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A fuzzy AHP approach to evaluating machine tool alternatives

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Abstract Selecting process of a machine tool has been very important issue for companies for years, because the improper selection of a machine tool might cause of many problems affecting negatively on productivity, precision, flexibility and company's responsive manufacturing capabilities. On the other hand, selecting the best machine tool from its increasing number of existing alternatives in market are multiple-criteria decision making (MCDM) problem in the presence of many quantitative and qualitative attributes. Therefore, in this paper, an analytic hierarchy process (AHP) is used for machine tool selection problem due to the fact that it has been widely used in evaluating various kinds of MCDM problems in both academic researches and practices. However, due to the vagueness and uncertainty on judgments of the decision-maker(s), the crisp pair wise comparison in the conventional AHP seems to insufficient and imprecise to capture the right judgments of decision-maker(s). That is why; fuzzy number logic is introduced in the pair wise comparison of AHP to make up for this deficiency in the conventional AHP. Shortly, in this study, an intelligent approach is proposed, where both techniques; fuzzy logic and AHP are come together, referred to as fuzzy AHP. First, the fuzzy AHP technique is used to weight the alternatives under multiple attributes; second Benefit/Cost (B/C) ratio analysis is carried out by using both the fuzzy AHP score and procurement cost, of each alternative. The alternative with highest B/C ratio is found out and called as the ultimate machine tool among others. In addition, a case study is also presented to make this approach more understandable for a decision-maker(s).

Keywords Machine tool selection · Fuzzy logic · Multiple-criteria decision making · Analytic hierarchy process (AHP) · Benefit/Cost (B/C) ratio analysis

Introduction

A proper machine tool selection has been very important issue for manufacturing companies due to the fact that improperly selected machine tool can negatively affect the overall performance of a manufacturing system. In addition, the outputs of manufacturing system (i.e. the rate, quality and cost) mostly depend on what kinds of properly selected and implemented machines tools are used. On the other hand, the selection of a new machine tool is a time-consuming and difficult process requiring advanced knowledge and experience and experience deeply. So, the process can be hard task for engineers and managers, and also for machine tool manufacturer or vendor, to carry out. For a proper and effective evaluation, the decision-maker may need a large amount of data to be analyzed and many factors to be considered. The decision-maker should be an expert or at least be very familiar with the specifications of machine tool to select the most suitable among the others. However, a survey conducted by Gerrard (1988a) reveals that the role of engineering staff in authorization for

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final selection is 6%, the rest belongs to middle and upper management (94%). Gerrard also indicated the need for a simplified and practical approach for the machine selection process.

Evaluating machine tool alternatives is a multiple-criteria decision making (MCDM) problem in the presence of many quantitative and qualitative attributes. So, we selected analytic hierarchy process (AHP) method developed by Saaty (1981), because it has been widely used for selecting the best alternative among others (Ayag, 2002). But, in the conventional AHP, the pair wise comparisons for each level with respect to the goal of the best alternative selection are conducted using a nine-point scale. So, the application of Saaty's AHP has some shortcomings as follows (Saaty, 1981); (1) The AHP method is mainly used in nearly crisp decision applications, (2) The AHP method creates and deals with a very unbalanced scale of judgment, (3) The AHP method does not take into account the uncertainty associated with the mapping of one's judgment to a number, (4) Ranking of the AHP method is rather imprecise, (5) The subjective judgment, selection and preference of decision-makers have great influence on the AHP results. In addition, a decision-maker's (i.e. manufacturing engineer or manager) requirements on evaluating machine tool alternatives always contain ambiguity and multiplicity of meaning. Furthermore, it is also recognized that human assesment on qualitative attributes is always subjective and thus imprecise. Therefore, conventional AHP seems inadequate to capture decision-maker's requirements explicitly. In order to model this kind of uncertainity in human preference, fuzzy sets could be incorporated with the pairwise comparison as an extension of AHP. The fuzzy AHP approach allows a more accurate description of the decision making process.

In this paper, a fuzzy AHP approach is proposed to make up the vagueness and uncertainty existing in the importance attributed to judgment of the decision-maker(s), because the crisp pair wise comparison in the conventional AHP seems to insufficient and imprecise to capture the degree of importance of decision-maker(s) on evaluating machine tool alternatives. So, fuzzy logic is introduced in the pairwise comparison of AHP. Furthermore, computer software is developed to make all calculations of the fuzzy AHP easily and quicker by using a data-driven user interface and related database. To reach to final solution, B/C ratio analysis is used carried out by using the fuzzy AHP score and procurement cost, of each machine tool alternative.

In final section, to prove the applicability of the proposed approach on a real-life system, a case study is presented to make the approach more understandable for a decision-maker(s). This case study was realized in a leading cutting tool manufacturer in Turkey, which designs and manufactures all kinds of cutting tools for many sectors.

Related research

The fuzzy set theory is a mathematical theory designed to model the vagueness or imprecision of human cognitive processes that pioneered by Zadeh (Lootsma, 1997). This theory is basically a theory of classes with unsharp boundaries. What is important to recognize is that any crisp theory can be fuzzified by generalizing the concept of a set within that theory to the concept of a fuzzy set (Zadeh, 1994). Fuzzy set theory and fuzzy logic have been applied in a great variety of applications, which are reviewed by several authors (Klir and Yuan, 1995; Zimmermann, 1996).

The key idea of fuzzy set theory is that an element has a degree of membership in a fuzzy set (Negoita, 1985; Zimmermann, 1996). The membership function represents the grade of membership of an element in a set. The membership values of an element vary between 1 and 0. Elements can belong to a set in a certain degree and elements can also belong to multiple set. Fuzzy set allows the partial membership of elements. Transition between membership and non-membership is gradually. Membership function maps the variation of value of linguistic variables into different linguistic classes. The adaptation of membership function for a given linguistic variable under a given situation is done in three ways; (a) experts previous knowledge about the linguistic variable; (b) using simple geometric forms having slopes (triangular, trapezoidal or s-functions) as per the nature of the variable; and (c) by trial and error learning process.

As one of the most commonly used MCDM methods, the AHP was first developed for decision making by Saaty (1981) and extended by Marsh, Moran, Nakui, & Hoffherr (1991) who have developed a more specific method directly for design decision-making. The Marsh's AHP has three steps ordering the factors (i.e. attributes) of a decision such that the most important ones receive greatest weight. Zahedi (1986) provided an extensive list of references on the AHP methodology and its applications.

In this study, both of the above-explained AHP and fuzzy logic methods (shortly referred to as fuzzy AHP) are integrated to use their advantages for machine tool selection problem. Next, a literature review regarding machine tool selection problem and the applications of fuzzy AHP are briefly presented.

Tabucanon, Batanov, and Verma (1994) developed a decision support system for multi-criteria machine selection problem for flexible manufacturing systems (FMS), and used the AHP technique for the selection process. Wang, Shaw, and Chen (2000) proposed a fuzzy multiple-attribute decision making model to assist the decision-maker to deal with the machine selection problem for a FMS. Machine selection from fixed number of available machines is also considered by Atmani and Lashkari (1998). They developed a model for

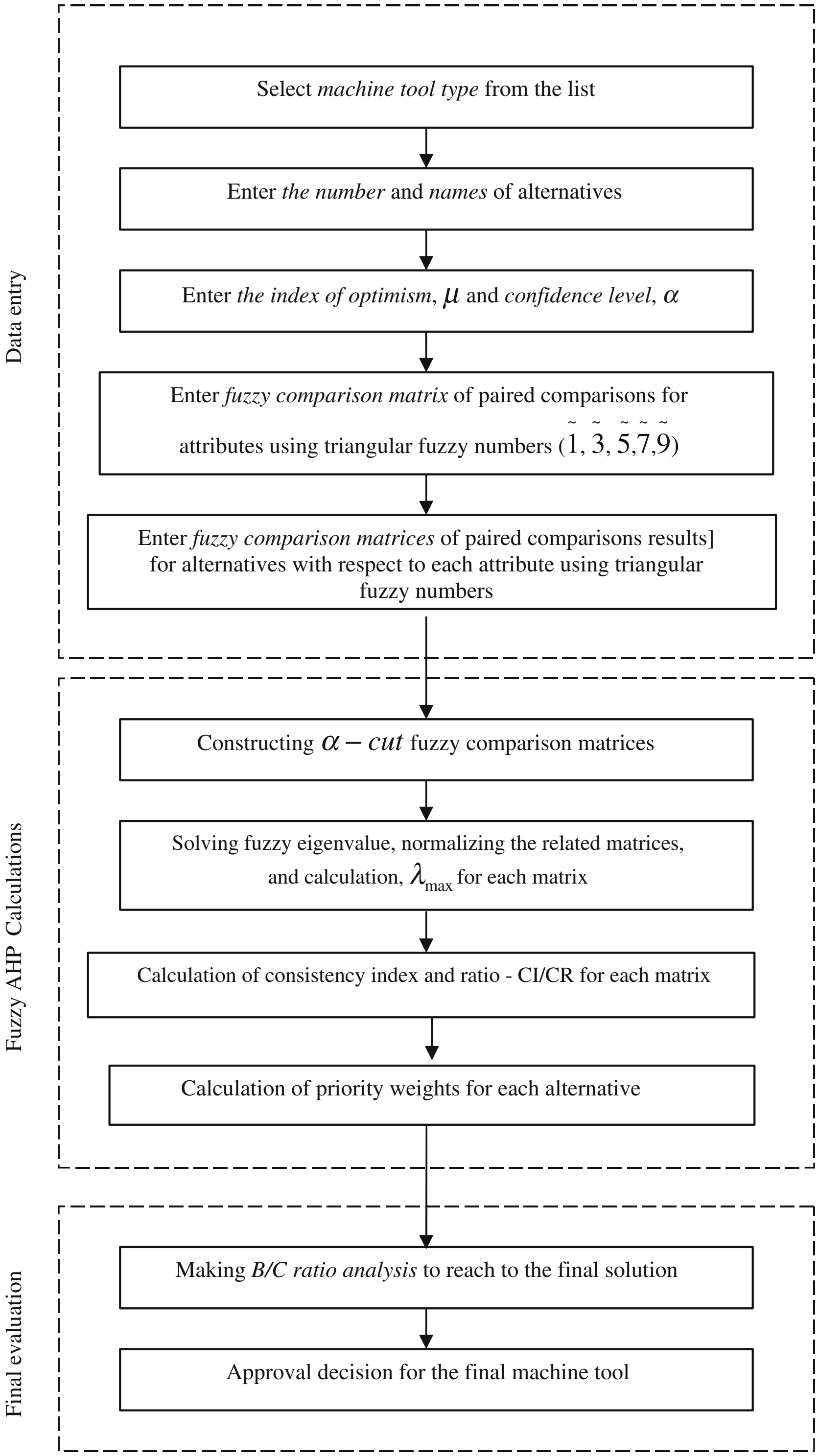
Table 1 The comparison of various fuzzy AHP methods (Buyukozkan et al., 2004)

Sources	Main characteristics	Advantages (A) /Disadvantages (D)
Van Laarhoven and Pedrycz (1983)	• Direct extension of Saaty's AHP method with triangular fuzzy numbers • Lootsma's logarithmic least square method is used to derive fuzzy weights and fuzzy performance scores	• (A) The opinions of multiple decision-makers can be modeled in the reciprocal matrix • (D) There is not always a solution to the linear equations • (D) The computational requirement is tremendous, even for a small problem • (D) It allows only triangular fuzzy numbers to be used
Buckley (1985)	• Direct extension of Saaty's AHP method with trapezoidal fuzzy numbers • Uses the geometric mean method to derive fuzzy weights and performance scores	• (A) It is easy to extend to the fuzzy case • (A) It guarantees an unique solution to the reciprocal comparison matrix • (D) The computational requirement is tremendous
Boender et al. (1989)	• Modifies van Laarhoven and Pedrycz's method • Presents a more robust approach to the normalization of the local priorities	• (A) The opinions of multiple decision-makers can be modeled • (D) The computational requirement is tremendous
Chang (1996)	• Synthetical degree values • Layer simple sequencing • Composite total sequencing	• (A) The computational requirement is relatively low • (A) It follows the steps of crisp AHP. It does not involve additional operations • (D) It allows only triangular fuzzy numbers to be used
Cheng (1996)	• Builds fuzzy standards • Represents performance scores by membership functions • Uses entropy concepts to calculate aggregate weights	• (A) The computational requirement is not tremendous • (D) Entropy is used when probability distribution is known. The method is based on both probability and possibility measures

machine tool selection and operation allocation in FMS. The model assumes that there is a set of machines with known processing capabilities. The AHP is also proposed by Lin and Yang (1994) to evaluate what type of machine tool the most appropriate for machining the certain part of a part is. Goh, Tung, and Cheng (1995) proposed a revised weighted sum decision model for robot selection by using weights assigned by a group of experts. Gerrard (1988b) also proposed a step-by-step methodology for the selection and introduction of new machine tools.

In addition, because of the accuracy of the fuzzy AHP method in the decision making process, it has been applied to many different areas. Here, some of its applications realized in various engineering fields are presented as follows; Kahraman, Cebeci, and Ulukan (2003) used fuzzy AHP to select the best supplier firm providing the most satisfaction for the attributes determined. Kuo, Chi, and Kao (2002) developed a decision support system using the fuzzy AHP to locate new convenience store. Murtaza (2003) presented a fuzzy version of AHP to country risk assessment problem. Kahraman, Cebeci, and Ruan (2004) developed an analytical tool using fuzzy AHP to select the best catering firm providing the most customer satisfaction. Weck, Klocke, Schell, and Ruenauver (1997) evaluated alternative production cycles using the extended fuzzy AHP method. Lee, Lau, Liu, and Tam (2001) proposed a fuzzy AHP approach in modular product design complemented with a case example to validate its feasibility in a real company. Ayag (2005a) also presented an integrated approach to evaluating conceptual design alternatives in a new product development (NPD) environment. Bozdag, Kahraman, and Ruan (2003) used fuzzy group decision making to evaluate CIM system alternatives. Piippo, Torkkeli, and Tuominen (1999) used group decision support system (GDSS) for a real-life CAD-system selection application for an industrial company. Ayag (2002) developed an AHP-based simulation model for implementation and analysis of computer-aided systems (CAx). Cheng and Mon (1994) evaluated weapon system by AHP based on fuzzy scales. Kwong and Bai (2002) suggested a fuzzy AHP approach to the determination of importance weights of customer requirements in quality function deployment (QFD). They proposed a new approach can improve the imprecise ranking of customer requirements which is based on the conventional AHP. Kwong and Bai (2002) also used the extent analysis method and the principles for the comparison of fuzzy numbers to determine the important weights for the customer requirements in QFD. The selection of advanced technology using the AHP can be merged with quantitative variables B/C and statistical analysis (Kengpol & O'Brien, 2001). In another study, Buyukozkan, Kahraman, and Ruan (2004) compared the fuzzy AHP methods in literature as seen in Table 1, which have important differences in their theoretical structures. This comparison includes advantages and disadvantages of each method.

Fig. 1 Fuzzy AHP approach for machine tool selection problem

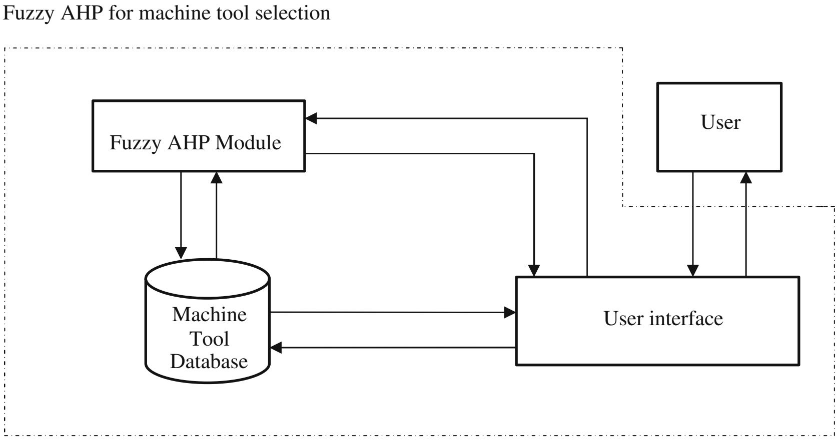


Proposed approach

In this paper, we propose a fuzzy AHP approach for machine tool selection problem as given in Fig. 1. First, the fuzzy AHP is used to weight the alternatives under multiple attributes; second B/C ratio analysis is carried out by using both the fuzzy AHP score and procurement cost, of each alternative. The alternative with highest B/C ratio is the best machine tool among others.

Figure 1 shows the steps of the proposed approach under three main sections (i.e. data entry, fuzzy AHP software calculations, final evaluation). As seen in figure, most of the steps require time-consuming calculations. Because many MCDM methods requires a lot of time to make all necessary

Fig. 2 Modular structure of fuzzy AHP software program for machine tool selection



calculations in order to reach to the final solution, depending upon the numbers of attributes and alternatives taken into consideration. In other words, as the number of attributes increases, the dimension of problem naturally expands such as an evaluation matrix with great deal of the columns and lines. This means too long and boring calculation process, especially if all calculations are done manually. On the other hand, the application fuzzy logic, in our study, with the AHP also requires a great deal of fuzzy matrix calculations. Therefore, we developed computer software to automatically carry out time-consuming steps defined in Fig. 1. This software allows a user (or decision-maker), who has not deep experience of the process to make all calculations of the fuzzy AHP easier and quicker. But, it is advised that he or she should know at least the basic principles of MCDM or the fuzzy AHP. The software was developed by using QBasic on PC platform, and is presented next.

The modular structure of fuzzy AHP software is given in Fig. 2. Its components (i.e. user interface, machine tool database and fuzzy AHP module) are explained more in detail next.

User interface and machine tool database

A database and user interface are designed and implemented. This database contains easily accessible data (i.e. main attributes and attributes) various kinds of conventional machine tools, as the user interface is an interactive data-driven tool to help the user both enter all necessary data and get the results of analysis. Input is taken through keyboard from the user to supply the fuzzy AHP analysis with the necessary information. User only enters all the requested data for the study (i.e. the number and names of alternatives, the index of optimism, μ, confidence level, α, fuzzy comparison matrices of paired comparisons for attributes, and alternatives with respect to each attribute) through a data-driven interactive tool (or user interface) in user friendly environment after reading the instructions given in detail on the screen. All data both entered via user interface and created during the analysis are kept in a database so that they can be easily reached for future studies. This database also contains easily accessible data (i.e. main attributes and attributes) for 24 kinds of conventional machine tools used for general purpose in market. This data can be updated anytime by user regarding the changes that might be during the analysis and rising from the real-life conditions of machine tool selection problem. Both the database and the user interface were tested and validated extensively for different cases. Some operational data are generated from the basic descriptions after the user completes data entry.

Fuzzy AHP software module

This module is used to automatically make all required calculations of the fuzzy AHP by using the related data from both the database and user such as; constructing pairwise comparison matrix for attributes, constructing pairwise comparison matrix of alternatives with respect to each attribute, automatically generating $\alpha - cut$ fuzzy comparison matrices, solving fuzzy eigenvalues, normalizing of priority weights for each matrix, calculating consistency index (CI) and ratio (CR) and calculating of priority weights for each alternative. The results of the process are presented to the user more in detail in an understandable format.

The evaluation attributes are emerged from various sources (i.e. a deep review of the literature, vendors and experts) on any kind of machine tool specification. For each kind of machine tool, main attributes and attributes are emerged as critical and kept in machine tool database so that they can be ready for further studies.

Fuzzy representation of pairwise comparison

The hierarchy of machine tool selection needs to be established before performing the pairwise comparison of AHP. After constructing a hierarchy, the decision-maker(s) is asked to compare the elements at a given level on a pair wise basis to estimate their relative importance in relation to the element at the immediate proceeding level. In conventional

Table 2 Definition and membership function of fuzzy number (Ayag, 2005b)

Intensity of Importance[a]	Fuzzy number	Definition	Membership function
1	$\tilde{1}$	Equally important/preferred	(1, 1, 2)
3	$\tilde{3}$	Moderately more important/preferred	(2, 3, 4)
5	$\tilde{5}$	Strongly more important/preferred	(4, 5, 6)
7	$\tilde{7}$	Very strongly more important/preferred	(6, 7, 8)
9	$\tilde{9}$	Extremely more important/preferred	(8, 9, 10)

[a] Fundamental scale used in pair wise comparison (Saaty, 1989)

AHP, the pairwise comparison is made by using a ratio scale. A frequently used scale is the nine-point scale (Saaty 1989, Table 2) which shows the participants' judgments or preferences among the options such as equally important, weakly more important, strongly more important, very strongly more important, and extremely more important preferred. Even though the discrete scale of 1–9 has the advantages of simplicity and easiness for use, it does not take into account the uncertainty associated with the mapping of one's perception or judgment to a number.

In this study, triangular fuzzy numbers, $\tilde{1}$–$\tilde{9}$, are used to represent subjective pairwise comparisons of selection process in order to capture the vagueness. A fuzzy number is a special fuzzy set $F = \{(x, \mu_{\mathrm{F}}(x)), x \in R\}$, where x takes it values on the real line, $R: -\infty < x < +\infty$ and $\mu_F(x)$ is a continuous mapping from R to the closed interval [0, 1]. A triangular fuzzy number denoted as $\tilde{M} = (l, m, u)$, where $l \leq m \leq u$, has the following triangular type membership function;

$$\mu_{\mathrm{F}}(x) = \begin{cases} 0 & x < l \\ x - l \big/ m - l & l \leq x \leq m \\ u - x / u - m & m \leq x \leq u \\ 0 & x > u \end{cases}$$

Alternatively, by defining the interval of confidence level α, the triangular fuzzy number can be characterized as

$$\forall \alpha \in [0,1] \quad \tilde{M}_\alpha = \left[l^\alpha, u^\alpha\right] = [(m-l)\alpha + l, -(u-m)\alpha + u]$$

Some main operations for positive fuzzy numbers are described by the interval of confidence, by Kaufmann and Gupta (1985) as given below

$$\forall m_L, m_R, n_L, n_R \in R^+, \tilde{M}_\alpha = \left[m_L^\alpha, m_R^\alpha\right],$$

$$\tilde{N}_\alpha = \left[n_L^\alpha, n_R^\alpha\right], \quad \alpha \in [0, 1]$$

$$\tilde{M} \oplus \tilde{N} = \left[m_L^\alpha + n_L^\alpha, m_R^\alpha + n_R^\alpha\right]$$

$$\tilde{M} \ominus \tilde{N} = \left[m_L^\alpha - n_L^\alpha, m_R^\alpha - n_R^\alpha\right]$$

$$\tilde{M} \otimes \tilde{N} = \left[m_L^\alpha n_L^\alpha, m_R^\alpha n_R^\alpha\right]$$

$$\tilde{M} / \tilde{N} = \left[m_L^\alpha / n_L^\alpha, m_R^\alpha / n_R^\alpha\right]$$

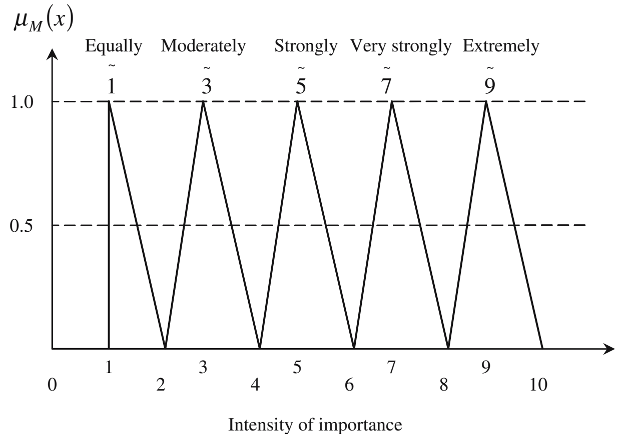


Fig. 3 Fuzzy membership function for linguistic values for attributes or alternatives

The triangular fuzzy numbers, $\tilde{1}$–$\tilde{9}$, are utilized to improve the conventional nine-point scaling scheme. In order to take the imprecision of human qualitative assessments into consideration, the five triangular fuzzy numbers are defined with the corresponding membership function as shown in Fig. 3.

The steps of fuzzy AHP approach

The AHP method is also known as an eigenvector method. It indicates that the eigenvector corresponding to the largest eigenvalue of the pairwise comparisons matrix provides the relative priorities of the factors, and preserves ordinal preferences among the alternatives. This means that if an alternative is preferred to another, its eigenvector component is larger than that of the other. A vector of weights obtained from the pairwise comparisons matrix reflects the relative performance of the various factors. In the fuzzy AHP triangular fuzzy numbers are utilized to improve the scaling scheme in the judgment matrices, and interval arithmetic is used to solve the fuzzy eigenvector (Cheng and Mon, 1994).

The four-step-procedure of this approach is given as follows;

Step 1. Comparing the performance score: Triangular fuzzy numbers $(\tilde{1},\tilde{3},\tilde{5},\tilde{7},\tilde{9})$ are used to indicate the relative strength of each pair of elements in the same hierarchy.

Step 2. Constructing the fuzzy comparison matrix: By using triangular fuzzy numbers, via pairwise comparison, the fuzzy judgment matrix $\tilde{A}\left(a_{ij}\right)$ is constructed as given below;

$$\tilde{A} = \begin{bmatrix} 1 & \tilde{a}_{12} & \ldots & \tilde{a}_{1n} \\ \tilde{a}_{21} & 1 & \ldots & \tilde{a}_{2n} \\ .. & .. & \ldots & .. \\ .. & .. & \ldots & .. \\ \tilde{a}_{n1} & \tilde{a}_{n2} & \ldots & 1 \end{bmatrix}$$

where, $\tilde{a}_{ij}^{\alpha} = 1$, if i is *equal* j, and $\tilde{a}_{ij}^{\alpha} = \tilde{1}, \tilde{3}, \tilde{5}, \tilde{7}, \tilde{9}$ or $\tilde{1}^{-1}, \tilde{3}^{-1}, \tilde{5}^{-1}, \tilde{7}^{-1}, \tilde{9}^{-1}$, if i is *not equal* j

Step 3. Solving fuzzy eigenvalue: A fuzzy eigenvalue, $\tilde{\lambda}$ is a fuzzy number solution to

$$\tilde{A}\tilde{x} = \tilde{\lambda}\tilde{x} \tag{1}$$

where is $n \times n$ fuzzy matrix containing fuzzy numbers $\tilde{a}_{ij}$ and $\tilde{x}$ is a non-zero $nx1$, fuzzy vector containing fuzzy number $\tilde{x}_i$. To perform fuzzy multiplications and additions by using the interval arithmetic and $\alpha - cut$, the equation $\tilde{A}\tilde{x} = \tilde{\lambda}\tilde{x}$ is equivalent to

$$\begin{aligned}&\left[a_{i1l}^{\alpha}x_{1l}^{\alpha}, a_{i1u}^{\alpha}x_{1u}^{\alpha}\right] \oplus \cdots \oplus \left[a_{inl}^{\alpha}x_{nl}^{\alpha}, a_{inu}^{\alpha}x_{nu}^{\alpha}\right] \\ &\quad = \left[\lambda x_{il}^{\alpha}, \lambda x_{iu}^{\alpha}\right]\end{aligned}$$

where,

$$\begin{aligned}&\tilde{A} = \left[\tilde{a}_{ij}\right], \quad \tilde{x}^{t} = \left(\tilde{x}_1, \ldots, \tilde{x}_n\right), \\ &\tilde{a}_{ij}{}^{\alpha} = \left[a_{ijl}^{\alpha}, a_{iju}^{\alpha}\right], \tilde{x}_i{}^{\alpha} = \left[x_{il}^{\alpha}, x_{iu}{}^{\alpha}\right], \tilde{\lambda}^{\alpha} = \left[\lambda_l^{\alpha}, \lambda_u^{\alpha}\right]\end{aligned} \tag{2}$$

for $0 < \alpha \leq 1$ and all i, j, where $i = 1, 2, \ldots, n, \quad j = 1, 2, \ldots, n$

$\alpha - cut$ is known to incorporate the experts or decision-maker(s) confidence over his/her preference or the judgments. Degree of satisfaction for the judgment matrix $\tilde{A}$ is estimated by the index of optimism μ. The larger value of index μ indicates the higher degree of optimism. The index of optimism is a linear convex combination (Lee, 1999) defined as

$$\tilde{a}_{ij}^{\alpha} = \mu a_{iju}^{\alpha} + (1 - \mu)\, a_{ijl}^{\alpha}, \quad \forall \mu \in [0, 1] \tag{3}$$

While α is fixed, the following matrix can be obtained after setting the index of optimism, μ, in order to estimate the degree of satisfaction.

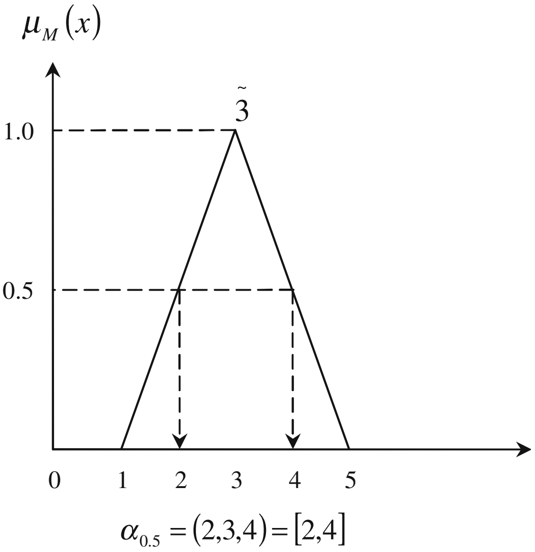


Fig. 4 $\alpha - cut$ operation on triangular fuzzy number

$$\tilde{A} = \begin{bmatrix} 1 & \tilde{a}_{12} & .. & .. & \tilde{a}_{1n}^{\alpha} \\ \tilde{a}_{21}^{\alpha} & 1 & .. & .. & \tilde{a}_{2n}^{\alpha} \\ .. & .. & .. & .. & .. \\ .. & .. & .. & .. & .. \\ \tilde{a}_{n1}^{\alpha} & \tilde{a}_{n2}^{\alpha} & .. & .. & 1 \end{bmatrix}$$

The eigenvector is calculated by fixing the μ value and identifying the maximal eigenvalue. $\alpha - cut$: It will yield an interval set of values from a fuzzy number. For example, $\alpha = 0.5$ will yield a set $\alpha_{0.5} = (2, 3, 4)$. The operation is presented by using Table 2 (Fig. 4).

Normalization of both the matrix of paired comparisons and calculation of priority weights (approx. attribute weights), and the matrices and priority weights for alternatives are also done before calculating $\lambda_{\max}$. In order to control the result of the method, the consistency ratio for each of the matrices and overall inconsistency for the hierarchy calculated. The deviations from consistency are expressed by the following equation CI, and the measure of inconsistency is called the CI,

$$\mathrm{CI} = \frac{\lambda_{\max} - n}{n - 1} \tag{4}$$

The consistency ratio (CR) is used to estimate directly the consistency of pairwise comparisons. The CR is computed by dividing the CI by a value obtained from a table of Random Consistency Index (RI);

$$\mathrm{CR} = \frac{\mathrm{CI}}{\mathrm{RI}} \tag{5}$$

If the CR less than 0.10, the comparisons are acceptable, otherwise not. RI is the average index for randomly generated weights (Saaty, 1981).

Table 3 Data entered by the user

- Machine tool type: CNC vertical turning center for general use
- Number of alternatives (m): 3 ($2 < m < 18$)
- Names of alternatives: Maho (m_1), Haas (m_2), Seiki (m_3)
- Index of optimism (μ): 0.5 (default value: 0.5, $0 < \mu < 1$)
- Confidence level (α): 0.5 (default value: 0.5), $0 < \alpha < 1$)
- Matrix of paired comparisons for the attributes using triangular fuzzy numbers ($n \times n = 19 \times 19$)
- Matrices of paired comparisons results for the alternatives (m_1, m_2, m_3) with respect to each attribute using triangular fuzzy numbers, respectively

Table 4 List of main attributes with their attributes for machine tool selection

#	Main attributes	#	Attributes
1	Productivity	A1	Spindle speed
		A2	Power
		A3	Cutting feed
		A4	Traverse speed
2	Flexibility	A5	Number of tools
		A6	Rotary table
3	Space	A7	Machine dimensions
4	Adaptability	A8	CNC type
		A9	Taper nr.
5	Precision	A10	Repeatability
		A11	Thermal deformation
6	Reliability	A12	Bearing failure rate
		A13	Reliability of drive system
7	Safety and environment	A14	Mist collector
		A15	Safety door
		A16	Fire extinguisher
		A17	Training
8	Maintenance and Service	A18	Repair service
		A19	Regular maintenance

Step 4. The priority weight of each alternative can be obtained by multiplying the matrix of evaluation ratings by the vector of attribute weights and summing over all attributes. Expresses in conventional mathematical notation;

$$\text{Weighted evaluation for alternative } k = \sum_{i=1}^{t} \left(\text{attribute weight}_i \times \text{ evaluation rating}_{ik} \right) \qquad (6)$$

for $i = 1, 2, \ldots, t$ (t: total number of attributes)

After calculating the weight of each alternative, the overall consistency index is calculated to make sure that it is smaller than 0.10 for consistency on judgments.

Case study

Above, a fuzzy AHP approach to evaluating of machine tool alternatives has been presented. In this section, a case study is realized to prove its applicability and validity to make this approach more understandable for the decision-maker(s). As case study, a new conventional machine tool (CNC vertical turning center for general use) investment decision of a leading cutting tool manufacturer in Turkey was taken into consideration.

The proposed approach was carried out by using the software defined in Fig. 2. The data entered by the user for the analysis are given in Table 3.

The main attributes and attributes for CNC vertical turning center called from the machine tool database are given in Table 4. In addition, Fig. 5 shows a diagram of the main attributes with their attributes used for this machine tool.

First, the fuzzy comparison matrix of pairwise comparisons for the attributes using triangular fuzzy numbers ($\tilde{1}, \tilde{3}, \tilde{5}, \tilde{7}, \tilde{9}$) is given in Table 5. And, the fuzzy comparison matrix of alternatives with respect to the attribute *spindle speed* of the main attribute *productivity* is shown in Table 6.

The lower limit and upper limit of the fuzzy numbers with respect to the α were defined as follows by applying Eq. 2;

$$\tilde{1}_\alpha = [1, 3 - 2\alpha],$$
$$\tilde{3}_\alpha = [1 + 2\alpha, 5 - 2\alpha], \quad \tilde{3}_\alpha^{-1} = \left[\frac{1}{5 - 2\alpha}, \frac{1}{1 + 2\alpha}\right],$$
$$\tilde{5}_\alpha = [3 + 2\alpha, 7 - 2\alpha], \quad \tilde{5}_\alpha^{-1} = \left[\frac{1}{7 - 2\alpha}, \frac{1}{3 + 2\alpha}\right],$$
$$\tilde{7}_\alpha = [5 + 2\alpha, 9 - 2\alpha], \quad \tilde{7}_\alpha^{-1} = \left[\frac{1}{9 - 2\alpha}, \frac{1}{5 + 2\alpha}\right],$$
$$\tilde{9}_\alpha = [7 + 2\alpha, 11 - 2\alpha], \quad \tilde{9}_\alpha^{-1} = \left[\frac{1}{11 - 2\alpha}, \frac{1}{7 + 2\alpha}\right].$$

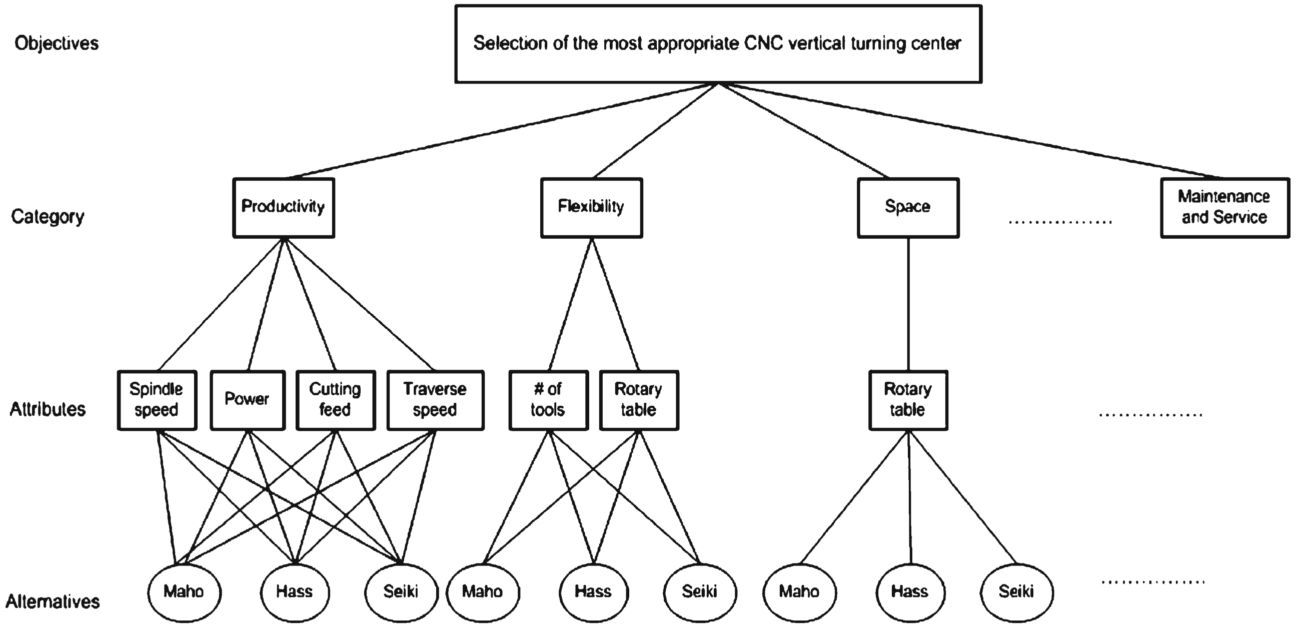


Fig. 5 Main attributes with their attributes for CNC vertical turning center (partly shown)

Table 5 Fuzzy comparison matrix of the attributes using triangular fuzzy numbers

Attribute	A1	A2	A3	A4	A5	…	…	A18	A19
A1	1	$\tilde{1}$	$\tilde{7}$	$\tilde{5}$	$\tilde{9}$	**…**	**…**	$\tilde{1}$	$\tilde{7}$
A2	$\tilde{1}^{-1}$	1	$\tilde{3}$	$\tilde{1}^{-1}$	$\tilde{3}$	…	…	$\tilde{3}$	$\tilde{7}$
A3	$\tilde{7}^{-1}$	$\tilde{3}^{-1}$	1	$\tilde{1}$	$\tilde{3}$	…	…	$\tilde{7}^{-1}$	$\tilde{3}$
A4	$\tilde{5}^{-1}$	$\tilde{1}$	$\tilde{1}^{-1}$	1	$\tilde{5}$	…	…	$\tilde{3}^{-1}$	$\tilde{5}$
A5	$\tilde{9}^{-1}$	$\tilde{3}^{-1}$	$\tilde{3}^{-1}$	$\tilde{5}^{-1}$	1	…	…	$\tilde{5}^{-1}$	$\tilde{3}$
…	…	…	…	…	…	…	…	…	…
…	…	…	…	…	…	…	…	…	…
A18	$\tilde{1}^{-1}$	$\tilde{3}^{-1}$	$\tilde{7}$	$\tilde{3}$	$\tilde{5}$	…	…	1	$\tilde{9}$
A19	$\tilde{7}^{-1}$	$\tilde{7}^{-1}$	$\tilde{3}^{-1}$	$\tilde{5}^{-1}$	$\tilde{3}^{-1}$	…	…	$\tilde{9}^{-1}$	1

Table 6 Fuzzy comparison matrix for alternatives with respect to the first attribute—*spindle speed* (A1) using triangular fuzzy numbers

Alternative	Maho	Hass	Seiki
Maho	1	$\tilde{1}$	$\tilde{5}$
Hass	$\tilde{1}^{-1}$	1	$\tilde{3}$
Seiki	$\tilde{5}^{-1}$	$\tilde{3}^{-1}$	1

Then, we substituted the values, $\alpha = 0.5$ and $\mu = 0.5$ above expression into fuzzy comparison matrices, and obtained all the $\alpha - cuts$ fuzzy comparison matrices (Tables 7, 8) (Eq. 3 was used to calculate eigenvectors for all comparison matrices) as follows;

Later, the eigenvectors for comparison matrices of both the attributes, and alternatives with respect to the first attribute—*spindle speed* were calculated by using Eq. 1 and shown in Tables 9, 10, respectively. For example, the CR for the matrix of pair wise comparisons of alternatives for the attribute—*spindle speed* was calculated by using the Eqs. 4 and 5 as follows (Table 10);

$$\mathrm{CI} = \frac{3.099 - 3}{2} = 0.050, \quad \mathrm{CR} = \frac{0.050}{0.58} = 0.086 < 0.10$$

For the fuzzy comparison matrices of three alternatives (Maho, Hass and Seiki) for the 18 remaining attributes, the CRs were calculated by using the same way, and it was clearly found

Table 7 $\alpha - cuts$ fuzzy comparison matrix for the attributes ($\alpha = 0.5, \mu = 0.5$)

Attribute	A1	A2	A3	A4	A5	…	…	A18	A19
A1	1	[1, 2]	[6, 8]	[4, 6]	[8, 10]	…	…	[1, 2]	[6, 8]
A2	[1/2, 1]	1	[2, 4]	[1/2, 1]	[2, 4]	…	…	[2, 4]	[6, 8]
A3	[1/8, 1/6]	[1/4, 1/2]	1	[1, 2]	[2, 4]	…	…	[1/8, 1/6]	[2, 4]
A4	[1/6, 1/4]	[1, 2]	[1/2, 1]	1	[4, 6]	…	…	[1/4, 1/2]	[4, 6]
A5	[1/10, 1/8]	[1/4, 1/2]	[1/4, 1/2]	[1/6, 1/4]	1	…	…	[1/6, 1/4]	[2, 4]
…	…	…	…	…	…	…	…	…	…
…	…	…	…	…	…	…	…	…	…
A18	[1/2, 1]	[1/4, 1/2]	[6, 8]	[2, 4]	[4, 6]	…	…	1	[8, 10]
A19	[1/8, 1/6]	[1/8, 1/6]	[1/4, 1/2]	[1/6, 1/4]	[1/4, 1/2]	…	…	[1/10, 1/8]	1

Table 8 The $\alpha - cuts$ fuzzy comparison matrix for alternatives with respect to the first attribute *–spindle speed (A1)* ($\alpha = 0.5, \mu = 0.5$)

Alternative	Maho	Hass	Seiki
Maho	1	[1, 2]	[4, 6]
Hass	[1/2, 1]	1	[2, 4]
Seiki	[1/6, 1/4]	[1/4, 1/2]	1

Table 9 The eigenvector for comparison matrix of the attributes

Attribute	A1	A2	A3	A4	A5	…	…	A18	A19	Priority vector
A1	1.000	1.500	7.000	5.000	9.000	…	…	1.500	7.000	0.102
A2	0.750	1.000	3.000	0.750	3.000	…	…	3.000	7.000	0.115
A3	0.146	0.375	1.000	1.500	3.000	…	…	0.146	3.000	0.057
A4	0.208	1.500	0.750	1.000	5.000	…	…	0.375	5.000	0.009
A5	0.113	0.375	0.375	0.208	1.000	…	…	0.208	3.000	0.012
…	…	…	…	…	…	…	…	…	…	…
…	…	…	…	…	…	…	…	…	…	…
A18	0.750	0.375	7.000	3.000	5.000	…	…	1.000	9.000	0.033
A19	0.146	0.146	0.375	0.208	0.375	…	…	0.113	1.000	0.021
									λ_{max}	22.11
									CI	0.173
									RI	1.77
									CR	0.098<0.1 Ok.

Table 10 The eigenvector for comparison matrix of the alternatives with respect to the first attribute—*spindle speed* (A1)

Alternative	Maho	Hass	Seiki	Priority vector
Maho	1.000	1.500	5.000	0.529
Hass	0.750	1.000	3.000	0.355
Seiki	0.208	0.375	1.000	0.116
			λ_{max}	3.099
			CI	0.050
			RI	0.58
			CR	0.086 < 0.1 Ok.

that they were all less than 0.10. Based on these calculations, the consistencies of the judgments in all comparison matrices were also acceptable.

Thus, the overall priority weights for Maho, Hass and Seiki, respectively were found out by using Eq. 6 as follows;

$$\sum_{i=1}^{19} \left(\text{attribute weight}_i \times \text{evaluation rating}_{ij} \right)$$
$$i = 1, 2, 3, \ldots, 19 \text{ and } j = 1, 2, 3$$

In addition, the overall consistency index was also calculated as 0.085. It shows all of the judgments are consistent. The results of the fuzzy AHP analysis is given in Table 11.

Finally, the B/C ratio analysis was carried out to find out the ultimate machine tool alternative and shown in Table 12. As seen in table, the final solution is *Hass CNC vertical turning center* with highest ratio, 0.185.

Conclusion

In this paper, a fuzzy AHP approach to evaluating machine tool alternatives was presented. In the approach, triangular fuzzy numbers were introduced into the conventional AHP in order to improve the degree of judgments of decision-maker(s). The central value of a fuzzy number is the corresponding real crisp number. The spread of the number is the estimation from the real crisp number. Equation 3 defines

Table 11 The final ranking of machine tool alternatives

		Alternative			
Attribute		Maho	Hass	Seiki	CR<=0.10
A1	0.102	0.529	0.355	0.116	0.086
A2	0.115	0.739	0.153	0.108	0.053
A3	0.057	0.660	0.249	0.091	0.078
A4	0.009	0.165	0.705	0.130	0.070
A5	0.012	0.153	0.739	0.108	0.053
A6	0.035	0.145	0.760	0.095	0.058
A7	0.074	0.064	0.274	0.662	0.070
A8	0.082	0.068	0.368	0.564	0.057
A9	0.101	0.058	0.397	0.544	0.047
A10	0.023	0.433	0.487	0.108	0.100
A11	0.033	0.368	0.068	0.564	0.057
A12	0.041	0.760	0.145	0.095	0.058
A13	0.008	0.165	0.705	0.130	0.070
A14	0.098	0.397	0.058	0.544	0.047
A15	0.067	0.153	0.739	0.108	0.053
A16	0.041	0.108	0.739	0.153	0.053
A17	0.048	0.274	0.064	0.662	0.070
A18	0.033	0.660	0.249	0.091	0.078
A19	0.021	0.662	0.064	0.274	0.070
Overall e-vector		0.358	0.324	0.319	

Table 12 B/C ratio analysis for machine tool alternatives

*HASS, the ultimate machine tool alternative with highest B/C ratio, 0.185

Alternative	Fuzzy AHP score (%)	Procurement cost (×1000$)	B/C ratio
Maho	35.8	200	0.179
Hass	32.4	175	0.185
Seiki	31.9	225	0.142

how the real crisp number, $\widetilde{a_{ij}}$ reacts to the real crisp number by adjusting the index of optimism, μ. The μ indicates the degree of optimism, which could be determined by manufacturing engineering team (manufacturing engineer and manufacturing manager).

Furthermore, a computer program was presented to make all time-consuming calculations of the fuzzy AHP easily and quicker for the decision-maker(s) (or user) by using a data-driven user interface and the related database. To reach to final solution, B/C ratio analysis is carried out by using both the fuzzy AHP score and procurement cost, of each machine tool alternative. Using of fuzzy AHP approach to evaluating machine tool alternatives results in the following two major advantages: (1) Fuzzy numbers are preferable to extend the range of a crisp comparison matrix of the conventional AHP method, as human judgment in the comparisons of selection criteria and machine tool alternatives is really fuzzy in nature, (2) Adoption of fuzzy numbers can allow decision-maker(s) to have freedom of estimation regarding the machine tool selection.

This approach aims to evaluating conventional machine tools, especially used for general use in manufacturing systems. The database contains data for 18 kinds of machine tools, any of which can be easily added, eliminated, and modified via data-driven user interface. The number of alternatives is limited to 24, because it was thought that it is enough for this study. The main attributes and their attributes are slightly different for each kind of machine tool. And, they stored in the database for each machine tool. Finally, a case study was presented to illustrate the applicability of the proposed approach. The case study was realized in a leading cutting tool manufacturer in Turkey.

For future research, the overall results of this study show that the combination of fuzzy decision making with AHP and expert systems could become a useful tool for selecting a machine tool.

References

Atmani, A., & Lashkari, R. S. (1998). A model of machine-tool selection and operation allocation in flexible manufacturing systems. *International Journal of Production Research*, *36*, 1339–1349

Ayag, Z. (2002). An analytic-hierarchy-process based simulation model for implementation and analysis of computer-aided systems. *International Journal of Production Research*, *40*, 3053–3073

Ayag, Z. (2005a). An integrated approach to evaluating conceptual design alternatives in a new product development environment. *International Journal of Production Research*, *43*, 687–713

Ayag, Z. (2005b). A fuzzy AHP-based simulation approach to concept evaluation in a NPD environment. *IIE Transactions*, *37*, 827–842

Boender, C. G. E., De Grann, J. G., & Lootsma, F. A. (1989). Multicriteria decison analysis with fuzzy pairwise comparison. *Fuzzy Sets and Systems*, *29*, 133–143

Bozdag, C. H., Kahraman, C., & Ruan, D. (2003). Fuzzy group decision making for selection among computer integrated manufacturing systems. *Computers in Industry*, *51*, 13–29

Buckley, J. J. (1985). Fuzzy hierarhical analysis. *Fuzzy Sets and Systems*, *17*, 233–247

Buyukozkan, G., Kahraman, C., & Ruan, D. (2004). A fuzzy multi-criteria decision approach for software development strategy selection. *International Journal of General Systems*, *33*, 259–280

Chang, D. Y. (1996). Applications of extent analysis method on fuzzy AHP. *European Journal of Operational Research*, *95*, 649–655

Cheng, C. H., & Mon, D. L. (1994). Evaluating weapon system by analytic hierarchy process based on fuzzy scales. *Fuzzy Sets and Systems*, *63*, 1–10

Cheng, C. H. (1996). Evaluating naval tactical missile systems by fuzzy AHP based on the grade value of membership function. *European Journal of Operational Research*, *96*, 343–350

Gerrard, W. (1988a). Selection procedures adopted by industry for introducing new machine tools. *Advances in Manufacturing Technology III. Proceedings of 4th National Conference on Production Research*, 525–531

Gerrard, W. (1988b). A strategy for selecting and introducing new technology/machine tools. *Advances in Manufacturing Technology III. Proceedings of 4th National Conference on Production Research*, 532–536

Goh, C. H., Tung, Y. C. A., & Cheng, C. H. (1995). A revised weighted sum decision model for robot selection. *Computers and Industrial Engineering*, *30*, 193–199

Kahraman, C., Cebeci, U., & Ulukan, Z. (2003). Multi-criteria supplier selection using fuzzy AHP. *Logistics Information Management*, *16*, 382–394

Kahraman, C., Cebeci,U., & Ruan, D. (2004). Multi-attribute comparison of catering service companies using fuzzy AHP: The case of Turkey. *International Journal of Production Economics*. *87*, 171–184

Kaufmann, A., & Gupta, M. M. (1985). *Introduction to fuzzy arithmetic: theory and applications*. New York: Van Nostrand Reinhold

Kengpol, A., & O'Brien, C. (2001). The development of a decision support tool for the selection of advanced technology to achieve rapid product development. *International Journal of Production Economics*, *69*, 177–191

Klir, G. J., & Yuan, B. (1995). *Fuzzy sets and fuzzy logic: theory and applications*. Prentice-Hall: Englewood Cliffs, NJ

Kuo, R. J., Chi, S. C., & Kao, S. S. (2002). A decision support system for selecting convenience store location through integration of fuzzy AHP and artificial neural network. *Computers in Industry*, *47*, 199–214

Kwong, C. K., & Bai, H. (2002). A fuzzy approach to the determination of important weights of customer requirements in quality function deployment. *Journal of Intelligent Manufacturing*, *13*, 367–377

Lee, A. R. (1999). Application of modified fuzzy AHP method to analyze bolting sequence of structural joints. *UMI Dissertation Services*, A Bell & Howell Company

Lee, W. B., Lau, H., Liu, Z. Z., & Tam, S. (2001). A fuzzy analytic hierarchy process approach in modular product design. *Expert Systems*, *18*, 32–42

Lin, Z. C., & Yang, C. B. (1994). Evaluation of machine selection by the AHP method. *Journal of Materials Processing Technology*, *57*, 253–258

Lootsma, F. A. (1997). *Fuzzy logic for planning and decision making* Dordrecht: Kluwer Academic Publisher

Marsh, S., Moran, J. V., Nakui, S., & Hoffherr, G. (1991). Facilitating and training in quality function deployment. Methuen, MA: GOAL/QPC

Murtaza, M. B. (2003). Fuzzy-AHP application to country risk assessment. *American Business Review*, *21*, 109–116

Negoita, C. V. (1985). *Expert systems and fuzzy systems*. Menlo Park, California: The Benjamin/Cummings

Piippo, P., Torkkeli, M., & Tuominen, M. (1999). Use of GDSS for selection of technology: new integrated CAD-system for an entire company, *International Conference on the Management of Engineering and Technology (PICMET'99),* Portland, USA, 25–29 July, p. 13

Saaty, T. L. (1981). *The Analytic Hierarchy Process*. McGraw-Hill: New York

Saaty, T. L. (1989). Decision Making, Scaling, and number crunching. *Decision Science*, *20*, 404–409

Tabucanon, M. T., Batanov, D. N., & Verma, D. K. (1994). Intelligent support system (DSS) for the selection process of alternative machines for flexible manufacturing systems. *Computers in Industry*, *25*, 131–134

Van Laarhoven, P. J. M., & Pedrycz. (1983). A fuzzy extension of Saaty's priority theory. *Fuzzy Sets and Systems*, *11*, 229–241

Wang, T. Y., Shaw, C. F., & Chen, Y. L. (2000). Machine selection in flexible manufacturing cell: A fuzzy multiple attribute decision making approach. *International Journal of Production Research*, *38*, 2079–2097

Weck, M., Klocke, F., Schell, H., & Ruenauver, E. (1997). Evaluating alternative production cycles using the extended fuzzy AHP method. *European Journal of Operational Research*, *100*, 351–366

Zadeh, L. A. (1994). Fuzzy Logic, neural network, and soft computing. *Communications of the ACM*, *37*, 77–84

Zahedi, F. (1986). The analytic hierarchy process: A survey of the method and its application. *Interfaces*, *16*, 96–108

Zimmermann, H. J. (1996). *Fuzzy set theory and its applications*. Massachusetts: Kluwer

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