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Linear expansions for frequency selective channels in OFDM

Habib Şenol[a], Hakan A. Çırpan[b,*], Erdal Panayırcı[c]

[a]Department of Computer Engineering, Kadir Has University, Cibali 34230, Istanbul, Turkey
[b]Department of Electrical Engineering, Istanbul University, Avcilar 34850, Istanbul, Turkey
[c]Department of Electronics Engineering, IŞIK University, Maslak 80670, Istanbul, Turkey

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Abstract

Modeling the frequency selective fading channels as random processes, we employ a linear expansion based on the Karhunen–Loeve (KL) series representation involving a complete set of orthogonal deterministic vectors with a corresponding uncorrelated random coefficients. Focusing on OFDM transmissions through frequency selective fading, this paper pursues a computationally efficient, pilot-aided linear minimum mean square error (MMSE) uncorrelated KL series expansion coefficients estimation algorithm. Based on such an expansion, no matrix inversion is required in the proposed MMSE estimator. Moreover, truncation in the linear expansion of channel is achieved by exploiting the optimal truncation property of the KL expansion resulting in a smaller computational load on the estimation algorithm. The performance of the proposed approach is studied through analytical and experimental results. We first exploit the performance of the MMSE channel estimator based on the evaluation of minimum Bayesian MSE. We also provide performance analysis results studying the influence of the effect of SNR and correlation mismatch on the estimator performance. Simulation results confirm our theoretical results and illustrate that the proposed algorithm is capable of tracking fast fading and improving performance.

Keywords: Bayesian channel estimation; OFDM systems; Linear expansions

1. Introduction

In a wireless orthogonal frequency division multiplexing (OFDM) systems over a frequency selective fading, channel variations arise mainly due to multipath effect [1]. Consequently, channel variations evolve in a progressive fashion and hence fit in some evolution model [2]. It appears that basis expansion approach could be natural way of modeling the channel variation [3]. Fourier, Taylor series, and polynomial expansion have played a prominent role in deterministic modeling [4,5]. As an alternative to the deterministic approaches, the variation in the channel can be captured by means of a stochastic modeling [3,4,6]. These random coefficient models are actually either used for identification of the model parameters which determine the evolution of the channel coefficients, or they are used for simulating fading channels with certain spectral characteristics [4]. Interestingly, random coefficient models used to simulate mobile fading channel can be obtained from the basis expansion model with random parameters [4,8]. Note that, the random process can be represented as a series expansion involving a complete set of deterministic vectors with corresponding random coefficients [7]. This expansion therefore provides a second-order characterization in terms of random variables and deterministic vectors. There are several such series that are widely in use. A commonly used series is the Karhunen–Loeve (KL) expansion [7,8]. The use of the KL

* Corresponding author.
E-mail address: hcirpan@istanbul.edu.tr (H.A. Çırpan).

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expansion with orthogonal deterministic basis vectors and uncorrelated random coefficients has generated interest because of its bi-orthogonal property, that is, both the deterministic basis vectors and the corresponding random coefficients are orthogonal. This allows for the optimal encapsulation of the information contained in the random process into a set of discrete uncorrelated random variables.

In this paper, we will focus on OFDM systems over frequency selective fading channel. Channel estimation for OFDM systems has attracted much attention with pioneering works of Edfords et al. [9] and Li et al. [10]. Numerous pilot-aided channel estimation methods for OFDM have been developed [9–11,13–15]. In particular, a low-rank approximation is applied to linear minimum mean square error (MMSE) estimator for the estimation of subcarrier channel attenuations by using the frequency correlation of the channel [9]. In [10], a MMSE channel estimator, which makes full use of the time and frequency correlation of the time-varying dispersive channel was proposed. Moreover, a low complexity MMSE-based doubly channel estimation approaches were presented in [11]. In [12], random phase introduced by Rayleigh fading in OFDM systems is modeled as a multichannel autoregressive (AR) process. Based on the proposed multichannel AR model, the Kalman filtering technique was applied for tracking the channel taps and maximum a posteriori (MAP) optimum detection technique was utilized for joint channel estimation and detection. In contrast, we will rely on the KL basis expansion of stochastic channel model to perform pilot-aided channel estimation. In the case of the KL series representation of stochastic channel model, a convenient choice of orthogonal basis set is one that makes the expansion coefficient random variables uncorrelated [16]. When these orthogonal bases are employed to characterize the variation of the channel impulse response, uncorrelated coefficients indeed represent the channel. Therefore, the KL representation allows one to tackle the estimation of correlated channel parameters as a parameter estimation problem of the uncorrelated coefficients. Exploiting the KL expansion, the main contribution of this paper is to propose a computationally efficient, pilot-aided MMSE channel estimation algorithms. Based on such representation, no matrix inversion is required in the proposed approach. Moreover, optimal rank reduction is achieved by exploiting the optimal truncation property of the KL expansion resulting in a smaller computational load on the estimation algorithm. The performance of the proposed batch approach is explored based on the evaluation of the Bayesian MSE for the random KL coefficients.

The rest of the paper is organized as follows. In Section 2, general model for OFDM systems is described and received signal model is presented. In Section 3, multipath channel statistics and its orthogonal series representation based on the KL expansion is presented. Basic and simplified MMSE-based expansion coefficients estimation algorithms are developed in Section 4. To show its efficiency, the performance bounds are analyzed and the performance degradation due to a mismatch of the estimator to the channel statistics as well as the SNR is demonstrated in Section 5. Some simulation examples are provided in Section 6. Finally, conclusions are drawn in Section 7.

2. OFDM system

OFDM has recently attracted considerable attention since it has been shown to be one of the most effective techniques for combating multipath delay spread over mobile wireless channels thereby improving the capacity and enhancing the performance of transmission. OFDM increases the symbol duration by dividing the entire channel into many narrowband subchannels and transmitting data in parallel. We now consider an OFDM system with N subcarriers signaling through a frequency selective fading channel. The channel response is assumed to be constant during one symbol duration. The block diagram in Fig. 1 describes of such OFDM system. The binary information data is grouped and mapped into multiphase signals.

In this paper the QPSK modulation is employed. An IDFT is then applied the QPSK symbols $\{X_k\}_{k=0}^{N-1}$, resulting in $\{x_n\}_{n=0}^{N-1}$, i.e.,

$$x_n = \mathrm{IDFT}\{X_k\} = \sum_{k=0}^{N-1} X_k \mathrm{e}^{\mathrm{j}2\pi kn/N}, \quad n = 0, \ldots, N-1. \tag{1}$$

In order to eliminate intersymbol interference arising due to multipath channel, the guard interval is inserted between OFDM frames. After pulse shaping and parallel to serial conversion, the signals are then transmitted through a frequency selective fading channel. At the receiver, after matched filtering and removing the guard interval, the time-domain received samples of an OFDM symbol is given by

$$y_n = x_n \otimes h_n + v_n = \sum_{k=0}^{L-1} h_n x_{n-k} + v_n, \tag{2}$$

where $\otimes$ represents the convolution operation, h_n is the channel impulse response, and v_n is the i.i.d. complex white Gaussian noise.

The received samples $\{y_n\}_{n=0}^{N-1}$, are then sent to the DFT block to demultiplex the multicarrier signals

$$Y_k = \mathrm{DFT}\{y_n\} = \frac{1}{N} \sum_{n=0}^{N-1} y_n \mathrm{e}^{-\mathrm{j}2\pi kn/N}, \quad k = 0, \ldots, N-1. \tag{3}$$

For OFDM systems with proper cyclic extensions and sample timing, the DFT output frequency domain subcarrier

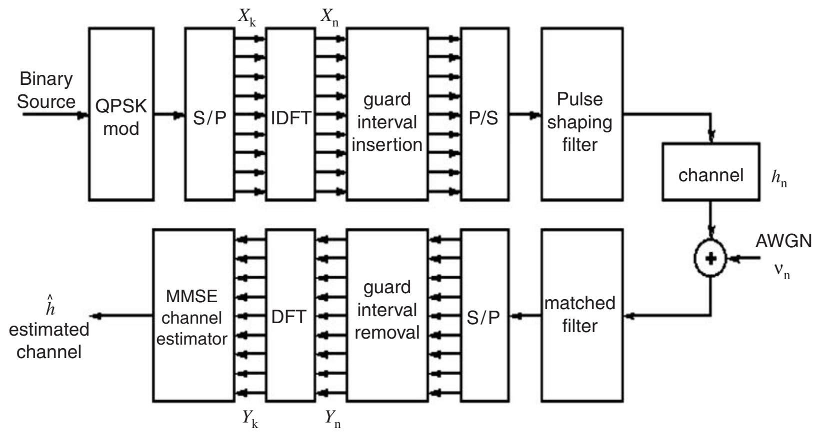

Fig. 1. OFDM system block diagram.

symbols can be expressed as

$$Y_k = X_k H_k + V_k, \tag{4}$$

where $V_k = \mathrm{DFT}\{v_n\},\ k = 0, 1, \ldots, N-1$ is frequency domain complex AWGN samples with zero mean and variance σ^2. H_k is the channel frequency response given by

$$H_k = \mathbf{w}^\dagger(k)\boldsymbol{h}, \quad k = 0, 1, \ldots, N-1, \tag{5}$$

where $\boldsymbol{h} = [h_0, h_1, \ldots, h_{L-1}]^{\mathrm{T}}$ contains the time response of all L taps, and $\mathbf{w}(k) = \left[1, \mathrm{e}^{-\mathrm{j}2\pi k/N}, \ldots, \mathrm{e}^{-\mathrm{j}2\pi k(L-1)/N}\right]^\dagger$ contains the corresponding DFT coefficients and $(\cdot)^\dagger$ denotes the Hermitian transpose. Substituting (5) into (4) yields

$$Y_k = X_k \mathbf{w}^\dagger(k)\boldsymbol{h} + V_k, \quad k = 0, \ldots, N-1. \tag{6}$$

If we focus at the received block $\mathbf{Y} = [Y_0, Y_1, \ldots, Y_{N-1}]^{\mathrm{T}}$, we can write the following from (6):

$$\mathbf{Y} = \mathbf{XW}\boldsymbol{h} + \mathbf{V}, \quad k = 0, \ldots, N-1. \tag{7}$$

where $\mathbf{X}$=diag$[X_0, X_1, \ldots, X_{N-1}]$ is a diagonal matrix with the data symbol entries, $\mathbf{W} = [\mathbf{w}(0), \ldots, \mathbf{w}(N-1)]^\dagger$ is the DFT matrix and similarly $\mathbf{V}$ is a zero-mean i.i.d. complex Gaussian vector.

Based on the model (7), our main objective in this paper is to develop a batch pilot-aided channel time response estimation algorithm according to MMSE criterion and then explore the performance of the resulting estimator. A proposed approach adapted herein explicitly models the random channel parameters by the KL series representation and estimates the uncorrelated expansion coefficients. Furthermore, the computational load of the proposed MMSE estimation technique is further reduced with the application of the KL expansion optimal truncation property [8]. In the following section the random channel model is introduced first.

3. Random channel model

The complex baseband representation of a fading multipath channel impulse response can be described as [11]

$$h(\tau) = \sum_l \alpha_l \delta(\tau - \tau_l T_{\mathrm{s}}), \tag{8}$$

where τ_l is the delay of the lth path and α_l is the corresponding complex amplitude with a power-delay profile $\theta(\tau_l)$. Note that α_l's are zero-mean, complex Gaussian random variables, which are assumed to be independent for different paths.

3.1. Channel statistics

We now briefly describe the channel statistics. The correlation function of the frequency response of the multipath fading channel for different frequencies is

$$c(f, f') \triangleq E[H(f)H^*(f')], \tag{9}$$

where

$$H(f) = \int_{-\infty}^{+\infty} h(\tau)\mathrm{e}^{-\mathrm{j}2\pi f\tau} = \sum_l \alpha_l \mathrm{e}^{-\mathrm{j}2\pi f\tau_l}. \tag{10}$$

It can be shown that (9) has the form [9]

$$c(f, f') = \sigma_g^2 c_f(f - f') = \sigma_g^2 c_f(\Delta f), \tag{11}$$

$$c_f(\Delta f) = (1/\sigma_g^2)\sum_l \sigma_l^2 \mathrm{e}^{-\mathrm{j}2\pi\Delta f\tau_l}, \tag{12}$$

where σ_l^2 is the average power of the lth path and σ_g^2 is the total average power of the channel impulse response defined as

$$\sigma_g^2 = \sum_l \sigma_l^2.$$

For an OFDM system with tone spacing Δf, the correlation function for different tones can be written more compactly as

$$c_{m,n} = E\{H_m H_n^*\}, \tag{13}$$

where $c_{m,n} = c_{m\Delta f, n\Delta f}$.

A more frequently used channel model could be explicitly derived in terms of an exponentially decaying power delay profile $\theta(\tau_l) = C\mathrm{e}^{-\tau_l/\tau_{\mathrm{rms}}}$ and delays τ_l that are uniformly and independently distributed over the length of guard interval. In [9], it is shown that the normalized exponential discrete channel correlation for different subcarriers is

$$c_{m,n} = \frac{1 - \exp\left(-L\left(\frac{1}{\tau_{\mathrm{rms}}} + \frac{2\pi \mathrm{j}(m-n)}{N}\right)\right)}{\tau_{\mathrm{rms}}\left(1 - \exp\left(-\frac{L}{\tau_{\mathrm{rms}}}\right)\right)\left(\frac{1}{\tau_{\mathrm{rms}}} + \frac{2\pi \mathrm{j}(m-n)}{N}\right)}. \tag{14}$$

Furthermore, the uniform channel correlation between the attenuations H_m and H_n can be obtained by letting $\tau_{\mathrm{rms}} \to \infty$ in (14), resulting in

$$c_{m,n} = \frac{1 - \exp\left(\frac{2\pi \mathrm{j} L(m-n)}{N}\right)}{\frac{2\pi \mathrm{j}(m-n)}{N}}. \tag{15}$$

Note that the correlation function of the channel taps for different frequencies depends, in general, only on the multipath delay spread and is separated from the effect of Doppler frequency. By only exploiting the frequency correlation in the channel estimation task, we are able to reduce complexity of the channel estimator.

3.2. Series expansion

The series expansion referred to as KL expansion provides a second moment characterization in terms of uncorrelated random variables and deterministic orthogonal vectors. In the KL expansion method the orthogonal deterministic basis vectors and its magnitude are respectively the eigenfunction and eigenvalue of the covariance matrix. Since channel impulse response $\boldsymbol{h}$ is a zero-mean Gaussian process with the covariance matrix $\mathbf{C}_{\boldsymbol{h}}$, the KL transformation rotates the vector $\boldsymbol{h}$ so that all its components are uncorrelated. Thus the vector $\boldsymbol{h}$, representing the channel impulse response during the OFDM block, can be expressed as a linear combination of the orthonormal basis vectors as follows:

$$\boldsymbol{h} = \sum_{l=0}^{L-1} g_l \psi_l = \boldsymbol{\Psi}\boldsymbol{g}, \tag{16}$$

where $\boldsymbol{\Psi} = [\psi_0, \psi_1, \ldots, \psi_{L-1}]$, ψ_l's are the orthonormal basis vectors, $\boldsymbol{g} = [g_0, g_1, \ldots, g_{L-1}]^{\mathrm{T}}$, and g_l is the l'th weight of the expansion. If we form the covariance matrix $\mathbf{C}_{\boldsymbol{h}}$ as

$$\mathbf{C}_{\boldsymbol{h}} = \boldsymbol{\Psi}\boldsymbol{\Lambda}_{\boldsymbol{g}}\boldsymbol{\Psi}^{\dagger}, \tag{17}$$

where $\boldsymbol{\Lambda}_{\boldsymbol{g}} = E\{\boldsymbol{g}\boldsymbol{g}^{\dagger}\}$, the KL expansion is the one in which $\boldsymbol{\Lambda}_{\boldsymbol{g}}$ of $\mathbf{C}_{\boldsymbol{h}}$ is a diagonal matrix (i.e., the coefficients are uncorrelated). If $\boldsymbol{\Lambda}_{\boldsymbol{g}}$ is diagonal, then the form $\boldsymbol{\Psi}\boldsymbol{\Lambda}_{\boldsymbol{g}}\boldsymbol{\Psi}^{\dagger}$ is called an *eigendecomposition* of $\mathbf{C}_{\boldsymbol{h}}$. The fact that only the eigenvectors diagonalize $\mathbf{C}_{\boldsymbol{h}}$ leads to the desirable property that the KL coefficients are uncorrelated. Furthermore, in Gaussian case, the uncorrelatedness of the coefficients renders them independent as well, providing additional simplicity. Thus, the channel estimation problem in this application is equivalent to estimating the i.i.d. complex Gaussian vector $\boldsymbol{g}$ KL expansion coefficients.

4. MMSE estimation of KL coefficients

A low-rank approximation to the frequency-domain linear MMSE channel estimator is provided by Edfords et al. [9] to reduce the complexity of the estimator. Optimal rank reduction is achieved in this approach by using the singular-value decomposition (SVD) of the channel attenuations covariance matrix $\mathbf{C}_{\mathbf{H}}$ of dimension $N \times N$. In contrast, here, we adapt the MMSE estimator for the estimation of multipath channel parameters $\boldsymbol{h}$ that uses covariance matrix of dimension $L \times L$. The proposed approach employs KL expansion of multipath channel parameters and reduces the complexity of the SVD used in *eigendecomposition* since L is usually much less than N. We will now develop MMSE batch estimator for pilot-assisted OFDM system in the sequel.

4.1. MMSE channel estimation

Pilot symbol-assisted techniques can provide information about an undersampled version of the channel that may be easier to identify. In this paper, we therefore address the problem of estimating multipath channel parameters by exploiting the distributed training symbols. Considering (7) and in order to include the pilot symbols in the output vector for the estimation purpose we focus on an under-sampled signal model. Assuming N_{p} pilot symbols are uniformly inserted at the known locations of the ith OFDM block, the $N_{\mathrm{p}} \times 1$ vector corresponding to the DFT output at the pilot locations becomes

$$\mathbf{Y}_{\mathrm{p}} = \mathbf{X}_{\mathrm{p}}\mathbf{W}_{\mathrm{p}}\boldsymbol{h} + \mathbf{V}_{\mathrm{p}}, \tag{18}$$

where $\mathbf{X}_{\mathrm{p}} = \mathrm{diag}[\mathbf{X}_i(0), \mathbf{X}_i(\Delta), \ldots, \mathbf{X}_i((N_{\mathrm{p}}-1)\Delta)]$ is a diagonal matrix with pilot symbol entries, Δ is pilot spacing interval, $\mathbf{W}_{\mathrm{p}}$ is an $N_{\mathrm{p}} \times L$ FFT matrix generated based on pilot indices, and similarly $\mathbf{V}_{\mathrm{p}}$ is the under-sampled noise vector.

For the estimation of $\boldsymbol{h}$, the new linear signal model can be formed by premultiplying both sides of (18) by $\mathbf{X}_{\mathrm{p}}^{\dagger}$ and assuming pilot symbols are taken from a QPSK constellation $\mathbf{X}_p^\dagger \mathbf{X}_p = \mathbf{I}_{N_p}$, then (18) takes the form

$$\mathbf{X}_p^\dagger \mathbf{Y}_p = \mathbf{W}_p \boldsymbol{h} + \mathbf{X}_p^\dagger \mathbf{V}_p,$$

$$\tilde{\mathbf{Y}} = \mathbf{W}_p \boldsymbol{h} + \tilde{\mathbf{V}}, \tag{19}$$

where $\tilde{\mathbf{Y}}$ and $\tilde{\mathbf{V}}$ are related to $\mathbf{Y}_p$ and $\mathbf{V}_p$ by the linear transformation, respectively. Furthermore, $\tilde{\mathbf{V}}$ is statistically equivalent to $\mathbf{V}_p$.

Eq. (19) offers a Bayesian linear model representation. Based on this representation, the minimum variance estimator for the time-domain channel vector $\boldsymbol{h}$ for the ith OFDM block, i.e., conditional mean of $\boldsymbol{h}$ given $\tilde{\mathbf{Y}}$, can be obtained using the MMSE estimator. We should clearly make the assumptions that $\boldsymbol{h} \sim \mathcal{N}(\mathbf{0}, \mathbf{C}_{\boldsymbol{h}})$, $\tilde{\mathbf{V}} \sim \mathcal{N}(\mathbf{0}, \mathbf{C}_{\tilde{\mathbf{V}}})$ and $\boldsymbol{h}$ is uncorrelated with $\tilde{\mathbf{V}}$. Therefore, the MMSE estimate of $\boldsymbol{h}$ is given by [18]

$$\hat{\boldsymbol{h}} = \left(\mathbf{W}_p^\dagger \mathbf{C}_{\tilde{\mathbf{V}}}^{-1} \mathbf{W}_p + \mathbf{C}_{\boldsymbol{h}}^{-1}\right)^{-1} \mathbf{W}_p^\dagger \mathbf{C}_{\tilde{\mathbf{V}}}^{-1} \tilde{\mathbf{Y}}. \tag{20}$$

Due to QPSK pilot symbol assumption together with the result $\mathbf{C}_{\tilde{\mathbf{V}}} = E\left[\tilde{\mathbf{V}}\tilde{\mathbf{V}}^\dagger\right] = \sigma^2 \mathbf{I}_{N_p}$, we can therefore express (20) by

$$\hat{\boldsymbol{h}} = \left(\mathbf{W}_p^\dagger \mathbf{W}_p + \sigma^2 \mathbf{C}_{\boldsymbol{h}}^{-1}\right)^{-1} \mathbf{W}_p^\dagger \tilde{\mathbf{Y}}. \tag{21}$$

Under the assumption that uniformly spaced pilot symbols are inserted with pilot spacing interval Δ and $N = \Delta \times N_p$, correspondingly, $\mathbf{W}_p^\dagger \mathbf{W}_p$ reduces to $\mathbf{W}_p^\dagger \mathbf{W}_p = N_p \mathbf{I}_L$. Then according to (21) and $\mathbf{W}_p^\dagger \mathbf{W}_p = N_p \mathbf{I}_L$, we arrive at the expression

$$\hat{\boldsymbol{h}} = \left(N_p \mathbf{I}_L + \sigma^2 \mathbf{C}_{\boldsymbol{h}}^{-1}\right)^{-1} \mathbf{W}_p^\dagger \tilde{\mathbf{Y}}. \tag{22}$$

Since the MMSE estimation still requires the inversion of $\mathbf{C}_{\boldsymbol{h}}$, it therefore suffers from a high computational complexity. However, it is possible to reduce complexity of the MMSE algorithm by diagonalizing channel covariance matrix with a linear KL expansion.

4.2. Estimation of KL coefficients

In contrast to (19) in which only $\boldsymbol{h}$ is to be estimated, we now assume the KL series expansion coefficients $\boldsymbol{g}$ is unknown. Substituting (16) in (19), the data model (19) is then rewritten for each OFDM block as

$$\tilde{\mathbf{Y}} = \mathbf{W}_p \boldsymbol{\Psi} \boldsymbol{g} + \tilde{\mathbf{V}} \tag{23}$$

which is also recognized as a Bayesian linear model, and that $\boldsymbol{g} \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Lambda}_{\boldsymbol{g}})$. As a result, the MMSE estimator of $\boldsymbol{g}$ is

$$\begin{aligned}\hat{\boldsymbol{g}} &= \boldsymbol{\Lambda}_{\boldsymbol{g}} (N_p \boldsymbol{\Lambda}_{\boldsymbol{g}} + \sigma^2 \mathbf{I}_L)^{-1} \boldsymbol{\Psi}^\dagger \mathbf{W}_p^\dagger \tilde{\mathbf{Y}} \\ &= \boldsymbol{\Gamma} \boldsymbol{\Psi}^\dagger \mathbf{W}_p^\dagger \tilde{\mathbf{Y}},\end{aligned} \tag{24}$$

where

$$\begin{aligned}\boldsymbol{\Gamma} &= \boldsymbol{\Lambda}_{\boldsymbol{g}} (N_p \boldsymbol{\Lambda}_{\boldsymbol{g}} + \sigma^2 \mathbf{I}_L)^{-1} \\ &= \operatorname{diag}\left\{\frac{\lambda_{g0}}{\lambda_{g0} N_p + \sigma^2}, \ldots, \frac{\lambda_{g_{L-1}}}{\lambda_{g_{L-1}} N_p + \sigma^2}\right\}\end{aligned} \tag{25}$$

and $\lambda_{g0}, \lambda_{g1}, \ldots, \lambda_{g_{L-1}}$ are the singular values of $\boldsymbol{\Lambda}_{\boldsymbol{g}}$.

It is clear that the complexity of the MMSE estimator in (22) is reduced by the application of the KL expansion. However, the complexity of the $\hat{\boldsymbol{g}}$ can be further reduced by exploiting the optimal truncation property of the KL expansion [8].

4.3. Truncated KL expansion

A truncated expansion $\boldsymbol{g}_r$ can be formed by selecting r orthonormal basis vectors among all basis vectors that satisfy $\mathbf{C}_{\boldsymbol{h}} \boldsymbol{\Psi} = \boldsymbol{\Psi} \boldsymbol{\Lambda}_{\boldsymbol{g}}$. The optimal selection of these vectors that yields the smallest average mean-squared truncation error $\frac{1}{L} E\left[\boldsymbol{\epsilon}_r^\dagger \boldsymbol{\epsilon}_r\right]$ is the one which chooses the orthonormal basis vectors associated with the first largest r eigenvalues as given by

$$\frac{1}{L} E\left[\boldsymbol{\epsilon}_r^\dagger \boldsymbol{\epsilon}_r\right] = \frac{1}{L} \sum_{i=r}^{L-1} \lambda_{g_i}, \tag{26}$$

where $\boldsymbol{\epsilon}_r = \boldsymbol{g} - \boldsymbol{g}_r$. For the problem at hand, truncation property of the KL expansion results in a low-rank approximation as well. Thus, a rank-r approximation to $\boldsymbol{\Lambda}_{\boldsymbol{g}_r}$ is defined as

$$\boldsymbol{\Lambda}_{\boldsymbol{g}_r} = \operatorname{diag}\{\lambda_{g0}, \lambda_{g1}, \ldots, \lambda_{g_{r-1}}, 0, \ldots, 0\}. \tag{27}$$

Since the trailing $L-r$ variances $\{\lambda_{gl}\}_{l=r}^{L-1}$ are small compared to the leading r variances $\{\lambda_{gl}\}_{l=0}^{r-1}$, the trailing $L-r$ variances are set to zero. However, in reality the pattern of the eigenvalues, of $\boldsymbol{\Lambda}_{\boldsymbol{g}}$ splits the eigenvectors into dominant and subdominant sets. Then the choice of r is more or less obvious. The optimal truncated KL (rank-r) estimator of (24) now becomes

$$\hat{\boldsymbol{g}}_r = \boldsymbol{\Gamma}_r \boldsymbol{\Psi}^\dagger \mathbf{W}_p^\dagger \tilde{\mathbf{Y}}, \tag{28}$$

where

$$\begin{aligned}\boldsymbol{\Gamma}_r &= \boldsymbol{\Lambda}_{\boldsymbol{g}_r} (N_p \boldsymbol{\Lambda}_{\boldsymbol{g}_r} + \sigma^2 \mathbf{I}_L)^{-1} \\ &= \operatorname{diag}\left\{\frac{\lambda_{g0}}{\lambda_{g0} N_p + \sigma^2}, \ldots, \frac{\lambda_{g_{r-1}}}{\lambda_{g_{r-1}} N_p + \sigma^2}, 0, \ldots, 0\right\}.\end{aligned} \tag{29}$$

5. Performance analysis

We turn our attention to analytical performance results of the MMSE approach. The performance of the MMSE channel estimator is exploited first based on the evaluation of the minimum Bayesian MSE.

5.1. Bayesian MSE

For the MMSE estimator $\hat{\boldsymbol{g}}$, the error is

$$\epsilon = \boldsymbol{g} - \hat{\boldsymbol{g}}. \tag{30}$$

Since the diagonal entries of the covariance matrix of the error represent the minimum Bayesian MSE, we now derive the covariance matrix $\mathbf{C}_\epsilon$ of the error vector. From *the Performance of the MMSE estimator for the Bayesian Linear model Theorem* [18, pp. 391], the error covariance matrix is obtained as

$$\begin{aligned}\mathbf{C}_\epsilon &= \left(\boldsymbol{\Lambda}_{\boldsymbol{g}}^{-1} + (\mathbf{F\Psi})^\dagger \mathbf{C}_{\tilde{\boldsymbol{\eta}}}^{-1}(\mathbf{F\Psi})\right)^{-1}\\ &= \sigma^2\left(N_\mathrm{p}\mathbf{I}_L + \sigma^2\boldsymbol{\Lambda}_{\boldsymbol{g}}^{-1}\right)^{-1}\\ &= \sigma^2\boldsymbol{\Gamma}\end{aligned} \tag{31}$$

and then the minimum Bayesian MSE of the full rank estimator becomes

$$\begin{aligned}\mathbf{B}_{\mathrm{MSE}}(\hat{\boldsymbol{g}}) &= \frac{1}{L}\,\mathrm{tr}(\mathbf{C}_\epsilon)\\ &= \frac{1}{L}\,\mathrm{tr}(\sigma^2\boldsymbol{\Gamma})\\ &= \frac{1}{L}\sum_{i=0}^{L-1}\frac{\lambda_{g_i}}{1 + N_\mathrm{p}\,\lambda_{g_i}\mathrm{SNR}},\end{aligned} \tag{32}$$

where $\mathrm{SNR} = 1/\sigma^2$ and tr denotes trace operator on matrices.

5.2. Mismatch analysis

Once the true frequency-domain correlation, characterizing the channel statistics and the SNR, are known the optimal channel estimator can be designed as indicated in Section 4. However, in mobile wireless communications, the channel statistics depend on the particular environment, for example, indoor or outdoor, urban or suburban, and change with time. Hence, it is important to analyze the performance degradation due to a mismatch of the estimator to the channel statistics as well as the SNR, and to study the choice of the channel correlation, and SNR for this estimator so that it is robust to variations in the channel statistics.

5.2.1. Bayesian MSE for truncated MMSE estimator under SNR mismatch

$\mathbf{B}_{\mathrm{MSE}}(\hat{\boldsymbol{g}})$ given in (32) can also be computed for the truncated (low-rank) case as follows. Substituting (23) in (28), the truncated MMSE KL estimator now becomes

$$\hat{\boldsymbol{g}}_r = N_\mathrm{p}\boldsymbol{\Gamma}_r\boldsymbol{g} + \boldsymbol{\Gamma}_r\boldsymbol{\Psi}^\dagger\mathbf{W}^\dagger\tilde{\mathbf{V}}. \tag{33}$$

The estimation error

$$\begin{aligned}\hat{\epsilon}_r &= \boldsymbol{g} - \hat{\boldsymbol{g}}_r\\ &= \boldsymbol{g} - (N_\mathrm{p}\boldsymbol{\Gamma}_r\boldsymbol{g} + \boldsymbol{\Gamma}_r\boldsymbol{\Psi}^\dagger\mathbf{W}_\mathrm{p}^\dagger\tilde{\mathbf{V}})\\ &= (\mathbf{I}_L - N_\mathrm{p}\boldsymbol{\Gamma}_r)\boldsymbol{g} - \boldsymbol{\Gamma}_r\boldsymbol{\Psi}^\dagger\mathbf{W}_\mathrm{p}^\dagger\tilde{\mathbf{V}}\end{aligned} \tag{34}$$

and then the average Bayesian MSE is

$$\begin{aligned}\mathbf{B}_{\mathrm{MSE}}(\hat{\boldsymbol{g}_r}) &= \frac{1}{L}\,\mathrm{tr}(\mathbf{C}_{\hat{\epsilon}_r})\\ &= \frac{1}{L}\,\mathrm{tr}(\boldsymbol{\Lambda}_{\boldsymbol{g}}(\mathbf{I}_L - N_\mathrm{p}\boldsymbol{\Gamma}_r)^2 + N_\mathrm{p}\tilde{\sigma}^2\boldsymbol{\Gamma}_r^2).\end{aligned} \tag{35}$$

In practice, the true channel correlations and true SNR denoted by $\widetilde{\mathrm{SNR}}$ are not known. If the MMSE channel estimator is designed to match the correlation of a multipath channel impulse response $\mathbf{C}_{\boldsymbol{h}}$ and SNR, but the true channel parameters $\tilde{\boldsymbol{h}}$ has the correlation $\mathbf{C}_{\tilde{\boldsymbol{h}}}$, then the average Bayesian MSE for the designed channel estimator is obtained as

$$\mathbf{B}_{\mathrm{MSE}}(\hat{\boldsymbol{g}_r}) = \frac{1}{L}\sum_{i=0}^{r-1}\frac{\lambda_{g_i}(\sigma^2)^2 + N_\mathrm{p}\lambda_{g_i}^2\tilde{\sigma}^2}{(N_\mathrm{p}\lambda_{g_i} + \sigma^2)^2} + \frac{1}{L}\sum_{i=r}^{L-1}\lambda_{g_i},$$

where $\sigma^2 = \dfrac{1}{\mathrm{SNR}}$, $\tilde{\sigma}^2 = \dfrac{1}{\widetilde{\mathrm{SNR}}}$

$$\begin{aligned}&= \frac{1}{L}\sum_{i=0}^{r-1}\frac{\lambda_{g_i}\left(1 + N_\mathrm{p}\lambda_{g_i}\frac{\mathrm{SNR}^2}{\widetilde{\mathrm{SNR}}}\right)}{(1 + N_\mathrm{p}\lambda_{g_i}\mathrm{SNR})^2}\\ &\quad + \frac{1}{L}\sum_{i=r}^{L-1}\lambda_{g_i}.\end{aligned} \tag{36}$$

Based on the result obtained in (36), Bayesian estimator performance can be further elaborated for the following scenarios:

- By taking $\widetilde{\mathrm{SNR}} = \mathrm{SNR}$, the performance result for the case of no SNR mismatch is

$$\mathbf{B}_{\mathrm{MSE}}(\hat{\boldsymbol{g}_r}) = \frac{1}{L}\sum_{i=0}^{r-1}\frac{\lambda_{g_i}}{1 + N_\mathrm{p}\lambda_{g_i}\mathrm{SNR}} + \frac{1}{L}\sum_{i=r}^{L-1}\lambda_{g_i}. \tag{37}$$

  Notice that, the second term in (37) is the sum of the powers in the KL transform coefficients not used in the truncated estimator. Thus, truncated $\mathbf{B}_{\mathrm{MSE}}(\hat{\boldsymbol{g}}_r)$ can be lower bounded by $\frac{1}{L}\sum_{i=r}^{L-1}\lambda_{g_i}$ which will cause an irreducible error floor in the SER results.
- As $r \to L$ in (35), $\mathbf{B}_{\mathrm{MSE}}(\hat{\boldsymbol{g}})$ under SNR mismatch results in the following Bayesian MSE:

$$\mathbf{B}_{\mathrm{MSE}}(\hat{\boldsymbol{g}}) = \frac{1}{L}\sum_{i=0}^{L-1}\frac{\lambda_{g_i}\left(1 + N_\mathrm{p}\lambda_{g_i}\frac{\mathrm{SNR}^2}{\widetilde{\mathrm{SNR}}}\right)}{(1 + N_\mathrm{p}\lambda_{g_i}\mathrm{SNR})^2}. \tag{38}$$

- Finally, the Bayesian MSE in the case of no SNR mismatch is also be obtained as,

$$\mathbf{B}_{\mathrm{MSE}}(\hat{\boldsymbol{g}}) = \frac{1}{L}\sum_{i=0}^{L-1}\frac{\lambda_{g_i}}{1 + N_\mathrm{p}\lambda_{g_i}\mathrm{SNR}}. \tag{39}$$

5.2.2. Bayesian MSE for truncated MMSE KL estimator under correlation mismatch

In this section, we derive the Bayesian MSE of the truncated MMSE KL estimator under correlation mismatch.

Although the real multipath channel $\tilde{\boldsymbol{h}}$ has the expansion correlation $\mathbf{C}_{\tilde{\boldsymbol{h}}}$, we designed the estimator for the multipath channel $\boldsymbol{h}=\boldsymbol{\Psi}\boldsymbol{g}$ with correlation $\mathbf{C}_{\boldsymbol{h}}$. To evaluate the estimation error $\tilde{\boldsymbol{g}}-\hat{\boldsymbol{g}}_r$ in the same space, we expand the $\tilde{\boldsymbol{h}}$ onto the eigenspace of $\boldsymbol{h}$ as $\tilde{\boldsymbol{h}}=\boldsymbol{\Psi}\tilde{\boldsymbol{g}}$ resulting in correlated expansion coefficients.

For the truncated MMSE estimator, the error is

$$\begin{aligned}\hat{\epsilon}_r &= \tilde{\boldsymbol{g}} - \hat{\boldsymbol{g}}_r \\ &= \tilde{\boldsymbol{g}} - (N_\mathrm{p}\boldsymbol{\Gamma}_r\boldsymbol{g} + \boldsymbol{\Gamma}_r\boldsymbol{\Psi}^\dagger\mathbf{W}_\mathrm{p}^\dagger\tilde{\mathbf{V}}) \\ &= \tilde{\boldsymbol{g}} - N_\mathrm{p}\boldsymbol{\Gamma}_r\boldsymbol{g} - \boldsymbol{\Gamma}_r\boldsymbol{\Psi}^\dagger\mathbf{W}_\mathrm{p}^\dagger\tilde{\mathbf{V}}. \end{aligned} \tag{40}$$

As a result, the average Bayesian MSE is

$$\begin{aligned}\mathbf{B}_{\mathrm{MSE}}(\hat{\boldsymbol{g}}_r) &= \frac{1}{L}\,\mathrm{tr}(\mathbf{C}_{\hat{\epsilon}_r}) \\ &= \frac{1}{L}\,\mathrm{tr}(\boldsymbol{\Lambda}_{\tilde{\boldsymbol{g}}} + N_\mathrm{p}^2\boldsymbol{\Gamma}_r^2\boldsymbol{\Lambda}_{\boldsymbol{g}} \\ &\quad + \sigma^2 N_\mathrm{p}\boldsymbol{\Gamma}_r^2 - 2N_\mathrm{p}\boldsymbol{\Gamma}_r\boldsymbol{\beta}) \\ &= \frac{1}{L}\sum_{i=0}^{r-1}\left[\tilde{\lambda}_{g_i} + \frac{N_\mathrm{p}\lambda_{g_i}(\lambda_{g_i}-2\beta_i)}{N_\mathrm{p}\lambda_{g_i}+\sigma^2}\right] \\ &\quad + \frac{1}{L}\sum_{i=r}^{L-1}\tilde{\lambda}_{g_i} \quad \text{and} \quad \sigma^2 = \frac{1}{\mathrm{SNR}} \\ &= \frac{1}{L}\sum_{i=0}^{r-1}\left[\tilde{\lambda}_{g_i} + \frac{N_\mathrm{p}\mathrm{SNR}\lambda_{g_i}(\lambda_{g_i}-2\beta_i)}{1+N_\mathrm{p}\mathrm{SNR}\lambda_{g_i}}\right] \\ &\quad + \frac{1}{L}\sum_{i=r}^{L-1}\tilde{\lambda}_{g_i} \\ &= \frac{1}{L}\sum_{i=0}^{r-1}\frac{\tilde{\lambda}_{g_i} + N_\mathrm{p}\mathrm{SNR}\lambda_{g_i}\left(\tilde{\lambda}_{g_i}+\lambda_{g_i}-2\beta_i\right)}{1+N_\mathrm{p}\mathrm{SNR}\lambda_{g_i}} \\ &\quad + \frac{1}{L}\sum_{i=r}^{L-1}\tilde{\lambda}_{g_i}, \end{aligned} \tag{41}$$

where $\boldsymbol{\beta}$ is the real part of $E\left[\tilde{\boldsymbol{g}}\boldsymbol{g}^\dagger\right]$ and β_i's are the diagonal elements of $\boldsymbol{\beta}$. With this result, we will now highlight some special cases:

- Letting $\beta_i=\lambda_{g_i}=\tilde{\lambda}_{g_i}$ in (41) for the case of no mismatch in the correlation of the KL expansion coefficients, the truncated Bayesian MSE is identical to that obtained in (37).
- As $r \to L$ in (41), the Bayesian MSE under correlation mismatch is obtained to yield

$$\mathbf{B}_{\mathrm{MSE}}(\hat{\boldsymbol{g}}) = \frac{1}{L}\sum_{i=0}^{L-1}\frac{\tilde{\lambda}_{g_i} + N_\mathrm{p}\mathrm{SNR}\lambda_{g_i}(\tilde{\lambda}_{g_i}+\lambda_{g_i}-2\beta_i)}{1+N_\mathrm{p}\mathrm{SNR}\lambda_{g_i}}. \tag{42}$$

- Under no correlation mismatch in (41) where $\beta_i=\lambda_{g_i}=\tilde{\lambda}_{g_i}$, the Bayesian MSE obtained from (41) is identical to that in (39).
- Also note that as SNR $\to \infty$, (41) reduces to MSE$(\tilde{\boldsymbol{g}}-\boldsymbol{g}_r)$.

6. Simulations

In this section, the merits of our channel estimators is illustrated through simulations. We choose average mean square error (MSE) as our figure of merit. The fading multipath channel with L paths given by (5) with an exponentially decaying power delay profile (14) is considered.

The scenario for our simulation study consists of a wireless QPSK OFDM system employing the pulse shape as a unit-energy Nyquist-root raised-cosine shape with rolloff $\alpha=0.2$, with a symbol period (T_s) of 0.120 µs, corresponding to an uncoded symbol rate of 8.33 Mbit/s. Transmission bandwidth (5 MHz) is divided into 1024 tones. We assume that the fading multipath channel has $L=40$ paths with an exponentially decaying power delay profile (5) with an $\tau_{\mathrm{rms}}=5$ sample (0.6 µs) long.

A QPSK-OFDM sequence passes through the channel taps and is corrupted by AWGN (10, 20, 30 and 40 dB, respectively). We use a pilot symbol for every twenty ($\Delta=20$) symbols. The MSE at each SNR point is averaged over 1000 realizations. We compare the experimental MSE performance and its theoretical Bayesian MSE of the proposed full-rank MMSE estimator with maximum-likelihood (ML) estimator and its corresponding Cramer–Rao bound (CRB). Fig. 2 confirms that MMSE estimator performs better than ML estimator at low SNR. However, the two approaches has comparable performance at high SNRs.

6.1. SNR design mismatch

In order to evaluate the performance of the proposed full-rank MMSE estimator to mismatch only in SNR design, the estimator is tested when SNRs of 10 and 30 dB are used in the design. The MSE curves for a design SNR of 10 and 30 dB are shown in Fig. 3. The performance of the MMSE estimator for high SNR (30 dB) design is better than low SNR (10 dB) design across a range of SNR values (0–30 dB). This results confirm that channel estimation error is concealed in noise for low SNR whereas it tends to dominate for high SNR. Thus, the system performance degrades especially for low SNR design.

6.2. Correlation mismatch

To analyze full-rank MMSE estimator's performance further, we need to study sensitivity of the estimator to design errors, i.e., correlation mismatch. We therefore designed the estimator for a uniform channel correlation which gives the worst MSE performance among all channels [9,13] and evaluated for an exponentially decaying power-delay profile. The uniform channel correlation between the

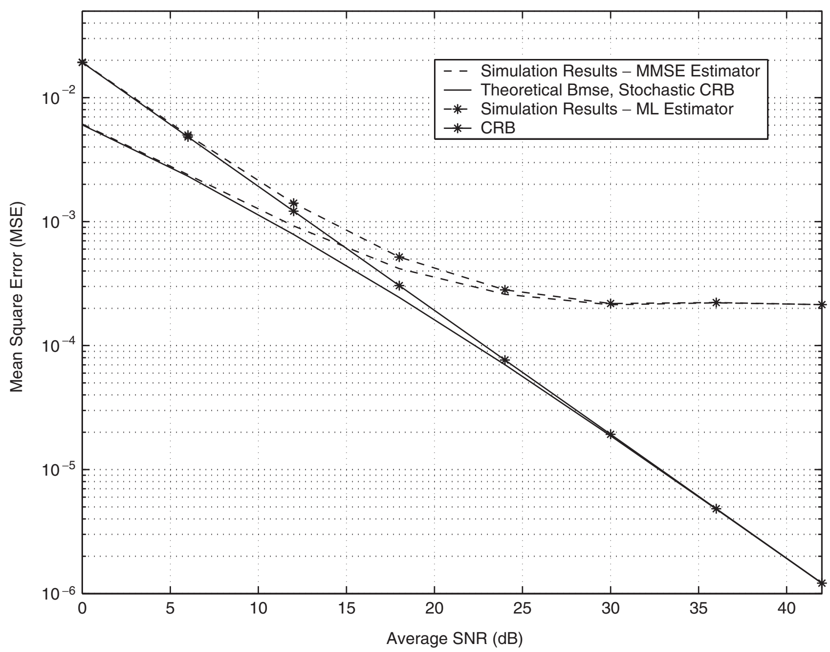

Fig. 2. Performance of proposed MMSE and MLE together with Bmse and CRB.

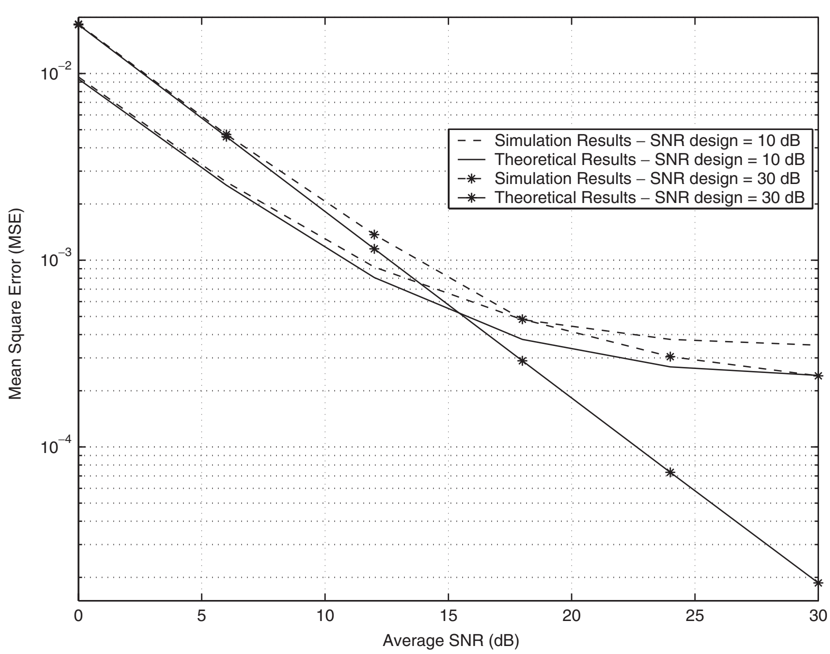

Fig. 3. Effects of SNR design mismatch on MSE.

attenuations can be obtained by letting $\tau_{\mathrm{rms}} \to \infty$ in (5), resulting in (15).

Fig. 4 demonstrates the estimator's sensitivity to the channel statistics in terms of the average MSE. As it can be seen from Fig. 4 only small performance loss is observed for low SNRs when the estimator is designed for mismatched channel statistics. This justifies the result that a design for worst correlation is robust to mismatch.

6.3. Performance of the truncated estimator

The truncated estimator performance is also studied as a function of the number of the KL coefficients. Fig. 5 presents the MSE result of the truncated MMSE estimator. If only a few expansion coefficients is employed to reduce the complexity of the proposed estimator, then the MSE between channel parameters becomes large. However, if the number

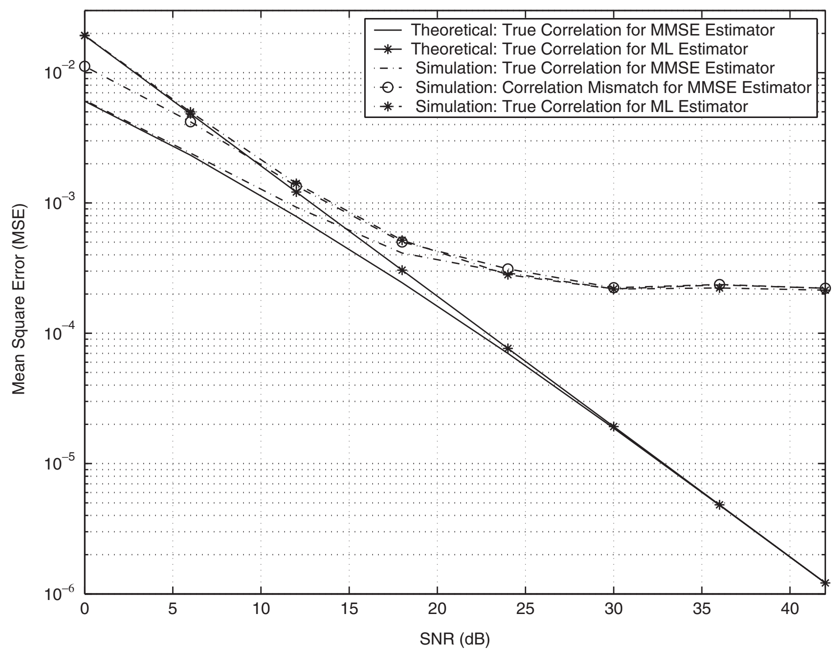

Fig. 4. Effects of correlation mismatch on MSE.

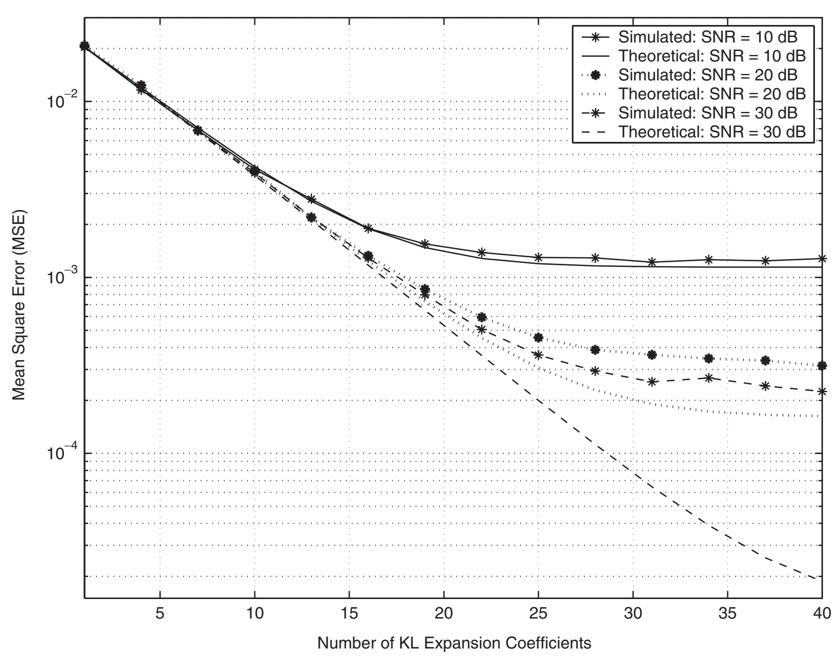

Fig. 5. MSE as a function of KL expansion coefficients.

of parameters in the expansion is increased, the irreducible error floor still occurs.

7. Conclusion

We consider the design of a low complexity MMSE channel estimator for OFDM systems in unknown wireless dispersive fading channels. We derive the batch MMSE estimator based on the stochastic orthogonal expansion representation of the channel via the KL transform. Based on such representation, we show that no matrix inversion is needed in the MMSE algorithm. Therefore, the computational cost for implementing the proposed MMSE estimator is low and computation is numerically stable. Moreover, the performance of our proposed batch method was studied through the derivation of minimum Bayesian MSE. Since the actual channel statistics and SNR may vary within OFDM block, we have also analyzed the effect of modeling mismatch on the estimator performance and shown both analytically and through simulations that the performance degradation due to such mismatch is negligible for low SNR values.

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Habib Şenol received the B.S. and M.S. degrees from the University of Istanbul in 1993 and in 1999, respectively. He is currently a Ph.D. student in the Department of Electronics Engineering at IŞIK University.
From 1996–1999, he was a Research Assistant with the University of Istanbul. In 1999, as a lecturer, he joined the faculty of the Department of Computer Engineering at Kadir Has University. His general research interests cover communication theory, estimation theory, statistical signal processing, and information theory. His current research activities are focused on wireless communication concepts with specific attention to channel estimation algorithms for multicarrier (OFDM) systems.

Hakan Ali Çırpan received the B.S. degree in 1989 from Uludag University, Bursa, Turkey, the M.S. degree in 1992 from the University of Istanbul, Istanbul, Turkey, and the Ph.D. degree in 1997 from the Stevens Institute of Technology, Hoboken, NJ, USA, all in Electrical Engineering.
From 1995–1997, he was a Research Assistant with the Stevens Institute of Technology, working on signal processing algorithms for wireless communication systems. In 1997, he joined the faculty of the Department of Electrical-Electronics Engineering at The University of Istanbul. His general research interests cover wireless communications, statistical signal and array processing, system identification and estimation theory. His current research activities are focused on signal processing and communication concepts with specific attention to channel estimation and equalization algorithms for space–time coding and multicarrier (OFDM) systems.
Dr. Cirpan received Peskin Award from Stevens Institute of Technology as well as Prof. Nazim Terzioglu award from the Research fund of The University of Istanbul. He is a Member of IEEE and Member of Sigma Xi.

Erdal Panayırcı received the Diploma Engineering degree in Electrical Engineering from Istanbul Technical University,Istanbul, Turkey in 1964 and the Ph.D. degree in Electrical Engineering and System Science from Michigan State University , East Lansing Michigan, USA, in 1970, between 1970 and 2000 he has been with the Faculty of

Electrical and Electronics Engineering at the Istanbul Technical University, where he was a Professor and Head of the Telecommunications Chair. Currently, he is a Professor and Head of the Electronics Engineering Department at IŞIK University, Istanbul, Turkey. He is engaged in research and teaching in digital communications and wireless Systems, equalization and channel estimation in multicarrier (OFDM) communication systems, and efficient modulation and coding techniques (TCM and turbo coding). He has been also part time consultant to the several leading companies in telecommunications in Turkey. He spent 2 years (1979–1981) with the Department of Computer Science, Michigan State University, as a Fulbright-Hays Fellow and a NATO Senior Scientist. Between 1983 and 1986 he served as a NATO Advisory Committee Member for the Special Panel on Sensory Systems for Robotic Control. From August 1990 to December 1991 he was with the Center for Communications and Signal Processing, New Jersey Institute of Technology, as a Visiting Professor, and took part in the research project on Interference Cancellation by Array Processing. Between 1998 and 2000, he was Visiting Professor at the Department of Electrical Engineering, Texas A&M University and took part in research on developing efficient synchronization algorithms for OFDM systems. Between 1995 and 1999, Prof. Panayırcı was an Editor for IEEE Transactions on Communications in the fields of Synchronization and Equalizations. He is currently Head of the Turkish Scientific Commission on Signals, Systems and Communications of URSI (International Union of Radio Science). He is a Fellow IEEE and Member of Sigma Xi.