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Space-Time Coded Systems with Joint Transmit and Receive Antenna Selection

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***Abstract*— This paper studies performance of space-time coded (STC) systems with joint transmit and receive antenna selection over multiple input multiple output (MIMO) flat and frequency-selective (FS) fading channels. Specifically, we first perform a pairwise error probability analysis over flat fading channels explicitly. Then, we comment on our expectations for the case of FS fading channels. We show that the joint transmit and receive antenna selection based on received power levels does not degrade the achievable diversity order when full rank STCs are employed. Simulation results are provided to verify our theoretical results for both full rank and rank-deficient codes.**

I. Introduction

Due to dramatically increased capacities promised by multi-input multi-output (MIMO) systems [1], [2], space-time coding (STC) techniques have recently become very popular [3], [4] as a means to exploit these capabilities, and to achieve low error rates and spatial diversity. A major concern for practical MIMO systems is the high cost of implementing multiple transceiver circuits. Due to the small size and low power requirements of mobile devices, it is a significant challenge to design many radio frequency (RF) parts (amplifiers, filters, converters, etc.) while achieving proper isolation and satisfactory performance. With the motivation of reducing the hardware complexity of MIMO systems, "antenna selection" [5] techniques have become highly desirable. When antenna selection is employed, a limited number of costly RF chains can be switched adaptively to a subset of many low-cost antennas, thus providing us with the benefits of spatial diversity with reduced cost.

Antenna selection schemes have been extensively studied in the literature. Antenna selection algorithms/analysis based on the minimization of error probability or maximization of channel capacity are provided in [6] and [7], respectively. Transmit antenna selection for uncoded spatial multiplexing systems is considered and several selection algorithms are proposed in [8], [9]. Performance of general STC systems with receive antenna selection which maximizes the received signal-to-noise ratio (SNR) is studied [10], [11], where it is shown that receive antenna selection can achieve full spatial diversity available provided the underlying space time code is full rank. Space-time trellis codes and space-time block codes with transmit antenna selection are shown to achieve full available diversity using simulations in [12], [13] without a thorough analysis for general STCs. For MIMO frequency selective fading channels, the amount of existing work on antenna selection is very limited. Space time block coding (STBC) [4] with antenna selection over FS fading channels have been studied in [14] and [15], however no error probability analysis is performed. For general STCs, receive antenna selection for MIMO frequency selective (FS) channels have been considered in [16]. To the best of our knowledge, in the literature, no results are available for the error probability of STCs with joint transmit and receive antenna selection in a general setting either for MIMO flat or FS fading channels until now.

In this paper, we consider joint selection of subset of receive antennas and transmit antennas in space-time coded MIMO communications, and study the achievable diversity orders. We assume that the antenna selection is based on the received SNRs, and there is a noiseless feedback from the receiver to the transmitter indicating which transmit antennas are to be employed. We first derive an upper bound for the pairwise error probability for space-time coding over MIMO flat fading channels. Then, based on our error probability derivations for the flat fading channels, we present the expected diversity orders for STC systems with joint transmit/receive selections over FS fading channels and verify them using simulations. We show that for full-rank space-time codes, joint transmit and receive antenna selection does not degrade the diversity gain compared to that of the full complexity system. Furthermore, we reveal that if the code does not achieve full diversity in full-complexity system, then performing antenna selection results in a loss of diversity order.

The paper is organized as follows: Section II describes the system model. Section III provides the pairwise-error probability (PEP) analysis for the STC systems with joint transmit and receive antenna selection over flat fading channels. Section IV discusses possible extensions to MIMO FS fading channels. Finally, Section V concludes the paper.

II. System Description

In this section, we provide the system model for STC systems over quasi-static flat fading channels. Figure 1 shows the $M \times N$ STC system with antenna selection at the transmitter and receiver. The channel is modeled as a quasi-static MIMO

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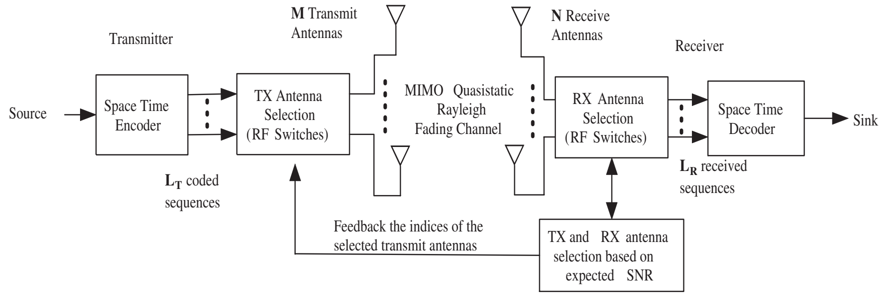

Fig. 1. Space-time coded MIMO system with joint transmit and receive antenna selection.

Rayleigh fading channel where the different sub-channels fade independently. We assume that the receiver knows the exact channel state information (CSI), i.e., the fading coefficients, however, the transmitter does not have access to this. At each frame, the received SNRs for joint transmit and receive antenna combinations can be obtained, and then the selection of L_T out of M transmit and L_R out of N receive antennas is done based on the largest SNRs. The receiver feeds back the indices of the best set of L_T antennas to the transmitter which requires at most M bits. The information sequence is encoded by the space-time encoder and then transmitted through the selected L_T antennas simultaneously. At the receiver, space-time decoding is performed using the signals from the selected L_R antennas.

For general STC-MIMO systems over flat fading channels, the received signal at the receive antenna n at time k, can be written as

$$y_n(k) = \sqrt{\frac{\rho}{L_T}} \sum_{m=1}^{L_T} h_{m,n} s_m(k) + w_n(k) \tag{1}$$

where $s_m(k)$ is the transmitted symbols from antenna m at time k, $h_{m,n}$ is the fading coefficient between transmit antenna m and receive antenna n, (which is assumed to remain constant for an entire frame, i.e., quasi-static fading), and $w_n(k)$ is the noise sample at the receive antenna n at time k, $(k = 1, \cdots, K)$. $h_{m,n}$ and $w_n(k)$ are i.i.d. circularly symmetric complex Gaussian random variables having zero mean and variance $1/2$ per dimension. ρ is the expected SNR at each receive antenna. After the antennas are selected, the received signals at all antennas can be stacked in matrix form as

$$\mathbf{Y} = \sqrt{\frac{\rho}{L_T}} \hat{\mathbf{H}} \mathbf{S} + \mathbf{W} \tag{2}$$

where the $L_R \times L_T$ channel coefficient matrix

$$\hat{\mathbf{H}} = \begin{pmatrix} h_{1,1} & \dots & h_{L_T,1} \\ \vdots & \ddots & \vdots \\ h_{1,L_R} & \dots & h_{L_T,L_R} \end{pmatrix},$$

is obtained from the original $N \times M$ channel coefficient matrix $\mathbf{H}$. The $L_T \times K$ codeword matrix $\mathbf{S}$ is

$$\mathbf{S} = \begin{pmatrix} s_1(1) & \dots & s_1(K) \\ \vdots & \ddots & \vdots \\ s_{L_T}(1) & \dots & s_{L_T}(K) \end{pmatrix} \tag{3}$$

and the $L_R \times K$ noise matrix $\mathbf{W}$ contains the noise samples, $w_n(k)$.

When the channel state information is known at the receiver, the PEP conditioned on the instantaneous CSI is the same as the one for the case of an AWGN channel. As in [10] for any given $\hat{\mathbf{H}}$, the PEP of erroneously receiving $\widehat{\mathbf{S}}$, when $\mathbf{S}$ was transmitted, is given by

$$P(\mathbf{S} \rightarrow \widehat{\mathbf{S}} | \hat{\mathbf{H}}) = \frac{1}{2} erfc \left(\sqrt{\frac{\rho}{4L_T}} \|\hat{\mathbf{H}}\mathbf{B}\| \right) \tag{4}$$

which can be upper bounded as

$$P(\mathbf{S} \rightarrow \widehat{\mathbf{S}} | \hat{\mathbf{H}}) \leq \exp \left(-\frac{\rho}{4L_T} \|\hat{\mathbf{H}}\mathbf{B}\|^2 \right) \tag{5}$$

where $\mathbf{B} = \mathbf{S} - \widehat{\mathbf{S}}$ is the codeword difference matrix. $\|.\|^2$ represents the Frobenius norm of a matrix, i.e., sum of magnitude squares of all entries. In order to find the unconditional PEP bound, we simply average the quantity in (5) over the fading statistics.

III. Joint Transmit and Receive Antenna Selection over Flat Fading Channels

In this section, we investigate the diversity order of STCs over flat fading channels with antenna selection both at the transmitter and the receiver. For analytical tractability, we assume that first the subset of receive antennas resulting in maximum SNR, and then the corresponding transmit antennas are selected. We note that this two step selection process (first the receive antennas, and then the transmit antennas) is in fact sub-optimal, i.e., it may not result in the optimal set of selected antennas that maximize the SNRs. However, it is practical since it can be done in faster selection by eliminating the search of all possible transmit and receive antenna combinations. Furthermore, as we will see later, it will not result in a degradation of the diversity order (under some conditions), thus it gives a near optimal performance.

Since derivation of the probability density function (pdf) of the selected channel coefficients is complicated for the general case, we first study a special case, and then extend our results.

A. *Selecting* 2×1 *from a* 3×2 *System*

We now consider a special case where there are $M = 3$ transmit antennas and $N = 2$ receive antennas. Our goal is to select two transmit antennas ($L_T = 2$), and one receive antenna ($L_R = 1$). We start with the selection of the row, $\mathbf{r}$, of $\mathbf{H}$ with the largest norm (or, SNR). Joint pdf of the selected row [10] is given by

$$f_{\mathbf{R}}(\mathbf{r}) = N\left(1 - e^{-\|\mathbf{r}\|^2}\sum_{m=0}^{M-1}\frac{\|\mathbf{r}\|^{2m}}{m!}\right)^{N-1}\frac{1}{\pi^M}e^{-\|\mathbf{r}\|^2} \quad (6)$$

where $\mathbf{r} = [c_1, c_2, c_3]$ containing 3 complex channel coefficients and $\mathbf{R}$ denotes the random vector with realization $\mathbf{r}$. With the definition $v_i = |c_i|^2$, $1 \leq i \leq 3$, and $v_{123} = v_1 + v_2 + v_3$, the above pdf can be written as

$$f_{\hat{C}_1,\hat{C}_2,\hat{C}_3}(c_1, c_2, c_3) = 2\left(1 - e^{-v_{123}}\left(1 + v_{123} + \frac{v_{123}^2}{2}\right)\right)\frac{1}{\pi^3}e^{-v_{123}}, \quad (7)$$

where $\hat{C}_i$ is the random variable with realization c_i. After selecting one row with 3 entries, we select 2 channel coefficients with the largest norms. The resulting joint pdf is then

$$f_{\hat{C}_1,\hat{C}_2}(c_1, c_2) = f_{C_1,C_2}(c_1, c_2|G) \quad (8)$$

where the event G is defined as the first two elements, C_1, C_2, having the largest norms. Using Bayes' rule, we get

$$f_{\hat{C}_1,\hat{C}_2}(c_1, c_2) = \frac{P(G|C_1 = c_1, C_2 = c_2)f_{C_1,C_2}(c_1, c_2)}{P(G)} \quad (9)$$

where $f_{C_1,C_2}(c_1, c_2)$ is the joint pdf of any two elements in the selected row and can be obtained as

$$f_{C_1,C_2}(c_1, c_2) = \int f_{C_1,C_2,C_3}(c_1, c_2, c_3)dc_3. \quad (10)$$

After using the pdf expression in (7) and taking the two dimensional integral with respect to angle and the magnitude of the complex number c_3, and defining $v_{12} = v_1 + v_2$, we obtain $f_{C_1,C_2}(c_1, c_2)$ as

$$f_{C_1,C_2}(c_1, c_2) = (\frac{1}{\pi^2})\left(2e^{-v_{12}} - e^{-2v_{12}}\left(\frac{7}{4} + \frac{3}{2}v_{12} + \frac{1}{2}v_{12}^2\right)\right). \quad (11)$$

The other term in (9) can be written as

$$P(G|C_1 = c_1, C_2 = c_2) = P(|c_3|^2 < |c_m|^2) \quad (12)$$

where $|c_m|^2 = min(|c_1|^2, |c_2|^2)$. With further simplification,

$$P(G|C_1 = c_1, C_2 = c_2) = P(|c_3| < |c_m|) = P(\sigma_3 < \sigma_m) \quad (13)$$

where $\sigma_3 = |c_3|$ and $\sigma_m = |c_m|$ and finally

$$P(G|C_1 = c_1, C_2 = c_2) = \int_0^{2\pi}\int_0^{\sigma_m} f_{C_3|C_1,C_2}(c_3|c_1, c_2)dc_3 \quad (14)$$

where we use $dc_3 = \sigma_3 d\sigma_3 d\theta_3$ for complex integration and $f_{C_3|C_1,C_2}(c_3|c_1, c_2)$ is the pdf of the third entry of the selected row when the first two entries $C_1 = c_1, C_2 = c_2$ are known. Using Bayes' rule, the conditional pdf can be written as

$$f_{C_3|C_1,C_2}(c_3|c_1, c_2) = \frac{f_{C_3,C_1,C_2}(c_3, c_1, c_2)}{f_{C_1,C_2}(c_1, c_2)}. \quad (15)$$

Since v_i and $|c_i|^2$ is interchangeable, for brevity of expressions, we note that

$$f_{\hat{C}_1,\hat{C}_2}(c_1, c_2) = \frac{1}{P(G)}\int_0^{v_m} f_{C_1,C_2,C_3}(v_1, v_2, v_3)dv_3. \quad (16)$$

Then, the final pdf of the selected two entries will be

$$\begin{aligned} f_{\hat{C}_1,\hat{C}_2}(c_1, c_2) = {} & (\frac{3}{\pi^2})e^{-2(v_{12}+v_m)}\left(\frac{7}{4} + \frac{3}{2}(v_{12} + v_m) + \frac{1}{2}(v_{12} + v_m)^2\right) \\ & + (\frac{3}{\pi^2})\left(2e^{-v_{12}} - e^{-2v_{12}}\left(\frac{7}{4} + \frac{3}{2}v_{12} + \frac{1}{2}v_{12}^2\right)\right) \\ & + \left(-\frac{6}{\pi^2}\right)e^{-(v_{12}+v_m)}. \end{aligned} \quad (17)$$

With the new channel matrix $\hat{\mathbf{H}}$ obtained from $\mathbf{H}$,

$$P(\mathbf{S} \to \widehat{\mathbf{S}}) \leq \int\int \exp\left(-\frac{\rho}{8}\|\hat{\mathbf{H}}\mathbf{B}\|^2\right) f_{\hat{C}_1,\hat{C}_2}(c_1, c_2)dc_1dc_2. \quad (18)$$

We can utilize eigenvalue decomposition of $\mathbf{BB}^* = \mathbf{U\Lambda U}^*$ where $\mathbf{U}$ is a unitary matrix and $\mathbf{\Lambda}$ is a diagonal matrix with eigenvalues of $\mathbf{B}$. Then, we note that

$$\|\hat{\mathbf{H}}\mathbf{B}\|^2 = tr\left((\hat{\mathbf{H}}\mathbf{U})\mathbf{\Lambda}(\hat{\mathbf{H}}\mathbf{U})^*\right) = \sum_{i=1}^{L_T}\lambda_i\|\mathbf{c}_i\|^2 \quad (19)$$

where $\mathbf{c}_i$ is the i^{th} column of $\hat{\mathbf{H}}\mathbf{U}$, and

$$\begin{aligned} \sum_{i=1}^{L_T}\|\mathbf{c}_i\|^2 &= tr\left((\hat{\mathbf{H}}\mathbf{U})(\hat{\mathbf{H}}\mathbf{U})^*\right) \\ &= tr\left(\hat{\mathbf{H}}\mathbf{U}\mathbf{U}^*\hat{\mathbf{H}}^*\right) \\ &= tr\left(\hat{\mathbf{H}}\hat{\mathbf{H}}^*\right) \\ &= \sum_{i=1}^{L_T}\|\mathbf{h}_i\|^2. \end{aligned} \quad (20)$$

In this derivation, we assume that we have a full-rank space-time code which means that the eigenvalues of the matrix $\mathbf{BB}^*$ are all positive (i.e., nonzero). We denote the minimum of $\lambda_1, ..., \lambda_{L_T}$ by $\hat{\lambda}$ and note that

$$\begin{aligned} \sum_{i=1}^{L_T}\lambda_i\|\mathbf{c_i}\|^2 &\geq \sum_{i=1}^{L_T}\hat{\lambda}\|\mathbf{c_i}\|^2 \\ &= \sum_{i=1}^{L_T}\hat{\lambda}\|\mathbf{h_i}\|^2. \end{aligned} \quad (21)$$

Using the derived pdf in (17), and the results from (19) and (21), the PEP bound can be written as

$$P(\mathbf{S} \to \widehat{\mathbf{S}}) \leq \frac{512 + 752\alpha + 52\alpha^2 + 346\alpha^3}{(\alpha + 4)^3(\alpha + 2)^4(\alpha + 1)^2} \approx 346 \times \alpha^{-6} \quad (22)$$

where $\alpha = (\rho/8)\hat{\lambda}$ and the last approximation is obtained by considering large SNRs. From this PEP bound, we observe that the maximum diversity order which is the exponent of the SNR term, ρ, is $MN = 6$, i.e., full spatial diversity is achieved with the use of a full rank STC. We note that in the PEP derivation for this special case we used numerical methods, however, when the number of antennas is increased, available numerical tools may not be helpful, thus we would like to derive other analytical PEP bounds as done in the following.

B. Selecting $L_T \times 1$ Antennas from an $M \times N$ System

Having studied a special case of joint transmit and receive antenna selection, we now consider selecting L_T of M transmit and only one of N receive antennas. The joint pdf of the row with the largest norm (or SNR), which is selected from $N \times M$ channel matrix $\mathbf{H}$, is

$$f_{\mathbf{R}}(\mathbf{r}) = N\left(1 - e^{-\|\mathbf{r}\|^2}\sum_{m=0}^{M-1}\frac{\|\mathbf{r}\|^{2m}}{m!}\right)^{(N-1)}\frac{1}{\pi^M}e^{-\|\mathbf{r}\|^2} \quad (23)$$

where $\mathbf{r} = [c_1, c_2, \ldots, c_M]$ containing M complex channel coefficients. With the definition $v_i = |c_i|^2$, $1 \leq i \leq M$ and $w = v_1 + v_2 + \ldots + v_M$ the pdf can be written as

$$f_{\hat{C}_1,\ldots,\hat{C}_M}(c_1,\ldots,c_M) = N\left(1 - e^{-w}\sum_{m=0}^{M-1}\frac{w^m}{m!}\right)^{(N-1)}\frac{1}{\pi^M}e^{-w}. \quad (24)$$

After finding the pdf for selected row which corresponds to channel coefficients for the selected receive antenna, we now select the L_T transmit antennas. For simplicity, we call the pdf for the largest L_T elements of the selected row $f_{\hat{C}_1,\hat{C}_2,\ldots,\hat{C}_{L_T}}(c_1, c_2, \ldots, c_{L_T})$ as f, then by extending the derived pdf (16), f can be written as

$$\begin{aligned} f &= \frac{N.M!}{(M-2)!2!}\sum_{p=1}^{L_T}\int_0^{v_p}\cdots\int_0^{v_p}\left(1 - e^{-w}\sum_{m=0}^{2}\frac{w^m}{m!}\right)^{(N-1)} \\ &\quad \frac{1}{\pi^{L_T}}e^{-w}I_{R_p}(v_1,\cdots,v_{L_T})\prod_{m=L_T+1}^{M}dv_m \end{aligned} \quad (25)$$

where $I_{R_p}(v_1, \cdots, v_{L_T})$ is the indicator function which is 1 if and only if the v_p is the minimum of all v_i for $1 \leq i \leq L_T$, otherwise, $I_{R_p}(v_1, \cdots, v_{L_T}) = 0$. We use the following result as in [10]

$$g(v) = 1 - e^{-v}\sum_{n=0}^{N-1}\frac{v^n}{n!} \leq \frac{v^N}{N!} \quad (26)$$

for $v > 0$, and thus, f can be written as

$$\begin{aligned} f &\leq \frac{N.M!}{(M-2)!2!}\sum_{p=1}^{L_T}\int_0^{v_p}\cdots\int_0^{v_p}\left(\frac{w^M}{M!}\right)^{(N-1)}\frac{1}{\pi^{L_T}} \\ &\quad e^{-w}I_{R_p}(v_1,\cdots,v_{L_T})\prod_{m=L_T+1}^{M}dv_m \end{aligned} \quad (27)$$

Similar to [10], we note that

$$\begin{aligned} (v_1 + \cdots + v_M)^{M(N-1)} &= \left(\sum_{i=1}^{M}v_i\right)^{M(N-1)} \\ &= \sum_{i_1=1}^{M}\cdots\sum_{i_M=1}^{M}v_{i_1}\cdots v_{i_{M(N-1)}} \end{aligned} \quad (28)$$

where the indexes i_k in v_{i_k}, $k \in \{1, \cdots, M(N-1)\}$, take values from the set $\mathcal{J} = \{1, \cdots, M\}$. Assume the subscript index j appears l_j times among the subscripts of the term $v_{i_1}\cdots v_{i_M}$. Then,

$$\begin{aligned} v_{i_1}\cdots v_{i_M} &= \prod_{k=1}^{M}v_{i_k} \\ &= \prod_{j=1}^{M}(v_j)^{l_j} \end{aligned} \quad (29)$$

such that $\sum_{j=1}^{M} l_j = M(N-1)$. Therefore, we can use

$$(v_1 + \cdots + v_M)^{M(N-1)} = \sum_{i_1=1}^{M}\cdots\sum_{i_M=1}^{M}\prod_{j=1}^{M}(v_j)^{l_j} \quad (30)$$

to obtain

$$\begin{aligned} f &\leq \beta\sum_{i_1=1}^{M}\cdots\sum_{i_{M(N-1)}=1}^{M}\sum_{p=1}^{L_T}\int_0^{v_p}\cdots\int_0^{v_p}\prod_{j=1}^{M}(v_j)^{l_j}e^{-(v_j)} \\ &= I_{R_p}(v_1,\cdots,v_{L_T})\prod_{m=L_T+1}^{M}dv_m \end{aligned} \quad (31)$$

where $\beta = \left(\frac{\frac{N.M!}{(M-2)!2!}}{(M!)^{(N-1)}}\right)$. Each integral can be written as follows

$$\begin{aligned} \int_0^{v_p}v_j^{l_j}e^{-v_j}dv_j &= l_j!\left[-e^{-v_p}\sum_{k=0}^{l_j}\frac{v_p^{(l_j-k)}}{(l_j-k)!}\right] \\ &= l_j!\left[-e^{-v_p}\sum_{m=0}^{l_j}\frac{v_p^m}{m!}\right] \leq l_j!\frac{v_p^{(l_j+1)}}{(l_j+1)!} \end{aligned} \quad (32)$$

where we used $m = l_j - k$ and the simplification in (26) for simpler expressions, and then pdf becomes

$$\begin{aligned} f &\leq \beta\sum_{i_1=1}^{M}\cdots\sum_{i_{M(N-1)}=1}^{M}\sum_{p=1}^{L_T}\prod_{k=1}^{L_T}e^{-v_k}(v_k)^{l_k} \\ &\quad \prod_{j=L_T+1}^{M}\frac{v_p^{(l_j+1)}}{(l_j+1)}I_{R_p}(v_1,\cdots,v_{L_T}). \end{aligned} \quad (33)$$

Using this pdf and similar to PEP derived for the special case, and by integrating over $0 \leq v_i \leq \infty$ region instead of integrating in region defined by I_{R_p}, which further loosens the upper bound of the PEP, we obtain

$$\begin{aligned} P(\mathbf{S}\to\hat{\mathbf{S}}) &\leq \int_0^{\infty}\cdots\int_0^{\infty}e^{-\gamma(v_1+\ldots+v_{L_T})}\beta\sum_{i_1=1}^{M}\cdots\sum_{i_{M(N-1)}=1}^{M} \\ &\quad \cdot\sum_{p=1}^{L_T}\prod_{j=L_T+1}^{M}\frac{v_p^{(l_j+1)}}{(l_j+1)}\prod_{k=1}^{L_T}e^{-v_k}(v_k)^{l_k}\prod_{k=1}^{L_T}dv_k \end{aligned} \quad (34)$$

where $\gamma = \frac{\rho}{4\times L_T}\hat{\lambda}$ and the integration with respect to angle canceled out the π^{L_T} term. After interchanging the summation and integration, and then, using $\int_0^{\infty}x^m e^{-\gamma x}dx = \frac{m!}{\gamma^{(m+1)}}$ for each integral with respect to each dv_i, we finally obtain

$$\begin{aligned} P(\mathbf{S}\to\hat{\mathbf{S}}) &\leq \sum_{i_1=1}^{M}\cdots\sum_{i_M=1}^{M}\left(\beta\prod_{j=L_T+1}^{M}\frac{1}{(l_j+1)}\right) \\ &\quad \cdot\frac{\left(l_p + (\sum_{j=L_T+1}^{M}(l_j+1))\right)!}{(\gamma+1)^{\left(l_p+1+(\sum_{j=L_T+1}^{M}(l_j+1))\right)}} \\ &\quad \cdot\prod_{k=1,k\neq p}^{L_T}\frac{l_k!}{(\gamma+1)^{(l_k+1)}}. \end{aligned} \quad (35)$$

We observe that the exponent of γ, and thus the exponent of SNR term ρ, will be $\sum_{j=1}^{M}(l_j+1) = MN$ which shows that full diversity is achieved even though only L_T transmit antennas and only a single receive antenna are used. This PEP expression can be used to design new STCs with joint transmit and receive antenna selection. In addition, we also note that a similar PEP derivation can be done for selecting 1 of M transmit antenna first then selecting L_R of N receive antennas.

Due to the complexity in deriving the joint pdf of selected channel coefficients for joint transmit antenna selection scheme for arbitrary number of antennas $L_T > 1$ and $L_R > 1$, we omit derivation of the PEP bound for this case. Clearly, since we have shown that full diversity is achievable with $L_R = 1$, selecting multiple receive antennas will also achieve full spatial diversity if a full-rank STC is used.

C. Joint Transmit and Receive Antenna Selection with Rank-Deficient Space-Time Codes

Until now, we considered the full-rank STCs and observed that they will achieve space diversity MN. To complete the picture, we consider the performance of rank deficient STCs with antenna selection.

For the PEP expression in (35) which is derived for $L_R = 1$, we note that if the underlying space-time code was a rank-deficient code with rank $q < L_T$, then the diversity order would be q since $N-q$ SNR terms would disappear due to zero eigenvalues. Although we do not have a PEP expression for arbitrary $L_T > 1$ and $L_R > 1$, considering the diversity results for transmit and receive antenna selection with rank deficient codes individually, we can expect that the diversity order for rank-deficient STCs in general case of selecting $L_T \times L_R$ from $M \times N$ system to be qL_R. This expectation is verified with numerical examples.

In order to verify our theoretical findings, Figure 2 shows the PEP plots for the system with several M, N, L_Rs. We used an arbitrary codeword pair from space-time block codes [4] with 4 input QPSK symbols ($[1, j, -1, -j]$ and $[1, 1, 1, 1]$ where $j = \sqrt{-1}$). Let us first consider a rank deficient STBC. With no antenna selection, this code achieves the diversity order of $q = 1$ for $M = 2$, $N = 1$ system although the available space diversity is 2. When $M = 3, N = 2$, or $M = 3, N = 3$ the diversity order remains as $qL_R = 1$. For $M = 3, N = 4$ system, if $L_R = 2$ receive antennas are used, then the diversity order becomes $qL_R = 2$. Similarly, if $L_R = 3$ receive antennas are used, then the diversity order becomes $qL_R = 3$. We also provide the PEP for a full rank STBC ($q = L_T = 2$) for comparison with the rank-deficient code. This code achieves the full diversity order of $MN = 6$ for $M = 3, N = 2$, although only 2 transmit and 1 receive antennas are used.

IV. Joint Transmit and Receive Antenna Selection over Frequency Selective Fading Channels

In this section, we deal with antenna selection over FS fading channels. For a general STC-MIMO system over FS fading channel having D intersymbol interference (ISI) taps, the received signal at the receive antenna n at time k can be written as

$$y_n(k) = \sqrt{\frac{\rho}{L_T D}} \sum_{d=0}^{D-1} \sum_{m=1}^{L_T} h_{m,n}^d s_m(k-d) + w_n(k), \qquad (36)$$

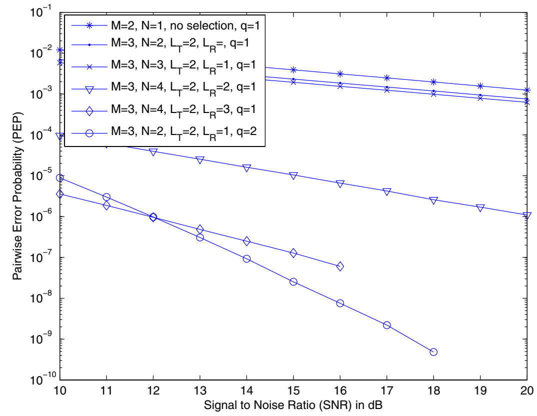

Fig. 2. PEP for STBC with joint transmit/receive antenna selection over flat fading channels, $D = 1$.

where $h_{m,n}^d$ is the fading coefficient between transmit antenna m and receive antenna n for the d^{th} ISI tap. We can still represent the received signals in matrix form as $\mathbf{Y} = \sqrt{\frac{\rho}{L_T D}} \hat{\mathbf{H}}\mathbf{S} + \mathbf{W}$ with the $L_R \times L_T D$ channel coefficient matrix $\hat{\mathbf{H}}$ (obtained from original $N \times MD$ channel matrix $\mathbf{H}$) as

$$\hat{\mathbf{H}} = \begin{pmatrix} h_{1,1}^0 & \dots & h_{1,1}^{D-1} & \dots & h_{L_T,1}^0 & \dots & h_{L_T,1}^{D-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ h_{1,L_R}^0 & \dots & h_{1,L_R}^{D-1} & \dots & h_{L_T,L_R}^0 & \dots & h_{L_T,L_R}^{D-1} \end{pmatrix},$$

Considering the channel and signal model, the MIMO FS fading channel with L_T antennas and D ISI taps can be considered as MIMO flat fading channel with $L_T D$ virtual transmit antennas, thus, similar proofs can be performed for the FS fading channels as well. However, since the pdf and PEP derivations are much more complicated, we only provide our expectation on the diversity orders using the extensions of the basic arguments of the previous section. We claim that for a STC-MIMO system with joint transmit and receive antenna selection over FS fading channels, the diversity order will be MND when a full rank STC is used and qL_R when a rank-deficient STC is used. We resort to simulations in order to verify these claims. Figure 3 shows the exact PEP plots for the system with $D = 2, L_T = 2, L_R = 1$. We use arbitrary codeword pairs from [17] with QPSK symbols and consider the MIMO systems with $N = 1, D = 2, L_T = 2$. The full rank codeword difference matrix used in the simulations is

$$\mathbf{B} = \begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 2 & 0 \\ 0 & 2 & 0 & 0 \end{pmatrix}.$$

With no antenna selection $M = 2, N = 1$, the scheme achieves a diversity order of 4. With joint transmit/receive antenna selection, the diversity order increases to 12 when $M = 2, N = 2$, and 18 when $M = 2, N = 3$. Figure 4 shows the PEP plots for the above system with a pair of codewords with rank deficient codeword difference matrix using [17]. We observe that when $M = 2, N = 1, D = 2$, and with no antenna selection, a diversity order $q = 3$ is achieved while the available space diversity is 4. When $M = 3, N = 2$

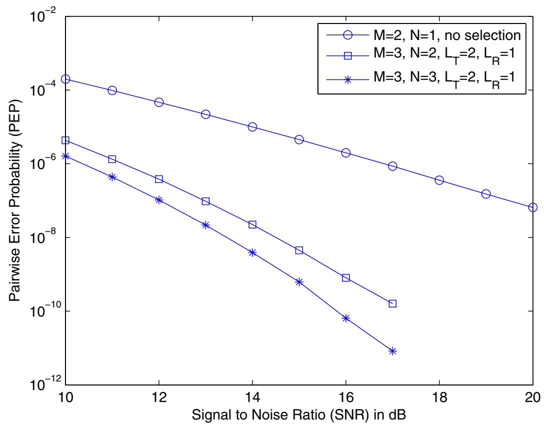

Fig. 3. PEP for full rank general delay diversity STC with joint transmit and receive antenna selection, $D = 2$.

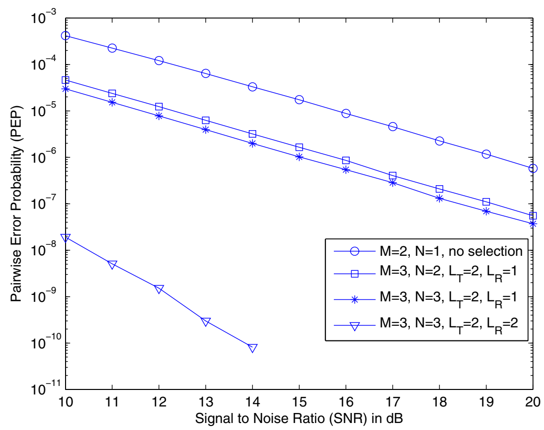

Fig. 4. PEP for rank deficient delay diversity STC with joint transmit and receive antenna selection, $D = 2$.

or when $M = 3, N = 3$, using only $L_T = 2$ transmit and $L_R = 1$ of them results in the same diversity order of $qL_R = 3$ as expected. However, when $M = 3, L_T = 2$ and $N = 3, L_R = 2$, the diversity order becomes $qL_R = 6$ which verifies our claims.

V. Conclusion

We have considered the joint selection of the transmit and receive antennas in a MIMO system in an attempt to reduce the required hardware complexity for both frequency flat and frequency-selective fading channels. We assume that the selection is based on the received signal to noise ratios. By performing appropriate pairwise error probability analysis, we have shown that if the underlying code is full rank, then there is no loss of diversity with joint transmit and receive antenna selection. This is true for both flat and frequency-selective fading channels, although for the case of frequency-selective fading channels we do not have formal proofs, and we resort to simulation results. However, if the code is not full rank, then the diversity order achieved reduces, and it depends on the number of selected receive antennas and the rank of the codeword difference matrix.

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