Broadband Single Matching with Lumped Elements

Metin Şengül

Kadir Has University, Istanbul, Turkey
msengul@khas.edu.tr

Abstract

While designing broadband single matching networks for microwave communication systems via commercially available software tools, it is necessary for these tools to select the proper matching network topology and element values. But how to make these selections is an unknown problem. But the proposed approach in this paper presents a practical method to generate single matching networks with good initial element values. After completing the design, the performance of the designed single matching network is trimmed employing the commercially available computer-aided design (CAD) tools. The approach is explained via the given example which shows that proposed approach provides very good initials for CAD tools.

1. Introduction

Broadband impedance matching can be considered as one of the most important problems for microwave engineers [1]. While designing matching networks, broadband matching analytic theory [2], [3] and computer aided design (CAD) tools can be used [4]-[6]. But analytic theory is not easy to implement. So usually CAD tools are preferred to design broadband matching networks. But these tools require the proper network topology and element values, and then optimize the performance of the matched system. After optimization, the element values of the broadband matching network are reached. Therefore, in this paper, a proper network topology and element value generator is proposed for these tools.

The broadband matching problem can be defined as the design of a lossless two-port network between a generator and complex load, in such a way that power transfer from the source to the load is maximized over a desired frequency band. The transferred power is best measured by means of the transducer power gain, which is the ratio of power delivered to the load to the available power from the generator.

The broadband matching problem can be grouped as single and double matching problems. In the first group, generator is purely resistive and the load is any arbitrary complex impedance. But in the second group, both the generator and the load are arbitrary complex impedances.

Let us consider the single matching problem depicted in Fig. 1. Transducer power gain (TPG) can be expressed in terms of the real and imaginary parts of the normalized load impedance $Z_L = R_L + jX_L$ and those of the normalized back-end impedance $Z_2 = R_2 + jX_2$, or in terms of the normalized generator resistance R_G ($X_G = 0$) and the real and imaginary parts of the normalized front-end impedance $Z_1 = R_1 + jX_1$ of the matching network as follows

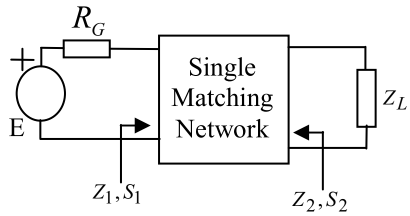

Fig. 1. Single matching arrangement

$$TPG(\omega) = \frac{4R_\alpha R_\beta}{(R_\alpha + R_\beta)^2 + (X_\alpha + X_\beta)^2} . \qquad (1)$$

Here if $\alpha = 1$, $\beta = G$, and if $\alpha = 2$, $\beta = L$.

The objective in single matching problems is to design a lossless matching network in such a way that the TPG given by (1) is maximized inside the desired frequency band. Since R_G and $Z_L = R_L + jX_L$ are given, the single matching problem is reduced to the determination of a realizable impedance function Z_1 or Z_2 . Once Z_1 or Z_2 are determined properly, the lossless single matching network can be easily synthesized.

In the methods exist in the literature [7-10], the matching network is expressed in terms of a set of free parameters by means of driving point impedance Z_2 . However, the matching problem can also be described by using any other set of parameters. In the real frequency scattering approach, which is referred to as the Simplified Real Frequency Technique (SRFT), the canonic polynomial representation of the scattering matrix is used to describe the matching network [11], [12].

In another method proposed in [13], the back-end impedance of the matching network Z_2 is modeled as a minimum reactance function, and then a Foster impedance is connected in series.

It can be concluded from the explanation above that the main objective is to express the back-end impedance Z_2 of the matching network in terms of any set of free parameters. Then gain performance of the matching network is optimized via (1). But the back-end impedance is obtained in very complicated manners in these methods. There is a very simple and obvious way to calculate the back-end impedance Z_2 or front-end impedance Z_1 of the matching network. This is the crux of the proposed method.

In the proposed approach, these impedances (Z_2 or Z_1) are determined via the scattering parameters of the lossless matching network, source and load reflection coefficients. In the next section, the canonic polynomial representation of the scattering parameters is briefly summarized, and then the rationale of the proposed approach is described.

2. Canonic Polynomial Representation of Scattering Matrix

By referring to the single matching configuration shown in Fig. 1, the scattering parameters of the lossless matching network can be written in terms of three real polynomials by using the Belevitch representation as follows:

$$\begin{aligned} S_{11}(p) &= h(p)/g(p), \quad S_{12}(p) = \mu f(-p)/g(p) \\ S_{21}(p) &= f(p)/g(p), \quad S_{22}(p) = -\mu h(-p)/g(p) \end{aligned} \qquad (2)$$

where $p = \sigma + j\omega$ is the complex frequency variable, g is a strictly Hurwitz polynomial, f is a real monic polynomial and μ is a constant ($\mu = \pm 1$). If the two-port is reciprocal, then the polynomial f is either even or odd and $\mu = f(-p)/f(p)$.

The polynomials $\{f, g, h\}$ are related by the Feldtkeller equation [14]

$$g(p)g(-p) = h(p)h(-p) + f(p)f(-p) . \qquad (3)$$

It is clear from (3) that the Hurwitz polynomial $g(p)$ is a function of $h(p)$ and $f(p)$. If the polynomials $f(p)$ and $h(p)$ are specified, then the scattering parameters of the two-port network, and then the network itself can be completely defined.

In almost all practical applications, the designer has an idea about the transmission zeros of the matching network. Hence the polynomial $f(p)$ which is constructed on the transmission zeros is usually defined by the designer. For practical problems, the designer may use the following form of $f(p)$

$$f(p) = p^{m_1} \prod_{i=0}^{m_2} (p^2 + a_i^2) \qquad (4)$$

where m_1 and m_2 are nonnegative integers and a_i 's are arbitrary real coefficients. This form corresponds to ladder type minimum phase structures, whose transmission zeros are on the imaginary axis of the complex p -plane.

3. Rationale of the Proposed Method

Consider the single matching arrangement shown in Fig. 1. The input reflection coefficient of the matching network when its output port is terminated in Z_L can be written in terms of scattering parameters of the matching network as

$$S_1 = S_{11} + \frac{S_{12} S_{21} S_L}{1 - S_{22} S_L} \qquad (5)$$

where S_L is the load reflection coefficient expressed as

$$S_L = \frac{Z_L - 1}{Z_L + 1} . \qquad (6)$$

Similarly, the output reflection coefficient of the matching network when its input port is terminated in R_G can be written in terms of scattering parameters of the matching network as

$$S_2 = S_{22} + \frac{S_{12} S_{21} S_G}{1 - S_{22} S_G} \qquad (7)$$

where S_G is the source reflection coefficient expressed as

$$S_G = \frac{R_G - 1}{R_G + 1} . \qquad (8)$$

So the input and output impedances of the matching network can be calculated via the following equations, respectively

$$Z_1 = \frac{1 + S_1}{1 - S_1} , \qquad (9a)$$

$$Z_2 = \frac{1 + S_2}{1 - S_2} . \qquad (9b)$$

As the result, the following algorithm can be proposed to solve single broadband matching problems with lumped elements. The modified version of the proposed approach has been given in [15] to solve double broadband matching problems.

4. Proposed Algorithm

Inputs:

- $Z_{L(measured)} = R_{L(measured)} + jX_{L(measured)}$, $R_{G(measured)}$: Measured load impedance and generator resistance data, respectively.
- $\omega_{i(measured)}$: Measurement frequencies, $\omega_{i(measurement)} = 2\pi f_{i(measurement)}$.
- f_{norm} : Normalization frequency.
- R_{norm} : Impedance normalization number in ohms.
- $h_0, h_1, h_2, \ldots, h_n$: Initial real coefficients of the polynomial $h(p)$. Here n is the degree of the polynomial which is equal to the number of lossless lumped elements in the matching network. The coefficients can be initialized as ±1 in an ad hoc manner, or the approach explained in [16] can be followed.
- $f(p)$: A monic polynomial constructed on the transmission zeros of the matching network. A practical form is given in (4).
- δ_c : The stopping criteria of the sum of the square errors.

Outputs:

- Analytic form of the input reflection coefficient of the lossless matching network given in the Belevitch form of $S_{11}(p) = h(p)/g(p)$. It is noted that this algorithm determines the coefficients of the polynomials $h(p)$ and $g(p)$, which in turn optimizes system performance.
- Circuit topology of the lossless matching network with element values: The circuit topology and element values are

obtained as the result of the synthesis of $S_{11}(p)$. Synthesis is carried out in the Darlington sense. That is, $S_{11}(p)$ is synthesized as a lossless two-port which is the desired matching network [17]. Also the synthesis process can be carried out by using impedance based Foster or Cauer methods via $Z_{11}(p)=(1+S_{11}(p))/(1-S_{11}(p))$ as explained in [18].

Computational Steps:

Step 1: Normalize the measured frequencies with respect to f_{norm} and set all the normalized angular frequencies

$\omega_i = f_{i(measured)} / f_{norm}$.

Normalize the measured load impedance and generator resistance data with respect to impedance normalization number R_{norm} ; $R_L = R_{L(measured)} / R_{norm}$, $X_L = X_{L(measured)} / R_{norm}$, $R_G = R_{G(measured)} / R_{norm}$ over the entire frequency band.

Step 2: Obtain the strictly Hurwitz polynomial $g(p)$ from (3). Then calculate scattering parameters via (2).

Step 3: Calculate load and source reflection coefficients S_L and S_G via (6) and (8), respectively.

Step 4: Calculate input and output reflection coefficients S_1 and S_2 via (5) and (7), respectively.

Step 5: Calculate input and output impedances Z_1 and Z_2 via (9a) and (9b), respectively.

Step 6: Calculate transducer power gain via (1)

Step 7: Calculate the error via $\varepsilon(\omega)=1-TPG(\omega)$, then $\delta=\sum|\varepsilon(\omega)|^2$.

Step 8: If δ is acceptable ($\delta \le \delta_c$), stop the algorithm and synthesize $S_{11}(p)$. Otherwise, change the initialized coefficients of the polynomial $h_l(p)$ via any optimization routine and return to step 2.

5. Example

In this section, a single matching example is presented for the design of a lossless broadband matching network. The normalized load impedance and generator resistance data are given in Table I. It should be noted that the given load data can easily be modeled as a capacitor $C_L = 4$ in parallel with a resistance $R_L = 1$ (i.e. $R_L // C_L$ type of impedance). Since the given data are already normalized, there is no need for a normalization step. The same example is solved here via SRFT.

The polynomial $h(p)$ is initialized as $h(p)=-p^4+p^3-p^2+p-1$ in an ad hoc manner. Also the polynomial $f(p)$ is selected as $f(p)=1$, since a low-pass matching network is desired. In the example, α and β are selected as $\alpha=2$, $\beta=L$. So back-end impedance Z_2 and load impedance Z_L are used in the TPG expression in Step 6. Then after running the proposed algorithm, the following scattering parameter of the matching network is obtained

Table 1. Given normalized load and generator data

ω	R_L	X_L	R_G
0.0	1.0000	0.0000	1.0000
0.1	0.8621	-0.3448	1.0000
0.2	0.6098	-0.4878	1.0000
0.3	0.4098	-0.4918	1.0000
0.4	0.2809	-0.4494	1.0000
0.5	0.2000	-0.4000	1.0000
0.6	0.1479	-0.3550	1.0000
0.7	0.1131	-0.3167	1.0000
0.8	0.0890	-0.2847	1.0000
0.9	0.0716	-0.2579	1.0000
1.0	0.0588	-0.2353	1.0000

$S_{11}(p)=\dfrac{h(p)}{g(p)}$ where

$h(p)=-1.4521p^4-0.8959p^3+0.3466p^2-1.3879p+0.4719$,

$g(p)=1.4521p^4+3.7552p^3+4.2326p^2+3.3106p+1.1058$.

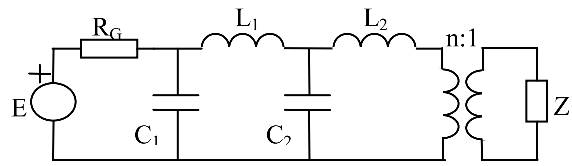

Fig. 2. Designed single matching network; proposed: $L_1=1.4791$, $L_2=1.5573$, $C_1=1.0157$, $C_2=1.9618$, $n=1.5791$, SRFT: $L_1=1.479$, $L_2=1.5479$, $C_1=1.0157$, $C_2=1.9633$, $n=1.5748$

After synthesizing the obtained scattering parameter or the corresponding impedance function, the matching network seen in Fig. 2 is obtained. The details can be found in [17].

As seen in from Fig. 3, initial performance of the matched system looks fairly good. The curves obtained via the proposed method and SRFT are very close to each other, and are nearly the same. However, it is further improved via optimization utilizing the commercially available design package called Microwave Office of Applied Wave Research Inc. (AWR) [4]. Thus, the final normalized elements values are given as $L_1=1.48$, $L_2=1.6862$, $C_1=0.8306$, $C_2=1.8153$, $n=1.6453$. For comparison purpose, both initial and the optimized performances of the matched system and the performance obtained via SRFT are depicted in Fig. 3 by using Microwave Office [4].

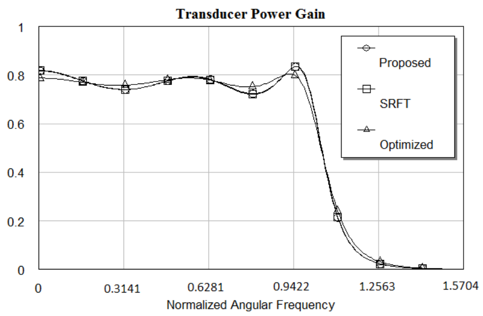

Fig. 3. Performance of the single matching network

The algorithm is implemented via Matlab. The elapsed time for this example is 3.20 seconds. It is 10.53 seconds via SRFT. Consequently, it can be said that the proposed method and SRFT have nearly the same performance, but the proposed method is faster than SRFT.

6. Conclusion

Usually commercially available computer-aided design tools are utilized to design broadband matching networks by microwave engineers. If the matching network topology is supplied, these packages are excellent tools to optimize system performance by working on the element values. So initial element values generation is very vital, since system performance is highly nonlinear in terms of the element values. Therefore, in this paper, an initialization approach is proposed for CAD tools to design single matching networks.

In the proposed approach, the back-end or front-end impedance of the matching network network is determined in terms of the scattering parameters of the matching network, source and load reflection coefficients. Then this impedance and one of the terminations (R_G or Z_L) are used to calculate the transducer power gain of the system. The scattering parameters of the matching network are optimized to be able to achieve maximum performance.

Finally, it is synthesized as a lossless two-port yielding the desired single matching network topology with initial element values. Eventually, the actual performance of the matched system is improved by means of a commercially available CAD tool.

An example has been presented here to construct a single matching network with lumped elements. It was shown that the proposed method generates very good initials to further improve the matched system performance by working on the element values. Therefore, it is expected that the proposed algorithm can be used as a front-end for commercially available CAD tools to design single matching networks for microwave communication systems.

7. References

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