Performance of Transmit and Receive Antenna Selection in the Presence of Channel Estimation Errors

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***Abstract*— This letter considers the effect of channel estimation errors on the performance of space-time coded (STC) systems with transmit and receive antenna selection over quasi-static flat fading channels. By performing pairwise error probability analysis and presenting numerical examples, we show that the diversity order achieved with perfect channel state information (CSI) is still achievable with imperfect CSI used both at the antenna selection and the space-time decoding processes. We note that our results apply to general STC systems with both transmit and/or receive antenna selection based on largest received powers which can be estimated by any channel estimator.**

***Index Terms*— Antenna selection, channel estimation errors, diversity, multiple antenna, space time coding.**

I. Introduction

ANTENNA selection [1] can be an effective technique to reduce the cost and complexity of space time coded (STC) [2] systems. With this technique, only a few of all transmit and/or receive antennas can be selected and switched to a reduced number of RF chains. There has been considerable research on the performance of STC systems with antenna selection, mostly considering the selection only at the receiver [3] or only at the transmitter [4] with the assumption of perfect channel state information (CSI) available at the receiver. However, it is more practical to consider both transmit and receive antenna selection with imperfect CSI. In the literature, antenna selection with imperfect CSI is addressed by only a few papers [5], [6], [7] studying specific space-time coding schemes employing antenna selection usually only at the receiver or transmitter.

In this letter, we study the performance of general STCs with transmit and/or receive antenna selection based on the largest received powers. Specifically, we derive an upper bound on the pairwise error probability of STC with joint transmit and receive selection. The derived bound and the simulation results show that the diversity order of STC systems with antenna selection is not degraded if erroneous channel estimates are used at the selection and the decoding processes.

The rest of the letter is organized as follows: Section II describes the system model. The pairwise error probability bound for joint transmit and receive selection when the receiver has imperfect channel estimates is derived in Section III. Numerical examples are provided in Section IV followed by the conclusions in Section V.

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II. System Description

We consider a STC system having M transmit and N receive antennas while at each frame only L_T transmit and L_R receive antennas are used after selection based on maximum received powers. We assume that only the receiver has the estimated channel coefficients and it feeds back only the indices of the selected L_T transmit antennas. The channel is modeled as a quasi-static flat Rayleigh fading where the channels for different transmit and receive antenna pairs fade independently and remain constant over the entire transmitted frame of symbols.

The received signals $y_n(k)$ from the receive antenna n ($n = 1, \cdots, L_R$) at time k ($k = 1, \cdots, K$) can be stacked in $L_R \times K$ matrix $\mathbf{Y}$ which can be written as

$$\mathbf{Y} = \sqrt{\frac{\rho}{L_T}}\mathbf{HS} + \mathbf{W}, \quad (1)$$

where $\mathbf{S}$ is the $L_T \times K$ transmitted space-time codeword matrix with elements $s_m(k)$ (transmitted symbol from antenna m ($m = 1, \cdots, L_T$) at time k). The channel matrix $\mathbf{H}$ contains the $L_R \times L_T$ fading coefficients, $h_{m,n}$, and $\mathbf{W}$ is the $L_R \times K$ noise matrix with noise samples $w_n(k)$. $h_{m,n}$ and $w_n(k)$ are i.i.d. complex Gaussian random variables having zero mean and variance $1/2$ per dimension. ρ is the expected signal to noise ratio (SNR) at each receive antenna.

When CSI is perfectly known at the receiver, the pairwise error probability (PEP) conditioned on the instantaneous $\mathbf{H}$, can be written as [8],

$$P(\mathbf{S} \to \widehat{\mathbf{S}}|\mathbf{H}) \leq \exp\left(-\frac{\rho}{4L_T}\|\mathbf{HB}\|^2\right), \quad (2)$$

where $\mathbf{B} = \mathbf{S} - \widehat{\mathbf{S}}$ is the codeword difference matrix when $\widehat{\mathbf{S}}$ is the $L_T \times K$ decoded codeword matrix. $\|.\|^2$ represents the sum of magnitude squares of all entries of a matrix.

In practice, the channel estimator at the receivers provides fading coefficient estimates, $\hat{h}_{m,n}$, which can be modeled as $\hat{h}_{m,n} = h_{m,n} + \epsilon_{m,n}$, where $\epsilon_{m,n}$ is a complex Gaussian random variable representing the channel estimation error independent of $h_{m,n}$, having zero mean and variance σ_e^2 [9]. $\hat{h}_{m,n}$ is a complex Gaussian random variable with zero mean, variance σ^2 per dimension and dependent on $h_{m,n}$ with the correlation coefficient, $\mu = \frac{1}{\sqrt{1+\sigma_e^2}}$. In general, σ_e^2 can be estimated using the SNR, the number of pilots, and the method of estimation.

In the presence of channel estimation errors, as in [9], when $\mathbf{S}$ is transmitted, the conditional mean of the received signal (complex Gaussian random variable $y_n(k)$) can be written as

$$E\{y_n(k)|\hat{h}_{m,n}, s_m(k)\} = \frac{\mu}{\sqrt{2}\sigma}\sqrt{\frac{\rho}{L_T}}\sum_{m=1}^{L_T}\hat{h}_{m,n}s_m(k),$$

and the conditional variance is as follows

$$Var\{y_n(k)|\hat{h}_{m,n}, s_m(k)\} = 1 + (1-|\mu|^2)\frac{\rho}{L_T}\sum_{m=1}^{L_T}|s_m(k)|^2.$$

We note that the distance term can be written as

$$d^2(\mathbf{S},\widehat{\mathbf{S}}) = \sum_{n=1}^{L_R}\sum_{k=1}^{K}\left|\sum_{m=1}^{L_T}\frac{\hat{h}_{m,n}}{\sqrt{2}\sigma}s_m(k)\right|^2 = \frac{1}{2\sigma^2}\|\hat{\mathbf{H}}\mathbf{B}\|^2, \quad (3)$$

where the $L_R \times L_T$ matrix $\hat{\mathbf{H}}$ contains the estimated channel coefficients, $\hat{h}_{m,n}$. Then, the PEP bound conditioned on $\hat{\mathbf{H}}$ can be obtained as,

$$P(\mathbf{S}\to\widehat{\mathbf{S}}|\hat{\mathbf{H}}) \leq \exp\left(-\frac{\tilde{\rho}}{2\sigma^2}\|\hat{\mathbf{H}}\mathbf{B}\|^2\right), \quad (4)$$

where we define

$$\tilde{\rho} \equiv \frac{\mu^2\frac{\rho}{L_T}}{4 + 4L_T(1-|\mu|^2)\frac{\rho}{L_T}}, \quad (5)$$

which approaches ρ at high SNRs (thus high μ). The unconditional PEP upper bound for any antenna selection scheme can be obtained by averaging the above conditional PEP using the statistics of the selected channel coefficients.

III. Antenna Selection in the Presence of Channel Estimation Errors

In this section, we obtain a bound on the pairwise error probabilities of space time coded systems with joint transmit and receive antenna selection over quasi-static flat fading channels. We first study a simple case in which only one antenna is selected both at the transmitter ($L_T = 1$) and the receiver ($L_R = 1$) by finding the largest channel coefficient $\hat{h}$ with the largest norm. Then, the size of the matrices $\mathbf{S}, \widehat{\mathbf{S}}, \mathbf{B}$ will be $1 \times K$, and thus, the upper bound on the conditional PEP can be written as

$$P(\mathbf{S}\to\widehat{\mathbf{S}}) \leq \int_{C^1}\exp\left(-\frac{\tilde{\rho_1}}{4}\frac{1}{2\sigma^2}\|\hat{h}\mathbf{B}\|^2\right)f(\hat{h})d\hat{h}, \quad (6)$$

where we define $\tilde{\rho_1} \equiv \mu^2\rho/\left(1+(1-|\mu|^2)\rho\right)$ and note that as μ approaches 1, $\tilde{\rho_1}$ approaches ρ. The integration is taken over the 1-dimensional complex space, C^1, and $f(\hat{h})$ denotes the probability density function (pdf) of $\hat{h}$ which is a zero mean complex Gaussian random variable with variance $2\sigma^2$ and it can be written as

$$f(\hat{h}) = MN\left(1-e^{-\frac{|\hat{h}|^2}{2\sigma^2}}\right)^{(MN-1)}\frac{1}{2\sigma^2\pi}e^{-\frac{|\hat{h}|^2}{2\sigma^2}}. \quad (7)$$

Since $\mathbf{BB}^*$ is 1×1, we can simply write $\mathbf{BB}^* = \lambda$ and $\|\hat{h}\mathbf{B}\|^2 = \lambda|\hat{h}|^2$. Moreover, by using the following result (as in [8])

$$g(x) = 1 - e^{-x}\sum_{n=0}^{N-1}\frac{x^n}{n!} \leq \frac{x^N}{N!},$$

for $x > 0$, we can write

$$P(\mathbf{S}\to\widehat{\mathbf{S}}) \leq MN\int_{C^1}\exp\left(-\frac{\tilde{\rho_1}}{4}\lambda\frac{|\hat{h}|^2}{2\sigma^2}\right)\left(-\frac{|\hat{h}|^2}{2\sigma^2}\right)^{(MN-1)}\frac{1}{2\sigma^2\pi}e^{-\frac{|\hat{h}|^2}{2\sigma^2}}d\hat{h}. \quad (8)$$

With the change of variable $\frac{\hat{h}}{\sqrt{2\sigma^2}} = \beta e^{j\theta}$, and thus, $d\hat{h} = \sqrt{2\sigma^2}\beta d\beta d\theta$, the complex integration can be converted into double integral,

$$P(\mathbf{S}\to\hat{\mathbf{S}}) \leq MN\int_0^{2\pi}d\theta\int_0^{\infty}e^{-\frac{\tilde{\rho_1}}{4}\lambda\beta^2}\left(\beta^2\right)^{(MN-1)}e^{-\beta^2}\frac{\sqrt{2\sigma^2}}{2\sigma^2\pi}\beta d\beta. \quad (9)$$

For further simplification, we use $v = \beta^2$ and $dv = 2\beta d\beta$,

$$P(\mathbf{S}\to\hat{\mathbf{S}}) \leq \frac{MN}{\sqrt{2\sigma^2}}\int_0^{\infty}e^{-\left(\frac{\tilde{\rho_1}}{4}\lambda+1\right)v}v^{(MN-1)}dv. \quad (10)$$

Then, after solving the integral, the simplified PEP expression can be obtained as

$$P(\mathbf{S}\to\hat{\mathbf{S}}) \leq \frac{MN}{\sqrt{2\sigma^2}}\frac{(MN-1)!}{\left(\frac{\tilde{\rho_1}}{4}\lambda+1\right)^{MN}}. \quad (11)$$

In this final PEP expression, we observe that the diversity advantage of MN (the exponent of $\tilde{\rho_1}$ and ρ at high SNRs) still can be achieved even with the use of single transmit and single receive antenna, while the antenna selection and the space time decoding use imperfect channel state information.

We note that the simple derivation above can also be performed for selecting more than one antenna at the receiver and the transmitter. For example, similar to [8], when L_T transmit antennas are selected after the selection of a single receive antenna, the pdf of the selected L_T channel coefficients can be written as

$$f = \frac{N.M!}{(M-L_T)!L_T!}\sum_{p=1}^{L_T}\int_0^{v_p}\cdots\int_0^{v_p}\left(1-e^{-w}\sum_{m=0}^{2}\frac{w^m}{m!}\right)^{(N-1)}\frac{1}{(\pi 2\sigma^2)^{L_T}}e^{-w}I_{R_p}(v_1,\cdots,v_{L_T})\prod_{m=L_T+1}^{M}dv_m, \quad (12)$$

where $w = v_1 + v_2 + \ldots + v_M$ with $v_i = \frac{|c_i|^2}{2\sigma^2}$, $1 \leq i \leq M$ and $I_{R_p}(v_1,\cdots,v_{L_T})$ is the indicator function which is 1 if and only if v_p is the minimum of all v_i for $1 \leq i \leq L_T$, otherwise, $I_{R_p}(v_1,\cdots,v_{L_T}) = 0$. The PEP upper bound can be obtained by averaging conditional PEP in expression (4) over the above joint pdf. Following the similar steps as in [8], the PEP bound can be simplified as

$$P(\mathbf{S}\to\hat{\mathbf{S}}) \leq \tilde{\kappa}\left(\frac{1}{\hat{\lambda}^{MN}}\right)\left(\frac{\tilde{\rho}}{4L_T}\right)^{-MN}, \quad (13)$$

where $\tilde{\kappa}$ is a constant which depends on the available and selected number of antennas, and $\hat{\lambda}$ is the minimum of eigenvalues of the square of the codeword difference matrix, $\mathbf{BB}^*$. In this final PEP result, we observe that the diversity order (the exponent of $\tilde{\rho}$) is maintained (still MN) although imperfect channel estimates are used at the receiver. When perfect CSI is available ($\mu = 1$), we note that $\tilde{\rho} = \rho$, therefore,

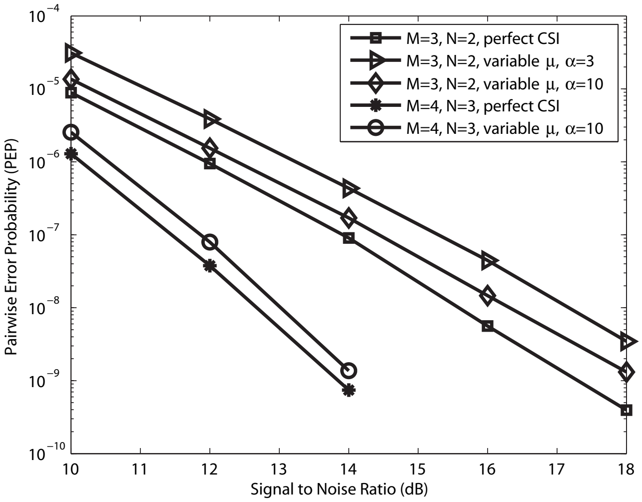

Fig. 1. PEP plots for STC from [10] with $L_T = 2, L_R = 1$, and channel estimation error variance depends on SNR (variable μ).

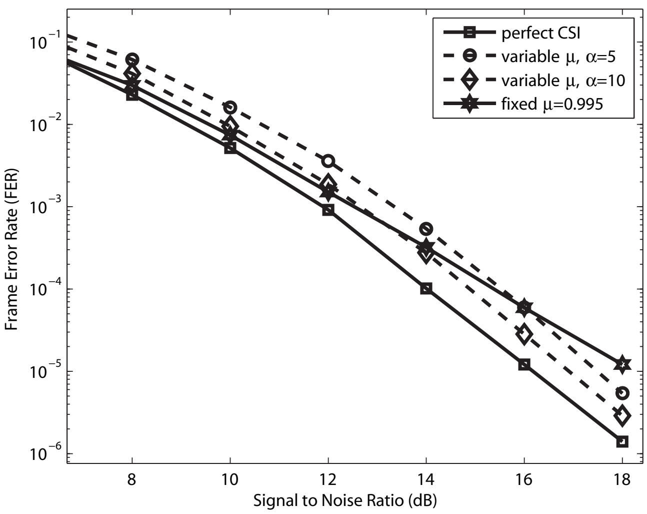

Fig. 2. FER plots of STC based on $(5, 7)_8$ convolutional coding with $M = 3, N = 2, L_T = 2, L_R = 1$, (considering both variable and fixed μ).

the effect of having imperfect channel estimates ($\mu < 1$) can be seen as reduction of SNR. Although not shown due to space limitations, similar PEP bounds for full rank or rank deficient space time coded systems employing only transmit [4] or only receive antenna selection [3] can be obtained.

IV. Simulation Results

In this section, the performance of STC systems with joint transmit and receive antenna selection using imperfect CSI is illustrated. We note that in the presence of channel estimation errors the decoding metric should be as described in [9] which is slightly different than the metric for perfect CSI scenario, however, we have observed that the performance difference is insignificant.

The PEP plots of the expression (4) and frame error rate (FER) plots of STC based on $(5, 7)_8$ convolutional coding with joint transmit/receive antenna selection $L_T = 2, L_R = 1$ are depicted in Figures 1 and 2, respectively. As in practical receivers, the channel estimation errors in these simulations are assumed to decrease with increasing SNR (variable μ), i.e., $\sigma_e^2 = 1/(\alpha.\rho)$, where the constant α depends on the number of pilots and estimation method. We observe that the simulated full rank codes can achieve full diversity even when imperfect CSI ($\mu < 1$) is used. Increasing α decreases error rates and the performance approaches to that of perfect CSI. FER plots with fixed correlation $\mu = 0.995$ ($\sigma_e^2 = 0.01$) shows the performance degradation at high SNRs. When the correlation μ is larger than 0.9995, the FER is almost the same as the FER with perfect CSI. When fixed μ is smaller than 0.995, the degradation becomes significant which suggests putting some restrictions on the mean square error performance of channel estimators to be used in these systems. Although not shown, similar performance curves are also obtained for rank deficient codes and for only transmit or only receive selection.

V. Conclusions

In this letter, the effect of imperfect channel estimates on the performance of STC systems with joint transmit and receive antenna selection is presented. Only the receiver is assumed to have the imperfect CSI and the antenna selection is based on maximum estimated received powers. The pairwise error probability analysis and the numerical examples have shown that the diversity order achievable with perfect CSI is not reduced when imperfect channel estimates are used in antenna selection and space time decoding. Therefore, we can claim that STC systems with antenna selection based on received powers are robust against channel estimation errors.

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