Design of Mixed-Element Networks via Modeling

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***Abstract*— A new method is introduced, to design mixed lumped and distributed element networks via modeling the data obtained from the driving point input reflectance of a lumped-element prototype. A mixed-element Chebyshev filter design is presented, to exhibit the utilization of the new method. It is expected that the new method will be employed, to design wideband communication networks manufactured using VLSI technology.**

I. INTRODUCTION

For many communications engineering applications, lumped-element networks are preferred up to X-band due to their compact sizes. However, in VLSI manufacturing process, interconnections of lumped circuit elements introduce metallic roads which may be considered as transmission lines. These undesirable connections destroy the idealized performance of the lumped-element network prototype. In this case, it would be wise, to use these connections as part of the design. Thus, designs with mixed lumped and distributed elements become inevitable. The common practice in designing mixed-element networks is to select the circuit topology. In this topology, lumped-element interconnections are regarded as idealized transmission lines, perhaps with fixed or variable lengths. Then, values of the lumped-elements (X_i) and characteristic impedances (Z_i) of the idealized transmission lines are determined by means of the optimization of the gain performance of the network. Although this approach seems straightforward, it presents serious difficulties. First, the optimization is heavily nonlinear in terms of X_i and Z_i that may result in local minima or may not converge at all. Secondly, there is no established process, to initialize the element values of the chosen circuit topology. Worst of all *the optimum choice of the circuit topology which best describes the filter network is in question*. Fortunately, these problems are overcome employing the design technique introduced in this paper. The new network design technique includes two major phases. In Phase I, lumped-element network prototype is constructed employing the well-established network design methods. Then, input reflection coefficient of the prototype is evaluated point by point over the passband. In Phase II, data generated from the input reflection coefficient is modeled using the analytic form of the input reflection coefficient described in two complex variables, which in turn results in the desired lossless network in two kinds of elements, namely lumped and distributed elements or so called commensurate transmission lines. In practice, commensurate transmission lines or equal length lines are used to connect lumped elements of the network.

In the following sections, first the analytic aspects of data modeling method in two variables are introduced. Then, the modeling algorithm is presented. Finally, a Chebyshev filter is built with mixed lumped and distributed elements, to exhibit the utilization of the proposed method. It is noted that the method introduced in this paper is the extension of the modeling technique proposed for one kind of elements [1-3].

II. THEORETICAL ASPECTS

In this paper, the modeling problem is defined as the generation of a realizable, two-variable bounded real (BR) reflectance function that best fits the given data. Eventually, this BR reflectance describes the lossless network in two kinds of elements in resistive termination which is called the Darlington representation of the input reflection function (Fig. 1).

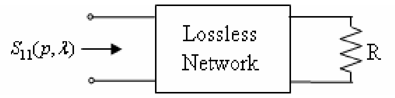

Fig. 1. Darlington representation of the modeled input reflectance function $S_{11}(p,\lambda)$.

Let $S(j\omega_i) = S_R(\omega_i) + jS_X(\omega_i)$ designate the given data obtained form the input reflectance of the lumped-element network prototype over the angular frequencies ω_i. Let $\{S_{kl}; k,l = 1,2\}$ designate the scattering parameters of the lossless network which consist of two kinds of elements. For a mixed lumped and distributed element, reciprocal, lossless two-port, the scattering parameters may be expressed in Belevitch form as follows [4-7]

$$S(p,\lambda)=\begin{bmatrix} S_{11}(p,\lambda) & S_{12}(p,\lambda) \\ S_{21}(p,\lambda) & S_{22}(p,\lambda) \end{bmatrix} = \frac{1}{g(p,\lambda)}\begin{bmatrix} h(p,\lambda) & \mu f(-p,-\lambda) \\ f(p,\lambda) & -\mu h(-p,-\lambda) \end{bmatrix} \quad (1a)$$

where $\mu = f(-p,-\lambda)/f(p,\lambda)$.

In Eq. (1a), $p=\sigma+j\omega$ is the usual complex frequency variable associated with lumped-elements, and $\lambda=\Sigma+j\Omega$ is the conventional Richards variable associated with equal length transmission lines or so called commensurate transmission lines ($\lambda=\tanh p\tau$, where τ is the commensurate one-way delay of the distributed elements). These three polynomials are related by the losslessness equation

$$g(p,\lambda)g(-p,-\lambda)=h(p,\lambda)h(-p,-\lambda)+f(p,\lambda)f(-p,-\lambda) \quad (1b).$$

The polynomials $h(p,\lambda)$ and $g(p,\lambda)$ can be divided in three parts as

$$h(p,\lambda)=h_L(p)+h_D(\lambda)+h_C(p,\lambda)\,, \quad (1c)$$

$$g(p,\lambda)=g_L(p)+g_D(\lambda)+g_C(p,\lambda)\,. \quad (1d)$$

where $h_L(p)$ and $g_L(p)$ represents the lumped subsection [L], $h_D(\lambda)$ and $g_D(\lambda)$ represents the distributed subsection [D], and $h_C(p,\lambda)$ and $g_C(p,\lambda)$ are the connectivity information of the components.

As far as the modeling problem is concerned, one has to generate the two-variable, realizable, BR scattering parameters of the lossless two-port of Fig. 1 in such a way that the input reflection coefficient $S_{11}(p,\lambda)$ is fit the computed data $S(j\omega)$ at each frequency point under consideration. This is not an easy task. In the following section however, a practical approach is presented, to build the models which guarantees the realizibility of the two-variable scattering parameters specified by Eq. (1).

III. A PRACTICAL MODELING APPROACH

Over the angular frequencies ω, let $\varepsilon(j\omega)=S(j\omega)-S_{11}(j\omega, j\tan(\omega\tau))$ be the error function defined as the difference between the given data and the analytic form of the input reflection coefficient of the network which will be constructed in two kinds of elements. Obviously, $|\varepsilon(j\omega)|^2$ is the function of both $h_L(p)$ and $h_D(\lambda)$. This functional relation can be expressed as

$$|\varepsilon|^2=\varepsilon\varepsilon^*=F(S_{11})\,;\; S_{11}=F(h_L,h_D) \quad (2)$$

where "*" represents the complex conjugate of a complex number.

Referring to Eq. (2), one can minimize the error $|\varepsilon|^2$ in the directions of the partial derivatives given by

$$\frac{\partial\varepsilon\varepsilon^*}{\partial h_L}=\frac{\partial|\varepsilon|^2}{\partial h_L}=\frac{-\varepsilon^*}{g(p,\lambda)} \quad (3a)$$

and

$$\frac{\partial\varepsilon\varepsilon^*}{\partial h_D}=\frac{\partial|\varepsilon|^2}{\partial h_D}=\frac{-\varepsilon^*}{g(p,\lambda)} \quad (3b)$$

In this case, an iterative method, perhaps the gradient method, may be employed, to minimize the error function $|\varepsilon|^2$ which in turn yields the polynomials h_L and h_D from the initialized coefficients as follows,

$$h_L^{(r)}=h_L^{(r-1)}-\frac{\partial|\varepsilon|^2}{\partial h_L}\Big|_{h_L^{(r-1)}} \quad (4a)$$

$$h_D^{(r)}=h_D^{(r-1)}-\frac{\partial|\varepsilon|^2}{\partial h_D}\Big|_{h_D^{(r-1)}} \quad (4b)$$

In Eq. (4), the subscript r designates the iteration index starting at $r=1$. And, $h_L^{(0)}(p)$ and $h_D^{(0)}(\lambda)$ are the initialized polynomials stem from the polynomials $h_L(p)$ and $h_D(\lambda)$, respectively. Thus, the following algorithm is proposed to design networks in two kinds of elements via modeling.

Algorithm: Generation of mixed-element network form the given reflectance data via modeling

Inputs:

- ω_i; $i=1,2,..,N_\omega$: Sample frequencies.
- N_ω: Total number of sample frequencies.
- $S(j\omega_i)=S_R(\omega_i)+jS_X(\omega_i)$; $i=1,2,..,N_\omega$: Sample points generated from the input reflection coefficient of the lumped-element network prototype.
- n_λ: Total number of distributed-elements in distributed network [D].
- $f_D(\lambda)$: A monic polynomial constructed on the transmission zeros of [D]. It is noted that for cascaded connection of UEs $f_D(\lambda)=(1-\lambda^2)^{n_\lambda/2}$ is selected.
- n_p: Total number of lumped elements in lumped network [L].
- k: Total number of transmission zeros at DC of the lumped network [L].
- $f_L(p)$: A monic polynomial constructed on the transmission zeros of [L]. In our modeling approach, all the transmission zeros are imbedded into the lumped ladder section [L] by choosing $f_L(p)=p^k$.

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- $h_{0D}^{(0)}, h_{1D}^{(0)}, h_{2D}^{(0)}, ..., h_{n_\lambda D}^{(0)}$ and $h_{0L}^{(0)}, h_{1L}^{(0)}, h_{2L}^{(0)}, ..., h_{n_p L}^{(0)}$: Initialized coefficients of the polynomials $h_D^{(0)}(\lambda)$ and $h_L^{(0)}(p)$, respectively.
- δ : The stopping criteria. For many practical problems, it is sufficient to choose $\delta = 10^{-3}$.

Computational Steps

Step 1: Set $r = 1$ and start the iterations.

Step 2: By using the $(r-1)^{th}$ initial coefficients $h_{0D}^{(r-1)}, h_{1D}^{(r-1)}, h_{2D}^{(r-1)}, ..., h_{n_\lambda D}^{(r-1)}$ and $h_{0L}^{(r-1)}, h_{1L}^{(r-1)}, h_{2L}^{(r-1)}, ..., h_{n_p L}^{(r-1)}$, compute the strictly Hurwitz polynomials $g_D^{(r-1)}(\lambda)$ and $g_L^{(r-1)}(p)$ employing the losslessness condition

Step 3: Synthesize lumped-element two-port [L], and distributed-element two-port [D], to obtain the component values.

Step 4: By using the component values, form the scattering transfer matrix for each element.

Step 5: According to the connection order, multiply the scattering transfer matrices, and obtain scattering transfer matrix of the mixed model, and then obtain $g(p,\lambda)$, $h(p,\lambda)$ and $f(p,\lambda)$ two-variable polynomials.

Step 6: Compute the error $\varepsilon^{(r-1)}(j\omega_i) = S(j\omega_i) - \dfrac{h^{(r-1)}(j\omega_i, j\tan(\omega_i\tau))}{g^{(r-1)}(j\omega_i, j\tan(\omega_i\tau))}$.

Step 7: Compute the sum of the square errors $\delta^{(r-1)} = \sum_{i=1}^{N_\omega} \left|\varepsilon^{(r-1)}(j\omega_i)\right|^2$. If $\delta^{(r-1)} \le \delta$, set $S_{11}(p,\lambda) = \dfrac{h^{(r-1)}(p,\lambda)}{g^{(r-1)}(p,\lambda)}$ and stop. Else, go to the next step.

Step 8: Compute the complex quantities over the sample frequencies for the given $(r-1)^{th}$ initials,

$$h_L^{(r)}(j\omega_i) = h_L^{(r-1)}(j\omega_i) + \frac{\varepsilon^{(r-1)}(j\omega_i)}{g^{(r-1)}(j\omega_i, j\tan(\omega_i\tau))} \quad \text{and}$$

$$h_D^{(r)}(j\tan(\omega_i\tau)) = h_D^{(r-1)}(j\tan(\omega_i\tau)) + \frac{\varepsilon^{(r-1)}(j\omega_i)}{g^{(r-1)}(j\omega_i, j\tan(\omega_i\tau))}.$$

Step 9: Separate the real and the imaginary parts of $h_L^{(r)} = h_{R,L}^{(r)} + jh_{X,L}^{(r)}$ and $h_D^{(r)} = h_{R,D}^{(r)} + jh_{X,D}^{(r)}$.

Step 10: Using the real and the imaginary parts of the above equations, find the coefficients of the polynomials $h_D^{(r)}(\lambda) = \sum_{i=0}^{n_\lambda} h_{iD}^{(r)}\lambda^i$ and $h_L^{(r)}(p) = \sum_{i=0}^{n_p} h_{iL}^{(r)} p^i$ by means of any linear interpolation or curve fitting routine.

Step 11: Set $r = r + 1$ and go to Step 2.

Referring to Step 10 of the above algorithm, it is crucial to generate the polynomials $h_D^{(r)}(\lambda)$ and $h_L^{(r)}(p)$ [8]. In the following, an example is worked out, to generate a model for a given lumped-element Chebyshev filter.

IV. EXAMPLE: MIXED-ELEMENT BUTTERWORTH FILTER

In this example, a two-lumped-element Butterworth filter prototype is transformed to a mixed-element counterpart via proposed modeling algorithm. Throughout the computations normalized elements are used. Lumped-element filter prototype, its transducer power gain (TPG) plot and the data generated from its input reflection coefficient are given in Fig. 2, Fig. 3 and Table 1, respectively.

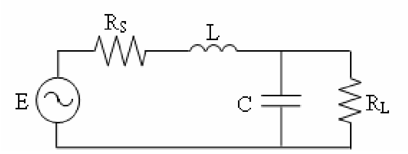

Fig 2. A two-element LC-ladder Butterworth filter (R_S=R_L=1, L=1.4142, C=1.4142).

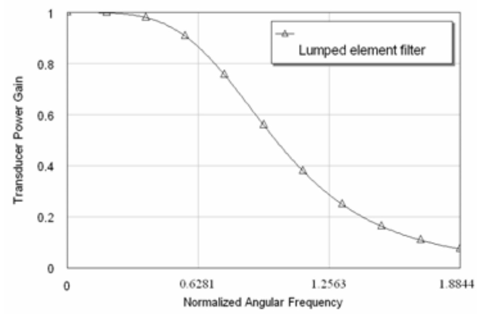

Fig 3. TPG curve of the normalized lumped-element filter.

Table 1. Reflection data for the lumped element Chebyshev filter.

ω_i	Re $\{S(j\omega_i)\}$	Im $\{S(j\omega_i)\}$	ω_i	Re $\{S(j\omega_i)\}$	Im $\{S(j\omega_i)\}$
0.0	0	0	0.9	-0.0929	0.6225
0.1	-0.0099	0.0014	1.0	0	0.7071
0.2	-0.0383	0.0113	1.1	0.1031	0.7639
0.3	-0.0812	0.0379	1.2	0.2061	0.7951
0.4	-0.1310	0.0883	1.3	0.3024	0.8057
0.5	-0.1765	0.1664	1.4	0.3886	0.8015
0.6	0.2040	0.2704	1.5	0.4639	0.7873
0.7	-0.2015	0.3912	1.6	0.5287	0.7669
0.8	-0.1635	0.5137	1.7	0.5841	0.7429

Close examination of Fig. 3 reveals that upper-edge or the cut-off frequency of the lumped prototype filter is at

$\omega_c = 1$ with minimum passband gain of $G_{\min} = 0.5$. In the mixed-element filter, equal length of the transmission lines is fixed as 90^0 (quarter wavelength) at the normalized frequency $f_0 = 0.6094$. That is, normalized delay length is fixed at $\tau = 0.4102$.

Since the lumped-element filter prototype has a low-pass nature, $f(p,\lambda)$ is selected as $f(p,\lambda) = f_L(p) f_D(\lambda) = 1 \cdot (1-\lambda^2)^2 = 1 - 2\lambda^2 + \lambda^4$. Then, applying the algorithm above, $S_{11}(p,\lambda)$ is computed with the coefficient matrices,

$$\Lambda_h = \begin{bmatrix} 0 & -0.1473 & 0.6416 \\ -0.0205 & 1.3143 & 0.6725 \\ 0.4469 & 0.3079 & 0 \end{bmatrix}$$

$$\Lambda_g = \begin{bmatrix} 1 & 2.0971 & 1.1881 \\ 0.9456 & 1.9800 & 0.6725 \\ 0.4469 & 0.3079 & 0 \end{bmatrix}$$

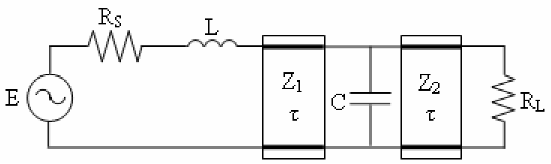

Fig. 4: Mixed element Butterworth filter (R_S=R_L=1, L=0.9251, C=0.9662, Z_1=1.2608, Z_2=0.6891, τ=0.4102).

Consequently, the transducer power gain performances of the lumped- and mixed-element filters are shown in Fig. 5. Close examination of this figure reveals that the mixed-element filter constructed via modeling exhibits excellent gain performance by preserving the minimum gain of the passband at $G_{\min} = 0.5$.

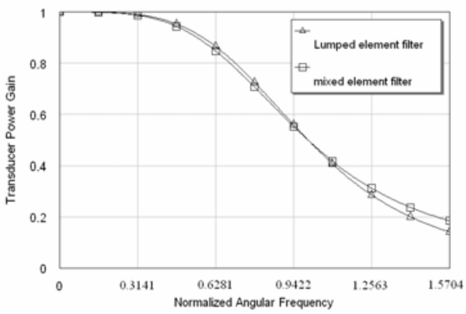

Fig. 5: TPG plots of lumped- and mixed-element filters.

V. CONCLUSION

In this paper, a new method is proposed, to construct networks in two kinds of elements, namely lumped and distributed elements, via modeling. The proposed method consists of two major phases. In Phase I, a lumped-element network prototype is designed using the classical techniques. Then, the input reflection coefficient of the prototype network is generated over the passband frequencies. In the second phase, reflectance data is modeled as a bounded real input reflection function in two complex variables, which in turn results in the desired mixed-element network with two kinds of elements. Application of the new procedure is exhibited by designing a mixed-element filter employing total number of four elements. In the design process, first a two-element Butterworth filter is constructed with minimum passband gain of $G_{\min} = 0.5$. Then, the mixed-element filter is designed by modeling the input reflection coefficient data obtained from the lumped-element prototype filter. In the mixed-element filter, two lumped-elements are used. These elements are connected with each other via equal length transmission lines one by one. Thus, the actual production of the mixed-element filter is facilitated by introducing the inevitable connections as part of the design. It is shown that mixed-element filter design preserves the minimum of the passband gain of the lumped-element prototype.

It is expected that the proposed method will find applications, to design networks for communication systems manufactured on a single chip using Si-Based VLSI technology.

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