Outage Diversity for Distributed Estimation over Parallel Fading Channels

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***Abstract*— In this paper we study the outage diversity in distributed estimation over parallel fading channels. We find tight upper and lower bounds on the diversity order and show that they are arbitrarily close under certain conditions. Our results show that the diversity order does not always equal to the number of sensors, but also depends on sensing quality of the sensors.**

I. Introduction

Research on distributed estimation in wireless sensor networks has been evolving very rapidly. The problem setting involves sensors observing an unknown parameter in noise which can be delivered to a fusion center (FC) by analog or digital transmission methods. The optimality of the analog amplify and forward method is described in [1], [2]. In [2], an amplify-and-forward approach is employed with an orthogonal multiple access fading channel, where the concept of estimation diversity is introduced, and shown to be given by the number of sensors. This seminal result which shows that the "estimation diversity" is given by the number of sensors is obtained under the assumption of asymptotically large number of sensors with statistically identical sensing quality, and large total transmission powers.

Similar to [2], we consider a parallel multiple access fading channel scenario, but obtain expressions for the outage diversity under a different asymptotic regime. Namely, we consider finitely many sensors and large total transmit power. Moreover, we assume that the sensors may have different sensing qualities. In contrast to [2], our results show that the diversity need not be equal to the number of sensors, and depends on both the sensing quality measured by the sensing signal-to-noise ratio (SNR), and the threshold used to define the outage.

II. System Model

Consider a distributed estimation problem (see Fig. 1) where the K sensor measurements x_k are related to the source parameter θ by

$$x_k = h_k\theta + n_k \quad k = 1, \dots, K \ , \tag{1}$$

where $n_k \sim \mathcal{CN}(0, \sigma^2_{n_k})$ is the sensing noise, and h_k is a parameter that controls the k^{th} sensor's SNR given by $\gamma_k := |h_k|^2/\sigma^2_{n_k}$. Without loss of generality, we will assume that $\gamma_1 \le \dots \le \gamma_K$. The sensors amplify and forward their measurements which are separately received by the fusion center over orthogonal channels:

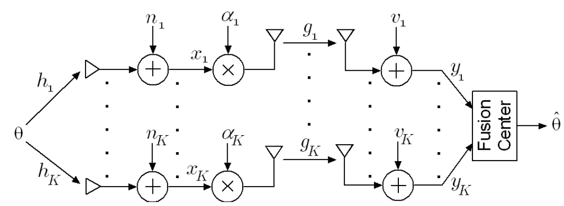

Fig. 1. Distributed estimation in wireless sensor networks

$$y_k = \alpha_k g_k(h_k\theta + n_k) + v_k \ , \quad k = 1, \dots, K \tag{2}$$

where $g_k \sim \mathcal{CN}(0, \sigma^2_{g_k})$ is the k^{th} channel coefficient, $v_k \sim \mathcal{CN}(0, \sigma^2_{v_k})$ is the receiver noise, and α_k is the amplification coefficient which controls the power of the k^{th} sensor. We assume that n_k, v_k and g_k are independent and identical distributed (i.i.d) random variables across sensor index k, respectively. We consider equal power transmission in the sequel, thus $|\alpha_k|$ is given by

$$|\alpha_k| = \sqrt{\frac{P_{tot}}{K(|h_k|^2\sigma^2_\theta + \sigma^2_{n_k})}} \ . \tag{3}$$

We assume that the fusion center employs maximal ratio combining before estimating the source parameter θ. Combining the received y_k's to get the maximum SNR amounts to multiplying with the conjugate of the coefficient of θ when the noise variances are equal [3]. Defining $\eta_k := |g_k|^2/\sigma^2_{v_k}$, the resulting SNR is equal to

$$\text{SNR} = \sum_{k=1}^{K} \frac{\eta_k\gamma_k}{\eta_k + \frac{K(\gamma_k\sigma^2_\theta+1)}{P_{tot}}} \ . \tag{4}$$

The SNR in (4) is random because the instantaneous SNR on the k^{th} channel, η_k, is random. The random variable η_k is assumed exponentially distributed with mean $\zeta_k := E[\eta_k] = \sigma^2_{g_k}/\sigma^2_{v_k}$.

III. Outage and Diversity

In distributed estimation of θ, the variance of the best linear unbiased estimator (BLUE) is given by SNR^{-1} [2]. We define the outage probability as

$$P_{out} := Pr\left[\sum_{k=1}^{K}\frac{\eta_k\gamma_k}{\eta_k+\frac{K(\gamma_k\sigma_\theta^2+1)}{P_{tot}}} < z\Big|\gamma_1,\cdots,\gamma_K\right] \quad (5)$$

where z is the threshold, the randomness of the SNR stems from the channels η_k, and γ_k are assumed deterministic. We now examine how fast the outage converges to zero as a function of the threshold z and the sensing SNRs γ_k by investigating outage diversity order defined as

$$d = \lim_{P_{tot}\to\infty} -\frac{\log P_{out}}{\log P_{tot}}\ . \quad (6)$$

This definition of diversity is in perfect analogy to the definitions of diversity for MIMO systems (see e.g., [4, eqn 3]).

In what follows, we find upper and lower bounds for the diversity order.

Theorem 1:

1) If $\sum_{k=1}^{i}\gamma_k < z$ for some $i \in \{0,\dots,K-1\}$ [1] then $d \le K-i$. Clearly, the upper bound on d is most useful if we find the largest such i.
2) If $z < \gamma_k, \forall k$ then $d = K$.

Proof: See Appendix I. ■

Theorem 2: If $\sum_{k=1}^{i}\gamma_k \le z$ for some $i \in \{0,\dots,K-1\}$, then

$$d \ge K - i - \frac{1}{\gamma_{i+1}}\left(z - \sum_{k=1}^{i}\gamma_k\right). \quad (7)$$

Proof: See Appendix II. ■

Combining the upper bound of Theorem 1 with the lower bound of Theorem 2, the diversity order is bounded as

$$K - i - \frac{1}{\gamma_{i+1}}\left(z - \sum_{k=1}^{i}\gamma_k\right) \le d \le K - i. \quad (8)$$

provided that $\sum_{k=1}^{i}\gamma_k < z$ by the assumption of Theorem 1 part (1). We now examine the tightness of the bounds. The threshold z falls in an interval of the form $\sum_{k=1}^{i}\gamma_k < z \le \sum_{k=1}^{i+1}\gamma_k$ for some $i \in \{0,\dots,K-1\}$. Therefore, the difference between the upper and lower bounds in (8) can be at most unity and arbitrarily close to zero, depending on the exact value of the threshold z, and its relationship with the sensing SNRs $\{\gamma_k\}_{k=1}^{K}$.

Let us now examine a corollary of Theorem 1 and Theorem 2 for the case of equal sensing SNRs ($\gamma_k = \gamma, \forall k$) to get simpler expressions.

Corollary 1: If the sensing SNRs are equal, ($\gamma_k = \gamma, \forall k$), then we have the following simple upper and lower bounds on d whenever z/γ is not an integer:

$$K - \frac{z}{\gamma} \le d \le K - \left\lfloor\frac{z}{\gamma}\right\rfloor\ . \quad (9)$$

If $z < \gamma$, we have the exact diversity order $d = K$.

Proof: Omitted due to space limitations. ■

[1]When $i = 0$, $\sum_{k=1}^{i}\gamma_k = 0$, by definition

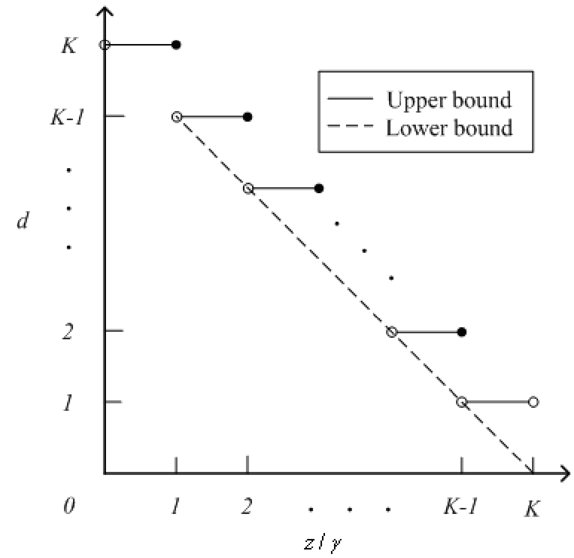

Fig. 2. Diversity order bounds when the sensing SNRs are equal

Note that in the case when z/γ is an integer, the same proof can be carried out with recognizing that the integer i can be chosen as $i = z/\gamma - 1$ which is less than z/γ. Using part (1) of Theorem 1 for this choice of i we obtain $d \le K - i = K - z/\gamma + 1$. Examining the tightness of the bounds in (9), we observe that, similar to the unequal sensing SNR case, the bounds can be apart at most by one. Fig. 2 illustrates the upper and lower bounds of the diversity order as a function of z/γ. It can be seen that $z \in (0, K\gamma)$. When $z < \gamma$, the diversity order is exactly K. When z/γ is an integer, the upper bound is $d \le K - z/\gamma + 1$ as per the discussion above, and it is exactly one more than the lower bound in (9). On the other hand, when z/γ is greater than, but sufficiently close to an integer, the difference between the upper and lower bounds becomes arbitrarily close to zero.

In this setting where the sensing SNRs are equal, the upper and lower bounds in (9) show that for a fixed z and γ, when a new sensor is added into the system, the bounds both increase by one. In fact, the diversity order increases like $d = \mathcal{O}(K)$ for large K. We note, however, that the growth of the diversity order with K applies when $\{\gamma_k\}_{k=1}^{K}$ are equal, and does not necessarily hold when $\{\gamma_k\}_{k=1}^{K}$ are unequal. In fact, examining the statement of Theorem 1, we see that it is possible to add new sensors with very small γ_k's such that the upper bound in Theorem 1 does not increase. To see this, suppose that the threshold z and set of sensing SNRs $\{\gamma_k\}_{k=1}^{K}$ are given. We add a new sensor whose sensing SNR is small enough to satisfy $\gamma_{new} < z - \sum_{k=1}^{i}\gamma_k$. This implies that we have $\sum_{k=1}^{i}\gamma_k + \gamma_{new} < z$. Using Theorem 1 with $i+1$ γ_k's, and $K+1$ sensors, we have $d \le (K+1) - (i+1) = K - i$, the same diversity order as when we had K sensors. Therefore, it is possible to add new sensors into the system without getting any diversity benefit. Note that the new sensor that was introduced had to have a sensing quality (measured by

γ_{new}) that was bad enough to not contribute to the diversity order. This example clearly illustrates that the diversity order depends on the sensing SNRs $\{\gamma_k\}_{k=1}^K$ and not just on the number of sensors.

The proofs of Theorems 1 and 2 which derive upper and lower bounds on the diversity order equations (20) and (30) depend on the distributions of the instantaneous channel SNR on the k^{th} sensor η_k, and therefore can be easily extended to cases where η_k is not exponentially distributed. In the next section, we extend these bounds to cases that involve line-of-sight between the sensors and the FC.

A. Diversity with Line of Sight

So far, we have assumed that the channel g_k is zero-mean complex Gaussian implying Rayleigh fading (exponential η_k). However, in the presence of line of sight between some or all of the sensors and the FC, distributions other than the exponential might be suitable for η_k. We first begin by considering a Ricean amplitude (i.e., $\sqrt{\eta_k}$ is Ricean), which means that the density function of η_k in this case is given by

$$f_{\eta_k}(x) = \frac{(1+\kappa)}{\zeta_k} e^{-\kappa} e^{-\frac{(\kappa+1)}{\zeta_k}x} I_0\left(2\sqrt{\frac{\kappa(\kappa+1)x}{\zeta_k}}\right) , \quad (10)$$

where κ is the Ricean factor, and $\zeta_k := E[\eta_k]$. Just like the exponential case, $f_{\eta_k}(0) \neq 0$, and $\lim_{a\to 0} a f'_{\eta_k}(a) = 0$, regardless of the value of κ, for the density function in (10). Reconsidering the lower bound in (20) and the upper bound in (30), we conclude that the bounds on the diversity in the Ricean case remain the same as the Rayleigh case.

Another widely used distribution for the channel envelope $\sqrt{\eta_k}$ in the presence of line of sight is the Nakagami distribution. The corresponding density function for η_k is given by

$$f_{\eta_k}(x) = \frac{m^m x^{m-1}}{\Gamma(m)\zeta_k^m} \exp\left(-\frac{mx}{\zeta_k}\right) , m > 1 , \quad (11)$$

where m is the Nakagami parameter, and $\zeta_k = E[\eta_k]$ as before. In this case, we now show that the bounds in (8) both scale by a factor of m:

Theorem 3: If $\sum_{k=1}^{i} \gamma_k \leq z \leq \sum_{k=1}^{i+1} \gamma_k$ and η_k are distributed as in (11) then

$$(K - i - 1)m \leq d \leq (K - i)m \quad (12)$$

Proof: Omitted due to space limitations. ■

Note that for the special case of $\gamma_k = \gamma \ \ \forall k$, the bounds can be obtained by multiplying the upper and lower bounds in (9) by m.

IV. SIMULATIONS

In this section, we provide simulation results to verify and illustrate our findings in previous sections. We assume that the variance of the source parameter $\sigma_\theta^2 = 0.1$ and the instantaneous channel SNRs $\{\eta_k\}_{k=1}^K$ are i.i.d exponential random variables with unit mean. The numerical results herein are obtained by generating over 10^9 runs, which is necessary since P_{out} is exceedingly small even for moderate values of K and P_{tot}.

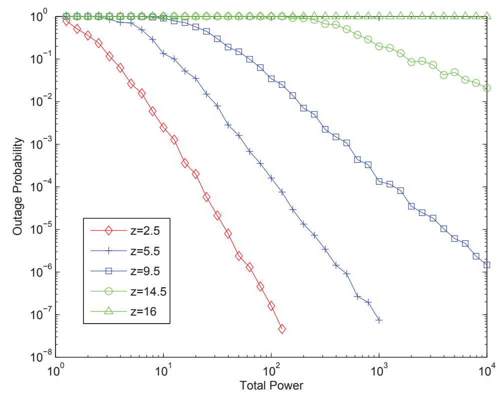

Fig. 3. Outage probability vs. total power for a set of fixed and unequal sensing SNRs $\{\gamma_k\} = \{1, 2, 3, 4, 5\}$

We first verify the outage and diversity for the case of fixed sensing SNRs $\{\gamma_k\}_{k=1}^K$, as in Theorems 1 and 2. Consider a case where there are 5 sensors with different sensing SNRs: $\gamma_1 = 1, \gamma_2 = 2, \gamma_3 = 3, \gamma_4 = 4$ and $\gamma_5 = 5$. We simulate the outage probabilities as a function of the total power for different thresholds where $z \in \{2.5, 5.5, 9.5, 14.5, 16\}$. The results are shown in Fig. 3 and Fig. 4. From Fig. 3 where the outage probability is plotted versus P_{tot}, we can see that the diversity order as seen from slopes decreases as the threshold z increases. When $z = 16$, the outage probability is always 1 since $z > \sum_{k=1}^{5} \gamma_k = 15$. To illustrate the results better, we also plot the estimated diversity order, given by $-\log P_{out}/\log P_{tot}$, in Fig. 4. Recall that from Theorem 1 and Theorem 2, the diversity order is bounded by $5 - i - 1 \leq d \leq 5 - i$ if $\sum_{k=1}^{i} \gamma_k < z \leq \sum_{k=1}^{i+1} \gamma_k$. Given the values of γ_k and z as above, using the above condition, we find that the appropriate i is given by $1, 2, 3, 4, 5$ corresponding to $2.5, 5.5, 9.5, 14.5, 16$, respectively. These theoretical diversity results are verified and illustrated clearly in Fig. 4 where it is shown that as P_{tot} increases the estimated diversity order converges into the correct region in Fig. 4. For example, when $z = 5.5$, we find that $i = 2$ since $\sum_{k=1}^{2} \gamma_k = 3 < z < \sum_{k=1}^{3} \gamma_k = 6$. Thus, the diversity order is expected to be bounded between $(2, 3)$ due to Theorem 1 and Theorem 2, and indeed seen to be correct in Fig. 4.

Fig. 5 and Fig. 6 show the outage probability and the diversity order for the case of fixed and equal sensing SNRs where $\gamma_k = 1, \forall k$, with $z \in \{1.2, 2.2, 3.2, 4.2, 5.2\}$. With the number of sensors $K = 5$, the theoretical diversity order is given by (9). Again, we observe that our simulation results match with the theoretical results: as z increases, the diversity order decreases. More importantly, all the aforementioned figures (Fig. 3 - Fig. 6) show that, given the sensing SNRs, the diversity order of the outage probability depends on not only the number of active sensors K in the system but also the comparative values of the outage threshold z and the sensing SNRs $\{\gamma_k\}_{k=1}^K$.

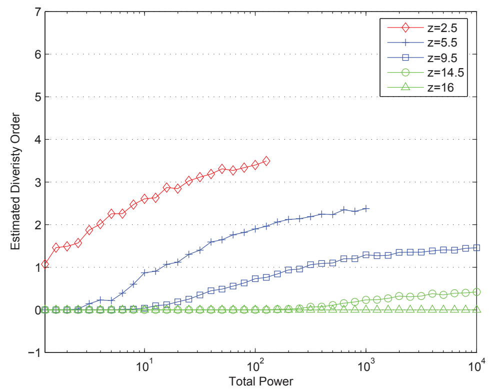

Fig. 4. Estimated diversity order vs. total power for a set of fixed and unequal sensing SNRs $\{\gamma_k\} = \{1, 2, 3, 4, 5\}$

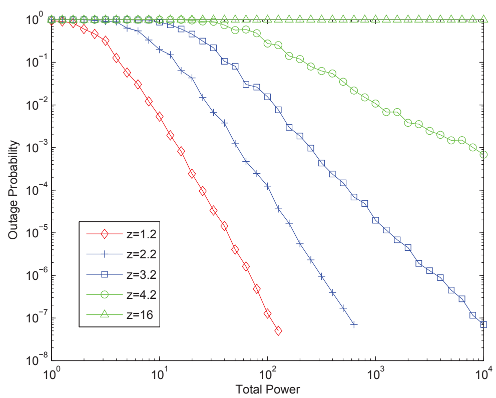

Fig. 5. Outage probability vs. total power for fixed and equal sensing SNRs $\gamma_k = 1, \forall k$

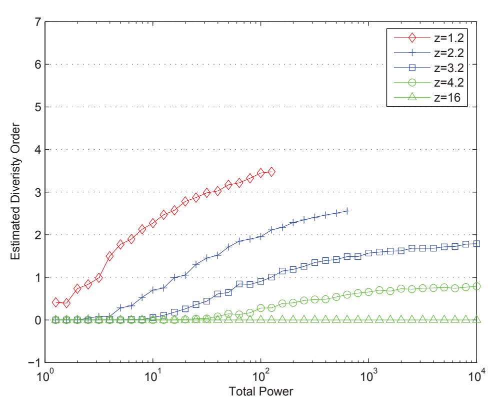

Fig. 6. Estimated diversity order vs. total power for fixed and equal sensing SNRs $\gamma_k = 1, \forall k$

V. CONCLUSION

We found upper and lower bounds on the diversity order of the distributed estimation problem, which are within unity of each other. We showed that the diversity order for a fixed K is not always given by K, and depends on the sensing SNRs γ_k and the threshold z. Our results suggest that the sensors with a bad sensing SNR should shut down to save energy and bandwidth since the system diversity gain by adding them is negligible.

APPENDIX I
PROOF OF THEOREM 1

Proof: We begin with part (1). Let $Z_k := \frac{\eta_k \gamma_k}{\eta_k + a c_k}$ be the k^{th} term of the sum in (5), where $a := P_{tot}^{-1}$ and $c_k := K(\gamma_k \sigma_\theta^2 + 1)$ for simplicity of notation. Recall that γ_k and c_k are deterministic, and η_k is exponentially distributed. Clearly $0 \le Z_k < \gamma_k$ with probability one.

For $\epsilon > 0$ sufficiently small we have

$$A_i := \{Z_1, \ldots, Z_K : Z_k \le \epsilon, k = i+1, \ldots, K\} \quad (13)$$
$$\subset \left\{ Z_1, \ldots, Z_K : \sum_{k=1}^{K} Z_k \le z \right\}, \quad (14)$$

because constraining $Z_{i+1}, \ldots, Z_K$ to be small ensures

$$\sum_{k=1}^{K} Z_k \le (K-i)\epsilon + \sum_{k=1}^{i} Z_k < (K-i)\epsilon + \sum_{k=1}^{i} \gamma_k , \quad (15)$$

where the second inequality follows because $Z_k < \gamma_k$. The rhs in (15) is smaller than z, by assumption of the Theorem for $\epsilon > 0$ sufficiently small. This establishes that (14) is correct.

Recalling that the set on the rhs of (14) is the outage event in (5), we have

$$Pr[A_i] \le P_{out} . \quad (16)$$

Since Z_k are independent,

$$Pr[A_i] = \prod_{k=i+1}^{K} Pr[Z_k \le \epsilon] \le P_{out}. \quad (17)$$

Keeping in mind that $Z_k = \frac{\eta_k \gamma_k}{\eta_k + a c_k}$ it is straightforward to verify that

$$Pr[Z_k \le \epsilon] = F_{Z_k}(\epsilon) = F_{\eta_k}\left(\frac{a \epsilon c_k}{\gamma_k - \epsilon}\right), \gamma_k > \epsilon \quad (18)$$

where $F_{Z_k}(\cdot)$ and $F_{\eta_k}(\cdot)$ are the cumulative distribution functions of Z_k and η_k, respectively. Taking the logarithm of both sides of (17), recalling $a = P_{tot}^{-1}$, and dividing by $\log P_{tot} = -\log a$ we obtain

$$d = \lim_{P_{tot} \to \infty} -\frac{\log P_{out}}{\log P_{tot}} \le \sum_{k=i+1}^{K} \lim_{a \to 0} \frac{\log F_{\eta_k}\left(\frac{a \epsilon c_k}{\gamma_k - \epsilon}\right)}{\log a} . \quad (19)$$

Using L'Hospital's rule twice, on the k^{th} term of the sum in (19), we have

$$d \le K - i + \sum_{k=i+1}^{K} \lim_{a \to 0} \frac{a f'_{\eta_k}\left(\frac{a c_k \epsilon}{\gamma_k - \epsilon}\right)}{f_{\eta_k}\left(\frac{a c_k \epsilon}{\gamma_k - \epsilon}\right)} . \quad (20)$$

Recall that η_k is exponential with $f_{\eta_k}(x) = \zeta_k^{-1}\exp(-x/\zeta_k)$ for $x \geq 0$, and therefore, each term of the sum in (20) is zero, establishing part (1) of the Theorem.

To prove part (2) we begin by recalling that $d \leq K$ due to part (1). To show $d \geq K$, note that

$$\left\{Z_1, \ldots, Z_K : \sum_{k=1}^{K} Z_k \leq z\right\} \subset \{Z_1, \ldots, Z_K : Z_k \leq z\} \quad (21)$$

because $Z_k \geq 0$. Therefore, the probabilities of the events in (21) are related as

$$P_{out} \leq Pr\left[Z_1 \leq z, \ldots, Z_K \leq z\right] = \prod_{k=1}^{K} Pr\left[Z_k \leq z\right] \ . \quad (22)$$

Using (18) and taking the logarithms of both sides, (22) can be written as

$$\log P_{out} \leq \sum_{k=1}^{K} \log F_{\eta_k}\left(\frac{azc_k}{\gamma_k - z}\right) , \quad (23)$$

where, we used $\gamma_k > z$ to write $Pr[Z_k \leq z]$ in terms of $F_{\eta_k}(\cdot)$. Dividing through by $-\log P_{tot} = \log a$ and taking the limit as $P_{tot} \to \infty$ $(a \to 0)$ we obtain

$$d = \lim_{P_{tot}\to\infty} -\frac{\log P_{out}}{\log P_{tot}} \geq \sum_{k=1}^{K} \lim_{a\to 0} \frac{\log F_{\eta_k}\left(\frac{azc_k}{\gamma_k - z}\right)}{\log a} \ . \quad (24)$$

Similar to (19) and (20), it is straightforward that each limit on the rhs of (24) is given by 1 using L'Hôspital's rule, which proves that $d \geq K$, and completes the proof. ■

Appendix II
Proof of Theorem 2

Proof: Using the Chernoff bound on the outage in (5) we obtain

$$P_{out} \leq \exp(\nu(a)z) \prod_{k=1}^{K} E\left[\exp\left(-\nu(a)\frac{\eta_k\gamma_k}{\eta_k + ac_k}\right)\right] , \quad (25)$$

where the expectation is with respect to η_k, and $\nu(a) > 0$ is an arbitrary but positive function of $a := P_{tot}^{-1}$, which we choose as $\nu(a) = -\beta \log a > 0$, for some constant $\beta > 0$, to be later specified, and for $a < 1$. Substituting $\nu(a)$ in (25), taking the logarithms of both sides, and expressing the expectation as an integral, we obtain

$$\log P_{out} \leq -z\beta \log a + \sum_{k=1}^{K} \log\left[\int_0^\infty f_{\eta_k}(x) a^{\frac{x\beta\gamma_k}{x+ac_k}} dx\right] \ . \quad (26)$$

Breaking up the integral in the k^{th} term of the sum for some function $g(a) > 0$, we have

$$\int_0^{g(a)} f_{\eta_k}(x) a^{\frac{x\beta\gamma_k}{x+ac_k}} dx + \int_{g(a)}^{\infty} f_{\eta_k}(x) a^{\frac{x\beta\gamma_k}{x+ac_k}} dx \leq \int_0^{g(a)} f_{\eta_k}(x)dx + a^{\frac{g(a)\beta\gamma_k}{g(a)+ac_k}} , \quad (27)$$

where we obtained upper bounds on both terms on the left hand side (lhs) of (27) by substituting the lower limits of both integrals for x in the exponent of a because $a^{\frac{x\beta\gamma_k}{x+ac_k}}$ is a monotonically decreasing function of x, and also used $\int_{g(a)}^{\infty} f_{\eta_k}(x)dx \leq 1$. Substituting $g(a) = a^{1-\delta}$, for some $0 < \delta < 1$, the exponent of the second term on the rhs of (27) can be written as

$$\frac{g(a)\beta\gamma_k}{g(a) + ac_k} = \beta\gamma_k - \frac{a^\delta\beta\gamma_k c_k}{1 + a^\delta c_k} \ . \quad (28)$$

Since the second term on the rhs of (28) is small for small a, $a^{\frac{g(a)\beta\gamma_k}{g(a)+ac_k}} \approx a^{\beta\gamma_k}$. Using this result with (27), the k^{th} term in (26) can be bounded for a sufficiently small as

$$\log\int_0^\infty f_{\eta_k}(x) a^{\frac{x\beta\gamma_k}{x+ac_k}} dx \leq \log\left[\int_0^{a^{1-\delta}} f_{\eta_k}(x)dx + a^{\beta\gamma_k}(1+\epsilon)\right]. \quad (29)$$

Recalling the definition of the diversity order (6), substituting (29) into (26), and using the L'Hospital's rule twice, we obtain

$$d \geq -z\beta + \sum_{k=1}^{K} \lim_{a\to 0} \frac{(1-\delta)^2(a^{1-\delta} f'_{\eta_k}(a^{1-\delta}) + f_{\eta_k}(a^{1-\delta}))}{(1-\delta)f_{\eta_k}(a^{1-\delta}) + \beta\gamma_k a^{\beta\gamma_k - 1+\delta}(1+\epsilon)} + \frac{(\beta\gamma_k)^2 a^{\beta\gamma_k-1+\delta}(1+\epsilon)}{(1-\delta)f_{\eta_k}(a^{1-\delta}) + \beta\gamma_k a^{\beta\gamma_k-1+\delta}(1+\epsilon)} \quad (30)$$

Substituting $f_{\eta_k}(a) = \zeta_k^{-1}\exp(-a/\zeta_k)$, we observe that $a^{1-\delta} f'_{\eta_k}(a^{1-\delta}) \to 0$ as $a \to 0$ and that $f_{\eta_k}(0) \neq 0$. Examining (30), it is clear that the limit depends on whether $\beta\gamma_k > 1 - \delta$ or not. Working out this limit, we have $d \geq -z\beta + \sum_{k=1}^{K} T_{1-\delta}(\beta\gamma_k)$, where $T_{1-\delta}(\beta\gamma_k)$ is the limit in the k^{th} term in (30), and $T_y(x) = y$ if $x \geq y$, and $T_y(x) = x$ if $x \leq y$. Since the above lower bound is most useful when it is larger, using the continuity of $T_y(x)$ with respect to y, we can take the supremum of $T_{1-\delta}(\beta\gamma_k)$ over $0 < \delta < 1$ which is obtained as $\delta \to 0$, yielding

$$d \geq -z\beta + \sum_{k=1}^{K} T_1(\beta\gamma_k) \ . \quad (31)$$

We can select any positive β for the Chernoff bound, which we choose as $\beta = 1/\gamma_{i+1}$ and substitute in (31) to obtain

$$d \geq (K - i) - \frac{1}{\gamma_{i+1}}\left(z - \sum_{k=1}^{i} \gamma_k\right) , \quad (32)$$

where to get the rhs, we used $0 < \gamma_1 \leq \gamma_2 \leq \ldots \gamma_K$, and the definition of $T_1(\cdot)$. This is completes the proof. ■

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