Performance of Space-Time Coded Systems with Transmit Antenna Selection

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***Abstract*— We deal with transmit antenna selection for space-time coded (STC) systems over multiple input multiple output (MIMO) channels. Using pairwise error probability analysis and simulation results, we show that transmit antenna selection based on received power levels does not reduce the achievable diversity order for full rank STCs. Initially, we prove the results for Rayleigh flat fading channels, and then we state our expectations for the case of frequency selective (FS) fading. In addition to the study of full rank codes, we also consider rank deficient space-time codes, and determine achievable diversity orders with transmit antenna selection.**

I. INTRODUCTION

Space-time coded (STC) systems have become popular since they offer increased data rates while achieving low error probabilities [1], [2], [3]. On the other hand, a major limitation in achieving the promised advantages in practical systems is the high cost of implementing multiple chains of radio frequency (RF) circuits (amplifiers, filters, etc.) at the transmitter and the receiver. A method to reduce the required hardware complexity is to employ antenna selection (at the transmitter and/or at the receiver). The idea is to use a small number of RF chains together with the selected subsets of available antennas for transmission, and still obtain the benefits of MIMO communications. In this paper, our focus is the transmit antenna selection based on feedback from the receiver.

Recently, there have been significant research on transmit and receive antenna selection for MIMO systems. A general overview of the capacity and performance of MIMO systems with antenna selection at the receiver is presented in [4]. Antenna selection algorithms and analysis techniques by considering the minimization of error probability of the STCs are studied in [5]. A set of near-optimal selection algorithms based on maximizing the channel capacity is presented in [6]. Antenna selection at the receiver based on maximizing the signal-to-noise ratio (SNR) over quasi-static flat fading channels is considered in [7] and [8]. The performance of STC systems when the MIMO subchannels experience correlated fading is studied in [9]. In [10], the authors demonstrate that transmit antenna selection combined with space-time trellis codes can achieve full available diversity using simulations. However, they do not perform an analytical error-rate analysis. In [11], performance analysis for space-time block codes using the Alamouti scheme with transmit antenna selection over Rayleigh fading channels is presented which basically proves that full diversity is achieved. It is shown that transmit antenna selection with maximum ratio combining at the receiver achieves full diversity [12]. Two adaptive transmit antenna selection criteria based on an upper bound for the conditional error probability of the space-time coded schemes are provided in [13]. Transmit antenna selection for uncoded spatial multiplexing systems is considered in [14]. Similarly, in [15], transmit antenna selection algorithms are proposed to maximize capacity or minimize error probability for spatial multiplexing.

When the transmission rates are increased, depending on the multipath spread of the channel, frequency selective (FS) fading channel model may be more suitable than the flat fading model. Although this is an important model for many practical applications, there is only some limited research on STC-MIMO systems with antenna selection over FS channels. Two suboptimal antenna subset selection schemes are proposed for direct-sequence code-division multiple access (CDMA) systems in [16]. Performance improvement with antenna selection over MIMO-FS fading channels has been presented in [17] and [18] where only space-time block codes are considered and no error probability analysis is performed. In [19], receive antenna selection for MIMO-FS fading channels is studied.

In this paper, we consider transmit antenna selection based on the received signal to noise ratios, and derive the diversity advantages of general space-time codes. First, using an approach similar to [7] (where receive antenna selection is considered), we perform a pairwise error probability analysis for the case of transmit selection over flat fading channels. Then, based on these results, we present our expectations on the offered diversity orders for STC systems over MIMO-FS fading channels. We do not have formal proofs for the latter. We show that for full-rank space-time codes, transmit antenna selection does not degrade the diversity gain compared to that of the full complexity system. Furthermore, we show that if the code does not achieve full diversity for the full-complexity system (i.e., it is rank deficient), then performing antenna selection results in a loss of overall diversity order. We note that the results are very general, and apply for different space-time codes as they are only based on pairwise

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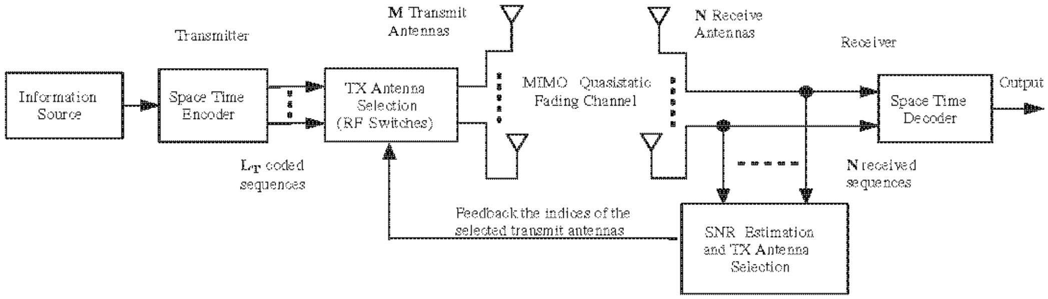

Fig. 1. Space-time coded MIMO system with transmit antenna selection based on received powers.

error probabilities. We corroborate our analytical results using extensive simulations.

The paper is organized as follows: Section II presents the system model. Section III provides the pairwise-error probability (PEP) analysis for STC systems with transmit antenna selection over flat fading channels. Section IV extends the results to the case of MIMO-FS channels. Finally, Section V concludes the paper.

II. System Description

In this section, we describe the system model for STC-MIMO systems. Figure 1 shows the STC system with antenna selection at the transmitter side. The channel is modelled as a quasi-static MIMO Rayleigh fading channel where the different sub-channels fade independently. In order to determine the antennas to be used, the pilot symbols can be transmitted from all available M transmit antennas, and then the SNR for each transmit antenna can be obtained at each frame. Once the selection of transmit antennas is done based on the largest received SNRs, the receiver can feedback the indices of the L_T transmit antennas to be used periodically. The feedback information about the selected transmit antennas only requires at most M bits, thus, it does not slow down the transmission rate significantly. After the selection of antennas is performed, the information sequence is encoded by a space-time encoder, and then, the coded sequence is multiplexed by a serial-to-parallel converter into several data streams. The resulting data streams are then modulated and transmitted through the selected L_T antennas simultaneously. At the receiver, space-time decoding is performed using the demodulated signals of the N receive antennas.

For a general MIMO system with M transmit and N receive antennas, and D intersymbol interference (ISI) taps, the received signal at antenna n at time k can be written as

$$y_n(k) = \sqrt{\frac{\rho}{MD}} \sum_{d=0}^{D-1} \sum_{m=1}^{M} h_{m,n}^{d}(k) s_m(k-d) + w_n(k) \quad (1)$$

where $h_{m,n}^{d}(k)$ is the fading coefficient at time k between transmit antenna m and receive antenna n for the d^{th} ISI tap, $s_m(k)$ is the transmitted symbol from antenna m at time k, and $w_n(k)$ is the noise term, $k = 1, \cdots, K$, where K is the frame length. Both fading channel coefficients, and noise terms are modeled as zero mean complex Gaussian random variables. The noise is assumed to be spatially and temporally white, and its variance is $1/2$ per dimension. The fading coefficients are spatially independent, but they are assumed to be constant over an entire frame (i.e., quasi-static fading), so the dependence on the variable k can be dropped. For the case of flat fading, $D = 1$, the fading coefficients are assumed to be identically distributed for different sub-channels with variance $1/2$ per dimension. For frequency selective fading, the multipath delay profiles need to be specified for all the sub-channels for a clear characterization, however, we assume that the for each sub-channel, the total power of the ISI channel is D, i.e., for uniform multipath delay profile, all the channel coefficients have a variance of $1/2$ per dimension. Signal constellation at each transmit antenna is normalized so that the average power of the transmitted signals is unity, and ρ is interpreted as the average SNR at each receive antenna. We assume that the receiver knows the channel state information (CSI) via some training symbols, however, the transmitter does not have access to this, thus it cannot use "waterfilling" type ideas, and it evenly splits its power across L_T transmit antennas used.

Assuming that L_T of the M available transmit antennas are selected at the transmitter side, the received signals can be stacked in a matrix form as

$$\mathbf{Y} = \sqrt{\frac{\rho}{L_T D}} \mathbf{HS} + \mathbf{W}$$

where the $N \times (K + D - 1)$ received signal matrix is

$$\mathbf{Y} = \begin{pmatrix} y_1(1) & \dots & y_1(K+D-1) \\ \vdots & \ddots & \vdots \\ y_N(1) & \dots & y_N(K+D-1) \end{pmatrix},$$

the $N \times L_T D$ channel coefficient matrix is

$$\mathbf{H} = \begin{pmatrix} h_{1,1}^{0} & \dots & h_{1,1}^{D-1} & \dots & h_{L_T,1}^{0} & \dots & h_{L_T,1}^{D-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ h_{1,N}^{0} & \dots & h_{1,N}^{D-1} & \dots & h_{L_T,N}^{0} & \dots & h_{L_T,N}^{D-1} \end{pmatrix},$$

the $L_T D \times (K + D - 1)$ codeword matrix is

$$
\mathbf{S} = \begin{pmatrix}
s_1(1) & \dots & s_1(K) & 0 & \dots & 0 \\
0 & s_1(1) & \dots & s_1(K) & \dots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & \dots & 0 & s_1(1) & \dots & s_1(K) \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
s_{L_T}(1) & \dots & s_{L_T}(K) & 0 & \dots & 0 \\
0 & s_{L_T}(1) & \dots & s_{L_T}(K) & \dots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & \dots & 0 & s_{L_T}(1) & \dots & s_{L_T}(K)
\end{pmatrix},
$$

and the $N \times (K + D - 1)$ noise matrix is

$$
\mathbf{W} = \begin{pmatrix}
w_1(1) & \dots & w_1(K+D-1) \\
\vdots & \ddots & \vdots \\
w_N(1) & \dots & w_N(K+D-1)
\end{pmatrix}.
$$

When the CSI is known at the receiver, the PEP conditioned on the instantaneous CSI is the same as the one for the case of a MIMO AWGN channel. For any given D and $\mathbf{H}$, the PEP of erroneously receiving $\widehat{\mathbf{S}}$, when $\mathbf{S}$ is transmitted, is given by,

$$
P(\mathbf{S} \to \widehat{\mathbf{S}}|\mathbf{H}) = \frac{1}{2} erfc\left(\sqrt{\frac{\rho}{4DM}}\|\mathbf{HB}\|\right) \tag{2}
$$

which can be upper bounded as

$$
P(\mathbf{S} \to \widehat{\mathbf{S}}|\mathbf{H}) \le \exp\left(-\frac{\rho}{4DM}\|\mathbf{HB}\|^2\right) \tag{3}
$$

where $\mathbf{B} = \mathbf{S} - \widehat{\mathbf{S}}$ is the codeword difference matrix. $\|.\|^2$ represents the sum of magnitude squares of all entries (i.e., $\|\mathbf{V}\|^2 = \sum_{i=1}^{I}\sum_{j=1}^{J}|v_{ij}|^2$ is the Frobenius norm of the $I \times J$ matrix $\mathbf{V}$, where v_{ij} is the entry of $\mathbf{V}$ at the i^{th} row and j^{th} column). To find the PEP over MIMO fading channels, we simply need to average this quantity in (3) over the fading statistics.

III. Transmit Antenna Selection over Flat Fading Channels

In this section, we study pairwise error probabilities for STCs with transmit antenna selection based on the received SNR levels over flat fading ($D = 1$) channels. First, we derive PEP bound for full rank codes, and then consider rank-deficient codes. Our approach is parallel to the one taken in [7] for receive antenna selection. We also present simulation results to verify the theoretical findings.

A. Transmit Antenna Selection with Full Rank Space-Time Codes

Let us denote the $N \times M$ channel transfer matrix by $\mathbf{H}'$, and its L_T columns having the largest norms by $\tilde{\mathbf{h}}_1, \tilde{\mathbf{h}}_2, \cdots, \tilde{\mathbf{h}}_{L_T}$, i.e., they form the equivalent channel matrix described in the previous section, $\mathbf{H}$. In order to derive an upper bound on the PEP, we first need to compute the joint probability density function (pdf) of the columns of $\mathbf{H}$.

Let us define the event $A = \{\mathbf{h}_1, \mathbf{h}_2, \cdots, \mathbf{h}_{L_T}$, the first L_T columns having the largest norms among all the columns of $\mathbf{H}'\}$. Then, the joint pdf of the columns of $\mathbf{H}$ is equivalent to the conditional pdf

$$
f_{\mathrm{H'}_1,\cdots,\mathrm{H'}_{L_T}}(\mathbf{h}_1, \cdots, \mathbf{h}_{L_T}|A) \tag{4}
$$

where $\mathbf{H'}_j$ denote random variables with the corresponding realization $\mathbf{h}_j$. For brevity, we will denote this joint pdf as f, which can be written as

$$
\begin{aligned}
f &= \frac{1}{P(A)} P(A|\mathbf{H'}_1 = \mathbf{h}_1, \cdots, \mathbf{H'}_{L_T} = \mathbf{h}_{L_T}) \\
&\quad . f_{\mathrm{H'}_1,\cdots,\mathrm{H'}_{L_T}}(\mathbf{h}_1, \cdots, \mathbf{h}_{L_T}).
\end{aligned} \tag{5}
$$

where $P(A) = \frac{1}{\alpha} = \frac{1}{\frac{M!}{(M-L_T)!L_T!}}$. Then the pdf becomes

$$
\begin{aligned}
f &= \alpha P(||\mathrm{H'}_{L_T+1}||^2 \le ||\mathbf{h}_{min}||^2, \cdots, ||\mathrm{H'}_M||^2 \le ||\mathbf{h}_{min}||^2) \\
&\quad . f_{\mathrm{H'}_1,\cdots,\mathrm{H'}_{L_T}}(\mathbf{h}_1, \cdots, \mathbf{h}_{L_T})
\end{aligned} \tag{6}
$$

where $||\mathbf{h}_{min}||^2 = \min\left\{||\mathbf{h}_1||^2, \cdots, ||\mathbf{h}_{L_T}||^2\right\}$, then the joint pdf can be written as,

$$
\begin{aligned}
f &= \alpha \left(\prod_{j=1}^{L_T} f_{\mathrm{H'}_j}(\mathbf{h}_j)\right) \sum_{l=1}^{L_T} I_{\mathcal{R}_l}(\mathbf{h}_1, \cdots, \mathbf{h}_{L_T}) \\
&\quad P(||\mathrm{H'}_{L_T+1}||^2 \le ||\mathbf{h}_l||^2, \cdots, ||\mathrm{H'}_M||^2 \le ||\mathbf{h}_l||^2)
\end{aligned} \tag{7}
$$

where $I_{\mathcal{R}_l}(\mathbf{h}_1, \cdots, \mathbf{h}_{L_T})$ is the indicator function

$$
I_{\mathcal{R}_l}(\mathbf{h}_1, \cdots, \mathbf{h}_{L_T}) = \begin{cases} 1 & \text{if } (\mathbf{h}_1, \cdots, \mathbf{h}_{L_T}) \in \mathcal{R}_l \\ 0 & \text{else} \end{cases}
$$

and the region $\mathcal{R}_l$ is defined as

$$
\mathcal{R}_l = \left\{\mathbf{h}_1, \cdots, \mathbf{h}_{L_T} : \|\mathbf{h}_l\| < \|\mathbf{h}_k\|,\ \ k = 1, \cdots, l-1, l+1, \cdots, L_T\right\}.
$$

Finally, using the Gaussian statistics, the joint pdf of the selected L_T columns can be written as

$$
\begin{aligned}
f &= \alpha \left(\sum_{l=1}^{L_T}\left[1 - e^{-\|\mathbf{h}_l\|^2}\sum_{n=0}^{N-1}\frac{\|\mathbf{h}_l\|^{2n}}{n!}\right]^{M-L_T} I_{\mathcal{R}_l}(\mathbf{h}_1, \cdots, \mathbf{h}_{L_T})\right) \\
&\quad . \frac{e^{-(\|\mathbf{h}_1\|^2 + \cdots + \|\mathbf{h}_{L_T}\|^2)}}{\pi^{NL_T}}.
\end{aligned} \tag{8}
$$

The PEP in (3) can be upper bounded by averaging over the selected columns having the joint pdf in (8), that is,

$$
\begin{aligned}
P(\mathbf{S} \to \hat{\mathbf{S}}) &\le \sum_{l=1}^{L_T}\int_{\mathcal{R}_l} e^{-\frac{\rho}{4L_T}\|\mathbf{HB}\|^2}\left[1 - e^{-\|\mathbf{h}_l\|^2}\sum_{n=0}^{N-1}\frac{\|\mathbf{h}_l\|^{2n}}{n!}\right]^{M-L_T} \\
&\quad . \alpha \frac{e^{-\sum_{i=1}^{L_T}\|\mathbf{h}_i\|^2}}{\pi^{NL_T}} d\mathbf{h}_1 \cdots d\mathbf{h}_{L_T}.
\end{aligned} \tag{9}
$$

We can utilize the eigenvalue decomposition of $\mathbf{BB}^* = \mathbf{U\Lambda U}^*$ where $\mathbf{U}$ is a unitary matrix and $\mathbf{\Lambda}$ is a diagonal matrix with eigenvalues of $\mathbf{BB}^*$. Then, we note that

$$\|\mathbf{HB}\|^2 = tr\left((\mathbf{HU})\mathbf{\Lambda}(\mathbf{HU})^*\right) = \sum_{i=1}^{L_T} \lambda_i \|\mathbf{c}_i\|^2 \tag{10}$$

where $\mathbf{c}_i$ is the i^{th} column of $\mathbf{HU}$, and

$$\begin{aligned}\sum_{i=1}^{L_T} \|\mathbf{c}_i\|^2 &= tr\left((\mathbf{HU})(\mathbf{HU})^*\right) \\ &= tr\left(\mathbf{HUU}^*\mathbf{H}^*\right) \\ &= tr\left(\mathbf{HH}^*\right) \\ &= \sum_{i=1}^{L_T} \|\mathbf{h}_i\|^2. \end{aligned} \tag{11}$$

At this point, let us assume that we have a full-rank space-time code which means that all the eigenvalues of the matrix $\mathbf{BB}^*$ are positive (i.e., nonzero). Later in this section, we will also consider the rank-deficient STCs (some of the eigenvalues of $\mathbf{BB}^*$ being zeros) as well. We denote the minimum of $\lambda_1, ..., \lambda_{L_T}$ by $\hat{\lambda}$ and note that

$$\begin{aligned}\sum_{i=1}^{L_T} \lambda_i \|\mathbf{c}_i\|^2 &\geq \sum_{i=1}^{L_T} \hat{\lambda} \|\mathbf{c}_i\|^2 \\ &= \sum_{i=1}^{L_T} \hat{\lambda} \|\mathbf{h}_i\|^2. \end{aligned} \tag{12}$$

Hence, the upper bound on the PEP can further be upper bounded as

$$\begin{aligned} P(\mathbf{S} \to \hat{\mathbf{S}}) \leq& \sum_{l=1}^{L_T} \int_{\mathcal{R}_l} \alpha e^{-\frac{\rho}{4L_T}\sum_{i=1}^{L_T} \hat{\lambda}\|\mathbf{h}_i\|^2} \frac{e^{-\sum_{i=1}^{L_T}\|\mathbf{h}_i\|^2}}{\pi^{NL_T}} \\ &\cdot \left[1 - e^{-\|\mathbf{h}_l\|^2} \sum_{n=0}^{N-1} \frac{\|\mathbf{h}_l\|^{2n}}{n!}\right]^{M-L_T} \prod_{m=1}^{L_T} d\mathbf{h_m}. \end{aligned} \tag{13}$$

To simplify this expression further, we use the following result (as also used in [7])

$$g(v) = 1 - e^{-v} \sum_{n=0}^{N-1} \frac{v^n}{n!} \leq \frac{v^N}{N!} \tag{14}$$

for $v > 0$, and write an upper bound to the l^{th} term of the summation as

$$\begin{aligned} \mathcal{I}_l \leq& \alpha \int_{\mathcal{R}_l} e^{-\frac{\rho}{4L_T}\sum_{i=1}^{L_T} \hat{\lambda}\|\mathbf{h}_i\|^2} \frac{e^{-\sum_{i=1}^{L_T}\|\mathbf{h}_i\|^2}}{\pi^{NL_T}} \\ &\cdot \left[\frac{\|\mathbf{h}_l\|^{2N}}{N!}\right]^{M-L_T} \prod_{m=1}^{L_T} d\mathbf{h_m} \end{aligned} \tag{15}$$

where $\|\mathbf{h}_l\|^2 = \sum_{n=1}^{N} |h_{l,n}|^2$. By making the change of variables, $h_{l,n} = \sigma_{ln} e^{\theta_{ln}}$ and $u_{ln} = \sigma_{ln}^2$, and taking the integral over the entire space (as opposed to each region $\mathcal{R}_l$), we can further upper bound this quantity as

$$\begin{aligned} \mathcal{I}_l =& \alpha \int_0^\infty \cdots \int_0^\infty e^{-\left(1+\frac{\rho\hat{\lambda}}{4L_T}\right)\sum_{m=1}^{L_T}\sum_{n=1}^{N} u_{mn}} \\ &\cdot \left(\frac{(\sum_{n=1}^{N} u_{ln})^N}{N!}\right)^{M-L_T} \prod_{m=1}^{L_T}\prod_{n=1}^{N} du_{mn}. \end{aligned} \tag{16}$$

Let us write $\mathcal{I}_l$ as $\mathcal{I}_l \leq \mathcal{I}_l^{(1)} \mathcal{I}_l^{(2)}$ with

$$\mathcal{I}_l^{(1)} = \alpha \int_0^\infty \cdots \int_0^\infty e^{-\left(1+\frac{\rho\hat{\lambda}}{4L_T}\right)\sum_{m=1, m\neq l}^{L_T}\left(\sum_{n=1}^{N} u_{mn}\right)} \prod_{m=1, m\neq l}^{L_T}\prod_{n=1}^{N} du_{mn}$$

$$\mathcal{I}_l^{(2)} = \int_0^\infty \cdots \int_0^\infty e^{-\left(1+\frac{\rho\hat{\lambda}}{4L_T}\right)\sum_{n=1}^{N} u_{ln}} \left(\frac{\left(\sum_{n=1}^{N} u_{ln}\right)^N}{N!}\right)^{(M-L_T)} \prod_{n=1}^{N} du_{ln}.$$

Using $\int_0^\infty e^{-kx} dx = \frac{1}{k}$, we obtain

$$\mathcal{I}_l^{(1)} = \alpha \left(\frac{1}{\prod_{i=1, i\neq l}^{L_T}\left(1 + \frac{\rho\hat{\lambda}}{4L_T}\right)}\right)^N. \tag{17}$$

For $\mathcal{I}_l^{(2)}$ we first use $v_n = u_{ln}$ and note that

$$\left(\sum_{n=1}^{N} v_n\right)^{NM-NL_T} = \sum_{n_1=1}^{N} \cdots \sum_{n_{NM-NL_T}=1}^{N} v_{n_1} \cdots v_{n_{NM-NL_T}} \tag{18}$$

and $v_{n_1} \cdots v_{n_{NM-NL_T}} = \prod_{n=1}^{N} (v_n)^{l_n}$ such that

$$\sum_{n=1}^{N} l_n = NM - NL_T.$$

Then we can write $\mathcal{I}_l^{(2)}$ as,

$$\begin{aligned} \mathcal{I}_l^{(2)} =& \left(\frac{1}{N!}\right)^{M-L_T} \int_0^\infty \cdots \int_0^\infty e^{-\sum_{n=1}^{N}\left(\frac{\rho\hat{\lambda}}{4L_T}+1\right)v_n} \\ & \sum_{n_1=1}^{N} \cdots \sum_{n_{NM-NL_T}=1}^{N} (v_n)^{l_n} dv_1 \cdots dv_N. \end{aligned} \tag{19}$$

Changing the order of summation and integration and using

$$\int_0^\infty x^m e^{-ax} dx = \frac{m!}{a^{m+1}}$$

results in

$$\mathcal{I}_l^{(2)} = \left(\frac{1}{N!}\right)^{M-L_T} \sum_{n_1=1}^{N} \cdots \sum_{n_{NM-NL_T}=1}^{N} \prod_{i=1}^{N} \frac{l_i!}{\left(\frac{\rho\hat{\lambda}}{4L_T}+1\right)^{(l_i+1)}} \tag{20}$$

Finally, at high SNRs, from $\mathcal{I}_l^{(1)}$ and $\mathcal{I}_l^{(2)}$ we obtain,

$$\begin{aligned} P(\mathbf{S} \to \hat{\mathbf{S}}) \leq& \frac{\alpha}{(N!)^{M-L_T}} \left(\frac{1}{\hat{\lambda}^{NM}}\right) \\ & \left(\sum_{n_1=1}^{N} \cdots \sum_{n_{NM-NL_T}=1}^{N} l_1! \cdots l_N!\right) \left(\frac{\rho}{4L_T}\right)^{-MN} \end{aligned} \tag{21}$$

This is our main result which shows that a diversity order of MN (i.e., full diversity available in the system) is achieved. The coding gain depends on the eigenvalues of the square of the codeword difference matrix, $\mathbf{BB}^*$. Obviously, the coding gain with antenna selection will be lower than that of full-complexity system. When a full-rank STC is used, $\hat{\lambda}$ will be

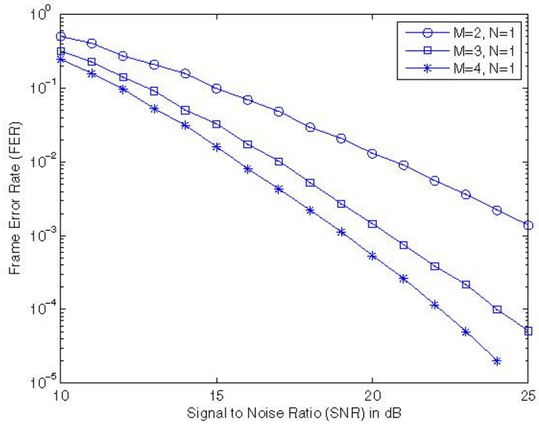

Fig. 2. FER for full rank 4 state STTC from [1] with transmit antenna selection.

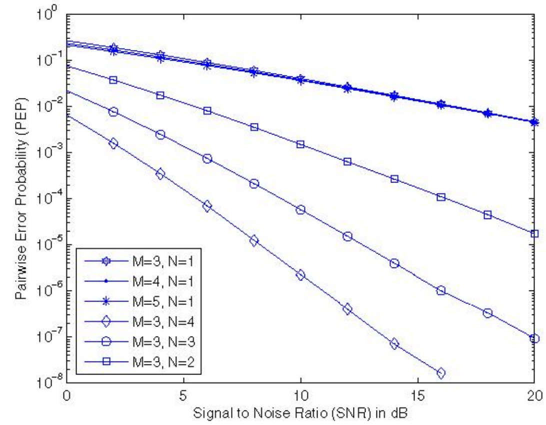

Fig. 3. PEP for rank-deficient STBC with transmit antenna selection.

nonzero and one way to design new codes suitable for transmit antenna selection would be maximizing the minimum value of $\hat{\lambda}$ of all codeword pairs.

Let us now provide several examples to illustrate the error rates of STC systems with transmit antenna selection. Figure 2 shows frame error rate (FER) plots for the M transmit and 1 receive antenna system (with $L_T = 2$) when the 4-state space-time trellis codes (STTC) from [1] with a frame length of 130 QPSK symbols are used. As seen from the plots, with no antenna selection, this full rank STTC achieves full space diversity of order 2 when $M = 2$ and $N = 1$. When the number of available transmit antennas is increased to $M = 3$ and $M = 4$, while still using $L_T = 2$ of them for transmission, the diversity order becomes 3 and 4, respectively.

B. Transmit Antenna Selection with Rank-Deficient Space-Time Codes

Until now, we considered the full-rank STCs and observed that they achieve space diversity of order MN. To complete the picture, in this section, we consider the performance of rank-deficient STCs with antenna selection.

For rank-deficient space-time codes, when $L_T > 1$ transmit antennas are selected, the derivation of the PEP will follow the same lines as full-rank codes ((9)-(20)). However, when rank-deficient space-time codes are used with the rank $q = rank(B) < L_T < M$, then $(L_T - q)$ eigenvalues (λ_i terms) will be zero. Therefore, $\mathcal{I}_l^{(1)}$ and $\mathcal{I}_l^{(2)}$ (expressions (17) and (20)) will be computed for only nonzero eigenvalues, where $\hat{\lambda}$ is the minimum of the nonzero eigenvalues. If the eigenvalue λ_l which corresponds to $i = l$ term in the overall $\mathcal{I}_l$ integral, is zero then the SNR term in $\mathcal{I}_l^{(2)}$ will disappear, on the other hand, SNR exponent in $\mathcal{I}_l^{(1)}$ will be Nq. If λ_l is nonzero then SNR exponent in $\mathcal{I}_l^{(2)}$ will be N while the exponent in $\mathcal{I}_l^{(1)}$ will be $N(q-1)$. From the summation of the SNR exponents for $\mathcal{I}_l^{(1)}$ and $\mathcal{I}_l^{(2)}$, we see that the diversity order for rank-deficient codes will be at least qN. We claim that this is the true diversity order as opposed to MN for full-rank codes. This is because, we can also derive a lower bound on the PEP that will result in the same diversity order. A similar argument is made in [7] for the case of receive antenna selection.

Let us now present several examples to verify our expectations. Figure 3 shows the PEP plots of the expression in (2) averaged over fading for the system with transmit antenna selection $L_T = 2$. We used an arbitrary codeword pair from space-time block codes [2] with 4 input QPSK symbols ($[1, j, -1, -j]$ and $[1, 1, 1, 1]$ where $j = \sqrt{-1}$). We observe that with this rank-deficient code with $q = 1$, the diversity order $qN = 1$ remains same for $N = 1$ and different numbers of available transmit antennas $M \in \{3, 4, 5\}$. The same rank-deficient codeword pair achieves diversity orders of 2, 3 and 4 when $M = 3, L_T = 2$ and N is 2, 3 and 4, respectively.

IV. Transmit Antenna Selection over Frequency Selective Fading Channels

In this section, we deal with transmit antenna selection over frequency-selective fading channels. Considering the channel and signal model described in Section II, it is clear that the MIMO FS fading channel with M antennas and D ISI taps can be considered as MIMO flat fading channel with MD virtual transmit antennas, thus, similar derivations can be performed for the FS fading channels as well. However, since the derivations of the joint pdf of the selected channel coefficients and the PEP bound are more complicated for this case, we only provide the expected diversity orders using the extensions of the basic arguments of the previous section.

Our claims are as follows. The diversity order for STCs with transmit antenna selection over FS fading channels will be MND if a full rank STC is used. If a STC with $q = rank(\mathbf{BB}^*) < MD$ is used, then similar to the flat fading case the diversity order for FS channel will be reduced to qN. We expect these claims to be valid regardless of the multipath delay profile of the underlying ISI channels, though the channel matrix could be easier to deal with for the uniform profile (as all the channel coefficients will be identically distributed). We do not have formal proofs of these claims, thus we resort to simulations to verify them.

We provide PEP plots of the expression in (2) for several STC systems with transmit antenna selection over FS fading

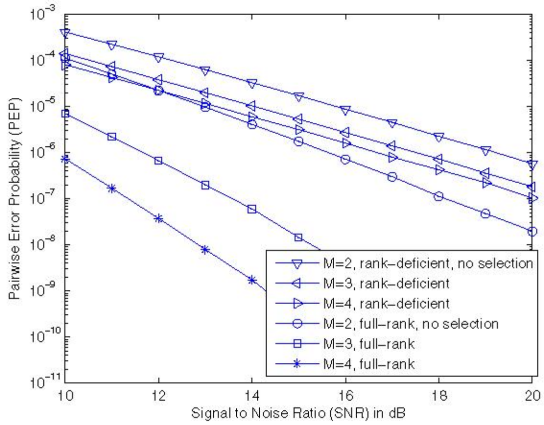

Fig. 4. PEP for full rank and rank deficient delay diversity STC with transmit antenna selection over FS fading channels.

channel in Figure 4. We use arbitrary codeword pairs from [20] with QPSK symbols and consider the MIMO systems with $N = 1, D = 2, L_T = 2$. The full rank codeword difference matrix used in the simulations is

$$\mathbf{B} = \begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 2 & 0 \\ 0 & 2 & 0 & 0 \end{pmatrix}.$$

For a full rank general delay diversity STC [20], the full rank STC achieves a diversity order of 4 when $M = 2$ with no antenna selection. With transmit antenna selection, $L_T = 2$, a diversity order of 6 is achieved when $M = 3$. Similarly, when $L_T = 2$ of $M = 4$ available transmit antennas are used, the diversity order becomes $MND = 8$. On the other hand, when a rank-deficient standard delay diversity STC [20] is used in $M = 2, D = 2, N = 1$ system with no antenna selection, the achieved diversity order is only 3, since the rank of the code is $q = 3$. When there are $M = 3$ or $M = 4$ available transmit antennas, using $L_T = 2$ of them results in the same diversity order of $qN = 3$ as expected. Having more transmit antennas only increases the coding gain.

V. Conclusion

In this paper, we studied the performance of STC-MIMO systems with transmit antenna selection over quasi-static fading channels. We considered transmit antenna selection based on the maximum received powers where only the receiver has knowledge of the channel state information. For flat fading channels, using pairwise error probability analysis and simulation results, we demonstrated that by employing antenna selection one can still achieve full available diversity provided that the underlying STC is full-rank. With rank-deficient STCs, the diversity order depends on the rank of the codeword difference matrix and the number of receive antennas. Based on our results for the transmit antenna selection schemes over flat fading channels, we have commented on the diversity orders of the STC-MIMO systems with transmit antenna selection over frequency-selective fading channels, and verified our expectations using simulations.

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