Simple outage probability bound for two-way relay networks with joint antenna and relay selection over Nakagami-*m* fading channels

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The performance of a multiple-input multiple-output amplify-and-forward two-way relay network with joint antenna and relay selection is analysed over Nakagami-*m* fading channels. Both approximate and asymptotic system outage probability expressions are derived, and diversity and coding gains for an arbitrary number of antennas, relays and fading severity are presented. Finally, the analytical findings are verified by numerical examples.

Introduction: In two-way relay networks (TWRNs), two terminals can concurrently transmit their messages to a relay in the first time slot and then the relay broadcasts the processed total signal in the second time slot, so that each terminal can obtain the transmitted message by subtracting its own message from the total signal. This technique has become popular since it can be a desirable solution for the loss of spectral efficiency occurring in one-way cooperative networks [1]. In an attempt to improve the advantages of TWRNs, multiple antennas and relays have been studied recently to explore enhanced performance. For example, in [2], Guo and Ge propose an amplify-and-forward (AF) TWRN with relay selection and derive the outage expression in Nakagami-*m* fading channels. In [3], two new joint transmit–receive antenna and relay selection strategies are proposed and outage probability (OP) is analysed for Rayleigh fading channels. Yang *et al.* [4] consider a single-relay multi-antenna TWRN with transmit–receive antenna selection, in which closed form and approximate system OPs for Nakagami-*m* fading channels are obtained. In general, exact system OP expressions are difficult to obtain, thus most papers have resorted to approximations [4, 5]. In [5], an opportunistic relay selection is studied for Rayleigh fading channels where approximate system OPs are derived by simplifying the overall CDF expression. We note that the outage expressions in recent TWRN studies (e.g. [3, 4]) are quite complicated, in general, which makes it difficult to gain insights about the system's behaviour.

In this Letter, we consider an AF MIMO TWRN with joint antenna and relay selection, and propose new simple upper bounds on e2e signal-to-noise ratios (SNRs). We then derive approximate and asymptotic system OPs for Nakagami-*m* fading channels, and obtain diversity and coding gains.

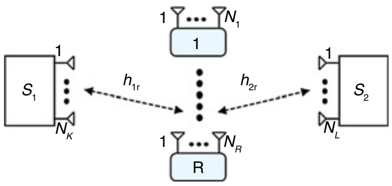

Fig. 1 *MIMO AF TWRN with multiple antennas and relays*

System model: We consider an AF MIMO TWRN consisting of two source terminals having N_K and N_L antennas communicating via R-relays having N_r antennas $\{r=1, \ldots, R\}$. The system block diagram is shown in Fig. 1. The direct link between two source terminals are assumed to be unavailable, e.g. due to heavy shadowing. We assume all transmit–receive antenna pairs between $S_1 \rightarrow r$ and $S_2 \rightarrow r$ hops are modelled as independent and identically distributed Nakagami-*m* with fading severity parameters m_1 and m_2, respectively. The communication between two terminals takes place in two time slots. In the first time slot, both sources transmit their signals x_1 and x_2 concurrently through their selected kth and lth antennas. As we assume equal power at S_1, S_2 and r, i.e. $P_1 = P_2 = P_r = P$, the received signal at the selected rth relay and jth antenna (best pairs) can be written as

$$y_r = \sqrt{P}h_{1r}^{(k,j)}x_1 + \sqrt{P}h_{2r}^{(l,j)}x_2 + n_r$$

where $h_{1r}^{(k,j)}$, $h_{2r}^{(l,j)}$ are the selected channel coefficients between $S_1 \rightarrow r$ and $S_2 \rightarrow r$ paths, respectively. n_r is the complex additive white Gaussian noise (AWGN) with zero mean and N_0 variance. Note that antennas and relays are selected to minimise the system OP which can be achieved by maximising the e2e SNR of the weakest source. In the second time slot, the rth relay amplifies the received signal with gain G_r and forwards to both source terminals. As $S \rightarrow r$ and $r \rightarrow S$ paths are assumed to be reciprocal in general TWRNs, the same antennas can be used. Hence, the received signal at S_1 and S_2 can be expressed as

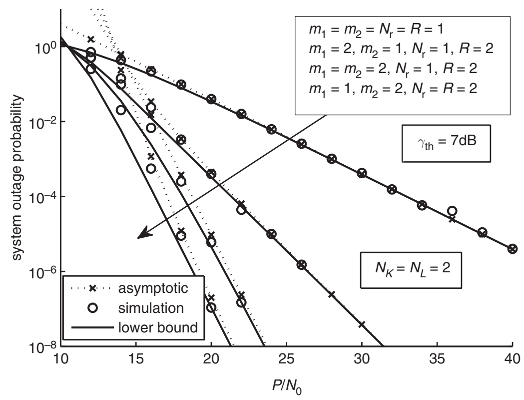

Fig. 2 *OP performance of MIMO AAF TWRN for different channel, antenna and relay configurations*

$$\begin{aligned} y_{S_1} &= \sqrt{P}G_r h_{1r}^{(k,j)} y_r + n_1 \\ y_{S_2} &= \sqrt{P}G_r h_{2r}^{(l,j)} y_r + n_2 \end{aligned} \tag{1}$$

where n_1, n_2 are the AWGN noises at S_1 and S_2 with zero mean and N_0 noise power. The amplifying gain is given as

$$G_r = \frac{1}{\sqrt{P|h_{1r}^{(k,j)}|^2 + P|h_{2r}^{(l,j)}|^2}} \tag{2}$$

Substituting (2) in (1) and after removing the self-interference term, the e2e SNR for both terminals can be written as follows:

$$\begin{aligned} \gamma_{S_1\rightarrow r\rightarrow S_2}^{(k,l,j)} &= \frac{(P/N_0)|h_{1r}^{(k,j)}|^2(P/N_0)|h_{2r}^{(l,j)}|^2}{2(P/N_0)|h_{1r}^{(k,j)}|^2 + (P/N_0)|h_{2r}^{(l,j)}|^2} = \frac{\gamma_{S_1}^{(k,j)}\gamma_{S_2}^{(l,j)}}{2\gamma_{S_1}^{(k,j)} + \gamma_{S_2}^{(l,j)}} \\ \gamma_{S_2\rightarrow r\rightarrow S_1}^{(k,l,j)} &= \frac{(P/N_0)|h_{1r}^{(k,j)}|^2(P/N_0)|h_{2r}^{(l,j)}|^2}{(P/N_0)|h_{1r}^{(k,j)}|^2 + 2(P/N_0)|h_{2r}^{(l,j)}|^2} = \frac{\gamma_{S_1}^{(k,j)}\gamma_{S_2}^{(l,j)}}{\gamma_{S_1}^{(k,j)} + 2\gamma_{S_2}^{(l,j)}} \end{aligned} \tag{3}$$

where

$$\gamma_{S_1}^{(k,j)} = \frac{P}{N_0}|h_{1r}^{(k,j)}|^2 \text{ and } \gamma_{S_2}^{(l,j)} = \frac{P}{N_0}|h_{2r}^{(l,j)}|^2$$

System OP: In TWRNs, system OP can be defined as the weakest e2e SNR falling below a certain threshold (γ_{th}), i.e. the $S_1 \rightarrow r \rightarrow S_2$ or $S_2 \rightarrow r \rightarrow S_1$ path is in outage. Mathematically, it can be expressed as follows:

$$P_{\text{out}} = \Pr\left[\max_{\substack{1\le k\le N_K,\, 1\le l\le N_L,\\ 1\le j\le N_r,\, 1\le r\le R}} \min\left(\gamma_{S_1\rightarrow r\rightarrow S_2}^{(k,l,j)},\ \gamma_{S_2\rightarrow r\rightarrow S_1}^{(k,l,j)}\right) \le \gamma_{\text{th}}\right] \tag{4}$$

where $\Pr[\cdot]$ denotes the probability of an event. It can be seen from [3, 4] that the analysis of (4) is difficult especially for MIMO TWRNs with antenna/relay selection in Nakagami-*m* fading channels. With the motivation of simplifying the analytical complexity and obtaining a simple outage expression, we start with a well-known inequality which is valid for AF-based relay networks

$$(\gamma_{S_1}^{(k,j)}\gamma_{S_2}^{(l,j)})/(\gamma_{S_1}^{(k,j)} + 2\gamma_{S_2}^{(l,j)}) \le \min\left(\frac{\gamma_{S_1}^{(k,j)}}{2},\ \gamma_{S_2}^{(l,j)}\right)$$

Using Monte Carlo simulations, we observe that

$$\frac{\gamma_{S_1}^{(k,j)}}{2} = \min\left(\frac{\gamma_{S_1}^{(k,j)}}{2},\ \gamma_{S_2}^{(l,j)}\right)$$

for $\sim$67% of the outcomes and

$$\frac{\gamma_{S_1}^{(k,j)}}{2} > \min\left(\frac{\gamma_{S_1}^{(k,j)}}{2},\ \gamma_{S_2}^{(l,j)}\right)$$

for $\sim$33% of the outcomes. Therefore, e2e SNRs can be approximately written as $\gamma_{S_1\to r\to S_2} \leq \gamma_{S_2}/2$ and $\gamma_{S_2\to r\to S_1} \leq \gamma_{S_1}/2$. Obviously this approximation simplifies the theoretical complexity in the derivation of the system OP in TWRNs and also performs quite well as can be seen in the 'Numerical examples' Sections below.

With the help of the above, (4) can be written as

$$P_{\text{out}} = \Pr\left[\max_{\substack{1\leq k\leq N_K,\, 1\leq l\leq N_L,\\ 1\leq j\leq N_r,\, 1\leq r\leq R}} \min\left(\frac{\gamma_{S_1}^{(k,j)}}{2},\ \frac{\gamma_{S_2}^{(l,j)}}{2}\right) \leq \gamma_{\text{th}}\right] \tag{5}$$

Using [6, eqn. 6] and after some manipulations, (5) can be expressed as

$$\begin{aligned} P_{\text{out}} &= \mathcal{F}_{\gamma}(\gamma_{\text{th}}) \\ \mathcal{F}_{\gamma}(\gamma) &= \prod_{r=1}^{R}\left[\mathcal{F}_{\gamma_{S_1}}(2\gamma) + \mathcal{F}_{\gamma_{S_2}}(2\gamma) - \mathcal{F}_{\gamma_{S_1}}(2\gamma)\mathcal{F}_{\gamma_{S_2}}(2\gamma)\right]^{N_r} \\ &\approx \prod_{r=1}^{R}\left[\mathcal{F}_{\gamma_{S_1}}(2\gamma) + \mathcal{F}_{\gamma_{S_2}}(2\gamma)\right]^{N_r} \end{aligned} \tag{6}$$

where $\mathcal{F}_{\gamma_{S_1}}(2\gamma) = \Pr[\gamma_{S_1} \leq 2\gamma]$ and $\mathcal{F}_{\gamma_{S_2}}(2\gamma) = \Pr[\gamma_{S_2} \leq 2\gamma]$. With the help of order statistics and [7, eqn. (8.352.6)], we can specify $\mathcal{F}_{\gamma_{S_i}}(2\gamma)$, $i = \{1, 2\}$ as follows:

$$\begin{aligned} \mathcal{F}_{\gamma_{S_i}}(2\gamma) &= \left(\frac{\Upsilon\left(m_i,\ 2m_i(\gamma/\Omega)\right)}{\Gamma(m_i)}\right)^{\mathcal{N}} \\ &= \left(1 - \mathrm{e}^{-2m_i(\gamma/\Omega_i)}\sum_{t=0}^{m_i-1}\left(2m_i\frac{\gamma}{\Omega}\right)^{t}\frac{1}{t!}\right)^{\mathcal{N}} \end{aligned} \tag{7}$$

where $\Upsilon(\cdot,\ \cdot)$ denotes the lower incomplete Gamma function [7, eqn. (8.350.1)] and $\Gamma(\cdot)$ stands for the Gamma function [7, eqn. (8.339.1)]. We denote $\Omega = P/N_0$ as the average SNR and $\mathcal{N} \in \{N_K,\ N_L\}$. By applying binomial [7, eqn. (1.111)] and multinomial expansions [7, eqn. (0.314)] $\mathcal{F}_{\gamma_{S_i}}(2\gamma)$ can be expressed as

$$\mathcal{F}_{\gamma_{S_i}}(2\gamma) = \sum_{a=0}^{\mathcal{N}}\sum_{t=0}^{a(m_i-1)}\binom{\mathcal{N}}{a}(-1)^{a}\gamma^{t}\,\mathrm{e}^{-2m_i a(\gamma/\Omega)}\mathcal{X}_t(a) \tag{8}$$

where the combination operation gives binomial coefficients and $\mathcal{X}_t(a)$ stands for multinomial coefficients which is written as

$$\chi_t(a) = \frac{1}{tz_0}\sum_{\rho=1}^{t}(a\rho - t + \rho)z_\rho\chi_{t-\rho}(a), \quad t \geq 1 \tag{9}$$

where $z_\rho = (2m_i(\gamma/\Omega))^{\rho}(1/\rho!)$ and $\mathcal{X}_0(a) = z_0^a = 1$. By substituting (8) in (6), the system OP can be obtained as

$$\begin{aligned} P_{\text{out}} = \prod_{r=1}^{R}\Bigg[&\sum_{a=0}^{N_K}\sum_{t=0}^{a(m_1-1)}\binom{N_K}{a}(-1)^{a}\gamma_{\text{th}}^{t}\,\mathrm{e}^{-2m_1 a(\gamma/\Omega)}\mathcal{X}_t(a) \\ &+ \sum_{a=0}^{N_L}\sum_{t=0}^{a(m_2-1)}\binom{N_L}{a}(-1)^{a}\gamma_{\text{th}}^{t}\,\mathrm{e}^{-2m_2 a(\gamma/\Omega)}\mathcal{X}_t(a)\Bigg]^{N_r} \end{aligned} \tag{10}$$

If both hops are balanced, i.e. $N_K = N_L = N_T$ and $m_1 = m_2 = m$, (10) can be written as

$$P_{\text{out}} = \prod_{r=1}^{R}\left[2\sum_{a=0}^{N_T}\sum_{t=0}^{a(m-1)}\binom{N_T}{a}(-1)^{a}\gamma_{\text{th}}^{t}\,\mathrm{e}^{-2ma(\gamma/\Omega)}\mathcal{X}_t(a)\right]^{N_r} \tag{11}$$

Diversity order and coding gain: Here we provide diversity ($\mathcal{G}_{\text{d}}$) and coding gains ($\mathcal{G}_{\text{c}}$) by deriving the system OP asymptotically as described in [8]. At high SNR, when $\Omega \to \infty$, the lower incomplete Gamma function can be asymptotically written as $\Upsilon(k,\ v \to 0) \to v^k/k$. Therefore, the asymptotic system OP can be expressed as

$$P_{\text{out}}^{\infty} = \prod_{r=1}^{R}\left(\left(\frac{(2m_1\gamma_{\text{th}})^{m_1}}{\Gamma(m_1+1)\Omega^{m_1}}\right)^{N_K} + \left(\frac{(2m_2\gamma_{\text{th}})^{m_2}}{\Gamma(m_2+1)\Omega^{m_2}}\right)^{N_L}\right)^{N_r} \tag{12}$$

Using [8, Prop. 5], P_{out}^{∞} can be obtained as

$$P_{\text{out}}^{\infty} = \prod_{r=1}^{R}(\mathcal{K})^{N_r}\left(\frac{\gamma_{\text{th}}}{\Omega}\right)^{\sum_{r=1}^{R}(N_r)\times\min(m_1N_K,\, m_2N_L)} + \text{H.O.T.} \tag{13}$$

where H.O.T. denotes high-order terms and $\mathcal{K}$ is given as

$$\mathcal{K} = \begin{cases} \left(\dfrac{(2m_1)^{m_1}}{\Gamma(m_1+1)}\right)^{N_K}, & m_1N_K < m_2N_L \\ \left(\dfrac{(2m_1)^{m_1}}{\Gamma(m_1+1)}\right)^{N_K} + \left(\dfrac{(2m_2)^{m_2}}{\Gamma(m_2+1)}\right)^{N_L}, & m_1N_K = m_2N_L \\ \left(\dfrac{(2m_2)^{m_2}}{\Gamma(m_2+1)}\right)^{N_L}, & m_1N_K > m_2N_L \end{cases} \tag{14}$$

As $P_{\text{out}} \approx (\mathcal{G}_{\text{c}}\Omega)^{-\mathcal{G}_{\text{d}}}$, the diversity order becomes $\mathcal{G}_{\text{d}} = \sum_{r=1}^{R}(N_r \times \min(m_1N_K,\ m_2N_L))$ and the coding gain is $\mathcal{G}_{\text{c}} = (\mathcal{K}\gamma_{\text{th}})^{-1/\mathcal{G}_{\text{d}}}$.

Numerical examples: Fig. 2 depicts the system OP against P/N_0 for various system parameters. As can be seen, the proposed lower bound matches almost perfectly with the simulations especially at the medium-to-high SNRs for all cases. In addition, the slopes of the curves 2, 4, 8 and 8 conform with the derived diversity orders. From Fig. 2, it is obvious that R and N_r improve outage performance much more than N_K, N_L or severity parameters.

Conclusion: In this Letter, the system OP of an AF MIMO TWRN with joint antenna and relay selection for i.n.i.d Nakagami-*m* fading channels is presented. Approximate and asymptotic outage expressions are obtained by simplifying e2e SNRs. Compared to previous studies, the derived outage expression is simpler and can be useful in the design of practical networks, for example, in wireless mesh or sensor networks. The system designer can obtain a quick idea about the performance without the need for simulations or prototyping.

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One or more of the Figures in this Letter are available in colour online.

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