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Iterative Channel Estimation Techniques for Uplink MC-CDMA Systems

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***Abstract*— *In this work, maximum likelihood (ML) channel estimation for uplink multicarrier code-division multiple-access (MC-CDMA) systems is considered in the presence of frequency fading channel. The expectation-maximization (EM)- and a space-alternating generalized expectation-maximization (SAGE) algorithm are introduced to avoid matrix inversion for the ML channel estimation problem. We compare the both algorithms in terms of the number of used iteration and show that the proposed algorithms converge the same performance of the ML estimator as the increasing number of iterations.*

Index Terms: MC-CDMA Systems, Channel estimation, Maximum Likelihood, Least Square, EM, SAGE.

I. Introduction

MC-CDMA scheme has gained considerable interest for Beyond 3G (B3G) mobile communication systems because it marries the best of the OFDM and CDMA worlds and consequently, it can supply good performance in frequency selective channels [1]. To evaluate the performance of these systems, ideal knowledge of transmission parameters is often assumed known. Since the channel information is required by the equalization algorithm, channel estimation is a crucial part of the receiver structure[2]. For the uplink problem, since the received signals are a superposition of signals transmitted from different user antennas, the simple channel estimation techniques used in single user systems cannot be used. Therefore channel estimation problem is a critical issue as well as detection of data symbols transmitted by users at the base station. Recently, receivers that use separate detection and estimation (SDE) and joint estimation and detection (JDE) methods have been investigated [3],[4]. In these works, it was also shown that channel estimation is a crucial part of the receiver and matrix inversion is necessary to estimate channel variations. Moreover, matrix inversion complexity increases by the number of active user and length of the channel.

The expectation-maximization (EM) algorithm [5] is an iterative approach which converges the maximum-likelihood (ML) estimate whose direct calculation is computationally prohibitive. Several iterative procedure based on EM algorithm were applied to channel estimation problem [6], [7]. EM is also considered for the channel estimation in OFDM receivers and compared with SAGE version [8]. For CDMA systems, Nelson and Poor [9] extended the EM and SAGE algorithms for detection rather than for estimation of continues parameters. Moreover, EM and SAGE based iterative receiver structures of tractable complexity for JDE of direct-sequence code-division multiple-access (DS-CDMA) signals were presented[10]. Recently, the work in [10] has been extended to MC-CDMA systems in the presence of frequency selective channels [3], [4].

In this paper, we apply the EM and SAGE algorithms to the problem of ML channel estimation of uplink MC-CDMA systems in the presence of frequency selective channels. In this way, we convert a multiple-input channel estimation problem into a number of single-input channel estimation problems which can be easily solved. Therefore, the computational cost for implementing the EM-based ML channel estimation is low and the computation is numerically stable. We show that the initialization of the algorithms are degraded for the increasing the number of active user and therefore the required number of iteration are increased and MSE performance is also degraded.

The rest of the paper is organized as follows. In Section II, the channel and the signal model of MC-CDMA systems considered in this work are given. In Section III, ML channel estimation based on EM and SAGE algorithms are presented. The performance of the algorithms proposed in the paper are assessed in Section IV by computer simulations. Finally, conclusions are drawn in Section V.

Notation: Vectors (matrices) are denoted by boldface lower (upper) case letters; all vectors are column vectors; $(.)^*$, $(.)^T$ $(.)^\dagger$ and $(.)^{-1}$ denote the conjugate, transpose, conjugate transpose and matrix inversion respectively; $\|.\|$ denotes the Frobenius norm; $\mathbf{I}_L$ denotes the $L \times L$ identity matrix.

II. Signal Model

We consider a baseband MC-CDMA uplink system with P sub-carriers and K mobile users which are simultaneously active. For the kth user, each transmit symbol is modulated in

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the frequency domain by means of a $P\times 1$ specific spreading sequence $\mathbf{c}_k$. After transforming by a P-point IDFT and parallel-to-serial (P/S) conversion, a cyclic prefix (CP) is inserted of length equal to at least the channel memory (L). In this work, to simplify the notation, it is assumed that the spreading factor equals to the number of sub-carriers and all users have the same spreading factor. Finally, the signal is transmitted through a multipath channel with impulse response

$$g_k(t)=\sum_{l=1}^{L} g_{k,l}\delta(t-\tau_{k,l}) \tag{1}$$

where L is the number of paths in the kth users channel; $g_{k,l}$ and $\tau_{k,l}$ are, respectively, the complex fading coefficient and the delay of lth path and P_k is the transmit power of the kth user. The fading process is assumed to be white. Note that the L-dimensional discrete channel impulse response vector $\mathbf{g}_k=[g_{k,1},g_{k,2},\cdots,g_{k,L}]^T$ and the transmission power P_k can be combined as $\mathbf{h}_k=\sqrt{P_k}\mathbf{g}_k$, since they can not be separated from each other.

In the receiver, the received signal is sampled at chip-rate, serial-to-parallel (S/P) converted, CP is removed, and DFT is then applied to the discrete time signal to obtain the received vector expressed as

$$\mathbf{y}(m) \;=\; \sum_{k=1}^{K} b_k(m)\mathbf{C}_k\mathbf{F}\mathbf{h}_k+\mathbf{w}(m) \;\; m=1,2,...,M \tag{2}$$

where $b_k(m)$ denotes data sent by the user k within the mth symbol; $\mathbf{C}_k=\text{diag}(\mathbf{c}_k)$ with $\mathbf{c}_k=[c_{k1},c_{k1},...,c_{kP}]^T$ where each chip, c_{ik}, takes values in the set $\{-\frac{1}{\sqrt{P}},\frac{1}{\sqrt{P}}\}$ denoting the kth users spreading code ; $\mathbf{F}\in\mathbb{C}^{P\times L}$ denotes the DFT matrix with the (k,l)th element given by $e^{-j2\pi kl/P}$; and $\mathbf{w}(m)$ is the $P\times 1$ zero-mean, i.i.d. complex Gaussian vector that models the additive noise in the P tones, with variance $\sigma^2/2$ per dimension.

Suppose M symbols are transmitted. We stack $\mathbf{y}(m)$ as $\mathbf{y}$=$[\mathbf{y}^T(1),...,\mathbf{y}^T(M)]^T$. Then the received signal model can be written as

$$\mathbf{y}=\begin{bmatrix} b_1(1)\mathbf{C}_1\mathbf{F} & \cdots & b_K(1)\mathbf{C}_K\mathbf{F} \\ \vdots & \ddots & \vdots \\ b_1(M)\mathbf{C}_1\mathbf{F} & \cdots & b_K(M)\mathbf{C}_K\mathbf{F} \end{bmatrix}\begin{bmatrix}\mathbf{h}_1\\ \vdots \\ \mathbf{h}_K\end{bmatrix}+\begin{bmatrix}\mathbf{w}(1)\\ \vdots \\ \mathbf{w}(M)\end{bmatrix} \tag{3}$$

and can be rewritten in more succinct form

$$\mathbf{y}=\mathbf{A}\mathbf{h}+\mathbf{w} \tag{4}$$

where $\mathbf{A}$ and $\mathbf{h}$ are $MP\times KL$ and $KL\times 1$ dimension matrices.

III. Maximum-likelihood (ML) Channel Estimation

Estimation of the channel impulse response vector is obtained by directly minimizing the following cost function

$$\hat{\mathbf{h}}=\arg\min_{\mathbf{h}}\{\|\mathbf{y}-\mathbf{A}\mathbf{h}\|^2\} \tag{5}$$

Assuming the channel model in the (2) is the correct channel model that ignores the leakage due to nonuniform channel tap spacing, the least-squares (LS) solution of (5) which is also the maximum-likelihood (ML) channel estimate (assuming known transmitted symbols) can be written ,while $\mathbf{A}$ is of full column rank[12], as follows

$$\hat{\mathbf{h}}_{\text{ML}}=(\mathbf{A}^\dagger\mathbf{A})^{-1}\mathbf{A}^\dagger\mathbf{y} \tag{6}$$

where

$$\mathbf{A}^\dagger\mathbf{A}\triangleq\mathbf{Q}=\begin{bmatrix} \mathbf{Q}_{1,1} & \mathbf{Q}_{1,2} & \cdots & \mathbf{Q}_{1,K} \\ \mathbf{Q}_{2,1} & \mathbf{Q}_{2,2} & \cdots & \mathbf{Q}_{2,K} \\ \vdots & & \ddots & \vdots \\ \mathbf{Q}_{K,1} & & \cdots & \mathbf{Q}_{K,K}\end{bmatrix} \tag{7}$$

Here, $\mathbf{Q}_{i,j}$ that is the matrix elements of $\mathbf{Q}$ can be evaluated as follows using the properties $b_k^*(m)b_k(m)=1$, $\mathbf{F}^\dagger\mathbf{F}=P\mathbf{I}_L$ and $\mathbf{C}_k^T\mathbf{C}_k=\frac{1}{P}\mathbf{I}_P$

$$\mathbf{Q}_{i,j}=\begin{cases} M\times\mathbf{I}_L & i=j \\ \sum\limits_{m=1}^{M}\mathbf{F}^\dagger\mathbf{C}_i b_i^*(m)b_j(m)\mathbf{C}_j\mathbf{F} & i\neq j\end{cases} \tag{8}$$

The problem of interest is the calculation of the inverse of the $KL\times KL$ square matrix in (6). The inverse of $\mathbf{Q}$ is of computational complexity $(O((KL)^3))$ and requires significant computation for large values of L and K. Especially, ongoing research with goal of increasing user capacity, the number of active user K will be increased enormously. Therefore, instead of directly minimizing (5), Expectation-Maximization (EM) and its generalized version SAGE algorithms will be proposed.

A. EM algorithm

The suitable approach for applying the EM algorithm for the problem at hand is to decompose the received signal in (2) into the sum [5] as follows

$$\mathbf{y}=\sum_{k=1}^{K}\mathbf{y}_k \tag{9}$$

where

$$\mathbf{y}_k=\begin{bmatrix} b_k(1)\mathbf{C}_k & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & b_k(M)\mathbf{C}_k\end{bmatrix}\begin{bmatrix}\mathbf{F}\\ \vdots \\ \mathbf{F}\end{bmatrix}[\mathbf{h}_k]+\begin{bmatrix}\mathbf{w}_k(1)\\ \vdots \\ \mathbf{w}_k(M)\end{bmatrix} \tag{10}$$

Here, $\mathbf{y}_k$ represents the received signal component transmitted by the kth user through the channel with impulse response $\mathbf{h}_k$. Note that $\mathbf{y}$ and $\mathbf{y}_k$ in (9) are treated as the complete and the incomplete data respectively in the EM approach employed.

Equation (10) can be written in more succinct form as follows,

$$\mathbf{y}_k=\mathbf{X}_k\tilde{\mathbf{F}}\mathbf{h}_k+\mathbf{w}_k \qquad 1\leq k\leq K \tag{11}$$

The Gaussian noise vector, $\mathbf{w}_k$ in (11) represents the portion of $\mathbf{w}$ in the decomposition defined by $\sum_{k=1}^{K}\mathbf{w}_k=\mathbf{w}$, whose variance is $\sigma^2\beta_k$. The coefficients β_k determine the part of the noise power of $\mathbf{w}$ assigned to $\mathbf{y}_k$, satisfying $\sum_{k=1}^{K}\beta_k = 1$, $0 \le \beta_k \le 1$.

At the qth iteration the EM algorithm computes in a first step, called *Expectation Step (E-Step)*. Following the EM technique presented in [8], the algorithm estimates the corresponding component in the received signal for each of the user links as follows,

E-Step: For k=1,2,...K, compute

$$\hat{\mathbf{z}}_k^{(q)} = \mathbf{X}_k\tilde{\mathbf{F}}\mathbf{h}_k^{(q)} \tag{12}$$

$$\hat{\mathbf{y}}_k^{(q)} = \hat{\mathbf{z}}_k^{(q)} + \beta_k\left[\mathbf{y} - \sum_{k=1}^{K}\hat{\mathbf{z}}_k^{(q)}\right] \tag{13}$$

M-Step: For k=1,2,...,K, compute

$$\hat{\mathbf{h}}_k^{(q)} = \arg\min_{\mathbf{h}_k}\left\{\left\|\hat{\mathbf{y}}_k^{(q)} - \mathbf{X}_k\tilde{\mathbf{F}}\mathbf{h}_k\right\|^2\right\} \tag{14}$$

Solving (14), we can calculate channel impulse response for the kth user as follows:

$$\hat{\mathbf{h}}_k^{(q+1)} = \tilde{\mathbf{F}}^{\dagger}\mathbf{X}_k^{-1}\mathbf{y}_k^{(q)} \tag{15}$$

In this step, as in the conventional OFDM scheme (single user), it divides the corresponding component by the reference symbols in the frequency domain and then multiply by $\tilde{\mathbf{F}}$ to obtain an updated estimate of the channel impulse response. The EM algorithm do not require any matrix inversion because $\mathbf{X}_k$ is a diagonal matrix.

B. SAGE algorithm

The SAGE algorithm proposed by Fessler et al. [11] is generalization of the EM algorithm. Rather than updating all parameters simultaneously at iteration q, updates only a subset of the elements of the parameter vector in each iteration. Following the SAGE technique presented in [8], the algorithm estimates the corresponding component in the received signal for each of the user links as follows, **Initialization**: For $1 \le k \le K$

$$\hat{\mathbf{z}}_k^{(0)} = \mathbf{X}_k\tilde{\mathbf{F}}\mathbf{h}_k^{(0)} \tag{16}$$

At the qth iteration (q=0,1,2,...): For $k = 1 + [q \, mod \, K]$, compute

$$\hat{\mathbf{y}}_k^{(q)} = \hat{\mathbf{z}}_k^{(q)} + \beta_k\left[\mathbf{y} - \sum_{k=1}^{K}\hat{\mathbf{z}}_k^{(q)}\right] \tag{17}$$

$$\hat{\mathbf{h}}_k^{(q+1)} = \tilde{\mathbf{F}}^{\dagger}\mathbf{X}_k^{-1}\mathbf{y}_k^{(q)} \tag{18}$$

$$\hat{\mathbf{z}}_k^{(q+1)} = \mathbf{X}_k\tilde{\mathbf{F}}\mathbf{h}_k^{(q+1)} \tag{19}$$

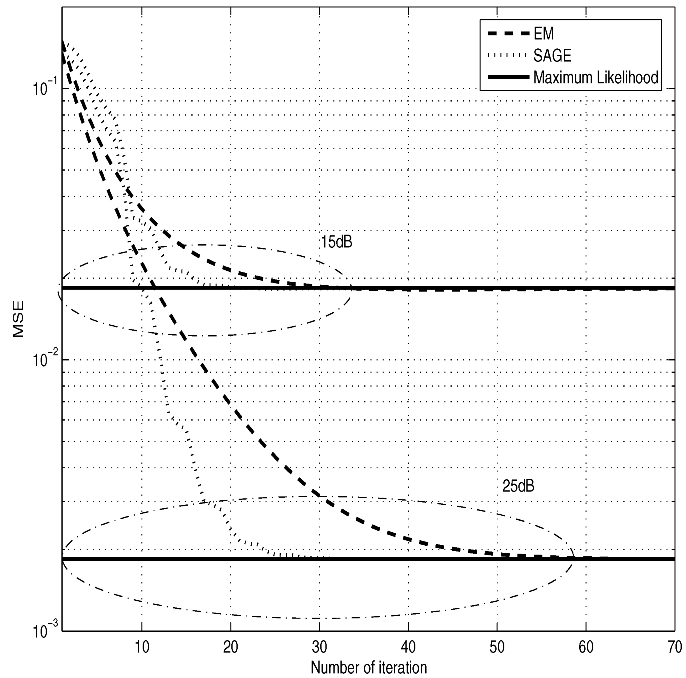

Fig. 1. Convergence of MSE with respect to number of iterations of the EM-type estimators compared with the MSE of the ML estimator.(L=4,T=8)

For $1 \le j \le K$ and $j \ne k$

$$\hat{\mathbf{z}}_k^{(q+1)} = \hat{\mathbf{z}}_k^{(q)} \tag{20}$$

Choosing initial values for the EM algorithm is an important issue for the convergence of speed of the both algorithm. We can obtain an initial estimate of the channel for the EM-type iteration as follows:

$$\hat{\mathbf{h}}_k^{(0)} = \tilde{\mathbf{F}}^{\dagger}\mathbf{X}_k^{-1}\mathbf{y}. \tag{21}$$

As expected from Eq.(21), increasing the number of active user will degrade the initialization performance of the algorithm. This in turn, will increase the number of iterations necessary for convergency as will be shown in the simulation section.

IV. Simulations

To demonstrate the performance of the proposed channel estimators, we simulate uplink MC-CDMA systems operating in the presence of frequency selective channels. In computer simulations, we assume that all users are received with the same power level (P_k=1). Orthogonal Walsh sequences selected as a spreading code and the processing gain is chosen equal to the number of subcarriers $P = 16$. Each user sends its data frame composed of T pilot symbols, and F data symbols, over mobile fading channel. Wireless channels between mobiles antennas and the receiver antenna are modeled based on a realistic channel model determined by COST-207 project in which Typical Urban(TU)channel model is considered having the channel length L and the channel tap gains are given in Table 1. QPSK signal modulation format is adopted with bandwidth is chosen as 1.228 MHz (Qual Comm-CDMA).

Table 1. Taps Power

Delay (μsn)	Linear	Logarithmic
0	0.6564	-1.8286
0.81	0.2086	-6.8072
1.62	0.0790	-11.0210
2.44	0.0560	-12.5171

At receiver, the initial ML channel estimate is obtained by using T preamble symbols. Fig.1 compares the mean-square error (MSE) performance of the EM-type algorithms as a function of the number of iterations for the number of active users is $K = 8$. For all simulations weight coefficients in (13) are chosen to be equal, i.e., β_k=$1/K$. It also includes comparisons with the MSE of the ML estimator. It is shown that the SAGE algorithm converges to the ML estimate within 16 iterations on average SNR 15 dB, while the EM algorithm converges to the ML estimate within 30 iterations. It was also shown that required the number of used iterations are increased to 24 and 60 for SAGE and EM respectively for SNR 25dB. On the other hand, since all the uplink channels are updated every K iterations while all of them are updated for every one iteration in the EM algorithm, we can count K iterations of the SAGE algorithm as if one iteration for EM algorithm as a function of complexity requirement. In this case, performance difference between SAGE and EM can be seen more clearly.

In Fig.2, increasing number of active users has been investigated. It was shown that performance of the ML channel estimator is degraded by the increasing the number of active user K because it needs to estimate more parameters $\mathbf{h}_k$ for constant pilot number T. Moreover, increasing the number of active user also affects the initialization of the EM and SAGE algorithm. Therefore, the number of required iterations for the EM and SAGE algorithm to converge the ML performance are also increase by the increasing of the number of active user. This fact can be demonstrated by the increasing the number of channel taps number,L.

In Fig 2, it is demonstrated that the required number of iteration are increased and MSE performance is also degraded for $K = 12$. It is also expected that this performance degradation continues for a full load system $K = 16$. Therefore, in Fig. 3, the number of pilot tones T are increased to improve channel estimation performance for $K = 16$. With increasing pilot tones, both MSE error and the number of iteration lead to decreasing as shown in Fig 3. It was concluded that increased observed vector length supply lower MSE error and initialization of both EM and SAGE are also improved.

V. CONCLUSIONS

The problem of maximum likelihood channel estimation for uplink MC-CDMA systems operating in the presence of frequency selective fading channels was investigated. We presented an iterative approach based on a version of the EM type

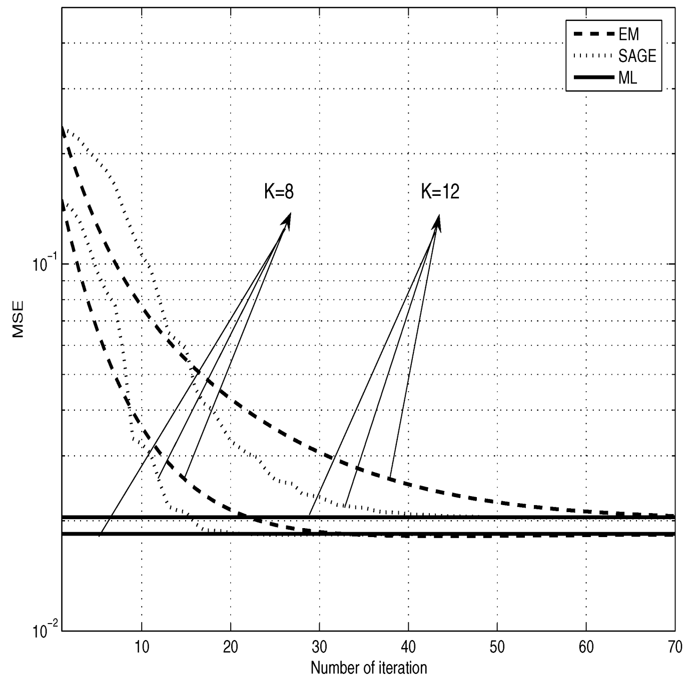

Fig. 2. MSE performances of EM type and ML estimator as a function of active user

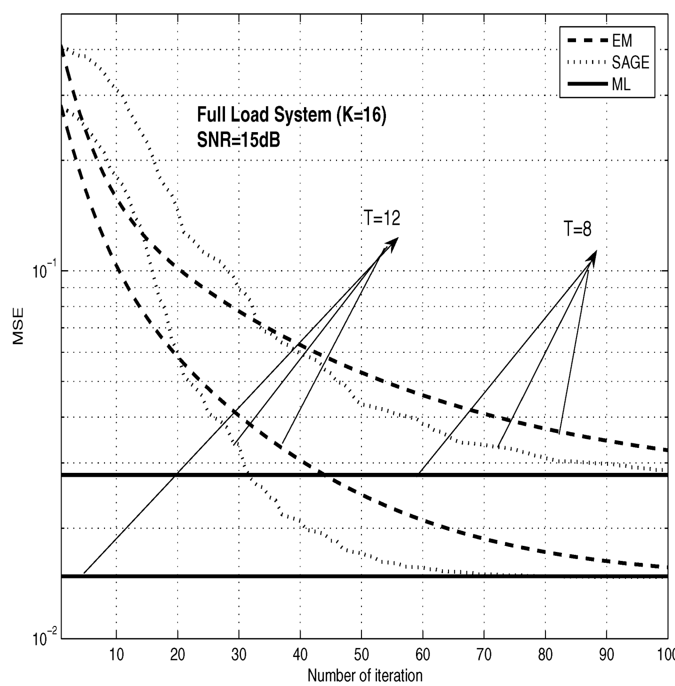

Fig. 3. MSE performances of EM type and ML estimator as a function of number of used pilot tones

algorithms suitable for superimposed signals. It was shown that the new channel estimation schemes allows to achieve ML estimator, when direct computation of the matrix inversion is too complex. In this work, it was shown that The SAGE channel estimator updates the parameters sequentially, while the EM channel estimator reestimates them simultaneously. Although SAGE can not use the benefits of parallelization, we demonstrated that it yields faster convergence than EM algorithm in channel estimation for MC-CDMA systems. Moreover, it was concluded that both algorithms require more iterations when the system capacity approaches to the full system capacity and this could be solved by increasing the number of pilot tones.

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