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Research Article

Evaluation of Gas-Fired Combi Boilers with HF-AHP-MULTIMOORA

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There are many alternative gas-fired combi boilers that can be used to heat residential homes. Evaluation and selection of gas-fired combi boilers for buildings is an intricate multi-criteria decision-making (MCDM) problem involving perhaps contradictory quantifiable and qualitative criteria. In this research, as the MCDM approach, hesitant fuzzy linguistic analytic hierarchy process (HF-AHP) and hesitant fuzzy linguistic "multiple objective optimization based on ratio analysis plus full multiplicative form (MULTIMOORA)" (HF-MULTIMOORA) are integrated to assess and rank combi boiler alternatives for buildings. First, with HF-AHP, fuzzy criteria weights are determined and then with HF-MULTIMOORA, boiler alternatives are ranked from best to worst. In this integrated HF-AHP-MULTIMOORA method, evaluations of decision-makers are combined with fuzzy envelope approach and then triangular fuzzy numbers are utilized. For comparison analysis, HF-AHP-TOPSIS method is also applied to the same problem. A case study in Turkey is presented where ten combi boiler alternatives are assessed based on fifteen criteria by five decision-makers. We have used various selection criteria for boilers ranging from maximum temperature, heating capacity up to environmental effects and decided on the best combi boiler for heating residential buildings in Turkey.

1. Introduction

Buildings and activities in buildings play a very critical role in energy consumption around the world. The amount and quality of energy used in the construction industry have a great impact on environmental and global activities, consumption of natural resources, emissions to the environment, and even human health. Building energy consumption continues to increase in recent years due to rapid population growth, global climate changes, and increasing demand for a healthy, comfortable, and efficient indoor environment. Nowadays, buildings are responsible for over one-third of global final energy consumption and around 40% of CO2 emissions.

Most of the energy use in buildings is to provide heating, ventilation and air conditioning (HVAC). Natural gas is one of the methods used in air conditioning and has become more popular recent years. Also natural gas consumption is one of the most critical sources of energy saving in building energy consumption.

After the initial use of natural gas, the heating sector met with combi boilers and the use of combi boilers became widespread. The combi boilers that have been produced since 1992 have been diversified according to the developing technology, today's needs, and customers' requests. Mistakes made in the selection of combi boiler devices, which have many differences in terms of their features, bring about heating problems; moreover, they may negatively affect fuel consumption.

Combi boilers are used to supply heat and hot water to households. These types of boilers are very popular in Turkish residential buildings because they are effective and take up little room. These boilers are usually placed on kitchen or bathroom walls. Combi boilers are energy efficient because they do not store water, and they heat water for baking and bathing as an open system and space heating as a closed system.

Another characteristic of the recent combi boilers that makes it more effective than conventional boilers is that the flue gas produced during the heating process is not completely lost with the combi boilers, as the gas is extracted and reused (condensation technique). Combi boilers do not require complex installation, and they are convenient to be installed. There is no dedicated tank for hot water as water is heated directly and used instantly. Combi boilers are directly connected to the main water source to supply hot water.

Condensation technique for combi heating was a major step as it reduces gas consumption in apartment houses and independent houses. Moreover, sensitive heat losses are significantly reduced due to the resulting low temperature of the flue gas. Condensing boilers are aimed to draw heat out of a given quantity of fuel as the temperature of the flue gases is reduced to a point where water vapor produced during combustion is condensed, releasing latent heat that would otherwise escape with the flue gases. Since the 1970s, condensing boilers have gone through many design and installation changes to improve their performance.

Natural gas has been used in Turkey since 1988. Around 25,000 m^3 natural gas is consumed per capita for individual heating in addition to central heating of the residential buildings. The individual heating in the form of combi boilers is commonly used for space and water heating. The life of the combi boiler unit is around 10 years. The two main factors are important for combi boiler selection for houses. The first one is the economic one that directly depends on the fuel consumption of combi boilers. The second one is related to the heating capacity of boilers that usually depends on the size of the house that needs to be heated. Combi boiler systems are installed to produce both hot water and space heating. Therefore, the maximum temperature of the hot water is an important criterion.

Condensing boilers utilize the latent heat contained in the flue gas. Therefore, the boiler efficiency is another important criterion. In the condensing combi boiler, the temperature of the heating medium must be as low as to create some temperature difference between the flow pipe and the return temperature. Therefore, the temperature of the heating medium and outdoor temperature are other important criteria. The supply and return temperatures vary according to the ambient temperature.

Heating requirements of residential homes, in particular space heating of residential homes, are generally a significant component of energy consumption in Turkey. Several heating appliance manufacturers are concerned about the search for fuel efficient boilers. Decisions on how to heat residential homes include many technological alternatives that involve many conflicting criteria. First of all, high life and capital costs are important economic indicators. Second, the solutions that are most beneficial for both capital and lifecycle costs cannot be implemented for noneconomic reasons. Another issue is decision-making: costs represent quantitative information, but there is also qualitative information that influences final choices. In order to make decisions where there are multiple criteria, a number of formal methods, known as MCDM methods, are available. A recent review of these methods and their application in the energy and building field can be found in [1]. Our study aims to show, through a case study, how MCDM can enable a conscious selection from among the combi boiler options for residential heating. In our paper, we will point out some distinctive features of choice of heating systems in a residential context.

In this study, as the MCDM method, HF-AHP is integrated with HF-MULTIMOORA. Here, first, HF-AHP is utilized to calculate the fuzzy criteria (importance) weights since HF-MULTIMOORA does not include a step to determine (fuzzy) weights and assumes readily available (fuzzy) weights. With HF-AHP, as a result of pairwise comparisons of criteria, consistent, therefore reliable fuzzy weights are obtained. Afterward, HF-MULTIMOORA is implemented to rank combi boiler alternatives since MULTIMOORA and its extensions are appraised as the most robust MCDM method with a wide range of real-life applications [2–4]. Here, in both HF-AHP and HF-MULTIMOORA methods, "hesitant fuzzy linguistic term sets" and "hesitant fuzzy set concepts" are employed with "fuzzy envelope approach" to "reflect the uncertainty, ambiguity, and hesitations DMs might have in their assessments" [5, 6] of criteria and combi boiler alternatives. Thus, the decision-making process becomes more flexible and "close to real" [5, 6]. As the novelty of the research, to the best of our knowledge, integration of HF-AHP with HF-MULTIMOORA (HF-AHP-MULTIMOORA) has never been studied, especially for the selection of combi boilers. Therefore, this study has the aim of attaining a consensus by the DMs about the best combi boiler among the alternatives considered, applying HF-AHP-MULTIMOORA. In the following section, associated literature review is presented. In section 3, details of HF-AHP-MULTIMOORA are given, along with the case study in Section 4 and conclusions in Section 5.

2. Literature Review

The selection of heating systems for the various buildings is a new area of application of MCDM methods. MCDM has been used extensively in the energy strategy [1], but there are few applications of MCDM methods to plan residential energy systems [1]. Multi-objective optimization tools for building design [7] or heating and cooling devices [8] have only been quantitatively investigated. Multi-objective optimization models are adapted to the solution of continuous problems [9]; on the other hand, multi-characteristic methods require a limited number of choices. Solving a multi-characteristic decision-making problem can be assumed to be a selection problem [9]. So far, only a few examples of heating system decision-making problems with multiple-characteristic approaches can be found in the literature. A widely used multi-characteristic approach, called ELECTRE method [10], was applied for the selection of the heating devices by Thiel and Mroz [11]: three alternative heating devices for a building based on quantitative primary energy consumption, total cost criteria, and qualitative criteria. Only conventional devices such as fossil fuels are considered in the study. Emphasis was placed on heat distribution systems and their interaction with the building structure. Kaklauskas et al. [12] applied their multi-characteristic method (COPRAS) to residential homes, focusing mainly on building design and renovation problems (e.g., Kaklauskas et al. [13]) to guide the selection of contractors to renovate buildings. Jaber et al. [14] and Alanne et al. [15] focused on policy designs: Jaber et al. [14] used the AHP method [16] and fuzzy sets [17, 18] to evaluate conventional and alternative energy sources for residential heating systems, and Alanne et al. [15] focused on combined heat and power systems and considered uncertainties in the decisions of decision-makers by means of the MCDM method [15]. Until now, the AHP method has been widely applied in the energy strategy field, especially for conventional and renewable power generation (see [19] and [20–22] for some more examples). However, the selection of combi boilers for residential heating has never been investigated in the literature from the point of view of the MCDM.

To capture the ambiguity and fuzziness in the decision-making process, fuzzy set theory concepts [18, 23] were integrated where AHP and fuzzy AHP were utilized in many MCDM applications. In this study, HF-AHP is favored instead of AHP or fuzzy AHP to determine fuzzy criteria weights since with HF-AHP, the ambiguity and vagueness on DMs' decisions are captured, and moreover, the degree of hesitancy DMs may have reflected with the usage of hesitant fuzzy set concepts and hesitant fuzzy linguistic term sets, which are introduced in [24, 25]. Overall, all these features provide flexibility in the decision-making. HF-AHP has been applied to a varied series of MCDM problems in the literature in order to determine (fuzzy) criteria weights such as evaluation of COVID-19 intervention strategies [6], CNC routers [26], innovation projects [27], power generation initiatives [28], summer sport schools [29], transformers in a power distribution project [30], performances of bank regions [31], and a cargo company [32].

After determination of fuzzy criteria weights with HF-AHP, in this study HF-MULTIMOORA is utilized to rank alternatives. Fuzzy MULTIMOORA (F-MULTIMOORA) is acclaimed as the most robust MCDM method and it is widely applied in real-life problems [2–4]. The parts of MULTIMOORA, the "ratio system," "reference point approach," and the "full multiplicative form" were first described by Brauers in 2004 [33]. Afterward, this method was named MOORA [34] and then MULTIMOORA was developed [35], which is combination of MOORA and the "full multiplicative form." The fuzzy extension, F-MULTIMOORA, was launched by Brauers et al. in 2011 [36]. MULTIMOORA and F-MULTIMOORA have been implemented in many MCDM problems such as assessment of efficient farming types (F-MULTIMOORA) [37], decisions about bank loans to buy property (MOORA) [36], evaluation of performance of smart-bike sharing programs (F-MULTIMOORA) [4], comparing EU member states in reaching Lisbon Strategy 2000–2008 goals [36] (F-MULTIMOORA), evaluation of personnel (F-MULTIMOORA with group decision-making) [38], selection of sustainable energy crop [39], and evaluation of innovative ability of universities (MULTIMOORA) [40]. An extensive overview of MULTIMOORA with theory and applications is given in Hafezalkotob et al.'s study [41].

Extension of MULTIMOORA with hesitant fuzzy sets was proposed by Li in 2014 [42]. Afterward, Liang et al. [43] developed the "dual hesitant fuzzy extended Bonferroni mean"-based MULTIMOORA and applied it to the renewable energy technology selection problem. Gou et al. [44] utilized the double hierarchy hesitant fuzzy linguistic MULTIMOORA to select the best city in China in terms of implementation of haze control measures. Liao et al. [45] applied unbalanced hesitant fuzzy linguistic MULTIMOORA to an investment problem regarding the shared bicycles. Later, Lia et al. [46] implemented the hesitant fuzzy linguistic Choquet integral-based MULTIMOORA to develop human resources and select talents. Zolfaghari and Mousavi [47] applied interval-valued hesitant fuzzy linguistic MULTIMOORA to rank healthcare system failures. Saraji et al. [48] worked on "stepwise weight assessment ratio analysis (SWARA)-MULTIMOORA" with hesitant fuzzy sets to assess defies of online education during COVID-19 and rank higher education institutions.

In the literature, there is no study focusing on integration of HF-AHP with HF-MULTIMOORA (HF-AHP-MULTIMOORA). Moreover, a MCDM method has never been implemented to combi boiler selection for residential heating. With HF-AHP-MULTIMOORA, reliable fuzzy criteria weights and a robust ranking are obtained. Here, first, HF-AHP is utilized to calculate fuzzy criteria weights, and afterward, HF-MULTIMOORA is used to rank combi boiler alternatives, applying acquired fuzzy weights. In the next section, details of the HF-AHP-MULTIMOORA are given.

3. HF-AHP-MULTIMOORA

3.1. Definitions. In the proposed HF-AHP-MULTIMOORA and HF-AHP-TOPSIS (applied for comparison analysis), due to its practicality, triangular fuzzy numbers (TFNs) are employed. A fuzzy number is a special fuzzy set $F = \{(x, \mu_F(x)), x \in R\}$. Here, $\mu_F(x)$ is a continuous mapping from R to [0, 1]. A TFN $\widetilde{M} = (l, m, u)$, where $l \leq m \leq u$, has the triangular-type membership function.

$$\mu_F(x) = \begin{cases} 0, & x < l, \\ \dfrac{x-l}{m-l}, & l \leq x \leq m, \\ \dfrac{u-x}{u-m}, & m \leq x \leq u, \\ 0, & x > u. \end{cases} \tag{1}$$

Basic operations between two positive TFNs $\widetilde{C} = (l_1, m_1, u_1)$, $\widetilde{D} = (l_2, m_2, u_2) l_1 \leq m_1 \leq u_1$, $l_2 \leq m_2 \leq u_2$ are given as [6, 27, 30, 49, 50]

$$\widetilde{C}+\widetilde{D}=(l_1+l_2,m_1+m_2,u_1+u_2),$$
$$\widetilde{C}-\widetilde{D}=(l_1-u_2,m_1-m_2,u_1-l_2),$$
$$\widetilde{C}*\widetilde{D}=(l_1*l_2,m_1*m_2,u_1*u_2),$$
$$\frac{\widetilde{C}}{\widetilde{D}}=\left(\frac{l_1}{u_2},\frac{m_1}{m_2},\frac{u_1}{l_2}\right),$$
$$\frac{\widetilde{C}}{\widetilde{D}}=\left(\min\left(\frac{l_1}{l_2},\frac{l_1}{u_2},\frac{u_1}{l_2},\frac{u_1}{u_2}\right),\frac{m_1}{m_2},\max\left(\frac{l_1}{l_2},\frac{l_1}{u_2},\frac{u_1}{l_2},\frac{u_1}{u_2}\right)\right). \quad (2)$$

$\widetilde{C}$ and $\widetilde{D}$ are TFNs (not essentially positive).

$$\widetilde{C^{-1}}=\left(\frac{1}{u_1},\frac{1}{m_1},\frac{1}{l_1}\right), \quad (3)$$

$$\mathrm{MAX}(\widetilde{C}+\widetilde{D})=(\max(l_1,l_2),\ \max(m_1,m_2),\ \max(u_1,u_2)),$$
$$\mathrm{MIN}(\widetilde{C}+\widetilde{D})=(\min(l_1,l_2),\ \min(m_1,m_2),\ \min(u_1,u_2)), \quad (4)$$

$$\mathrm{Crisp}(\widetilde{C})=\frac{(4m_1+l_1+u_1)}{6}. \quad (5)$$

(graded mean integration approach [51])

$$\mathrm{d}(\widetilde{C},\widetilde{D})=\sqrt{\left(\frac{1}{3}\right)\left[(l_1-l_2)^2+(m_1-m_2)^2+(u_1-u_2)^2\right]}, \quad (6)$$

$$\widetilde{C}^{\widetilde{D}}\cong\left(l_1^{l_2},m_1^{m_2},u_1^{u_2}\right). \quad (7)$$

Hesitant fuzzy set (HFS) concept determines the membership degree of an element when a DM has hesitation in evaluations, since there might be more than one potential value [25, 52]. The "membership degree" of an element to a given set is articulated by numerous likely TFNs in triangular fuzzy HFS.

If X is a fixed set, the HFS on X returns a subset of $[0, 1]$. Here, $h_E(x)$ is the possible membership degrees of element $x \in X$ to set E by taking values in $[0, 1]$.

$$E=\{\langle x,h_E(x)\rangle | x\in X\}. \quad (8)$$

The lower and upper bounds are determined as

$$h^-_{(x)}=\min h(x),$$
$$h^+_{(x)}=\max h(x). \quad (9)$$

HFS basic operations with h_1 and h_2 are presented as

$$h_1^{\lambda}=\bigcup_{\gamma\in h_1}\{\gamma^{\lambda}\},$$
$$\lambda h_1=\bigcup_{\gamma\in h_1}\{1-(1-\gamma)^{\lambda}\},$$
$$h_1\pm h_2=\bigcup_{\gamma 1\in h1,\gamma 2\in h2}\{\gamma^1+\gamma^2-\gamma^1\gamma^2\},$$
$$h_1\cap h_2=\bigcup_{\gamma 1\in h1,\gamma 2\in h2}\min\{\gamma^1,\gamma^2\},$$
$$h_1\cup h_2=\bigcup_{\gamma 1\in h1,\gamma 2\in h2}\max\{\gamma^1,\gamma^2\},$$
$$h_1\otimes h_2=\bigcup_{\gamma 1\in h1,\gamma 2\in h2}\{\gamma^1\gamma^2\}. \quad (10)$$

Table 1: Scale for criteria evaluation in HF-AHP.

Linguistic terms	TFN
"Absolutely strong (AS)"	"(2, 5/2, 3)"
"Very strong (VS)"	"(3/2, 2, 5/2)"
"Fairly strong (FS)"	"(1, 3/2, 2)"
"Slightly strong (SS)"	"(1, 1, 3/2)"
"Equal (E)"	"(1, 1, 1)"
"Slightly weak (SW)"	"(2/3, 1, 1)"
"Fairly weak (FW)"	"(1/2, 2/3, 1)"
"Very weak (VW)"	"(2/5, 1/2, 2/3)"
"Absolutely weak (AW)"	"(1/3, 2/5, 1/2)"

Table 2: Scale for of alternative evaluations in HF-MULTIMOORA.

Linguistic terms	TFN
"Very poor (VP)"	"(0, 0, 1)"
"Poor (P)"	"(0, 1, 3)"
"Medium poor (MP)"	"(1, 3, 5)"
"Fair (F)"	"(3, 5, 7)"
"Medium good (MG)"	"(5, 7, 9)"
"Good (G)"	"(7, 9, 10)"
"Very good (VG)"	"(9, 10, 10)"

An ordered weighting averaging (OWA) operator can be applied as below:

$\mathrm{OWA}(\mathrm{d}_1,\mathrm{d}_2,\ldots,\mathrm{d}_n)=\sum_{j=1}^{n} w_j e_j$, e_j is the jth largest of $\mathrm{d}_1,\mathrm{d}_2,\ldots,\mathrm{d}_n$ [53],

$$w_j\in[0,1],\quad \forall j,$$
$$\sum_{j=1}^{n} w_j=1. \quad (11)$$

In the presented HF-AHP-MULTIMOORA, DM assessments are combined with fuzzy envelope approach [6, 27, 30, 54]. Here, s_k and s_b are the lowest and highest scales for the assessments. If the assessments are between s_i and s_j, then $s_k\le s_i\le s_j\le s_b$. Based on the hesitant fuzzy linguistic term sets, linguistic expressions can be represented by a $\widetilde{C}=(d,e,f)$, where d, e, and f are

$$\mathrm{d}=\min\left\{\mathrm{d}_L^i,\mathrm{d}_M^i,\mathrm{d}_M^{i+1},\ldots,\mathrm{d}_M^j,\mathrm{d}_R^j\right\}=\mathrm{d}_L^i,$$
$$e=\begin{cases}\mathrm{d}_M^i, & \text{if } i+1=j,\\ \mathrm{OWA}_W\left\{\mathrm{d}_M^i,\mathrm{d}_M^{i+1},\ldots,\mathrm{d}_M^j\right\}, & \text{otherwise,}\end{cases}$$
$$f=\max\left\{\mathrm{d}_L^i,\mathrm{d}_M^i,\mathrm{d}_M^{i+1},\ldots,\mathrm{d}_M^j,\mathrm{d}_R^j\right\}=\mathrm{d}_R^j. \quad (12)$$

Weight vector in OWA operator is defined as [55]

$$w_1=\partial^{n-1},$$
$$w_2=(1-\partial)\partial^{n-2},\ldots,w_n=(1-\partial),$$
$$\text{where } \partial=\frac{b-j+i}{b-1}. \quad (13)$$

Here, b is the number of terms in DM's assessment scale (in Tables 1 or 2), j is the rank of the highest, and i is the rank of the lowest assessment value. i and j can be ranks from 0 to b and $n=j\text{-}i$. [53].

3.2. HF-AHP. In HF-AHP-MULTIMOORA, first fuzzy criteria weights are calculated with HF-AHP. DMs compare criteria (pairwise) with the linguistic terms in Table 1 [6, 30, 56].

Steps of HF-AHP [6, 27, 30] are presented as

(1) For DMs' assessments with hesitant linguistic terms, fuzzy envelope approach is applied with equations (11)–(13) and corresponding TFNs are obtained. Since there are K DMs, $\widetilde{x}_{ij} = (1/K)(\widetilde{x}_{ij}^1 (+)\widetilde{x}_{ij}^2 (+)\cdots(+)\widetilde{x}_{ij}^K)$ are calculated. Here, $\widetilde{x}_{ij}^K = (a_{ij}^K, b_{ij}^K, c_{ij}^K)\ \forall i, j, k$ is the corresponding TFN of the Kth DM. There are n criteria, so $nxn\widetilde{X}$ matrix with elements $\widetilde{x}_{ij} = (a_{ij}, b_{ij}, c_{ij})$ is normalized and $\widetilde{Y} = \begin{bmatrix} \widetilde{y_{11}} & \widetilde{y_{12}} & .. & .. & \widetilde{y_{1n}} \\ y_{21} & y_{22} & .. & .. & y_{2n} \\ .. & .. & .. & .. & .. \\ .. & .. & .. & .. & .. \\ \widetilde{y_{n1}} & \widetilde{y_{n2}} & .. & .. & \widetilde{y_{nn}} \end{bmatrix}$ with elements $\widetilde{y}_{ij} = (a_{ij}/\sum_i c_{ij}, b_{ij}/\sum_i b_{ij}, c_{ij}/\sum_i a_{ij})$ is determined. Fuzzy criteria weight vector $\widetilde{w_c} = (\widetilde{w_1}, \widetilde{w_2}, \ldots, \widetilde{w_n})$ is calculated by taking the average of the entries at each row of $\widetilde{Y}$.

(2) $\widetilde{X}$ is defuzzified with equation (5), and $w_{cr} = (w_1, w_2, \ldots, w_n)$ is the normalized principal eigenvector of X. Principal eigenvalue $\Delta_{\max}$ is obtained with $Xw_{cr} = \Delta_{\max} w_{cr}$.

(3) "Consistency index" (CI) is calculated with ,$\mathrm{CI} = \Delta_{\max} - (n/n-1)$ and "consistency ratio" (CR) is $\mathrm{CR} = \mathrm{CI}/\mathrm{RI}$, where RI is "random index." If $\mathrm{CR} < 0.1$, the comparisons are "consistent" [57].

$\widetilde{w_c} = (\widetilde{w_1}, \widetilde{w_2}, \ldots, \widetilde{w_n})$ obtained with HF-AHP is then utilized in HF-MULTIMOORA.

3.3. Hesitant Fuzzy Linguistic MULTIMOORA (HF-MULTIMOORA). In HF-MULTIMOORA, DMs assess alternatives with respect to each criterion with the linguistic terms in Table 2 [6, 30, 56].

Steps of HF-MULTIMOORA [34, 36, 41, 42, 58, 59] are given as follows:

(1) For DMs' assessments with hesitant linguistic terms, fuzzy envelope approach is applied with equations (11)–(13) and corresponding TFNs are calculated. Determine $\widetilde{z}_{ij} = (1/K)(\widetilde{z}_{ij}^1 (+)\widetilde{z}_{ij}^2 (+)\cdots(+)z_{ij}^K)$ where $\widetilde{z}_{ij}^K = (a_{ij}^K, b_{ij}^K, c_{ij}^K)\ \forall i, j, k$ is the corresponding TFN of the Kth DM. Since there are m alternatives and n criteria, mxn fuzzy evaluation matrix $\widetilde{Z} = \begin{bmatrix} \widetilde{z_{11}} & \widetilde{z_{12}} & .. & .. & \widetilde{z_{1n}} \\ z_{21} & z_{22} & .. & .. & z_{2n} \\ .. & .. & .. & .. & .. \\ .. & .. & .. & .. & .. \\ \widetilde{z_{n1}} & \widetilde{z_{n2}} & .. & .. & \widetilde{z_{mn}} \end{bmatrix}$ with positive TFN elements $\widetilde{z}_{ij} = (a_{ij}, b_{ij}, c_{ij})\ \forall i, j$ is determined.

(2) $\widetilde{Z} = (\widetilde{z}_{ij})_{\mathrm{mxn}}$ is normalized with the following equations [60, 61]:

$$a_{ij}^* = \frac{a_{ij}}{\sqrt{(1/3)\sum_{i=1}^m \left[a_{ij}^2 + b_{ij}^2 + c_{ij}^2\right]}}, \tag{14}$$

$$b_{ij}^* = \frac{b_{ij}}{\sqrt{(1/3)\sum_{i=1}^m \left[a_{ij}^2 + b_{ij}^2 + c_{ij}^2\right]}}, \tag{15}$$

$$c_{ij}^* = \frac{c_{ij}}{\sqrt{(1/3)\sum_{i=1}^m \left[a_{ij}^2 + b_{ij}^2 + c_{ij}^2\right]}}, \tag{16}$$

where $\widetilde{z_{ij}^*} = (a_{ij}^*, b_{ij}^*, c_{ij}^*)$ is the normalized version of the $\widetilde{z}_{ij} = (a_{ij}, b_{ij}, c_{ij})\ \forall i, j$. Here, $0 \le a_{ij}^* \le b_{ij}^* \le c_{ij}^* \le 1$.

(3) Ranking based on fuzzy *ratio system (RS):* To compute the utility of the fuzzy RS, fuzzy weighted normalized ratings are added for benefit (maximization) criteria and deducted for cost (minimization) criteria for each alternative i.

$$\widetilde{y}_i = \sum_{j=1}^{g} \widetilde{w}_j \widetilde{z_{ij}^*} - \sum_{j=g+1}^{n} \widetilde{w_j z_{ij}^*}, \quad \forall i. \tag{17}$$

$\widetilde{y}_i$ is defuzzified with equation (5), and y_i is determined for each alternative i. Based on the fuzzy RS, the best alternative is the one with the maximum utility y_i. The ranking is obtained for alternatives $A1$, $A1, \ldots Am$ in descending order as

$$R_{RS} = \left\{A_{i|\max_i y_i} \gg \cdots \gg A_{i|\min_i y_i}\right\}. \tag{18}$$

(4) Ranking based on fuzzy reference point approach (RP) :RP is based on Tchebycheff min-max metric [36, 37, 41]. In fuzzy RP, first, the maximal objective reference point vector is determined as

$$\widetilde{r}_j = \begin{cases} \widetilde{z}_j^+ = \left(\max_i a_{ij}^*, \max_i b_{ij}^*, \max_i c_{ij}^*\right) j \le g, \\ \widetilde{z}_j^+ = \left(\min_i a_{ij}^*, \min_i b_{ij}^*, \min_i c_{ij}^*\right) j > g. \end{cases} \tag{19}$$

Then, ranking of alternatives is done based on deviation from the reference point vector and this is determined with the Tchebycheff min-max metric as

$$s_i = \left(\operatorname{Max}_j \mathrm{d}\left(\widetilde{w_j}\widetilde{r}_j, \widetilde{w_j z_{ij}^*}\right)\right), \quad \forall i. \tag{20}$$

In equation (20), distance (deviation) is calculated with the vertex method (equation (7)). Based on the fuzzy RP, the best alternative has the minimum s_i. The ranking is obtained for alternatives $A_1, A_1, \ldots, A_m$ in ascending order as

$$R_{RP} = \left\{A_{i|\min_i s_i} \gg \cdots \gg A_{i|\max_i s_i}\right\}. \tag{21}$$

(5) Ranking based on fuzzy full multiplicative form (FMF): Utility of fuzzy FMF is determined

by dividing the product of fuzzy weighted normalized alternatives ratings on maximization criteria by the product of fuzzy weighted normalized alternative ratings on minimization criteria as follows:

$$\tilde{p}_i = \frac{\prod_{J=1}^{g} \left(\widetilde{z_{ij}^*}\right)^{\widetilde{w_j}}}{\prod_{J=g+1}^{n} \left(\widetilde{z_{ij}^*}\right)^{\widetilde{w_j}}}, \quad \forall i. \tag{22}$$

$\tilde{p}_i$is defuzzified with equation (5), and p_i is obtained. Based on the fuzzy FMF, the best alternative is the one with the maximum p_i. The ranking is obtained for alternatives $A1, A1, \ldots A_m$ in descending order as

$$R_{FMF} = \left\{A_{i|\max_i p_i} \gg \cdots \gg A_{i|\min_i p_i}\right\}. \tag{23}$$

(6) After obtaining ranking of alternatives with these methods ($R_{\mathrm{RS}}, R_{\mathrm{RP}}, R_{\mathrm{FMF}}$), these rankings are fused to determine the final ranking of alternatives. Here, improved Borda rule (IMB) is used for this since it is superior to dominance theory in the sense that it integrates both utilities and rankings [41, 59].

First, vector normalization [34, 41] is used to normalize y_i, s_i, p_i values as shown below:

$$y_i^* = \frac{y_i}{\sqrt{\sum_{i=1}^{M} (y_i)^2}},$$
$$s_i^* = \frac{s_i}{\sqrt{\sum_{i=1}^{M} (s_i)^2}}, \tag{24}$$
$$p_i^* = \frac{p_i}{\sqrt{\sum_{i=1}^{M} (p_i)^2}}, \quad \forall i.$$

Then, assessment value of IMB (IMB_i) for each alternative i is calculated as

$$\mathrm{IMB}_i = y_i^* \frac{m - R(y_i) + 1}{m(m+1)/2} - s_i^* \frac{m - R(s_i) + 1}{m(m+1)/2} + p_i^* \frac{m - R(p_i) + 1}{m(m+1)/2}, \quad \forall i, \tag{25}$$

where $R(y_i), R(s_i), R(p_i)$ are the rankings of fuzzy RS, RP, and FMF [41, 59].

As the final ranking of alternatives (R_{final}), the best alternative is the one with the maximum IMB_i. The ranking is obtained for alternatives A1, A1, . . .Am in descending order as

$$R_{\mathrm{final}} = \left\{A_{i|\max_i \mathrm{IMB}_i} \gg \cdots \gg A_{i|\min_i \mathrm{IMB}_i}\right\}. \tag{26}$$

Summary of the steps of HF-AHP-MULTIMOORA is given in Figure 1.

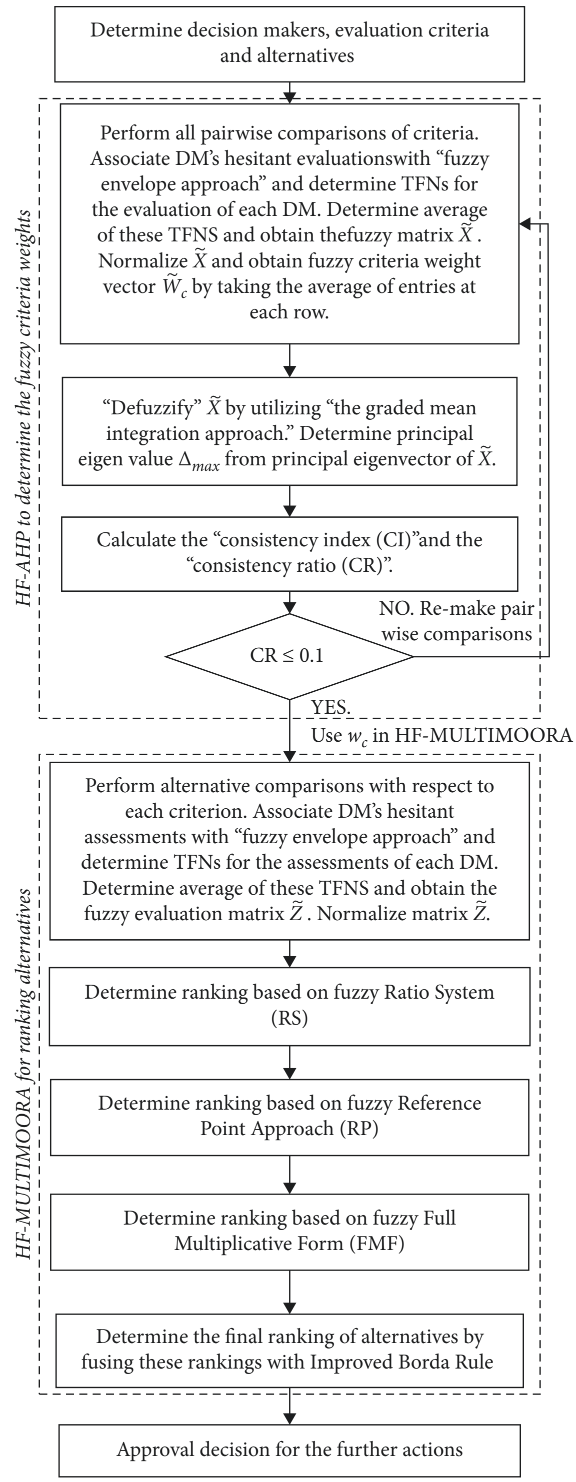

Figure 1: Steps of HF-AHP-MULTIMOORA.

4. HF-AHP-TOPSIS

Steps of HF-AHP-TOPSIS are as follows [30, 62]:

(1) After fuzzy evaluation matrix $\tilde{Z}$ is determined (in Step 1 of HF-MULTIMOORA), elements of $\tilde{Z}$ are defuzzified with equation (5) and Z with elementsz_{ij} is obtained. "Normalized decision matrix" D with

Table 3: Evaluation criteria for combi boilers.

	Criteria	Explanations
C1	Space and water heating capacity	The spatial heating capacity of the combi boiler
C2	Maximum space and water heating temperature	The maximum space and water temperature that the combi boiler provides
C3	Natural gas and electricity consumption reduction	The natural gas and electricity consumption level of the product
C4	Noise level reduction	The noise level of the product
C5	NOx emission reduction	The NOx emission level of the product
C6	Condensing boiler	Whether the combi boiler is condensing one or not
C7	Price efficiency	The price of the product compared to similar products in the market
C8	Product design	The product design and its attractiveness
C9	Digital level of the product	How advanced the digital level of the product
C10	Security level	Gas and water security level of the product
C11	Warranty period	Warranty period that manufacturing company provides
C12	Reliable brand	The brand perception among customers
C13	End-user satisfaction	Customer satisfaction
C14	Customer service	Customer service provided by the manufacturing company
C15	After-sales service	Service provided by the company after sale

Table 4: Pairwise comparisons of criteria by 5 DMs.

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13	C14	C15
C1	E	SS	SW	FS	SS	E	SW	VS	AS	SW	E	SS	E	VS	E
	E	FS	FS	AS	FS	FS	SS	AS	FS	E	AS	AS	FS	FS	FS
	E	AS	SW-FW	AS	AS-VS	AS	AS-VS	FS	AS	SS	AS	SW	SW-FW	AS	AS-VS
	E	VS	SW	VS	FS	FW	SW	AS	AS	E	SW	FW	VW	SW	AW
	E	VS-FS	SW	VS	FS	SS	FS	AS-VS	AS	SW-FW	VS	FW-VW	SW	FS	FS
C2		E	SW	SS	SS	SS	E	VS	AS	SW	SS-E	SS	E	SS	E
		E	E	FS	SS	SS	E	SS	SS	FW	SS	FS	SS	E	E
		E	SS	FS	AS	AS-VS	FS	FS-SS	SS	SS	VS-FS	SW-FW	FW	VS	AS
		E	SW	E-SW	FS-SS	FW-VW	SS	SS	VS	SS-E	SW	SW	SW	E	SS
		E	FS-SS	FS	SS	FS-SS	FS	AS-VS	VS	FW	FS	VW	FW	FS	FS
C3			E	VS	SS	E	SS	AS	AS	E	AS	AS	SS	VS	VS
			E	AS	AS	SS	E	AS	FS	SW	SS	FS	FS	FS	FS
			E	FS-SS	SS	VS-FS	FS-SS	SS	AS	SS	AS	SW	FW	AS	VS
			E	VS	FS	SW	FS-SS	SW	FS	FW-VW	FW	E	SW	FS-SS	VS
			E	AS	FS	VS	VS	AS	AS	SW-FW	FS	FW-VW	SW	VS	VS
C4				E	SW	AW	E	VS	E	VW	SW	E	E	E	E
				E	SW	SW	VW	E	E	AW	E	SW	SW	SW	SW
				E	FS-SS	SW-FW	FS-SS	VS-SS	AS	SW	AS-VS	SW	SW	VS	AS
				E	SS-E	VW	FS-SS	SS	E	E	SW-FW	SS	FW	E-SW	SS
				E	SW	SW-FW	FS	FS	VS-FS	FW-VW	VS	VW	SW	VS	AS
C5					E	E	SW	SS	SS	VW	SW	E	SS	SS	SS
					E	SS	SW	SS	E	AW	E	SW	E	E	E
					E	FS	FS	FS	FS-SS	SW	VS-SS	FW-VW	SW-FW	VS-FS	SS
					E	SW	FW	SS	FS	FW	E-SW	SS	FW	FW	SW
					E	FS	FS	AS	AS	FW-VW	SS-FS	FW	FS-SS	FS	SW-FW
C6						E	SS	VS	AS	VW	E	E	E	E	E
						E	FW	SW	E	AW	SS	SS	E	SS	SS
						E	AS-VS	AS	AS-VS	SW-FW	VS	SW	SW-FW	VS-FS	SS
						E	FS-SS	VS	AS	E	FS	SS	VS-AS	FS	AS
						E	VS-FS	AS	AS-VS	SW	VS-FS	FW-VW	FW	FS	VS-FS

Table 4: Continued.

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13	C14	C15
C7							E	VS	VS	VW	SS-E	E	E	E	E
							E	FS	FS	FW	FS	SS	SS	FS	SS
							E	AS-VS	AS	SS	AS	VW	FW	AS	VS
							E	E-SS	FS	FW	SW	FW	VW	SS	VW
							E	AS-VS	FS	FW	SS	VW	FW	SS	FW
C8								E	E	VW	VW	VW	VW	VW	VW
								E	E	VW	SW	E	SW	SW	SW
								E	AS	SS	VS	FW	SW	FS	SS
								E	FW	VW	FW-VW	E	SW	SW	E-SW
								E	SS	AW	SW	AW	VS	FS	VS-FS
C9									E	AW	VW	VW	VW	VW	SW
									E	AW	E	E	SW	SW	E
									E	FS	AS-FS	VW	SW	VS	FS
									E	FW	FW-VW	SW	VW	VW	E-SW
									E	VW-AW	SW	FW-AW	VW	SW	VW
C10										E	VS	SS	VS	VS	AS
										E	AS	FS	FS	FS	AS
										E	AS	FW-VW	SW-FW	AS	FS
										E	SS	FS	E	SS	VS
										E	VS-FS	SS	VS	AS	FS
C11											E	E	E	E	SS
											E	E	SW	SW	E
											E	SW	SS	AS	SS
											E	E-SW	E	SS	VS
											E	SW-FW	SS	SW	SS
C12												E	E	E	SS
												E	SS	SW	E
												E	SS	VS	SS
												E	E-SW	E	FS
												E	FS-SS	VS	FW
C13													E	E	E
													E	E	SS
													E	VS-FS	SS
													E	VS-FS	SS
													E	VS-FS	SW
C14														E	SW
														E	SS
														E	FS
														E	SW-FW
														E	SW
C15															E
															E
															E
															E
															E

Table 5: The fuzzy evaluation matrix ($\widetilde{X}$).

	C1	*C2*	*C3*	*C4*	*C5*
C1	(1.000, 1.000, 1.000)	(1.300, 1.700, 2.300)	(0.702, 1.033, 1.200)	(1.600, 2.100, 2.600)	(1.100, 1.500, 2.100)
C2	(0.460, 0.614, 0.834)	(1.000, 1.000, 1.000)	(0.868, 1.000, 1.300)	(0.933, 1.300, 1.700)	(1.200, 1.300, 1.900)
C3	(0.900, 0.934, 1.500)	(0.834, 0.933, 1.200)	(1.000, 1.000, 1.000)	(1.600, 2.000, 2.600)	(1.200, 1.500, 2.000)
C4	(0.392, 0.494, 0.668)	(0.634, 0.802, 1.100)	(0.392, 0.493, 0.668)	(1.000, 1.000, 1.000)	(0.802, 1.000, 1.300)
C5	(0.501, 0.682, 0.933)	(0.568, 0.813, 0.900)	(0.534, 0.748, 0.900)	(0.833, 0.933, 1.300)	(1.000, 1.000, 1.000)
C6	(0.700, 0.914, 1.100)	(0.635, 0.913, 1.233)	(0.694, 0.800, 1.034)	(1.300, 1.500, 2.200)	(0.734, 0.868, 1.100)
C7	(0.701, 0.814, 1.133)	(0.734, 0.868, 1.000)	(0.614, 0.767, 0.934)	(0.800, 1.001, 1.300)	(0.800, 0.968, 1.400)
C8	(0.379, 0.474, 0.667)	(0.515, 0.713, 0.867)	(0.532, 0.640, 0.800)	(0.594, 0.824, 0.934)	(0.568, 0.814, 0.900)
C9	(0.364, 0.454, 0.600)	(0.494, 0.680, 0.768)	(0.398, 0.508, 0.700)	(0.746, 0.780, 0.900)	(0.600, 0.747, 0.900)
C10	(0.934, 1.000, 1.300)	(0.867, 1.200, 1.500)	(0.934, 1.100, 1.600)	(1.300, 1.600, 2.100)	(1.300, 1.700, 2.300)
C11	(0.612, 0.660, 0.834)	(0.647, 0.834, 1.100)	(0.566, 0.794, 1.000)	(0.747, 0.780, 1.167)	(0.780, 0.924, 1.200)
C12	(0.800, 1.080, 1.500)	(0.934, 1.134, 1.600)	(0.766, 0.914, 1.300)	(1.034, 1.200, 1.500)	(0.934, 1.200, 1.600)
C13	(1.000, 1.134, 1.600)	(0.934, 1.200, 1.500)	(0.834, 1.034, 1.400)	(1.000, 1.100, 1.500)	(0.834, 1.033, 1.400)
C14	(0.546, 0.648, 0.934)	(0.714, 0.834, 0.934)	(0.426, 0.547, 0.768)	(0.760, 0.800, 1.068)	(0.714, 0.934, 1.200)
C15	(0.867, 1.048, 1.333)	(0.700, 0.814, 0.900)	(0.420, 0.534, 0.736)	(0.666, 0.760, 0.900)	(0.868, 1.000, 1.300)

	C6	*C7*	*C8*	*C9*	*C10*
C1	(1.100, 1.334, 1.700)	(0.968, 1.300, 1.700)	(1.600, 2.100, 2.700)	(1.800, 2.300, 2.800)	(0.834, 0.933, 1.100)
C2	(0.980, 1.100, 1.800)	(1.000, 1.200, 1.500)	(1.200, 1.400, 2.100)	(1.400, 1.700, 2.200)	(0.734, 0.868, 1.200)
C3	(1.034, 1.300, 1.700)	(1.100, 1.200, 1.800)	(1.534, 1.900, 2.300)	(1.600, 2.100, 2.600)	(0.714, 0.833, 1.100)
C4	(0.480, 0.647, 0.834)	(0.880, 1.000, 1.534)	(1.100, 1.486, 1.900)	(1.200, 1.400, 1.700)	(0.560, 0.680, 0.834)
C5	(0.934, 1.200, 1.500)	(0.768, 1.134, 1.400)	(1.200, 1.400, 1.900)	(1.200, 1.400, 1.900)	(0.460, 0.614, 0.834)
C6	(1.000, 1.000, 1.000)	(1.000, 1.234, 2.000)	(1.534, 2.000, 2.400)	(1.600, 2.000, 2.600)	(0.580, 0.713, 0.834)
C7	(0.581, 0.813, 1.133)	(1.000, 1.000, 1.000)	(1.300, 1.700, 2.400)	(1.300, 1.800, 2.300)	(0.580, 0.702, 1.034)
C8	(0.492, 0.560, 0.768)	(0.447, 0.594, 0.801)	(1.000, 1.000, 1.000)	(1.100, 1.234, 1.500)	(0.506, 0.580, 0.802)
C9	(0.465, 0.520, 0.667)	(0.446, 0.582, 0.834)	(0.800, 0.980, 1.100)	(1.000, 1.000, 1.000)	(0.499, 0.674, 0.933)
C10	(1.300, 1.500, 2.000)	(1.034, 1.500, 1.900)	(1.434, 1.900, 2.300)	(1.400, 1.834, 2.400)	(1.000, 1.000, 1.000)
C11	(0.594, 0.734, 0.934)	(0.633, 0.814, 1.000)	(0.980, 1.200, 1.734)	(0.980, 1.200, 1.700)	(0.426, 0.560, 0.734)
C12	(0.868, 1.100, 1.400)	(1.134, 1.500, 1.800)	(1.300, 1.600, 1.900)	(1.200, 1.686, 2.100)	(0.668, 0.968, 1.300)
C13	(0.867, 0.980, 1.333)	(1.034, 1.400, 1.700)	(0.980, 1.100, 1.534)	(1.300, 1.600, 2.100)	(0.660, 0.734, 1.068)
C14	(0.614, 0.768, 1.000)	(0.634, 0.814, 0.900)	(0.900, 1.068, 1.500)	(1.080, 1.300, 1.734)	(0.446, 0.594, 0.734)
C15	(0.614, 0.780, 0.900)	(0.914, 1.200, 1.434)	(0.914, 1.100, 1.500)	(1.000, 1.134, 1.500)	(0.412, 0.528, 0.734)

	C11	*C12*	*C13*	*C14*	*C15*
C1	(1.434, 1.800, 2.100)	(0.914, 1.134, 1.500)	(0.714, 0.933, 1.134)	(1.234, 1.700, 2.100)	(0.966, 1.280, 1.700)
C2	(0.934, 1.200, 1.700)	(0.714, 0.933, 1.234)	(0.734, 0.868, 1.100)	(1.100, 1.300, 1.600)	(1.200, 1.400, 1.700)
C3	(1.300, 1.634, 2.100)	(1.014, 1.300, 1.600)	(0.768, 1.034, 1.300)	(1.400, 1.800, 2.400)	(1.400, 1.900, 2.400)
C4	(1.034, 1.333, 1.700)	(0.748, 0.900, 1.034)	(0.702, 0.934, 1.000)	(1.067, 1.400, 1.600)	(1.334, 1.600, 1.900)
C5	(0.867, 1.186, 1.500)	(0.714, 0.834, 1.100)	(0.800, 0.867, 1.300)	(0.900, 1.134, 1.600)	(0.834, 0.933, 1.200)
C6	(1.100, 1.400, 1.900)	(0.814, 0.900, 1.200)	(0.900, 1.067, 1.400)	(1.000, 1.300, 1.800)	(1.200, 1.400, 1.900)
C7	(1.134, 1.400, 1.800)	(0.660, 0.734, 0.968)	(0.680, 0.768, 1.034)	(1.200, 1.400, 1.800)	(0.880, 1.034, 1.334)
C8	(0.728, 1.000, 1.234)	(0.646, 0.714, 0.834)	(0.782, 1.100, 1.234)	(0.748, 1.100, 1.334)	(0.747, 1.000, 1.334)
C9	(0.694, 1.086, 1.334)	(0.561, 0.729, 0.868)	(0.508, 0.700, 0.802)	(0.728, 1.000, 1.168)	(0.747, 1.000, 1.134)
C10	(1.500, 1.900, 2.500)	(0.880, 1.100, 1.600)	(1.100, 1.433, 1.800)	(1.500, 1.900, 2.400)	(1.500, 2.000, 2.500)
C11	(1.000, 1.000, 1.000)	(0.767, 0.933, 1.000)	(0.934, 1.000, 1.200)	(1.068, 1.300, 1.500)	(1.100, 1.200, 1.600)
C12	(1.000, 1.000, 1.400)	(1.000, 1.000, 1.000)	(0.933, 1.000, 1.400)	(1.134, 1.400, 1.600)	(0.900, 1.034, 1.400)
C13	(0.868, 1.000, 1.100)	(0.768, 0.933, 1.100)	(1.000, 1.000, 1.000)	(1.000, 1.300, 1.900)	(0.934, 1.000, 1.300)
C14	(0.800, 0.880, 1.100)	(0.760, 0.800, 0.968)	(0.640, 0.700, 1.000)	(1.000, 1.000, 1.000)	(0.768, 1.033, 1.300)
C15	(0.682, 0.900, 0.934)	(0.768, 1.034, 1.200)	(0.802, 1.000, 1.100)	(0.834, 0.934, 1.400)	(1.000, 1.000, 1.000)

Table 6: Fuzzy criteria weight vector $\widetilde{w_c} = (\widetilde{w_1}, \widetilde{w_2}, \ldots, \widetilde{w_n})$.

Criteria	Fuzzy weights	Criteria	Fuzzy weights	Criteria	Fuzzy weights
C1	(0.055, 0.090, 0.137)	C6	(0.047, 0.072, 0.116)	C11	(0.038, 0.057, 0.088)
C2	(0.046, 0.070, 0.114)	C7	(0.041, 0.063, 0.101)	C12	(0.047, 0.074, 0.116)
C3	(0.055, 0.086, 0.136)	C8	(0.031, 0.050, 0.075)	C13	(0.046, 0.069, 0.110)
C4	(0.038, 0.060, 0.092)	C9	(0.029, 0.047, 0.069)	C14	(0.034, 0.052, 0.081)
C5	(0.038, 0.061, 0.096)	C10	(0.058, 0.092, 0.145)	C15	(0.037, 0.057, 0.085)

Table 7: 5 DMs' evaluations of combi boilers with respect to each criterion.

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13	C14	C15
	MG	MG	F	MG	F	VG	P	F	F	G	MP	G	G	G	MG
	MG	MG	P	G	F	P	P	F	G	VG	VG	VG	G	G	MG
A1	G	G	MG	MG	F	G	F	G	VG	F	F	MG	VG	G	VG
	MG	VG	VG	MG	G	G	P	F	MG	G	G	G	G	MG	F
	F	F	MG	G	F	VG	P	VG	VG	VG	F	G	VG	G	MG
	MG	MG	MP	MP	MP	VG	MG	F	MG	G	MG	G	F	MP	MG
	VG	MG	G	F	P	G	G	F	MG	MG	VG	MG	G	MG	G
A2	MG	MG	P	P	P	G	VP-F	MG	MG	G	F	VG	MG	F	F
	VG	VG	P	P	G	G	G	G-VG	G-VG	G-VG	G	G	MG	G	MG
	MG	VG	MP	F	MP	G	MG	G	G	VG	VG	G	F	F	MG
	MG	MG	P	F	MP	VG	F	F	G	F	MP	MP	P	F	G
	MG	MG	MG	G	P	G	F	F	G	MG	VG	MG	G	MG	MG
A3	VG	VG	VP-F	G	VP-F	P	P	VP-F	VP-F	G	G	VP-F	P	VG	VG
	MG	MG	MP	MP	MP	G	MP	F-MG	MG	MG	MG	MG	MP	MG	G
	MG	G	MP	MG	MP	F	F	VP	VP	MG	F	VP	MP	MG	VG
	MG	MG	F	F	P	VG	G	F	MP	F	MG	G	MP	MP	MG
	MG	MG	F	G	F	G	VG	F	MG	MG	VG	MG	G	MG	G
A4	VG	G	G	MG	G	G	MG	VG	G	VG	VG	VG	G	G	MG
	F	MG	G	MP	MP	G	G	MG	MG	MG-G	G	G	MG	G	MG
	MG	F	MG	MG	F	G	VG	G	G	F-G	VG	G	F	F	MG
	F	F	F	F	P	VG	F	F	F	P	MP	VG	P	F	F
	MG	G	F	G	F	F	F	F	MG	F	G	MG	G	MG	G
A5	G	F	VP-F	G	P	VG	G	P	P	P	MP	MP	P	VP-F	G
	G	MG	MP	MG	MG	G	G	MG	F	MG	MG	MG	MG	MG	MG
	VG	G	P	MG	F	G	F	VP	F	MP	F	F	MP	MP	F
	F	F	F	MG	P	VG	MP	F	MG	G	MP	P	MG	F	P
	MG	F	P	MG	F	P	G	F	MG	VG	G	VG	G	F	MG
A6	VG	VG	G	VG	G	VG	P	P	VP	VP	VP	F	P	P	VP-F
	MG	G	F	MG	G	G	MG	MG	MG	G	G	G	G	G	MG
	G	F	F	G	F	VG	MP	VP	P	VG	F	VG	F	F	VP
	MG	MG	F	G	G	VG	MG	F	MP	MG	MP	P	MG	MG	MP
	MG	G	P	VG	VG	MG	MG	F	G	G	G	P	F	F	F
A7	VP-F	G	VG	G	G	VG	VG	G	F	F	G	F	G	VG	P
	G	G	G	G	P	G	G	G	G	G	G	G	MG-G	MG-G	MG
	MP	G	VG	VG	VG	G	G	VG	VG	G	F	F	G	MG	MP
	MG	MG	G	G	MP	VG	P	F	MP	MG	MG	P	G	G	MP
	VG-G	G	VG	VG	P	MG	G	F	MG	G	G	P	F	F	F
A8	F	F	F	MG	P	F	MP	VG	F	F	MG	MG	G	G	P
	MG	MG	F	G	MG	G	G	G	G	G	G	G	MG-G	MG-G	MG
	VG	MG	MG	VG	MP	G	MP	VG	VG	G	VG	F	G	MG	MP
	MG	MG	F	MG	MP	VG	MG	F	G	F	MG	VG	P	P	VG
	MG	MG	G	MG	P	MG	MG	F	G	MG	G	G	F	G	G
A9	P	MP	MP	G	MP	VG	VG	F	F	VG	VG	MP	P	G	VG
	MG	MG	MG	F	MG	G	G	MG	MG	MG	MG	MG	MG	MG	MG
	G	MG	MP	G	MP	VG	G	VP	VP	MG	VG	VP	MP	MG	VG
	MG	MG	MG	G	F	VG	MG	F	VG	G	MG	VG	F	MG	G
	MG	G	VG	VG	VP	VG	G	MG	G	VG	VG	VG	G	G	G
A10	MG	MG	P	MG	VG	VG	MG	G	G	MG	VG	F	MG	VG	MG
	G	G	G	G	MG	G	G	G	MG-G	G	G	G	G	G	G
	VG	MG	F	VG	F	F	G	VG	VG	VG	VG	G	MG	G	VG

elements $d_{ij} = z_{ij}/(\sum_i z_{ij}^2)^{1/2}$ is calculated. Given $w_{cr} = (w_1, w_2, \ldots, w_n)$ (calculated in Step 2 of HF-AHP), "weighted normalized decision matrix" E is obtained with elements $e_{ij} = w_{crj} d_{ij}$.

(2) The "positive ideal" A^* and "negative ideal" A^- solutions are determined. $A^* = (e_1^*, \ldots, e_n^*)$ where $e_j^* = \max_i (e_{ij})$ *for max. criteria and* $e_j^* = \min_i (e_{ij})$ *for min. criteria.* $A^- = (e_1^-, \ldots, e_n^-)$, $e_j^- = \min_i (e_{ij})$ *for max. criteria and* $e_j^- = \max_i (e_{ij})$ *for min. criteria.*

(3) The "separation measures" of ach alternative are calculated. "Separation from ideal solution is" $S_i^* = (\sum_j (e_j^* - e_{ij})^2)^{1/2}$, and "separation from negative ideal solution" is $S_i^- = (\sum_j (e_j^- - e_{ij})^2)^{1/2}$.

Table 8: Fuzzy evaluation matrix ($\widetilde{\mathbf{Z}}$).

	C1	C2	C3	C4	C5
A1	(5.000, 7.000, 8.800)	(5.800, 7.600, 9.000)	(4.400, 6.000, 7.600)	(5.800, 7.800, 9.400)	(3.800, 5.800, 7.600)
A2	(6.600, 8.200, 9.400)	(6.600, 8.200, 9.400)	(1.800, 3.400, 5.200)	(1.400, 3.000, 5.000)	(1.800, 3.400, 5.200)
A3	(5.800, 7.600, 9.200)	(6.200, 8.000, 9.400)	(1.400, 3.384, 5.800)	(4.600, 6.600, 8.200)	(0.600, 2.584, 5.000)
A4	(5.400, 7.200, 8.800)	(5.000, 7.000, 8.800)	(5.000, 7.000, 8.600)	(4.200, 6.200, 8.000)	(2.800, 4.600, 6.400)
A5	(6.200, 8.000, 9.200)	(5.000, 7.000, 8.600)	(1.400, 3.384, 5.800)	(5.400, 7.400, 9.000)	(2.200, 3.800, 5.800)
A6	(5.800, 7.600, 9.000)	(5.000, 6.800, 8.200)	(3.200, 5.000, 6.800)	(6.200, 8.000, 9.400)	(4.000, 5.800, 7.400)
A7	(3.600, 5.784, 8.000)	(6.600, 8.600, 9.800)	(5.600, 7.000, 8.000)	(7.800, 9.400, 10.00)	(6.400, 7.800, 8.600)
A8	(5.800, 7.600, 9.000)	(5.000, 7.000, 8.800)	(5.400, 7.200, 8.600)	(7.400, 9.000, 9.800)	(1.400, 3.000, 5.000)
A9	(4.400, 6.200, 8.000)	(4.200, 6.200, 8.200)	(3.400, 5.400, 7.200)	(5.400, 7.400, 9.000)	(1.600, 3.400, 5.400)
A10	(6.200, 8.000, 9.400)	(5.800, 7.800, 9.400)	(4.800, 6.400, 7.800)	(7.400, 9.000, 9.800)	(4.000, 5.400, 6.800)

	C6	C7	C8	C9	C10
A1	(6.400, 7.800, 8.600)	(0.600, 1.800, 3.800)	(5.000, 6.800, 8.200)	(6.600, 8.200, 9.200)	(7.000, 8.600, 9.400)
A2	(7.400, 9.200, 10.00)	(4.800, 6.984, 9.000)	(5.000, 7.000, 8.600)	(5.800, 7.800, 9.400)	(7.000, 8.800, 9.800)
A3	(5.200, 6.800, 8.000)	(2.000, 3.800, 5.800)	(1.800, 3.584, 6.200)	(3.800, 5.584, 7.400)	(5.000, 7.000, 8.800)
A4	(7.400, 9.200, 10.00)	(7.400, 9.000, 9.800)	(5.400, 7.200, 8.600)	(5.000, 7.000, 8.600)	(5.000, 7.520, 9.200)
A5	(7.000, 8.600, 9.400)	(4.600, 6.600, 8.200)	(2.200, 3.600, 5.400)	(2.800, 4.600, 6.600)	(1.800, 3.400, 5.400)
A6	(6.800, 8.000, 8.600)	(2.800, 4.600, 6.400)	(2.200, 3.600, 5.400)	(3.000, 4.400, 6.200)	(6.400, 7.600, 8.200)
A7	(7.400, 9.000, 9.800)	(6.600, 8.400, 9.600)	(5.800, 7.600, 8.800)	(5.400, 7.200, 8.400)	(5.800, 7.800, 9.200)
A8	(6.200, 8.000, 9.200)	(3.200, 5.000, 6.600)	(6.200, 7.800, 8.800)	(5.000, 6.800, 8.200)	(5.800, 7.800, 9.200)
A9	(7.800, 9.200, 9.800)	(6.600, 8.400, 9.600)	(2.800, 4.400, 6.200)	(4.400, 6.000, 7.400)	(5.400, 7.200, 8.800)
A10	(7.400, 8.800, 9.400)	(6.200, 8.200, 9.600)	(6.200, 8.000, 9.200)	(7.400, 9.000, 10.00)	(7.400, 9.000, 9.800)

	C11	C12	C13	C14	C15
A1	(4.600, 6.400, 7.800)	(7.000, 8.800, 9.800)	(7.800, 9.400, 10.00)	(6.600, 8.600, 9.800)	(5.400, 7.200, 8.800)
A2	(6.600, 8.200, 9.200)	(7.000, 8.800, 9.800)	(4.600, 6.600, 8.400)	(3.800, 5.800, 7.600)	(5.000, 7.000, 8.800)
A3	(5.000, 6.800, 8.200)	(2.200, 3.984, 6.200)	(1.800, 3.400, 5.200)	(5.400, 7.200, 8.800)	(7.400, 9.000, 9.800)
A4	(7.800, 9.200, 9.800)	(7.000, 8.800, 9.800)	(4.600, 6.600, 8.200)	(4.600, 6.600, 8.200)	(5.400, 7.400, 9.200)
A5	(3.400, 5.400, 7.200)	(4.600, 6.400, 8.000)	(2.600, 4.200, 6.000)	(2.800, 4.984, 7.400)	(5.000, 7.000, 8.600)
A6	(3.600, 5.200, 6.600)	(5.600, 7.000, 8.000)	(4.400, 6.200, 7.800)	(3.200, 5.000, 6.800)	(2.000, 3.584, 5.800)
A7	(5.000, 7.000, 8.400)	(2.600, 4.200, 6.000)	(5.400, 7.400, 9.200)	(5.400, 7.200, 9.000)	(2.000, 3.800, 5.800)
A8	(6.600, 8.400, 9.600)	(3.000, 4.600, 6.400)	(5.800, 7.800, 9.400)	(5.400, 7.400, 9.200)	(2.000, 3.800, 5.800)
A9	(7.000, 8.600, 9.600)	(4.400, 5.800, 7.000)	(1.800, 3.400, 5.400)	(4.800, 6.600, 8.200)	(7.800, 9.200, 9.800)
A10	(7.800, 9.200, 9.800)	(7.000, 8.600, 9.400)	(5.400, 7.400, 9.000)	(7.000, 8.800, 9.800)	(7.000, 8.800, 9.800)

(4) "Closeness index values" C_i for each alternative are calculated as $C_i = S_i^- / S_i^- + S_i^*$. Alternatives are ranked based on C_i (highest C_i is best).

5. Case Study

In this research, 10 combi boiler alternatives that are frequently used in Turkey (A1, A2,. . ., A10) are evaluated with respect to 15 benefit (maximization) criteria (C1, C2, . . ., C15) by 5 decision-makers (DM1, DM2, . . ., DM5). Evaluation criteria are listed in Table 3 with explanations. The combi boiler alternatives that are ranked in this study are Vaillant ecoTEC Pro VUW 286/5-3 (A1), Baymak Brötje Novadens 42 Fi (A2), E.C.A. Proteus Premix 28 HM (A3), Baymak Brötje Novadens 24 Fi (A4), Demirdöküm Nitromix P 28 (A5), Viessmann Vitodens 050-W 24 (A6), Buderus Logamax Plus GB012-25K V2 (A7), Buderus Logamax Plus GB122i 24KD (A8), E.C.A. Calora Premix 24 (A9), and Bosch Condens 2200i W 24 (A10). DMs are thermal engineering graduate students from Istanbul Technical University (Turkey) and Middle East Technical University (Turkey).

In HF-AHP-MULTIMOORA, fuzzy criteria weight vector $\widetilde{w_c} = (\widetilde{w_1}, \widetilde{w_2}, \ldots, \widetilde{w_n})$ is determined with HF-AHP. Here, DMs' pairwise comparisons of criteria with the linguistic terms in Table 1 are presented in Table 4.

For DMs' assessments with hesitant linguistic terms, fuzzy envelope approach is applied with equations (11)–(13) and corresponding TFNs are obtained. Afterward, average of 5 DMs TFNs is taken and fuzzy evaluation matrix $\widetilde{X}$ in Table 5 is determined. $\widetilde{Y}$ is obtained by normalizing $\widetilde{X}$. Then, $\widetilde{w_c} = (\widetilde{w_1}, \widetilde{w_2}, \ldots, \widetilde{w_n})$ in Table 6 is calculated by taking the average of each row of $\widetilde{Y}$.

$\widetilde{X}$ is defuzzified with equation (5), and $w_{cr} = (0.089, 0.071, 0.086, 0.060, 0.061, 0.073, 0.064, 0.050, 0.046, 0.092, 0.057, 0.074, 0.070, 0.052, 0.057)$ is calculated. $\Delta_{\max} = 15.91$, $CI = (15.91 - 15)/14 = 0.065$, random index $(RI) = 1.59$ for $n = 15$, and $CR = CI/RI = 0.065/1.59 = 0.041 < 0.1$, so the comparisons are consistent.

$\widetilde{w_c} = (\widetilde{w_1}, \widetilde{w_2}, \ldots, \widetilde{w_n})$obtained with HF-AHP is then utilized in HF-MULTIMOORA to rank combi boilers. DMs' assessments of alternatives with respect to each criterion using the linguistic terms in Table 2 are presented in Table 7.

For DMs' assessments with hesitant linguistic terms, fuzzy envelope approach is applied with equations (11)–(13) and TFNs are obtained. Then, 5 DMs TFNs are aggregated by taking average and fuzzy evaluation matrix $\widetilde{Z}$ in Table 8 is

Table 9: Normalized fuzzy evaluation matrix ($\widetilde{\mathbf{Z}}^*$).

	C1	C2	C3	C4	C5
A1	(0.214, 0.299, 0.376)	(0.246, 0.322, 0.381)	(0.242, 0.329, 0.417)	(0.243, 0.327, 0.394)	(0.240, 0.366, 0.480)
A2	(0.282, 0.351, 0.402)	(0.280, 0.347, 0.398)	(0.099, 0.187, 0.285)	(0.059, 0.126, 0.210)	(0.114, 0.215, 0.328)
A3	(0.248, 0.325, 0.393)	(0.263, 0.339, 0.398)	(0.077, 0.186, 0.318)	(0.193, 0.277, 0.344)	(0.038, 0.163, 0.316)
A4	(0.231, 0.308, 0.376)	(0.212, 0.297, 0.373)	(0.275, 0.384, 0.472)	(0.176, 0.260, 0.336)	(0.177, 0.290, 0.404)
A5	(0.265, 0.342, 0.393)	(0.212, 0.297, 0.364)	(0.077, 0.186, 0.318)	(0.227, 0.310, 0.378)	(0.139, 0.240, 0.366)
A6	(0.248, 0.325, 0.385)	(0.212, 0.288, 0.347)	(0.176, 0.275, 0.373)	(0.260, 0.336, 0.394)	(0.252, 0.366, 0.467)
A7	(0.154, 0.247, 0.342)	(0.280, 0.364, 0.415)	(0.307, 0.384, 0.439)	(0.327, 0.394, 0.420)	(0.404, 0.492, 0.543)
A8	(0.248, 0.325, 0.385)	(0.212, 0.297, 0.373)	(0.296, 0.395, 0.472)	(0.310, 0.378, 0.411)	(0.088, 0.189, 0.316)
A9	(0.188, 0.265, 0.342)	(0.178, 0.263, 0.347)	(0.187, 0.296, 0.395)	(0.227, 0.310, 0.378)	(0.101, 0.215, 0.341)
A10	(0.265, 0.342, 0.402)	(0.246, 0.330, 0.398)	(0.264, 0.351, 0.428)	(0.310, 0.378, 0.411)	(0.252, 0.341, 0.429)
	C6	C7	C8	C9	C10
A1	(0.244, 0.297, 0.328)	(0.028, 0.085, 0.179)	(0.251, 0.342, 0.412)	(0.305, 0.379, 0.426)	(0.293, 0.360, 0.394)
A2	(0.282, 0.350, 0.381)	(0.227, 0.330, 0.425)	(0.251, 0.352, 0.432)	(0.268, 0.361, 0.435)	(0.293, 0.369, 0.410)
A3	(0.198, 0.259, 0.305)	(0.094, 0.179, 0.274)	(0.090, 0.180, 0.312)	(0.176, 0.258, 0.342)	(0.209, 0.293, 0.369)
A4	(0.282, 0.350, 0.381)	(0.349, 0.425, 0.463)	(0.271, 0.362, 0.432)	(0.231, 0.324, 0.398)	(0.209, 0.315, 0.385)
A5	(0.267, 0.328, 0.358)	(0.217, 0.312, 0.387)	(0.111, 0.181, 0.271)	(0.130, 0.213, 0.305)	(0.075, 0.142, 0.226)
A6	(0.259, 0.305, 0.328)	(0.132, 0.217, 0.302)	(0.111, 0.181, 0.271)	(0.139, 0.204, 0.287)	(0.268, 0.318, 0.343)
A7	(0.282, 0.343, 0.373)	(0.312, 0.397, 0.453)	(0.291, 0.382, 0.442)	(0.250, 0.333, 0.389)	(0.243, 0.327, 0.385)
A8	(0.236, 0.305, 0.350)	(0.151, 0.236, 0.312)	(0.312, 0.392, 0.442)	(0.231, 0.315, 0.379)	(0.243, 0.327, 0.385)
A9	(0.297, 0.350, 0.373)	(0.312, 0.397, 0.453)	(0.141, 0.221, 0.312)	(0.204, 0.278, 0.342)	(0.226, 0.302, 0.369)
A10	(0.282, 0.335, 0.358)	(0.293, 0.387, 0.453)	(0.312, 0.402, 0.462)	(0.342, 0.416, 0.463)	(0.310, 0.377, 0.410)
	C11	C12	C13	C14	C15
A1	(0.194, 0.270, 0.329)	(0.319, 0.401, 0.447)	(0.375, 0.452, 0.481)	(0.299, 0.389, 0.443)	(0.243, 0.325, 0.397)
A2	(0.279, 0.346, 0.389)	(0.319, 0.401, 0.447)	(0.221, 0.317, 0.404)	(0.172, 0.262, 0.344)	(0.225, 0.315, 0.397)
A3	(0.211, 0.287, 0.346)	(0.100, 0.182, 0.283)	(0.087, 0.163, 0.250)	(0.244, 0.326, 0.398)	(0.334, 0.406, 0.442)
A4	(0.329, 0.389, 0.414)	(0.319, 0.401, 0.447)	(0.221, 0.317, 0.394)	(0.208, 0.299, 0.371)	(0.243, 0.334, 0.415)
A5	(0.144, 0.228, 0.304)	(0.210, 0.292, 0.365)	(0.125, 0.202, 0.288)	(0.127, 0.225, 0.335)	(0.225, 0.315, 0.388)
A6	(0.152, 0.220, 0.279)	(0.255, 0.319, 0.365)	(0.212, 0.298, 0.375)	(0.145, 0.226, 0.308)	(0.090, 0.162, 0.261)
A7	(0.211, 0.296, 0.355)	(0.119, 0.192, 0.274)	(0.260, 0.356, 0.442)	(0.244, 0.326, 0.407)	(0.090, 0.171, 0.261)
A8	(0.279, 0.355, 0.405)	(0.137, 0.210, 0.292)	(0.279, 0.375, 0.452)	(0.244, 0.335, 0.416)	(0.090, 0.171, 0.261)
A9	(0.296, 0.363, 0.405)	(0.201, 0.265, 0.319)	(0.087, 0.163, 0.260)	(0.217, 0.299, 0.371)	(0.352, 0.415, 0.442)
A10	(0.329, 0.389, 0.414)	(0.319, 0.392, 0.429)	(0.260, 0.356, 0.433)	(0.317, 0.398, 0.443)	(0.315, 0.397, 0.442)

Table 10: HF-AHP-MULTIMOORA results and rankings.

	Fuzzy RS			Fuzzy RP		Fuzzy FMF			Results	
	$\widetilde{\mathbf{y}}_\mathbf{i}$	$\mathbf{y}_\mathbf{i}$	Rank ($\mathbf{R}_\mathbf{RS}$)	$\mathbf{s}_\mathbf{i}$	Rank ($\mathbf{R}_\mathbf{RP}$)	$\widetilde{\mathbf{p}}_\mathbf{i}$	$\mathbf{p}_\mathbf{i}$	Rank ($\mathbf{R}_\mathbf{FMF}$)	$\mathbf{IMB}_\mathbf{i}$	Rank ($\mathbf{R}_\mathbf{final}$)
A1	(0.160, 0.330, 0.610)	0.348	4	0.022	10	(0.386, 0.314, 0.224)	0.311	4	0.012	7
A2	(0.146, 0.310, 0.591)	0.329	5	0.019	6	(0.365, 0.298, 0.215)	0.295	6	0.025	5
A3	(0.110, 0.255, 0.528)	0.276	9	0.020	8	(0.293, 0.244, 0.181)	0.242	10	−0.039	9
A4	(0.160, 0.337, 0.631)	0.357	2	0.012	2	(0.405, 0.334, 0.242)	0.331	2	0.106	2
A5	(0.109, 0.254, 0.524)	0.275	10	0.021	9	(0.304, 0.246, 0.179)	0.244	9	−0.049	10
A6	(0.129, 0.277, 0.537)	0.296	8	0.013	3	(0.346, 0.270, 0.187)	0.269	8	0.018	6
A7	(0.160, 0.331, 0.616)	0.350	3	0.016	4	(0.394, 0.321, 0.229)	0.318	3	0.078	3
A8	(0.144, 0.308, 0.590)	0.328	6	0.018	5	(0.369, 0.299, 0.215)	0.297	5	0.034	4
A9	(0.137, 0.292, 0.566)	0.312	7	0.020	7	(0.356, 0.285, 0.203)	0.283	7	−0.002	8
A10	(0.187, 0.370, 0.658)	0.387	1	0.009	1	(0.452, 0.369, 0.259)	0.364	1	0.135	1

determined. Afterward, $\widetilde{Z}$ is normalized with equations (14)–(16) and Table 9 is obtained.

In steps 3, 4, and 5 of HF-MULTIMOORA, alternatives' rankings based on the fuzzy RS, fuzzy RP, and fuzzy FMF are determined, based on y_i, s_i, and p_i values, respectively. For y_i and p_i, bigger values are better, and for s_i smaller values are better in terms of rankings. In the last step, final ranking of alternatives is obtained applying the improved Borda rule. Alternatives are ranked based on IMB_i (higher values are better). Consequently, based on HF-AHP-MULTIMOORA, alternatives are ranked from best to worst as Bosch Condens 2200i W 24 (A10), Baymak Brötje Novadens 24 Fi (A4), Buderus Logamax Plus GB012-25K V2 (A7), Buderus Logamax Plus GB122i 24KD (A8), Baymak Brötje Novadens 42 Fi (A2), Viessmann Vitodens 050-W 24 (A6), Vaillant ecoTEC Pro VUW 286/5-3 (A1), E.C.A. Calora Premix 24 (A9), E.C.A. Proteus Premix 28 HM (A3), and Demirdöküm Nitromix P 28 (A5). These results are presented in Table 10.

For comparison analysis, HF-AHP-TOPSIS is implemented to the same problem. First, fuzzy evaluation matrix

Table 11: Weighted normalized decision matrix E, A^* and A^- for HF-AHP-TOPSIS.

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13	C14	C15
A1	0.027	0.023	0.029	0.020	0.023	0.021	0.006	0.017	0.017	0.033	0.015	0.029	0.031	0.020	0.019
A2	0.031	0.025	0.017	0.008	0.014	0.025	0.021	0.018	0.017	0.034	0.020	0.029	0.022	0.014	0.018
A3	0.029	0.024	0.017	0.017	0.011	0.018	0.012	0.009	0.012	0.027	0.016	0.014	0.012	0.017	0.023
A4	0.027	0.021	0.034	0.016	0.018	0.025	0.027	0.018	0.015	0.029	0.022	0.029	0.022	0.016	0.019
A5	0.030	0.021	0.017	0.019	0.015	0.023	0.020	0.009	0.010	0.013	0.013	0.022	0.014	0.012	0.018
A6	0.029	0.020	0.024	0.020	0.023	0.022	0.014	0.009	0.010	0.029	0.012	0.024	0.021	0.012	0.010
A7	0.022	0.026	0.034	0.023	0.031	0.024	0.025	0.019	0.015	0.030	0.017	0.014	0.025	0.017	0.010
A8	0.029	0.021	0.035	0.022	0.012	0.022	0.015	0.020	0.014	0.030	0.020	0.016	0.026	0.018	0.010
A9	0.024	0.019	0.026	0.019	0.014	0.025	0.025	0.011	0.013	0.028	0.021	0.020	0.012	0.016	0.023
A10	0.030	0.023	0.031	0.022	0.022	0.024	0.025	0.020	0.019	0.034	0.022	0.029	0.025	0.021	0.022
A^*	0.031	0.026	0.035	0.023	0.031	0.025	0.027	0.020	0.019	0.034	0.022	0.029	0.031	0.021	0.023
A^-	0.022	0.019	0.017	0.008	0.011	0.018	0.006	0.009	0.010	0.013	0.012	0.014	0.012	0.012	0.010

Table 12: Separation measures (S^*, S^-), closeness indices (C_i), and ranking of alternatives with HF-AHP-TOPSIS.

	S^*	S^-	C_i	Rank
A1	0.026	0.042	0.619	4
A2	0.033	0.037	0.534	6
A3	0.044	0.024	0.356	10
A4	0.021	0.043	0.672	2
A5	0.043	0.024	0.358	9
A6	0.034	0.030	0.474	8
A7	0.025	0.045	0.645	3
A8	0.031	0.037	0.545	5
A9	0.034	0.034	0.500	7
A10	0.012	0.049	0.797	1

$\widetilde{Z}$ in Table 8 is defuzzified with equation (5) and normalized. Then, after multiplication with the weights (w_{cr}), "weighted normalized decision matrix" E is determined as in Table 11. "Positive and negative ideal solutions" (A^* and A^-) are also presented in Table 11. "Separation measures" (S^*, S^-) and "closeness index of each alternative" (C_i) and ranking of alternatives based on C_i values are given in Table 12. As shown in Tables 10 and 12, HF-AHP-TOPSIS ranking is the same as the ranking obtained in fuzzy full multiplicative form (FMF) of HF-AHP-MULTIMOORA and the best alternative is determined by both HF-AHP-MULTIMOORA and HF-AHP-TOPSIS as Bosch Condens 2200i W 24 (A10).

6. Conclusions

As the proposed combi boiler alternatives for building heating were so diverse, our objective was to help select the most suitable option for residential customers; therefore, our aim was not to design the system or suggest further alternatives, but to evaluate the available ones. Basically, we can group heat distribution systems into two classes, depending on the temperature of the flowing heat carrier, as high and low temperature systems. These systems are both radiant heating systems which fundamentally consist of a burner unit coupled with a heat exchanger tube where combustion flue gases flow; however, their heat capacities differ. In this study, we targeted low temperature systems.

On the technical side, yearly energy requirements can be calculated before deciding on the combi boiler by estimating the local degree days, the indoor design temperature, and an operating time. The difference between the indoor design temperature and the outdoor design temperature is also an important criterion before selecting the combi boiler. However, one still needs to consider nontechnical criteria for selection of a combi boiler for residential heating.

The goal in this research was to choose the best combi boiler for residential heating according to the technical and nontechnical criteria listed in Table 3. These criteria were elicited from the expert decision-makers through a number of interviews, firstly defining a wider set of criteria, then identifying the most important through a tentative evaluation of criteria weights by pairwise comparisons, and removing the more minor with the aim of obtaining no more than fifteen final criteria.

In the end, the following fifteen criteria were retained: 1. space and water heating capacity is the spatial heating capacity of the combi boiler; 2. maximum space and water heating temperature is the maximum space and water temperature that the combi boiler provides; 3. natural gas and electricity consumption reduction is the natural gas and electricity consumption level of the product; 4. noise level reduction is the noise level of the product; 5. NOx emission reduction is the NOx emission level of the product; 6. condensing boiler is whether the combi boiler is condensing one or not; 7. price efficiency is the price of the product compared to similar products in the market; 8. product design is the product design and its attractiveness; 9. digital level of the product is how advanced the digital level of the product; 10. security level is the gas and water security level of the product; 11. warranty period is the warranty period that manufacturing company provides; 12. reliable brand is the brand perception among customers; 13. end-user satisfaction is the customer satisfaction level; 14. customer service is the customer service provided by the manufacturing company; and 15. after-sales service is the service provided by the company after sale. With HF-AHP, the relative importance of these criteria was assessed by pairwise comparisons elicited from expert decision-makers and fuzzy criteria weights were obtained.

As a result of HF-AHP-MULTIMOORA method and HF-AHP-TOPSIS method, the best alternative is found as Bosch Condens 2200i W 24 (A10). This combi boiler has a good customer service and reputation together with a good price compared to its heating capacity. This brand is well known in Turkey, and its other household appliances have good end-user satisfaction. Its natural gas consumption is low compared to other alternatives, and its space and water heating capacity is higher compared with other alternatives.

In this paper, we showed that HF-AHP-MULTIMOORA method can be successfully used to select building energy systems by evaluating technical and nontechnical selection criteria.

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Data Availability

All the data needed are included in the manuscript.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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