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Analytic solution of the Feldtkeller equation

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Abstract

In every reflectance-based application like broadband matching, circuit modeling, etc., a nonlinear equation following from energy conservation, the Feldtkeller equation, must be solved, in order to obtain real networks. In the literature, however, there is no analytic solution available but only numerical solutions. Consequently, the resulting error depends on the accuracy of the numerical tools. In this paper, an analytic solution is proposed, which is based on the modified *ABCD*-parameters of a lossless reciprocal two-port network. An algorithm is presented and examples are included to illustrate the implementation of the analytical method.

Keywords: Feldtkeller equation; Lossless circuits; Two-port circuits; Reciprocal circuits

1. Introduction

A significant and practical simplification of the characterization of lossless two-port networks was achieved by *Belevitch* showed that the scattering coefficients can be expressed using only three polynomials $\{g,\ h$ and $f\}$ and a unimodular constant $\{\mu = \pm 1\}$ [1]. These polynomials are related by $gg_* = hh_* + ff_*$, an equation known as Feldtkeller equation (see also Eq. (4) below), where "*" denotes para-conjugation (sometimes also termed *Hurwitz* conjugation).

In every passive lossless two-port network design approach based on scattering coefficients expressed in Belevitch form, the Feldtkeller equation must be satisfied. For example, in the design process of a broadband matching network based on the simplified real frequency technique [2], the polynomial f is constructed from the transmission zeros, h is selected as optimization parameter and g, eventually, is formed by using the left half-plane (LHP) roots of gg_*. Also in reflectance-based modeling approaches [3–7], the Feldtkeller equation is used to obtain g from LHP roots.

In all these techniques, a numerical root-search algorithm is necessary. So the accuracy of the synthesized polynomial g depends on the accuracy of the search algorithm. In literature, there is no analytic solution of the Feldtkeller equation. In this paper, an analytic method based on modified *ABCD*-parameters of the passive lossless two-port is presented to solve the equation.

2. Formulation of the problem

Let us first briefly describe *ABCD*- and *S*-parameters of a two-port network. The *ABCD*-matrix is defined in terms of the total voltages V_i and currents I_i at port i, for a two-port network like the one depicted in Fig. 1 [8]:

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}. \tag{1}$$

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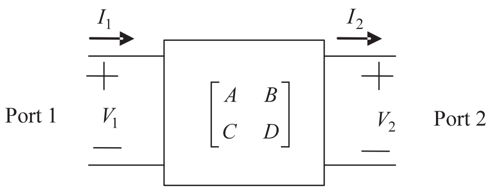

Fig. 1. A two-port network with *ABCD*-matrix.

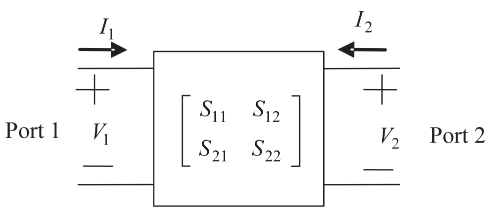

Fig. 2. A two-port network with scattering matrix.

If the two-port includes only lumped elements, the elements of the *ABCD*-matrix are rational functions in the complex frequency $p=\sigma+\mathrm{j}\omega$:

$$ABCD=\begin{bmatrix} A(p) & B(p) \\ C(p) & D(p) \end{bmatrix}. \tag{2}$$

The scattering parameters of a lossless two-port (c.f., Fig. 2) consisting of lumped elements only can be described by [9]

$$S(p)=\begin{bmatrix} S_{11}(p) & S_{12}(p) \\ S_{21}(p) & S_{22}(p) \end{bmatrix}=\frac{1}{g(p)}\begin{bmatrix} h(p) & \mu f(-p) \\ f(p) & -\mu h(-p) \end{bmatrix}. \tag{3}$$

Here $\mu=f(-p)/f(p)$ is a constant. If the two-port is reciprocal, the polynomial $f(p)$ must be either even or odd, so that $\mu=+1$ if $f(p)$ is even and $\mu=-1$ if $f(p)$ is odd. The functions $f(p)$, $g(p)$ and $h(p)$ are real polynomials with coefficients f_r, g_r and h_r for $r\geqslant 0$. Hence, if $f(p)$ is even or odd, all odd or even coefficients f_r vanish, respectively. The degrees of the three polynomials m_f, m_g, and m_h meet the requirement $m_h\leqslant m_g$ and $m_f\leqslant m_g$ [9]. The difference m_g-m_f defines the number of transmission zeros at infinity. The degree m_g of the polynomial $g(p)$ is referred to as the degree of the two-port; mathematically $g(p)$ is a strictly Hurwitz polynomial. The losslessness of the two-port leads to an important additional condition, which links the functions $f(p)$, $g(p)$ and $h(p)$: the Feldtkeller equation:

$$g(p)g(-p)=h(p)h(-p)+f(p)f(-p). \tag{4}$$

In every reflectance-based application, Eq. (4) must be solved in order to obtain real networks [2–4]. Hence, we can distinguish two cases. *Case* 1: If $f(p)$ and $h(p)$ are known, e.g., f is constructed from the transmission zeros of the two-port and h is selected as a free optimization parameter defined by the designer, $g(p)$ has to be constructed on the basis of Eq. (4). *Case* 2: If $f(p)$ and $g(p)$ are known, $h(p)$ has to be found. In the literature, only numerical approaches are available for case 1 [2–4]; for case 2, not even numerical solutions are found.

In this paper, we derive analytic solutions for both cases. The solution strategy is explained in the next section. Later on, an algorithm is presented and applied to selected examples to illustrate the implementation of the proposed method.

3. Analytic solution

In order to construct either $g(p)$ or $h(p)$ (Cases 1 or 2, respectively), Eq. (4) is converted into a set of nonlinear equations in terms of the *S*- and *ABCD*-parameters, which are subsequently solved.

Let us start by noting that the *ABCD*-parameters can be expressed by *S*-parameters and vice versa, as described in many textbooks (e.g., [8]). In detail, we find

$$\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}=\frac{1}{A+B+C+D}\cdot\begin{bmatrix} A+B-C-D & 2(AD-BC) \\ 2 & -A+B-C+D \end{bmatrix} \tag{5}$$

and

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}=\frac{1}{2S_{21}}\cdot\begin{bmatrix} (1+S_{11})(1-S_{22})+S_{12}S_{21} & (1+S_{11})(1+S_{22})-S_{12}S_{21} \\ (1-S_{11})(1-S_{22})-S_{12}S_{21} & (1-S_{11})(1+S_{22})-S_{12}S_{21} \end{bmatrix}. \tag{6}$$

Substituting Eq. (3) into Eq. (6), the *ABCD*-parameters can be expressed in terms of the polynomials f, g, and h:

$$A(p)=\frac{1}{2}\frac{g(p)g(p)+h(p)g(p)+\mu h(-p)g(p)}{g(p)f(p)}+\frac{1}{2}\frac{\mu h(p)h(-p)+\mu f(p)f(-p)}{g(p)f(p)}, \tag{7a}$$

$$B(p)=\frac{1}{2}\frac{g(p)g(p)+h(p)g(p)-\mu h(-p)g(p)}{g(p)f(p)}+\frac{1}{2}\frac{-\mu h(p)h(-p)-\mu f(p)f(-p)}{g(p)f(p)}, \tag{7b}$$

$$C(p)=\frac{1}{2}\frac{g(p)g(p)-h(p)g(p)+\mu h(-p)g(p)}{g(p)f(p)}+\frac{1}{2}\frac{-\mu h(p)h(-p)-\mu f(p)f(-p)}{g(p)f(p)}, \tag{7c}$$

Table 1. Definition of indices i and j in Eq. (12) for all possible cases

	$\mu=+1$ m_g even	$\mu=+1$ m_g odd	$\mu=-1$ m_g even	$\mu=-1$ m_g odd
$i=$	$0,2,\ldots,m_g$	$0,2,\ldots,m_g-1$	$1,3,\ldots,m_g-1$	$1,3,\ldots,m_g$
$j=$	$1,3,\ldots,m_g-1$	$1,3,\ldots,m_g$	$0,2,\ldots,m_g$	$0,2,\ldots,m_g-1$

$$D(p)=\frac{1}{2}\frac{g(p)g(p)-h(p)g(p)-\mu h(-p)g(p)}{g(p)f(p)}+\frac{1}{2}\frac{\mu h(p)h(-p)+\mu f(p)f(-p)}{g(p)f(p)}. \tag{7d}$$

Substituting Eq. (4) into Eq. (7), we arrive at the following equations:

$$A(p)=\frac{1}{2}\frac{g(p)+h(p)+\mu(h(-p)+g(-p))}{f(p)}, \tag{8a}$$

$$B(p)=\frac{1}{2}\frac{g(p)+h(p)-\mu(h(-p)+g(-p))}{f(p)}, \tag{8b}$$

$$C(p)=\frac{1}{2}\frac{g(p)-h(p)+\mu(h(-p)-g(-p))}{f(p)}, \tag{8c}$$

$$D(p)=\frac{1}{2}\frac{g(p)-h(p)-\mu(h(-p)-g(-p))}{f(p)}. \tag{8d}$$

At this point, if $ABCD$-matrix is multiplied by $f(p)/2$, a modified matrix, called $ABCD_m$-matrix with polynomial elements, is obtained,

$$ABCD_m=\frac{f(p)}{2}ABCD=\begin{bmatrix}A_m(p) & B_m(p)\\ C_m(p) & D_m(p)\end{bmatrix}. \tag{9}$$

Let the qth coefficients of $A_m(p)$, $B_m(p)$, $C_m(p)$ and $D_m(p)$ be designated as a_q, b_q, c_q and d_q, and let the degrees of the four polynomials be denoted by n_a, n_b, n_c and n_d, so that, for example,

$$A_m(p)=a_0+a_1p+a_2p^2+a_3p^3+\cdots+a_{n_a}p^{n_a} \tag{10}$$

and so on.

On the other hand, from Eqs. (8) and (9) and keeping in mind that $\mu=\pm 1$ is a unimodular constant, it can be concluded that if $\mu=+1$, the polynomials A_m and D_m are even, while B_m and C_m are odd. In the opposite case, $\mu=-1$, A_m and D_m are odd, while B_m and C_m are even. Also the following relations can be written from Eqs. (8) and (9):

$$g(p)=A_m(p)+B_m(p)+C_m(p)+D_m(p) \tag{11a}$$

and

$$h(p)=A_m(p)+B_m(p)-C_m(p)-D_m(p). \tag{11b}$$

Since the polynomials A_m, B_m, C_m and D_m are either even or odd, the coefficients of $g(p)$ and $h(p)$ can be expressed by the coefficients of $A_m(p)$, $B_m(p)$, $C_m(p)$, $D_m(p)$

$$g_i=a_i+d_i, \tag{12a}$$

$$g_j=b_j+c_j \tag{12b}$$

and

$$h_i=a_i-d_i, \tag{12c}$$

$$h_j=b_j-c_j, \tag{12d}$$

with the indices i and j as given in Table 1.

Eq. (12) defines the first part of the desired set of equations.

Now let us obtain the remaining equations. If g and h as given by Eq. (11) are substituted into Eq. (4), we arrive at

$$A_m(p)D_m(p)-B_m(p)C_m(p)=\frac{f(p)f(-p)}{4}. \tag{13}$$

Eq. (13) can be converted into corresponding equations expressed by the coefficients of A_m, B_m, C_m and D_m (see Section 2) and arranged according to the following cases:

- if $f(p)$ is an even polynomial and m_g is even,

$$\sum_{\substack{i=0,2,\ldots,m_g\\ j=0,2,\ldots,m_g\\ i+j=k}}\frac{f_if_j}{4}=\sum_{\substack{i=0,2,\ldots,m_g\\ j=0,2,\ldots,m_g\\ i+j=k}}a_id_j-\sum_{\substack{l=1,3,\ldots,m_g-1\\ m=1,3,\ldots,m_g-1\\ l+m=k}}b_lc_m, \tag{14a}$$

- if $f(p)$ is an even polynomial and m_g is odd,

$$\sum_{\substack{i=0,2,\ldots,m_g-1\\ j=0,2,\ldots,m_g-1\\ i+j=k}}\frac{f_if_j}{4}=\sum_{\substack{i=0,2,\ldots,m_g-1\\ j=0,2,\ldots,m_g-1\\ i+j=k}}a_id_j-\sum_{\substack{l=1,3,\ldots,m_g\\ m=1,3,\ldots,m_g\\ l+m=k}}b_lc_m, \tag{14b}$$

- if $f(p)$ is an odd polynomial and m_g is even,

$$-\sum_{\substack{l=1,3,\ldots,m_g-1\\ m=1,3,\ldots,m_g-1\\ l+m=k}} \frac{f_l f_m}{4} = \sum_{\substack{i=1,3,\ldots,m_g-1\\ j=1,3,\ldots,m_g-1\\ i+j=k}} a_i d_j - \sum_{\substack{l=0,2,\ldots,m_g\\ m=0,2,\ldots,m_g\\ l+m=k}} b_l c_m, \quad (14c)$$

- if $f(p)$ is an odd polynomial and m_g is odd,

$$-\sum_{\substack{l=1,3,\ldots,m_g\\ m=1,3,\ldots,m_g\\ l+m=k}} \frac{f_l f_m}{4} = \sum_{\substack{i=1,3,\ldots,m_g\\ j=1,3,\ldots,m_g\\ i+j=k}} a_i d_j - \sum_{\substack{l=0,2,\ldots,m_g-1\\ m=0,2,\ldots,m_g-1\\ l+m=k}} b_l c_m, \quad (14d)$$

where $k = 0, 2, \ldots, 2m_g$. Eq. (14) completes the set of non-linear equations.

The solution follows from a sequence of two steps. First, the coefficients of A_m, B_m, C_m and D_m are derived for a given polynomial $f(p)$, according to Eq. (14). The polynomials $g(p)$ or $h(p)$ can then be constructed on the basis of Eq. (12). For case 1 [$f(p)$ and $h(p)$ are given], a unique Hurwitian solution for $g(p)$ is reached in the following manner: Since the equation set is nonlinear, there will be many solutions, so many $g(p)$. But either only one solution will be a Hurwitz polynomial or all the solutions are the same Hurwitz polynomial. Also we know that $g(p)$ must be a Hurwitz polynomial. As a result, the acceptable solution for $g(p)$ is always unique. For case 2 [$f(p)$ and $g(p)$ are given], all possible solutions for $h(p)$ are found. In case 2, there may be more than one solution; each solution corresponds to different networks with the same polynomials $f(p)$ and $g(p)$.

So far, we assumed that the two-port consisted of lumped elements only. If the two-port contains only distributed-elements instead, the formulation remains valid still. In this case, the complex frequency p has to be replaced by the so-called Richards variable λ, where $\lambda = \Sigma + \mathrm{j}\,\Omega$ is associated with the equal-length transmission lines or the so-called commensurate transmission lines [10].

Fig. 3 summarizes the algorithm for the construction of an analytic solution of the Feldtkeller equation.

After synthesis, normalized element values are obtained. Actual values can be calculated by de-normalization. In this case, they are given by actual capacitance = normalized capacitance$/2\pi f_{\mathrm{N}} R_{\mathrm{N}}$, actual inductance = normalized inductance. $R_{\mathrm{N}}/2\pi f_{\mathrm{N}}$, actual impedance = normalized impedance. R_{N}, where f_{N} and R_{N} are normalization frequency and resistance, respectively.

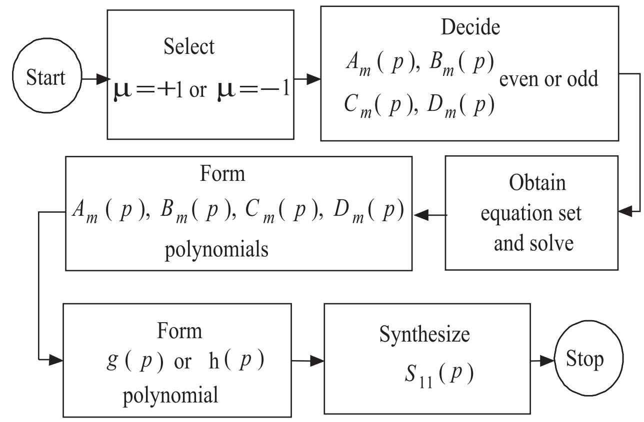

Fig. 3. Flowchart of the algorithm to solve Eqs. (12) and (14).

4. Examples

In this section, two examples are presented to illustrate the implementation of the proposed method.

4.1. Example for case 1

Let the given polynomials be $f(p) = 6p$ and $h(p) = 120p^4 + 36p^3 + 29p^2 - 4p + 1$. In this case, a unique polynomial $g(p)$ must be determined. Since $f(p)$ is odd, $\mu = -1$. Therefore, $A_m(p)$ and $D_m(p)$ are odd, and $B_m(p)$ and $C_m(p)$ are even:

$$ABCD_m = \begin{bmatrix} A_m(p) & B_m(p) \\ C_m(p) & D_m(p) \end{bmatrix} = \begin{bmatrix} a_3p^3 + a_1p & b_4p^4 + b_2p^2 + b_0 \\ c_4p^4 + c_2p^2 + c_0 & d_3p^3 + d_1p \end{bmatrix}.$$

From Eqs. (12) and (14) follows that $h_4 = 120 = b_4 - c_4$, $h_3 = 36 = a_3 - d_3$, $h_2 = 29 = b_2 - c_2$, $h_1 = -4 = a_1 - d_1$, $h_0 = 1 = b_0 - c_0$, $-b_4c_4 = 0$, $a_3d_3 - b_4c_2 - b_2c_4 = 0$, $a_1d_3 + a_3d_1 - b_2c_2 - b_4c_0 - b_0c_4 = 0$, $a_1d_1 - b_2c_0 - b_0c_2 = -9$ and $-b_0c_0 = 0$. After solving this set of equations, eighth possible formulations for the $ABCD_m$-parameters, are obtained:

- Solution 1

$$ABCD_m = \begin{bmatrix} -3.05135p^3 - 1.01777p & -2.21354p^2 \\ 120p^4 + 55.2135p^2 + 1 & 87.0514p^3 + 11.0178p \end{bmatrix}.$$

- Solution 2

$$ABCD_m = \begin{bmatrix} 87.0514p^3 + 11.0178p & -2.21354p^2 \\ 120p^4 + 55.2135p^2 + 1 & -3.05135p^3 - 1.01777p \end{bmatrix}.$$

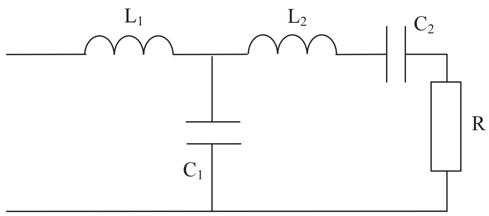

Fig. 4. Obtained network, $L_1=5$, $L_2=2$, $C_1=4$, $C_2=3$, $R=1$, (normalized values).

- Solution 3

$$ABCD_m = \begin{bmatrix} 54.3081p^3+0.465422p & 13.4376p^2 \\ 120p^4+39.5624p^2+1 & 29.6919p^3+9.53458p \end{bmatrix}.$$

- Solution 4

$$ABCD_m = \begin{bmatrix} 29.6919p^3+9.53458p & 13.4376p^2 \\ 120p^4+39.5624p^2+1 & 54.3081p^3+0.465422p \end{bmatrix}.$$

- Solution 5

$$ABCD_m = \begin{bmatrix} -3.05135p^3-1.01777p & 120p^4+55.2135p^2+1 \\ -2.21354p^2 & 87.0514p^3+11.0178p \end{bmatrix}.$$

- Solution 6

$$ABCD_m = \begin{bmatrix} 87.0514p^3+11.0178p & 120p^4+55.2135p^2+1 \\ -2.21354p^2 & -3.05135p^3-1.01777p \end{bmatrix}.$$

- Solution 7

$$ABCD_m = \begin{bmatrix} 54.3081p^3+0.465422p & 120p^4+39.5624p^2+1 \\ 13.4376p^2 & 29.6919p^3+9.53458p \end{bmatrix}.$$

- Solution 8

$$ABCD_m = \begin{bmatrix} 29.6919p^3+9.53458p & 120p^4+39.5624p^2+1 \\ 13.4376p^2 & 54.3081p^3+0.465422p \end{bmatrix}.$$

These $ABCD_m$-matrices are substituted in Eq. (11a), leading to the unique Hurwitian solution for the polynomial $g(p)$, which satisfies Eq. (4): $g(p)=120p^4+84p^3+53p^2+10p+1$.

After synthesizing $S_{11}(p)=h(p)/g(p)$, the two-port network depicted in Fig. 4 is obtained.

If the same example is solved numerically, the following computations must be made:

$hh_* + ff_* = (14400p^8+5664p^6+1369p^4+42p^2+1)+(-36p^2)=gg_*$. Next, the roots of the polynomial gg_* are $-0.2348\pm0.5011i$, $0.2348\pm0.5011i$, $-0.1152\pm0.1181i$, $0.1152\pm0.1181i$ (here, i is the imaginary unit). By choosing the LHP roots, we arrive at the same $g(p)$ that resulted from the analytical solution.

4.2. Example for case 2

Let the given polynomials be $f(p)=1$ and $g(p)=3p^2+2.5p+1$. In this case, there may be more than one solution, but all of them must describe different synthesizable networks. Since $f(p)$ is even, $\mu=+1$. Therefore, $A_m(p)$ and $D_m(p)$ are even and $B_m(p)$ and $C_m(p)$ are odd:

$$ABCD_m = \begin{bmatrix} A_m(p) & B_m(p) \\ C_m(p) & D_m(p) \end{bmatrix} = \begin{bmatrix} a_2p^2+a_0 & b_1p \\ c_1p & d_2p^2+d_0 \end{bmatrix}.$$

From Eqs. (12) and (14) follows that $g_2=3=a_2+d_2$, $g_1=2.5=b_1+c_1$, $g_0=1=a_0+d_0$, $a_2d_2=0$, $a_2d_0+a_0d_2-b_1c_1=0$, $a_0d_0=0.25$. After solving this set of equations, four different $ABCD_m$-parameters and $h(p)$ polynomials are obtained, all of which satisfy Eq. (4):

- Solution 1

$$\begin{bmatrix} A_m(p) & B_m(p) \\ C_m(p) & D_m(p) \end{bmatrix} = \begin{bmatrix} 3p^2+0.5 & p \\ 1.5p & 0.5 \end{bmatrix} \Rightarrow$$

$$h(p)=A_m(p)+B_m(p)-C_m(p)-D_m(p) = 3p^2-0.5p.$$

- Solution 2

$$\begin{bmatrix} A_m(p) & B_m(p) \\ C_m(p) & D_m(p) \end{bmatrix} = \begin{bmatrix} 3p^2+0.5 & 1.5p \\ p & 0.5 \end{bmatrix} \Rightarrow$$

$$h(p)=A_m(p)+B_m(p)-C_m(p)-D_m(p) = 3p^2+0.5p.$$

- Solution 3

$$\begin{bmatrix} A_m(p) & B_m(p) \\ C_m(p) & D_m(p) \end{bmatrix} = \begin{bmatrix} 0.5 & p \\ 1.5p & 3p^2+0.5 \end{bmatrix} \Rightarrow$$

$$h(p)=A_m(p)+B_m(p)-C_m(p)-D_m(p) = -3p^2-0.5p.$$

- Solution 4

$$\begin{bmatrix} A_m(p) & B_m(p) \\ C_m(p) & D_m(p) \end{bmatrix} = \begin{bmatrix} 0.5 & 1.5p \\ p & 3p^2+0.5 \end{bmatrix} \Rightarrow$$

$$h(p)=A_m(p)+B_m(p)-C_m(p)-D_m(p) = -3p^2+0.5p.$$

After synthesizing $S_{11}(p)=h(p)/g(p)$, the two-port networks displayed in Fig. 5 are obtained.

Let us try to solve the same example via Eq. (4) numerically:

$gg_* - ff_* = (9p^4-0.25p^2+1)-(1)=hh_*$. Next, the roots of the polynomial hh_* are 0, 0, 0.1667, −0.1667. At this

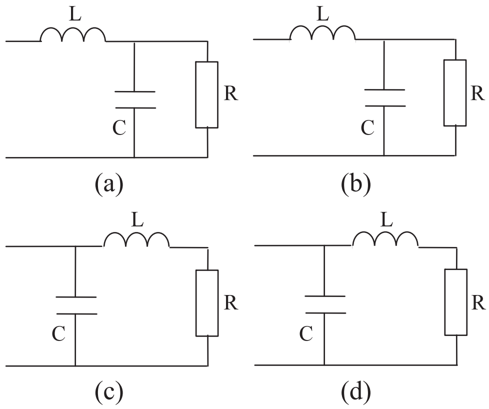

Fig. 5. Four possible networks: (a) $L=2$, $C=3$, $R=1$, (b) $L=3$, $C=2$, $R=1$, (c) $C=3$, $L=2$, $R=1$, (d) $C=2$, $L=3$, $R=1$ (normalized values).

step, there is no root-choice procedure. But it can be seen that there are three possibilities, since the problem is simple. Two of them give the networks a and b found analytically. But the last one is completely different. If this solution and f are used to obtain the given polynomial g, it is seen that the same polynomial cannot be obtained. Also, it is impossible to obtain the other possible solutions, the networks c and d. So there is no well-defined numerical method to solve the second case.

5. Conclusions

An analytic procedure was derived to solve the nonlinear Feldtkeller equation. Two cases were defined. Case 1: If $f(p)$ and $h(p)$ were known, $g(p)$ had to be constructed on the basis of the Feldtkeller equation. Case 2: If $f(p)$ and $g(p)$ were known, $h(p)$ had to be formed. In the literature, only numerical approaches are available for case 1 with the accuracy of the used numerical tools; for case 2, not even numerical solutions are found. We have shown that both cases can be solved analytically without numerical error. As a result, an analytic solution method is presented, which is very simple to implement in a large variety of network design problems.

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