP.

Lemma 2. *In the constant cost case, if* $\frac{c_p}{c_f} < \frac{1}{2}(1 - c_X^2 - \frac{1}{\mu})$*, then the optimal block time is finite.*

Proof. The long-run average cost under BRP with block time t, $g(t) \leq g(\infty)$ for t large enough, which follows from $\lim_{t\to\infty} \left(H(t) - \frac{t}{\mu}\right) = \frac{1}{2}(c_X^2 - 1 + \frac{1}{\mu})$, (Feller, 1949, Eq. (6.7)), where $H(\cdot)$ is the discrete renewal function. □

If we maintain preventively at the cheapest moment once every m years in the time-varying cost case, $\min_{i_1}\{c_p(i_1)\}$ is incurred as PM cost instead of $\bar{c}_p$. The total CM cost over a period of length t are however, no longer $\bar{c}_f H(t)$ as in the constant cost case, since they depend on the periods in which failures occur. We, therefore, need an adjustment term, which expresses the relative difference in total costs when starting in period i_1 compared to the total costs in the constant cost case. We denote the expected number of failures at time t in the discrete renewal process by $h(t) = H(t) - H(t-1)$. As we allow only one failure per period, $h(t)$ is the probability of failure at time t. Let

$$b(i_1) = \limsup_{n\to\infty} \left\{ \sum_{t=1}^{nN} h(t) \left(\frac{c_f(i_1 + t)}{\bar{c}_f} - 1 \right) \right\}, \tag{8}$$

where the lim sup is taken over numbers $n \in \mathbb{N}^+$ only and i_1 is any starting period in the cycle. The parameter $b(i_1)$ depends on the failure distribution but is finite for example for Weibull distributions. As long as the renewal density converges to $\frac{1}{E(X)}$ fast enough, $b(i_1)$ is finite for all $i_1 \in \mathcal{I}_1$. The convergence needs to be of the order $\mathcal{O}(t^{-a})$ for some $a > 1$ for the limit to exist. Note that the parameter $b(i_1)$ can be negative or positive. Using $b(i_1)$, we give an adjusted condition for the existence of an optimal policy within the class of p-BRPs in Theorem 4, which is proved in Appendix.

Theorem 4. *If there exists a period* $i_1^0 \in \mathcal{I}_1$ *satisfying* $\frac{c_p(i_1^0)}{\bar{c}_f} + b(i_1^0) < \frac{1}{2}(1 - c_X^2 - \frac{1}{\mu})$*, then there exists an optimal p-BRP.*

If the limit exists for all periods $i_1 \in \mathcal{I}_1$, then $\sum_{i_1 \in \mathcal{I}_1} b(i_1) = 0$. This implies that the condition in Theorem 4 is weaker than the one for the constant cost case, as we require it only for one period, rather than for all periods. If PM and CM costs are constant over the year, then $b(i_1) = 0$ for all $i_1 \in \mathcal{I}_1$.

Theorem 5. *If* $\frac{\bar{c}_p}{\bar{c}_f} < \frac{1}{2}(1 - c_X^2 - \frac{1}{\mu})$ *and* $b(i_1)$ *is finite for all* $i_i \in \mathcal{I}_1$*, then there exists an optimal p-BRP.*

The idea behind the proof of Theorem 5 is as follows. If the condition in Theorem 5 holds, then there exists a period $i_1^0 \in \mathcal{I}_1$ satisfying the condition in Theorem 4. In this case, there exists a p-BRP which is cheaper than the BRP that is optimal for the constant cost case.

We can calculate the optimal parameters of the p-BRP with a mixed integer linear programming (MILP) formulation, developed with the introduction of some extra constraints to the model (7). To our opinion, this formulation is new and avoids explicit calculation of the renewal function $H(t)$.

We define variables indicating in which periods to perform PM by

$$y_{i_1} = \begin{cases} 1, & \text{if we maintain preventively in period } i_1 \in \mathcal{I}_1, \\ 0, & \text{else.} \end{cases} \tag{9}$$

The following MILP leads to the optimal p-BRP policy with respect to the long-run average cost criterion.

$$\text{(p-BRP)} \quad \text{minimize} \quad \sum_{i=(i_1,i_2)\in\mathcal{I}\setminus\mathcal{I}^b} c_p(i_1)x_{i,1} + \sum_{i=(i_1,i_2)\in\mathcal{I}^b} c_f(i_1)x_{i,1} \tag{10a}$$

s.t.:

$$\sum_{a\in\mathcal{A}(i)} x_{i,a} - \sum_{j\in\mathcal{I}} \sum_{a\in\mathcal{A}(j)} \pi_{ji}(a)x_{j,a} = 0 \qquad \forall i = (i_1, i_2) \in \mathcal{I} \tag{10b}$$

$$x_{i,0} + y_{i_1} \leq 1 \qquad \forall i = (i_1, i_2) \in \mathcal{I} : i_2 > 0 \tag{10c}$$

$$x_{i,1} - y_{i_1} \leq 0 \qquad \forall i = (i_1, i_2) \in \mathcal{I} : i_2 > 0 \tag{10d}$$

$$\sum_{i_2\in\mathcal{I}_2} \sum_{a\in\mathcal{A}(i)} x_{i,a} = \frac{1}{mN} \qquad \forall i_1 \in \mathcal{I}_1 \tag{10e}$$

$$x_{i,a} \geq 0 \qquad \forall i = (i_1, i_2) \in \mathcal{I},\ a \in \mathcal{A} \tag{10f}$$

$$y_{i_1} \in \{0, 1\} \qquad \forall i_1 \in \mathcal{I}_1 \tag{10g}$$

For p-BRP, the maintenance decision depends on the period within the cycle and it is independent of the age of the component. If we maintain preventively ($a = 1$) in a period $i_1 \in \mathcal{I}_1$ (or do not maintain preventively), we are not allowed to take the other action $a = 0$ (or $a = 1$) for any age $i_2 > 0$. We enforce this by constraints (10c) and (10d).

If the condition in Theorem 4 holds, the policy parameters found are optimal. Otherwise, one has to compare the long-run average cost of the p-BRP found with those of the CM policy. The optimal p-BRP policy can be obtained in a similar way as for LP (7).

It may be optimal to plan preventive maintenance on a yearly basis, then it suffices to consider $m = 1$ only. This depends on the cost and distribution parameters of the problem. Considering a larger m may improve the solution, but does increase computation time, since the number of decision variables increases. One should therefore check if the optimal policy remains the same for larger m.

Note that p-BRP is a special case of p-ARP, where the critical maintenance age is 1 for the periods that we maintain and infinity for other periods. The long-run average cost induced by a p-BRP is thus at least the cost induced by the optimal p-ARP.

4.3. Modified block replacement policies

The advantage of block over age replacement is that it is easier to plan the maintenance ahead and to coordinate maintenance of multiple components. Yet it may also replace a component at a relatively young age. Berg & Epstein (1976) proposed a modification of block replacement in which preventive replacements are possible at fixed times, like in block replacement, but the actual replacement is performed only if the component's age is larger than a certain threshold. We will extend this policy to the time-varying cost case. The periodic MBRP is formally defined in Definition 3.

Definition 3 (p–MBRP)**.** A p-MBRP is a policy with a finite cycle of m years, in which PM can only be performed at times $T_1, T_2, \dots, T_n \le mN$, for some $n \in \mathbb{N}^+$. PM is performed at occasion k if a critical maintenance age $t_{(k)}$ has been reached, where $t_{(k)} \le T_k - T_{k-1}$ for $k = 2, 3, \dots n$ and $t_{(1)} \le T_1 - T_n + mN$.

The MBRP with constant PM, CM costs c_p and c_f in which the block size is constant is referred to as the standard MBRP. The requirement that $t_{(k)} \le T_k - T_{k-1}$ is added to differentiate the policy from ARP, as choosing $T_k = k, k = 1, 2, \dots, mN$ would yield an ARP policy. In the sequel we use another, equivalent specification of the MBRP, where to each period in $\mathcal{I}_1$, we associate a threshold age, denoted by t_k for $k \in \mathcal{I}_1$, of when to perform PM at period k. So, if $T_k = i_1$, then $t_{i_1} = t_{(k)}$ which is the minimum age at which we maintain in period $i_1 \in \mathcal{I}_1$. The periods in which PM is not performed get a threshold larger than the maximum age M. Notice that in this way the policy looks similar to a p-ARP. In fact, a p-ARP policy which maintains only in one period per year and thus has thresholds ∞ in all other periods is a p-MBRP.

Similar to the p-ARP and p-BRP, we develop sufficient conditions for MBRP to be better than the CM policy for some finite cycle m and finite opportunities T_k and critical ages t_k. Archibald & Dekker (1996) state that the necessary conditions for block times to be finite in the constant cost case are sufficient for the standard MBRP problem, but also observe that these conditions may be too strict. Here, we confirm that conjecture by stating that the weaker conditions for ARP are sufficient for the MBRP and p-MBRP. The proofs are in Appendix.

Theorem 6. *If $c_f > c_p$ and $p_\infty > \frac{1}{\mathbb{E}(X)} \frac{c_f}{c_f - c_p}$, then the optimal standard MBRP has a finite optimal critical maintenance age t and block time T.*

A similar condition holds for the time-varying costs case.

Theorem 7. *If $\bar{c}_f > \bar{c}_p$ and $p_\infty > \frac{1}{\mathbb{E}(X)} \frac{\bar{c}_f}{\bar{c}_f - \bar{c}_p}$, then the optimal p-MBRP has finite cycle m, block times $T_1, T_2, \dots, T_n$, and critical ages $t_{(1)}, t_{(2)}, \dots, t_{(n)}$.*

The proofs rely on the observation that a MBRP and p-MBRP are just versions of ARP and p-ARP where PM is restricted to certain periods. Under finiteness conditions, we again develop a MILP formulation to determine the optimal parameters of p-MBRP. We first introduce extra variables that decide for which period and which age we maintain.

$$z_i = z_{i_1,i_2} = \begin{cases} 1, & \text{if we maintain for age } i_2 \in \mathcal{I}_2 \text{ in period } i_1 \in \mathcal{I}_1, \\ 0, & \text{else.} \end{cases} \tag{11}$$

The following MILP leads to the optimal p-MBRP policy with respect to the long-run average cost criterion.

$$\text{(p-MBRP)} \qquad \text{minimize} \sum_{i=(i_1,i_2)\in\mathcal{I}\setminus\mathcal{I}^b} c_p(i_1)x_{i,1} + \sum_{i=(i_1,i_2)\in\mathcal{I}^b} c_f(i_1)x_{i,1} \tag{12a}$$

s.t.

$$\sum_{a\in\mathcal{A}(i)} x_{i,a} - \sum_{j\in\mathcal{I}} \sum_{a\in\mathcal{A}(j)} \pi_{ji}(a)x_{j,a} = 0 \qquad \forall i = (i_1, i_2) \in \mathcal{I} \tag{12b}$$

$$\sum_{i_2\in\mathcal{I}_1} \sum_{a\in\mathcal{A}(i)} x_{i,a} = \frac{1}{mN} \qquad \forall i_1 \in \mathcal{I}_1 \tag{12c}$$

$$z_{i_1,i_2} - y_{i_1} \le 0 \qquad \forall i_1 \in \mathcal{I}_1, \forall i_2 \in \mathcal{I}_2 \tag{12d}$$

$$z_{i_1,i_2} - z_{i_1,j_2} \le 0 \qquad \forall (i_1, i_2) \in \mathcal{I}, \forall j_2 \in \mathcal{I}_2 : i_2 < j_2 \tag{12e}$$

$$t_{i_1} + j_1 y_{j_1} + mN y_{j_1} \le mN + i_1 \qquad \forall i_1, j_1 \in \mathcal{I}_1 : j_1 < i_1 \tag{12f}$$

$$t_{i_1} + j_1 y_{j_1} \le mN + i_1 \qquad \forall i_1, j_1 \in \mathcal{I}_2 : j_1 > i_1 \tag{12g}$$

$$My_{i_1} - Mz_{i_1,i_2} - t_{i_1} \le M - 1 - i_2 \qquad \forall i_1 \in \mathcal{I}_1,\ i_2 \in \mathcal{I}_2 \tag{12h}$$

$$Mz_{i_1,i_2} + t_{i_1} \le M + i_2 \qquad \forall i_1 \in \mathcal{I}_1,\ i_2 \in \mathcal{I}_2 \tag{12i}$$

$$x_{i,a} \ge 0 \qquad \forall i \in \mathcal{I},\ a \in \mathcal{A} \tag{12j}$$

$$z_i \in \{0, 1\} \qquad \forall i \in \mathcal{I} \tag{12k}$$

$$y_{i_1} \in \{0, 1\} \qquad \forall i_1 \in \mathcal{I}_1 \tag{12l}$$

$$t_{i_1} \in \mathbb{N}^+ \qquad \forall i_1 \in \mathcal{I}_1 \tag{12m}$$

In constraint (12d), y_{i_1} is as defined in (9). Constraint (12d) ensures that if we maintain for age i_2 in a certain period i_1, then we must maintain for all ages j_2 with $j_2 > i_2$ in this period. Constraints (12f) and (12g) ensure that the condition on the critical maintenance age holds. When $y_{i_1} = 0$, t_{i_1} can take any value, since maintenance is never performed in period $i_1 \in \mathcal{I}_1$. If $y_{i_1} = 1$, then t_{i_1} is less than or equal to the time since the previous opportunity, where we take the repetition of the cycle at mN into account.

Table 2
Yearly costs, in thousands of €, savings with respect to constant cost model, and maintenance months for p-ARP, p-BRP and p-MBRP. We let $\bar{c}_f = 50$, $\bar{c}_p = 10$, $\Delta_f = \Delta \cdot \bar{c}_f$ and $\Delta_p = \Delta \cdot \bar{c}_p$ for different Δ. We let $\alpha = 1$ year and $\beta = 2$.

	p-ARP		p-BRP ($m = 1$)			p-MBRP ($m = 1$)			
Δ	costs	savings	costs	savings	mos.	costs	savings	mos.	ages
0%	40.098		41.501			40.311			
10%	40.035	0.16%	41.420	0.20%	6, 11	40.263	0.12%	6, 11	4, 4
20%	39.701	0.99%	40.933	1.37%	6, 11	39.855	1.13%	6, 11	4, 4
30%	39.224	2.18%	40.361	2.75%	6, 10	39.338	2.41%	6, 10	5, 3
40%	38.461	4.08%	39.439	4.97%	6, 10	38.556	4.35%	6, 10	5, 3
50%	37.635	6.14%	38.466	7.31%	7, 10	37.773	6.30%	6, 10	5, 3
CPU	< 1 s		< 1 s			3 s			

*Optimal age for $\Delta = 0$ is $t^* = 6$ months with costs 40.098. **Optimal block for $\Delta = 0$ is $T^* = 6$ months with costs 41.501. ***Optimal modified block for $\Delta = 0$ is $(t^*, T^*) = (4, 6)$ with costs 40.311. *** Costs for not maintaining at all is $g(\infty) = 53.885$.

Table 3
Yearly costs, in thousands of €, savings with respect to constant cost model, and maintenance months for p-ARP, p-BRP and p-MBRP. We let $\bar{c}_f = 50$, $\bar{c}_p = 10$, $\Delta_f = \Delta \cdot \bar{c}_f$ and $\Delta_p = \Delta \cdot \bar{c}_p$ for different Δ. We let $\alpha = 3$ years and $\beta = 2$.

	p-ARP		p-BRP (m=3)			p-MBRP (m=3)			
Δ	costs	savings	costs	savings	mos.	costs	savings	mos.	ages
0%	13.530		14.173			13.622			
10%	13.252	2.05%	13.828	2.43%	6, 21	13.338	2.08%	6, 21	11, 9
20%	12.707	6.08%	13.135	7.32%	7, 19, 31	12.707	6.72%	7, 19, 31	12, 12, 12
30%	11.779	12.94%	12.114	14.53%	7, 19, 31	11.779	13.53%	7, 19, 31	10, 10, 10
40%	10.844	19.85%	11.093	21.73%	7, 19, 31	10.844	20.39%	7, 19, 31	8, 8, 8
50%	9.900	26.83%	10.072	28.94%	7, 19, 31	9.900	27.32%	7, 19, 31	7, 7, 7
CPU	< 1 s		2 s			17 s			

*Optimal age for $\Delta = 0$ is $t^* = 19$ months with costs 13.530. **Optimal block for $\Delta = 0$ is $T^* = 18$ months with costs 14.173. ***Optimal modified block for $\Delta = 0$ is $(t^*, T^*) = (11, 18)$ with costs 13.622. ****Costs for not maintaining at all is $g(\infty) = 18.516$.

In addition, maintenance times for all $i_1 \in \mathcal{I}_1$ and $i_2 \in \mathcal{I}_2$ should satisfy:

$$z_{i_1,i_2} = \begin{cases} y_{i_1}, & \text{if } i_2 \geq t_{i_1}, \\ 0, & \text{elsewhere.} \end{cases} \tag{13}$$

Using the relatively large number M (the maximum age), we rewrite (13) as linear constraints (12h) and (12i).

5. Numerical results

We are interested in the effect of the yearly weather cycle on the optimal maintenance interval and the cost savings due to the coordination of preventive maintenance with low wind periods. In this section we present the results of our numerical experiments for several components with different lifetime and cost parameters. We assume that the component's lifetime X follows a discretized Weibull distribution, with cdf $F(x) = 1 - exp(\frac{x}{\alpha})^{\beta}$ and $P(X = x) = F(x) - F(x-1)$ for $x \in \bar{\mathbb{N}}$, where $\alpha > 0$ is the scale parameter and $\beta > 0$ is the shape parameter of the Weibull distribution. Hence, the probability mass is shifted to the right and the distribution is strictly positive. Moreover, we assume that both preventive and corrective maintenance costs are time-varying with a yearly cycle. That is,

$$c_p(i_1) = \bar{c}_p + \Delta_p \ \cos\left(\frac{2\pi i_1}{N} + \phi\right), \qquad c_f(i_1) = \bar{c}_f + \Delta_f \ \cos\left(\frac{2\pi i_1}{N} + \phi\right).$$

We will express time i_1 in months, hence $N = 12$ and $i_1 = 1$ indicates January. The constant ϕ is chosen such that the lowest costs are obtained in July and highest in winter months. Hence $\phi = \frac{-2\pi}{12}$. This typically models Northern European wind patterns. The parameter m is given in each table. We solve the MILPs using CPLEX 12.8.0 in Java.

Tables 2 and 3 show the results for two cases, Weibull scale parameters α being either 1 or 3 years (the corresponding mean lifetimes $\mathbb{E}(X)$ are 0.9 and 2.7 years respectively), Weibull shape parameter β being 2, and $\bar{c}_f$ being 5 times larger than $\bar{c}_p$. Note the difference in behaviour of the two cases as the mean lifetime is either shorter or longer than a year.

Cost savings are calculated as the relative cost difference under period-dependent policies p-ARP, p-BRP, and p-MBRP compared to ARP, BRP, and MBRP, respectively. ARP, BRP, and MBRP policies are obtained considering the constant cost case. Table 2 shows that the cost savings are larger when there are larger cost fluctuations (i.e. a higher Δ) over the year. Increasing the maintenance cycle horizon, m, from 1 to $2, 3, \ldots, 8$ does not lower the costs and gives the same policy for p-BRP and p-MBRP as obtained with $m = 1$. Hence, these results are omitted. The savings in Table 2 (with $\mathbb{E}(X) = 0.9$) are relatively small, since we maintain multiple times a year and it is therefore difficult to maintain only at cheap moments. Fig. 1 shows that for the constant cost case, the optimal critical replacement age is 6 months. As Δ increases the optimal critical age becomes more dependent on the period of the year and it varies between 3 and M months. In this way, one prevents PM in winter months and allows earlier PM in summer and fall months.

Table 3 (with $\alpha = 3, \mathbb{E}(X) = 2.7$ year) also shows that the cost savings are larger when there are larger fluctuations over the year. We found that an increase in m to $m = 4, \ldots, 8$ does not give lower costs, and hence these results are omitted. The savings in Table 3 are much larger than in Table 2, since we do PM mostly on a yearly basis and we are therefore able to maintain preventively at cheap moments only. For $\Delta = 10\%$ the p-BRP and p-MBRP policies maintain in June (month 6), then 15 months later in September (month 21), then 21 months later in June and so forth. In this way we save 2.43% and 2.08% with respect to maintaining every

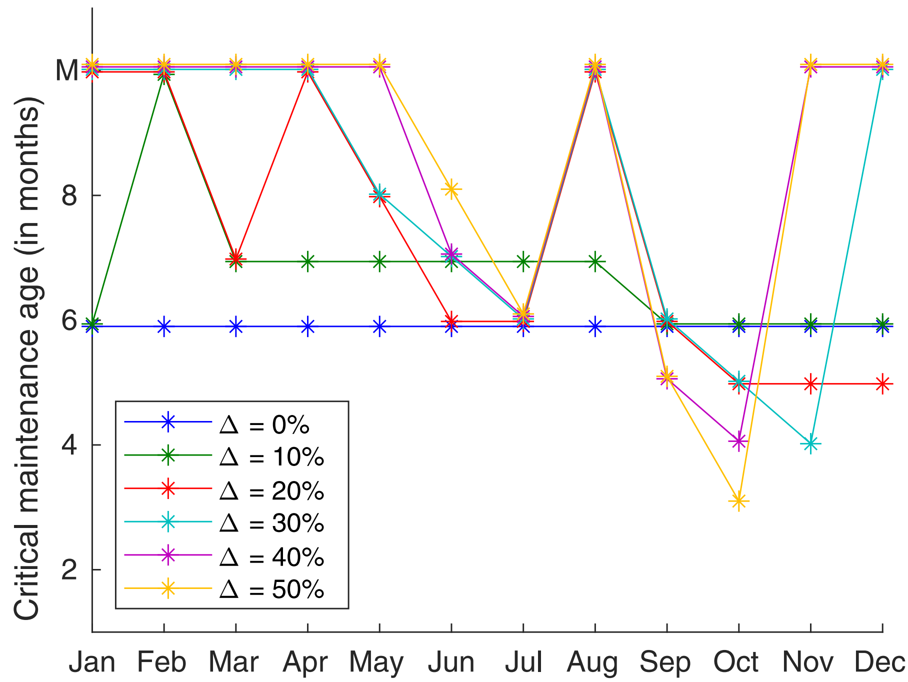

Fig. 1. Critical maintenance ages for p-ARP with $\alpha = 1$ year, $\beta = 2$, $\bar{c}_f = 50$, $\bar{c}_p = 10$.

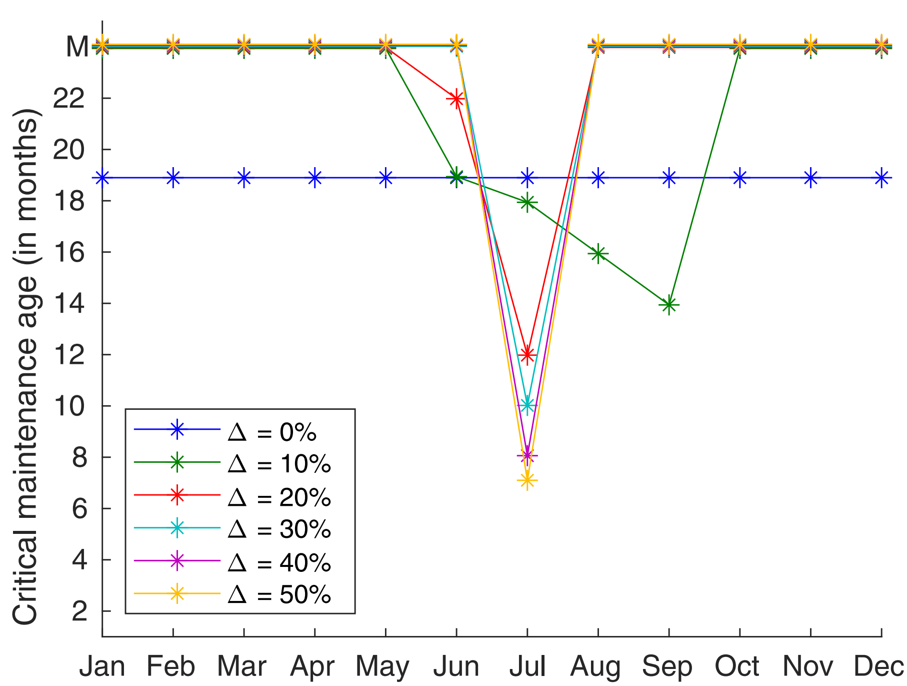

Fig. 2. Critical maintenance ages for p-ARP with $\alpha = 3$ year, $\bar{c}_f = 50$ and $\bar{c}_p = 10$.

18 months, as dictated by BRP and MBRP policies for the constant cost case. As the fluctuations increase to $\Delta = 20\%, \ldots, 50\%$, maintenance in summer only yields the lowest cost rates and the interval reduces to 12 months. As a result maintenance is performed on a yearly basis in July for these scenarios. The minimal age for the p-MBRP decreases for increasing fluctuations, since maintenance is relatively cheap in summer for large cost fluctuations.

In Fig. 2, we show how the critical p-ARP ages change if the cost fluctuations (Δ) increase. We observe that PM is performed in July only for $\Delta \geq 30\%$. Thus, the p-ARP policy reduces to an p-MBRP policy. The explanation is that the savings of doing PM in July versus other months outweigh the costs of having a smaller interval compared to the constant cost optimum of 19 months. Indeed, average cost curves for ARP or BRP are often quite flat around the optimum. Moreover, an increasing Δ implies a higher ratio of PM costs in summer versus CM costs in winter, implying a lower age threshold for PM in summer.

The savings depend heavily on the parameters of the model. We should consider many parameters to be able to assess the added value of considering the cost fluctuations. We, therefore, consider $\alpha = 1, 3, 5, 8, 10$ years, $\beta = 2, 3$ and $\frac{c_f}{c_p} = 5, 10$ and compute the percentage savings for each scenario. Table 4 gives the summarized

Table 4
Average percentage savings of the p-ARP, p-BRP, p-MBRP policies compared to the constant cost model. The average savings are given for each parameter, averaged over the other parameter settings.

Δ	Method	$\alpha=1$	$\alpha=3$	$\alpha=5$	$\alpha=8$	$\alpha=10$	$\beta=2$	$\beta=3$	$c=5$	$c=10$	Average
	ARP	0.06%	3.80%	2.73%	5.15%	5.23%	2.95%	3.85%	3.06%	3.73%	3.40%
10%	BRP	0.14%	4.47%	4.45%	5.42%	5.48%	3.28%	4.70%	3.55%	4.43%	3.99%
	MBRP	0.14%	4.30%	4.06%	5.02%	5.15%	2.96%	4.51%	3.22%	4.24%	3.73%
	ARP	0.66%	9.25%	7.76%	10.73%	10.74%	6.82%	8.84%	7.39%	8.26%	7.83%
20%	BRP	0.87%	10.45%	10.65%	11.28%	11.28%	7.49%	10.32%	8.51%	9.30%	8.91%
	MBRP	0.83%	9.86%	9.99%	10.69%	10.70%	6.98%	9.85%	7.82%	9.01%	8.41%
	ARP	1.62%	16.55%	13.48%	16.92%	17.12%	11.52%	14.76%	12.95%	13.33%	13.14%
30%	BRP	1.94%	18.05%	17.14%	17.89%	18.17%	12.45%	16.83%	14.64%	14.64%	14.64%
	MBRP	1.83%	17.26%	16.36%	17.09%	17.31%	11.75%	16.19%	13.67%	14.27%	13.97%
	ARP	2.95%	24.10%	20.98%	23.80%	23.94%	16.69%	21.61%	19.21%	19.10%	19.15%
40%	BRP	3.33%	25.65%	24.36%	24.93%	25.09%	17.84%	23.51%	21.02%	20.32%	20.67%
	MBRP	3.16%	24.73%	23.39%	23.94%	24.05%	16.90%	22.81%	19.85%	19.86%	19.86%
	ARP	4.78%	31.68%	28.61%	31.14%	31.32%	22.18%	28.83%	25.71%	25.30%	25.51%
50%	BRP	5.34%	33.25%	31.57%	32.45%	32.60%	23.51%	30.57%	27.62%	26.46%	27.04%
	MBRP	5.00%	32.25%	30.46%	31.30%	31.54%	22.39%	29.82%	26.30%	25.92%	26.11%

results for the p-ARP, p-BRP, and p-MBRP policies respectively for these parameters. In each column the average of the savings for the value of all other parameters is given, i.e. in the column for $\alpha = 1$ the savings are given over the constant cost model, averaged over the savings for the parameters $\beta = 2, 3$ and $c = \frac{c_f}{c_p} = 5, 10$.

Table 4 shows that the savings are very small for $\alpha = 1$ year and significantly larger for higher α values. The behavior is not monotonic in α. First, we see an increase (between $\alpha = 1, 3$ years), then a decrease (between $\alpha = 3, 5$ years) and then again an increase (between $\alpha = 5, 8$ years). The differences between $\alpha = 8$ and $\alpha = 10$ years are very small. The explanation is that the change of maintenance interval (such that we maintain in summers only) leads to a relatively smaller change in case of a large lifetime, dictated by a large α value. The non-monotonic behavior is due to averaging over different parameters. If we fix β and c, the figures become almost monotonic (the rare differences are small in magnitude). In those cases, the non-monotonicity can be explained by a discretization effect.

All three policies have more savings for $\beta = 3$ than for $\beta = 2$ for all Δ considered. The explanation is that for a larger β, the average cost curves become more peaked and there are less failures before the optimum maintenance moment. Hence, a coordination with seasons is easier to achieve and costly failures in the winter season can be avoided. For $c = 10$ the savings are slightly larger than for $c = 5$ for low cost fluctuations (i.e. $\Delta \leq 30\%$). For large cost fluctuations (i.e. $\Delta \geq 40\%$) the savings for $c = 5$ are mostly larger than for $c = 10$, yet the differences are small and may be caused by the particular behavior in case of $\alpha = 1$. Changing $c = 5$ into $c = 10$ then forces BRP to do PM 3 times a year instead of 2 times, with much lower savings compared to the constant cost model. Note that the savings of the three policies are comparable, in Table 4 the p-BRP has often the highest savings, next p-MBRP and least p-ARP, but in other cases the savings order can be different (see Schouten, 2019).

6. Wind turbine gearbox example

In this section, we will apply our methods on a realistic example for illustration: a gearbox within a Vestas V164 9.5 MW wind turbine in the North Sea. The data we give below are assembled from various literature sources and are scaled up, as today's (2020) wind turbines are much bigger than those of 10 years ago. The cost data are obtained through a scaling procedure by Fingersh, Hand, & Laxson (2006) from the published costs of a 2.0 MW wind turbine. The cost for every component in a wind turbine is approximated as a function of the power output and the size of the turbine. For further details of our approximation and scaling scheme, we refer to Schouten (2019).

We use data from Tian, Jin, Wu, & Ding (2011) for the lifetime distribution, which is based on a 2.0 MW offshore wind turbine. For the gearbox the time to failure can be modeled by a Weibull distribution with a scale parameter $\alpha = 80$ months and $\beta = 3$, implying an average time to failure of 71 months.

Maintenance costs consist of the following components: manpower, material, and lost production costs due to downtime for preventive or corrective maintenance. The lost production costs vary over the year. We will relate the lost production costs to the weekly average wind speed at sea level, corrected for the height of the rotor. The lost production costs can be computed using the expected downtime of the components, the power output curve, and historical wind speed data. For each maintenance action, we assume that we know the downtime of the component.

Note that the average power output cannot be computed from the mean wind speed over the month, since the relation between power output and wind speed is non-linear. Fig. 3 shows such a relation for the V164 9.5 MW turbine, the newest and most powerful Vestas wind turbine to date (2018). The data is obtained from (Netherlands Commission for Environmental Assessment (NCEA), 2016), which contains the power output for a V164 8.0 MW turbine which we scaled up to the 9.5 MW turbine.

The daily data from (Royal Netherlands Meteorological Institute, 2018) will be used to estimate the power output on every day of the year. This contains the daily average wind speeds in IJmuiden, a coastal city in the Netherlands, from 1971 to 2017. For every day, we approximate the power output by $P(\bar{v})$ in which $\bar{v}$ is the mean wind speed over the day. Taking the average over all years and within each week yields the average historical power output for each week of the year. To these rather fluctuating values we fitted a cosine approximation. This historic power output and cosine fit are given in Fig. 4. This figure also indicates in the bottom curve per week the lowest daily wind speed, averaged over the 47 years, also with a cosine fit.

Fig. 4 indicates that there is a big difference in the mean and minimum over the week. The intra-week fluctuations are thus significant and if we would be able to predict the wind accurately beforehand for each day in a week, we will be able to save a lot in the downtime costs. These downtime costs for a complete day of downtime is given in Eq. (14), where we use a constant electricity price of € 0.06/kWh. This is the guaranteed price in a 2018 German project. This leads to the following downtime cost estimates per day (in euros, where t is in weeks and the shift term in the cosine has therefore changed from the value in months given

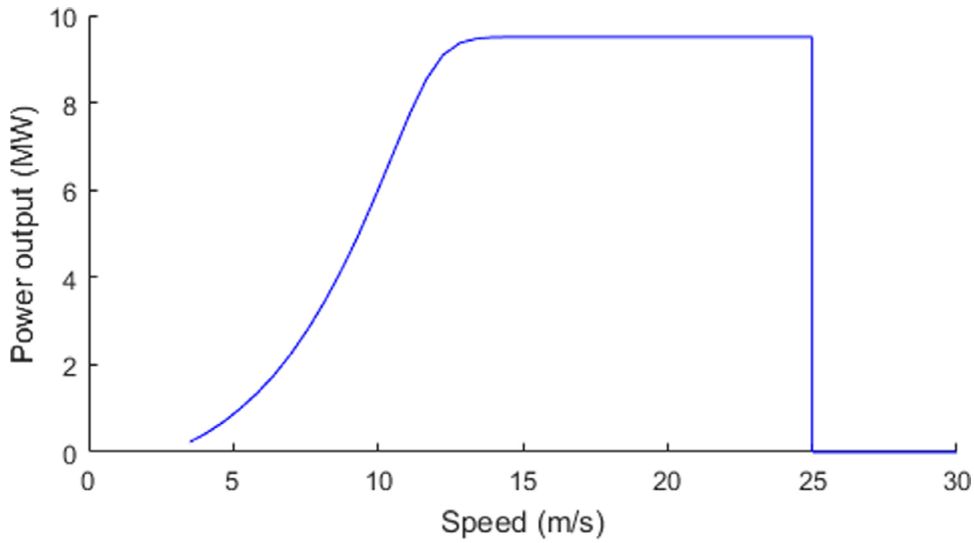

Fig. 3. This figure shows the approximated power output of a Vestas V164-9.5 MW wind turbine as a function of the wind speed.

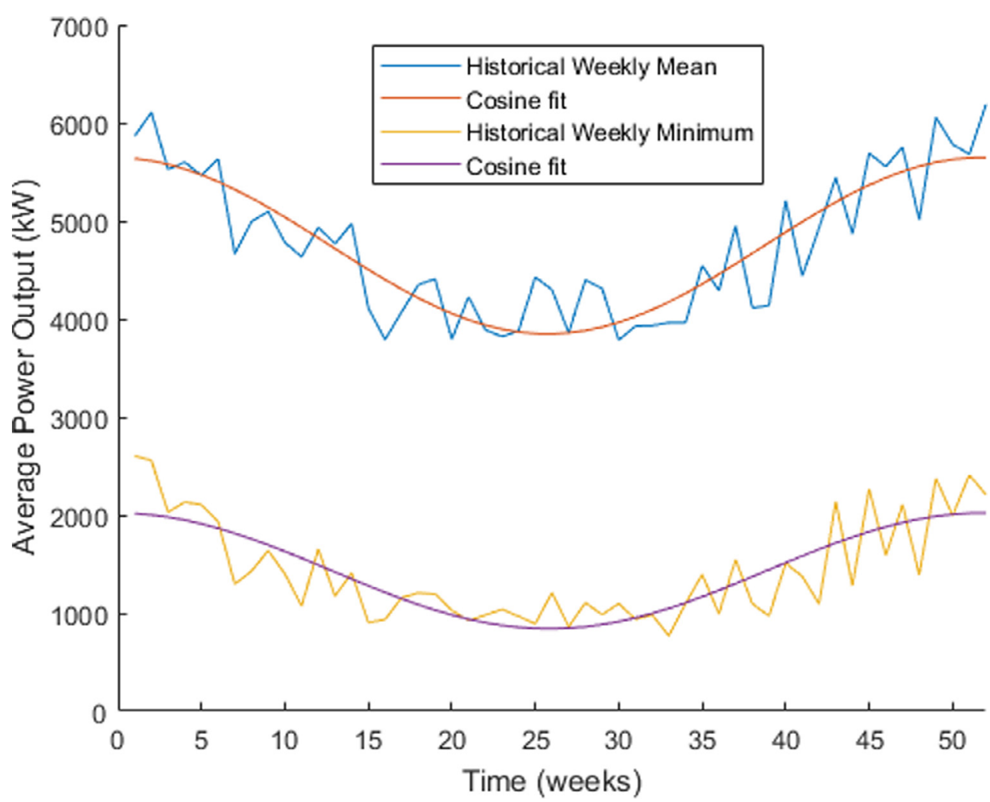

Fig. 4. This figure shows the approximated power output of a Vestas V164-9.5 MW wind turbine as function of the time of the year in IJmuiden (a coastal city in The Netherlands). The lower data and fit represent the minimum (over all days) power output for each week averaged over all years. The upper data and line represent the average output per week average over all years.

earlier).

$$C_{\text{mean}}(t) = 6836 + 1295\cos\left(\frac{2\pi t}{52} + 0.034\right), \tag{14a}$$

$$C_{\min}(t) = 2061 + 850\cos\left(\frac{2\pi t}{52} + 0.020\right). \tag{14b}$$

Tian et al. (2011) state that the lead time for corrective maintenance is a month. Papatzimos, Dawood, & Thies (2018) state that some 10 days are needed for the preventive maintenance of a gearbox for a 2.3 MW turbine and therefore we use a downtime of 40 days (lead time plus 10 days) for corrective maintenance. Hence, the manpower and material costs for preventive and corrective maintenance of the gearbox are estimated as 148.20 and 592.80 thousand euros respectively. Adding the downtime costs for PM of 10 days (in total 68.36 thousand euros) and 40 days for CM (in total 273.44 thousand euros) to these numbers we arrive at the following cost values in thousand euros for the average wind speeds.

$$c_p(i_1) = 216.56 + 12.9\cos\left(\frac{2\pi i_1}{52} + 0.034\right), \tag{15}$$

Table 5
Annual costs under the different models.

	p-ARP	p-BRP	p-MBRP
Constant cost model	89.740	95.331	89.958
Model using mean wind speed (14a)	87.369	92.984	87.996
Model using lowest wind speed (14b)	68.456	73.019	69.081
Average savings using (14a)	2.64%	2.46%	2.18%
Optimistic savings using (14b)	23.72%	23.40%	23.21%

Table 6
Critical maintenance ages (in weeks) for the p-ARP.

Week	27	28	29	30	31
Average using (14a)	204	192	180	167	165
Optimistic using (14b)	205	195	185	175	165

$$c_f(i_1) = 866.24 + 51.6\cos\left(\frac{2\pi i_1}{52} + 0.034\right). \tag{16}$$

For the case we maintain preventively at the days with the lowest wind speed we get

$$c_p^{low}(i_1) = 168.81 + 8.5\cos\left(\frac{2\pi i_1}{52} + 0.020\right), \tag{17}$$

while the failure cost remain the same. Using these cost functions we can compute the optimal p-ARP, p-BRP and p-MBRP using formulations (7), (10), and (12) and using $m = 4$. If we do not allow for short-term planning and have to maintain at the start of the week that we plan far ahead, we pay the costs of $C_{\text{mean}}(t)$ for downtime. Using our model we obtain the average costs that are somewhat pessimistic estimates. If we are able to forecast the weather in the chosen week perfectly and can plan the maintenance on the days with the lowest wind speed in that week, we could incur downtime costs of $C_{\min}(t)$ per day and hence $c_p^{low}(i_1)$ for preventive maintenance. This function therefore gives an optimistic estimate on the costs we incur. The results are given in Table 5.

Since we take $m = 4$, we consider 4×52 periods in the model. In the p-BRP, for both average and optimistic settings, we maintain in week 131 and this corresponds to a cycle of three years and maintaining the component approximately the first week of July. In the p-MBRP, for both average and optimistic settings, we maintain in the exact same week if the age is 121 weeks or above. The standard ARP with average settings, maintains at 191 weeks, the standard BRP at 187 weeks. Taking the varying wind conditions into account therefore leads to a much shorter maintenance interval. In Table 6, we give the critical maintenance ages for the p-ARP, depending on the week of the year.

7. Conclusion

In this paper, we generalize the standard ARP, BRP and MBRP policies to time-varying maintenance costs case, which is appropriate for wind farm operations. Using time-discretization, we formulate a Markov decision process describing a component's maintenance optimization problem. We derive conditions for the existence of finite replacement policies and find that these are weaker than those for the case with constant costs. The optimal time-dependent p-ARP can be obtained from an LP formulation and the optimal time-dependent p-BRP and p-MBRP policies from MILP formulations. Our numerical results indicate that considering the cost fluctuations in maintenance optimization can save a significant percentage of costs (i.e. up-to 23%) compared to the constant cost policies.

For the time-varying cost case, the p-ARP is the optimal policy. But this policy can complicate long-term planning as the planned maintenance execution time changes every time a component fails. The p-BRP results in somewhat higher costs, but maintenance actions can be planned in advance and are never canceled, which is convenient for maintenance teams and spare parts provisioning. The p-MBRP is slightly more expensive than the p-ARP and cheaper than the p-BRP. Under p-MBRP, maintenance can still be planned in advance, but can be canceled if the component has not reached a specific age at the preventive maintenance moment. Our numerical experiments reveals that the p-MBRP policy equals the p-ARP policy in case of high cost fluctuations. The p-BRP and p-MBRP policies have the advantage that they can easily be extended to multi-component situations with economic dependencies, while this is more complicated for p-ARP policies.

In this paper, we consider time-varying costs, which we assume driven by the energy output rate of generators. Considering the value of the output rate might require the dynamics of energy prices and demand-supply balance in the market. This requires further research and is left to future studies.

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Appendix

Theorem 1. *There exists an optimal maintenance policy under time-varying costs that is a p-ARP policy.*

Proof. This theorem holds due to the following. For $i_1^0 \in \mathcal{I}_1$ and $i_2^* \in \mathcal{I}_2$, if it is optimal to perform a PM in state (i_1^0, i_2^*), then it is optimal to perform a PM in all states (i_1^0, i_2) with $i_2 \geq i_2^*$, which is equivalent to a p-ARP policy.

The proof follows similar lines as that in Ross (1970) p.129,147. We start with the α-discounted case and apply value iteration to prove that the value increases with age. Next we let $\alpha \to 1$ and obtain the results for the average cost case. Hence, let $v_{(i_1,i_2)}^{\alpha,0} = 0$ for all states $i = (i_1, i_2) \in \mathcal{I}$. In the following we keep i_1 fixed. Next we determine:

$$v_{(i_1,i_2)}^{\alpha,1} = \min_{a\in\mathcal{A}(i_1,i_2)}\left(c_{(i_1,i_2)}(a) + \alpha\sum_{j_1,j_2}\pi_{(i_1,i_2)(j_1,j_2)}v_{(j_1,j_2)}^{\alpha,0}\right) = \begin{cases} 0, & i_2 \neq 0 \\ c_f(i_1), & i_2 = 0 \end{cases}$$

Hence $v_{(i_1,i_2)}^{\alpha,1}$ is increasing in $i_2 \leq 1$ and $v_{(i_1,i_2)}^{\alpha,1} \geq v_{(i_1,0)}^{\alpha,1} = c_f(i_1) \quad \forall (i_1, i_2) \in I$. Next assume that $v_{(i_1,i_2)}^{\alpha,n-1}$ is increasing in $i_2 \geq 1$ and $v_{(i_1,i_2)}^{\alpha,n-1} \leq v_{(i_1,0)}^{\alpha,n-1} \quad \forall (i_1, i_2) \in I$ for some $n > 1$. We then have

$$\begin{aligned} v_{(i_1,i_2)}^{\alpha,n} &= \min_{a\in\mathcal{A}(i_1,i_2)}\left(c_{(i_1,i_2)}(a) + \alpha\sum_{j_1,j_2}\pi_{(i_1,i_2)(j_1,j_2)}v_{(j_1,j_2)}^{\alpha,n-1}\right) \\ &= \min\Big(p_{i_2}\alpha v_{(i_1+1,0)}^{\alpha,n-1} + (1-p_{i_2})\alpha v_{(i_1+1,i_2+1)}^{\alpha,n-1}, c_p(i_1) \\ &\quad + p_0\alpha v_{(i_1+1,0)}^{\alpha,n-1} + (1-p_0)\alpha v_{(i_1+1,1)}^{\alpha,n-1}\Big) \end{aligned}$$

Notice that the second term in the minimum is constant in i_2. The first term we can rewrite into

$$\alpha\Big(p_{i_2}v_{(i_1+1,0)}^{\alpha,n-1} + (1-p_{i_2})v_{(i_1+1,i_2)}^{\alpha,n-1}\Big) + (1-p_{i_2})\big(v_{(i_1+1,i_2+1)}^{\alpha,n-1} - v_{(i_1+1,i_2)}^{\alpha,n-1}\big)\Big),$$

which is larger than

$$\alpha\Big(p_{i_2-1}v_{(i_1+1,0)}^{\alpha,n-1} + (1-p_{i_2-1})v_{(i_1+1,i_2-1)}^{\alpha,n-1}\Big),$$

which is the left term in the minimum equation for $v^{\alpha,n-1}_{(i_1,i_2-1)}$. Hence $v^{\alpha,n}_{(i_1,i_2)}$ is increasing in i_2. It is easy to see that $v^{\alpha,n}_{(i_1,i_2)} \le v^{\alpha,n}_{(i_1,0)}$.

The following step is to let $n \to \infty$ and because of the boundedness of all terms $v^{\alpha,n}_{(i_1,i_2)} \to v^{\alpha}_{(i_1,i_2)}$. We obtain that $v^{\alpha}_{(i_1,i_2)}$ is increasing in $i_2 > 0$ and $v^{\alpha}_{(i_1,i_2)} \le v^{\alpha}_{(i_1,0)}$. Since the right term in the definition of $v^{\alpha}_{(i_1,i_2)}$ is constant it means that if for some (i_1, i_{2_0}) the left term exceeds the right term, it does so for all (i_1, i_2) with $i_2 \ge i_{2_0}$. The result for the average cost criterion is obtained by letting $\alpha \uparrow 1$ and observing that all terms are bounded, which means we can invoke Theorem 6.18 from (Ross, 1970) implying that for some sequence $\alpha_k \uparrow 1$ there exist a function

$$h_{(i_1,i_2)} = \lim_{k\to\infty} v^{\alpha_k}_{(i_1,i_2)} - v^{\alpha_k}_{(1,0)} \tag{A.1}$$

and $\lim_{\alpha\uparrow 1}(1-\alpha)v^{\alpha}_{(1,0)} = g$, where g and $h_{(i_1,i_2)}$ satisfy the average optimality equations and $h_{(i_1,i_2)}$ is increasing in i_2. This implies our result. □

Lemma 1. *Let $g(T)$ denote the long-run average maintenance cost for a p-ARP policy, where we maintain at age $T \in \bar{\mathbb{N}} \setminus \{0\}$ in all periods $i_1 \in \mathcal{I}_1$. Then, we have $g(T) = \bar{g}(T)$, where $\bar{g}(T)$ is the long-run average cost for the problem where the time-varying maintenance costs are replaced by the averages over the year (i.e. $\bar{c}_p$ and $\bar{c}_f$).*

Proof. Let $T \in \bar{\mathbb{N}} \setminus \{0\}$ arbitrary. Let A denote the limiting distribution of the age of the component at any period. This limiting distribution exists under the unichain result and is time-invariant, since the lifetime distribution is strictly positive and the policy has constant maintenance age. The expected cost for Markov decision processes can be computed using the long-run distribution as follows.

$$\begin{aligned} g(T) &= \frac{1}{N}\sum_{i_1\in\mathcal{I}_1}\left\{c_p(i_1)\mathbb{P}(A=T) + c_f(i_1)\mathbb{P}(A=0)\right\} \\ &= \bar{c}_p\mathbb{P}(A=T) + \bar{c}_f\mathbb{P}(A=0) = \bar{g}(T). \end{aligned} \tag{A.2}$$

This proves the desired result. □

Theorem 3. *Suppose $c_f(i_1) = \bar{c}_f$ for all $i_1 \in \mathcal{I}_1$. If there exists $i_1^0 \in \mathcal{I}_1$ satisfying $\bar{c}_f > c_p(i_1^0)$ and $p_\infty > \frac{1}{\mathbb{E}(X)}\frac{\bar{c}_f}{\bar{c}_f - c_p(i_1^0)}$, then there exists a finite optimal p-ARP.*

Proof. Consider the case where $c_p(i_1) = c_p(i_1^0)$ for all $i_1 \in I_1$. This is a constant cost case and by Nakagawa (1984) there exists a finite optimal maintenance policy.

We now compare the MDP for the constant cost case with only age i_2 as state variable with the extended MDP for the time-varying cost case, i.e. with period i_1 and age i_2 as state variables. Note that there is a correspondence between the extended states (i_1, i_2) and the original state i_2. The transitions and cost in the extended MDP only depend on the i_2 part of the state in this case. Consider now the CM policy in both models. For the steady-state probabilities ω_i for the extended MDP we have $\omega_{(i_1,i_2)}(\infty) = \frac{1}{N}\omega_{i_2}(\infty)$ under the CM policy. The long-run average cost under the corrective maintenance policy for both MDPs are the same. Note further that the value functions v are the same in both models, as the period information does not influence costs nor transition probabilities. In other words, we have $v_{(i_1,i_2)}(\infty) = v_{i_2}(\infty)$ for all $i_2 \in \mathcal{I}_2$ under the CM policy.

As in the constant cost case there exists a finite optimal maintenance policy, it means that some policy improvement equations w.r.t the corrective maintenance policy have negative solutions, i.e. there exists a $i_2^* \in \mathcal{I}_2$ for which we have

$$\min_{a\in\mathcal{A}(i_2)} c_{i_2}(a) + \sum p_{i_2,j_2}(a)v_{j_2}(\infty) - v_{i_2}(\infty) - g(\infty) < 0, \forall i_2 \ge i_2^*, \in \mathcal{I}_2, \tag{A.3}$$

where

$$\mathcal{A}(i_2) = \begin{cases} \{1\} & \text{if } i_2 = 0, \\ \{0,1\} & \text{otherwise.} \end{cases}$$

Let us now turn to the case with time-varying PM costs and the extended MDP. Note that the value function for the CM policy remains the same. Moreover, the policy improvement equations remain the same for the (i_1^0, i_2) states. This means that the policy which takes the PM action for the states (i_1^0, i_2) with $i_2 \ge i_2^*$ and the no PM action for all other states has average costs which are less than those under the CM policy. □

Theorem 4. *If there exists period $i_1^0 \in \mathcal{I}_1$ satisfying $\frac{c_p(i_1^0)}{\bar{c}_f} + b(i_1^0) < \frac{1}{2}(1 - c_X^2 - \frac{1}{\mu})$, then there exists an optimal p-BRP.*

Proof. Suppose in period $i_1^0 \in \mathcal{I}_1$ we do preventive maintenance at costs $c_p(i_1^0)$, while the component is at age t_0 and never again. The cost $d(n)$ of such a policy over nN periods with $n \in \mathbb{N}^+$ is as follows:

$$d(n) = \frac{1}{nN}\left\{c_p(i_1^0) + \sum_{t=1}^{nN} c_f(i_1^0 + t)h(t)\right\}. \tag{A.4}$$

The desired result is obtained by showing that there exists an n such that $d(n) < \frac{\bar{c}_f}{\mathbb{E}[X]}$ where the right-hand-side is the cost rate of the CM policy. Suppose that such an n does not exist, i.e. $d(n) \ge \frac{\bar{c}_f}{\mathbb{E}[X]}$ for all $n \in \mathbb{N}$, and the following inequality holds.

$$\frac{c_p(i_1^0)}{\bar{c}_f} + b(i_1^0) < \frac{1}{2}\left(1 - c_X^2 - \frac{1}{\mu}\right) \tag{A.5}$$

For $d(n) - \frac{\bar{c}_f}{\mathbb{E}[X]}$ we can write the following equations.

$$d(n) - \frac{\bar{c}_f}{\mathbb{E}(X)} = \frac{1}{nN}\left\{c_p(i_1^0) + \sum_{t=1}^{nN} c_f(i_1^0 + t)h(t)\right\} - \frac{\bar{c}_f}{\mathbb{E}(X)}, \tag{A.6}$$

$$= \frac{1}{nN}\left\{c_p(i_1^0) + \sum_{t=1}^{nN}[c_f(i_1^0 + t) - \bar{c}_f]h(t)\right\} + \frac{1}{nN}\sum_{t=1}^{nN}\bar{c}_f h(t) - \frac{\bar{c}_f}{\mathbb{E}(X)}, \tag{A.7}$$

$$= \frac{\bar{c}_f}{nN}\left\{\frac{c_p(i_1^0)}{\bar{c}_f} + \sum_{t=1}^{nN}\left[\frac{c_f(i_1^0 + t)}{\bar{c}_f} - 1\right]h(t)\right\} + \frac{\bar{c}_f H(Nn)}{nN} - \frac{\bar{c}_f}{\mathbb{E}(X)}, \tag{A.8}$$

where the second equality follows from $h(i_1^0 + t) = h(t)$ for $t \ge 1$ when there is a PM at period i_1^0. The definition of $b(i_1^0)$ in (8) implies that

$$d(n) - \frac{\bar{c}_f}{\mathbb{E}(X)} \le \frac{\bar{c}_f}{nN}\left(\frac{c_p(i_1^0)}{\bar{c}_f} + b(i_1^0)\right) + \frac{\bar{c}_f H(Nn)}{nN} - \frac{\bar{c}_f}{\mathbb{E}(X)}.$$

Due to Feller (1949, Eq. (6.7)),

$$\lim_{t\to\infty}\left[H(t) - \frac{t}{\mathbb{E}[X]}\right] = \frac{\mathbb{E}[X^2]}{2\mathbb{E}[X]^2} - 1 + \frac{1}{2E[X]}.$$

This implies that for any real number ϵ there exists a $k \in \mathbb{N}^+$ such that

$$H(kN) < \frac{kN}{\mathbb{E}[X]} + \frac{1}{2}\left(c_X^2 - 1 + \frac{1}{\mu}\right) + \epsilon.$$

Writing (A.8) for k leads to

$$d(k) - \frac{\bar{c}_f}{\mathbb{E}(X)} \leq \frac{\bar{c}_f}{kN}\left(\frac{c_p(i_1^0)}{\bar{c}_f} + b(i_1^0)\right) + \frac{\bar{c}_f H(kN)}{kN} - \frac{\bar{c}_f}{\mathbb{E}(X)}$$
$$< \frac{\bar{c}_f}{kN}\left(\frac{c_p(i_1^0)}{\bar{c}_f} + b(i_1^0) - \frac{1}{2}\left(1 - c_X^2 - \frac{1}{\mu}\right) + \epsilon\right).$$

Since ϵ is arbitrary, (A.5) implies that $d(k) - \frac{\bar{c}_f}{\mathbb{E}(X)}$ is negative which is a contradiction. Hence there exists a finite n that leads to a cost rate lower than the pure corrective maintenance policy. □

Theorem 5. *If* $\frac{\bar{c}_p}{\bar{c}_f} < \frac{1}{2}(1 - c_X^2 - \frac{1}{\mu})$ *and* $b(i_1)$ *is finite for all* $i_1 \in \mathcal{I}_1$, *then there exists an optimal p-BRP.*

Proof. Let $\bar{b} = \frac{1}{N}\sum_{i_1=1}^{N} b(i_1)$ and notice that we already established that $\bar{b} = 0$ (see (8) and the text thereafter). As $\frac{\bar{c}_p}{\bar{c}_f} + \bar{b} < \frac{1}{2}(1 - c_X^2 - \frac{1}{\mu})$,

$$\frac{1}{N}\sum_{i_1=1}^{N}\left(\frac{c_p(i_1)}{\bar{c}_f} + b(i_1)\right) < \frac{1}{2}(1 - c_X^2 - \frac{1}{\mu}).$$

One can show by contradiction that there will be a period $i_1^0 \in \mathcal{I}_1$ for which the condition of Theorem 4 holds. □

Theorem 6. *If* $c_f > c_p$ *and* $p_\infty > \frac{1}{\mathbb{E}(X)}\frac{c_f}{c_f - c_p}$, *then the optimal standard MBRP has a finite optimal critical maintenance age* t *and block time* T.

Proof. Since condition (5) is met, we know that there exists a finite optimal critical age for the standard ARP. Denote this optimal age by i^*. Consider next the MDP for age replacement, where state i indicates the age of the component. Now we extend the state space with a counter variable, T. This construction and the remainder of the proof are similar to the proof of Theorem 3. We obtain a MBRP by restricting preventive maintenance actions to states with counter variable $T = i^*$ only. □

Theorem 7. *If* $\bar{c}_f > \bar{c}_p$ *and* $p_\infty > \frac{1}{\mathbb{E}(X)}\frac{\bar{c}_f}{\bar{c}_f - \bar{c}_p}$, *then the optimal p-MBRP has finite cycle* m, *block times* $T_1, T_2, \ldots, T_n$, *and critical ages* $t_{(1)}, t_{(2)}, \ldots, t_{(n)}$.

Proof. This proof follows similar lines as that of Theorem 6, by starting with the original MDP being p-ARP from Theorem 2, so with states (i_1, i_2) indicating age and period and extending it with a counter T. □

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