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Detection of Jointly Active Primary Systems

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Abstract: Recent studies in cognitive radios consider the detection of multiple bands for a better utilization of the spectrum. If the cognitive radio (CR) is an ultra wideband (UWB) or a wideband system, then the CR should ensure all the coexisting primary systems in these bands are detected before the CR can start data transmission. In this work, we study the primary system detection performance of a wideband CR assuming that there are multiple coexisting primary systems and that these primary systems may be jointly active. Accordingly, we consider the implementation of (i) a maximum a posteriori (MAP) based detection (i.e., joint detection) that takes into account the statistics of simultaneously operating systems in independent bands, and (ii) a Neyman-Pearson (NP) test based detection that optimizes the threshold values independently in each band (i.e., independent detection). In addition to obtaining the probabilities of false alarm and detection expressions for these two methods, we use the threshold values obtained from joint detection so as to achieve the optimum NP test based independent detection results with a simpler implementation. We also provide the performance comparison of joint and independent detection for various practical scenarios when there are multiple active primary systems.

Keywords: Cognitive radios, ultra wideband (UWB) systems, detect-and-avoid, wideband spectrum sensing, joint detection

1. Introduction

As the spectrum becomes more and more crowded, cognitive radios (CRs) [1] and ultra wideband (UWB) systems [2] have been widely accepted as alternative technologies for communication. From the perspective of a licensed primary system, the major concern for the implementation of either CRs or UWB systems is the possible interference they may cause to primary systems. Hence, many regulatory agencies worldwide have mandated detect-and-avoid (DAA) techniques in various bands [3]. Accordingly, CRs and UWB systems have to perform spectrum sensing before they can communicate.

The most commonly used spectrum sensing method is the energy detection. There is a comprehensive literature on energy detection in a single frequency band with further improvements using multiple antennas, multiple observations, time-domain diversity schemes, or collaboration among secondary users [4]–[6]. On the other hand, the literature on energy detection in *multiple frequency bands* is rather new. This concept is indeed quite important as it is more desired to assess the availability of a wider spectrum for better utilization. Moreover, the CRs may be UWB or wideband systems, and therefore, they should ensure all the coexisting primary systems in common bands are detected before they can start data transmission. In [7]–[9], energy detection in multiple bands was considered so as to make a joint decision on these bands. The common assumption in these studies was that the primary systems in different bands were independent. On the other hand, if the licensed systems in different bands are dependent, the detection performance can be further improved. In [10], the primary system detection performance was assessed for $M = 2$ interdependent bands using a maximum a posteriori (MAP) based detection method and the detection gain over independent detection was quantified. Here, $M = 2$ could be an example of frequency division duplex uplink-downlink communications.

In this paper, motivated by quantifying the detection performance gain when there are $M > 2$ interdependent systems, we generalize the work in [10] to multiple bands. Here, $M > 2$ could be an example of M systems in independent frequency bands with known activity statistics. For example, the statistics might indicate that two systems are jointly active 40% of the time, while three systems are jointly active 50% of the time. For that we consider the implementation of a MAP based detection (i.e., joint detection) and a Neyman-Pearson (NP) test based detection that optimizes the threshold values independently in each band (i.e., independent detection). Different from [10], we (i) generalize the probability of false alarm and detection expressions for $M > 2$ for both joint and independent detection, (ii) use the threshold values obtained from joint detection so as to achieve the optimum NP test based independent detection results with a simpler implementation, and (iii) provide practical examples to quantify the performance gain of joint detection over independent detection for various cases. The results are important as they are of interest to researchers working on CRs and UWB systems that employ DAA.

The rest of the paper is organized as follows. In Section 2, the receiver model is presented. In Section 3, the implementations of joint detection and independent detection are presented. In Section 4, numerical results are presented for the comparison of the considered detection methods under different scenarios. Concluding remarks are given in Section 5.

2. Receiver Model

We assume that there are M primary systems coexisting with a wideband CR in the same frequency band. These systems may be active or passive depending on the time of the day. The received signals are filtered using ideal zonal bandpass filters with bandwidths W_m at each frequency band to eliminate the out-of-band noise. Accordingly, the two hypotheses corresponding to the absence and presence of the filtered signal received from the mth system, respectively, are

$$H_{0,m}: \quad r_m(t) = n_m(t) \tag{1}$$
$$H_{1,m}: \quad r_m(t) = A_m\, e^{j\theta_m}\, s_m(t - \tau_m) + n_m(t) \tag{2}$$

where each signal $s_m(t)$ passes through a channel with amplitude A_m and phase θ_m uniformly distributed over $[0, 2\pi)$, τ_m is the timing offset between the two systems, and $n_m(t)$ is band-limited additive white Gaussian noise (AWGN) with variance $\sigma_{n_m}^2 = N_0 W_m$. Using a square-law detector and normalizing the output with the two-sided noise power spectral density $N_0/2$, the decision variable for the mth system can be obtained as

$$d_m = \frac{2}{N_0} \int_0^{T_m} |r_m(t)|^2 \, dt \tag{3}$$

where T_m is the integration time for the mth system and $|\cdot|$ is the absolute value operator. For either hypothesis, it can be shown that d_m can be modeled using χ^2

distribution with $N_m = 2T_mW_m$ degrees of freedom [10], where the variance term is $\sigma_m^2 = \frac{\sigma_{nm}^2}{N_0W_m} = 1$ for a passive system. For an active system, the variance term is $\sigma_m^2 = \gamma_m + 1$, where the signal-to-noise-ratio (SNR) is defined as $\gamma_m = \frac{A_m^2\sigma_s^2}{N_0W_m}$ with σ_s^2 being the variance of the primary signal samples.

2.1 Detection of a Single System

In conventional detection, the decision variable d_m is compared to a pre-selected threshold value λ_m in order to make a decision for the mth system. The performance measures, probability of false alarm and probability of detection can be respectively expressed as

$$P_{f,m} = \Pr[d_m > \lambda_m | H_{0,m}] \quad (4)$$

$$P_{d,m} = \Pr[d_m > \lambda_m | H_{1,m}] \quad (5)$$

where (4) and (5) can be simplified to $P_{x,m} = Q\left(\frac{N_m}{2}, \frac{\lambda_m}{2\sigma_m^2}\right) = \frac{\Gamma\left(\frac{N_m}{2}, \frac{\lambda_m}{2\sigma_m^2}\right)}{\Gamma\left(\frac{N_m}{2}\right)}$, $x \in \{f, d\}$ with the corresponding σ_m^2 values for $H_{0,m}$ and $H_{1,m}$, and $Q(a, b)$ is the upper incomplete Gamma function [11].

2.2 Detection of Multiple Systems

If the CR is a wideband or a UWB system, then it has to assess the presence of all coexisting primary systems before it can communicate. Accordingly, the hypotheses have to be redefined as $\mathbf{H} = \left\{[H_{x_M,M}, \ldots, H_{x_2,2}, H_{x_1,1}] \,|\, x_m \in \{0,1\}\right\}$. Since there are M primary systems, there are 2^M possible combinations of hypotheses. Accordingly, the CR can only transmit if $x_m = 0$, $\forall m$, which can be represented by $\mathbf{H}_0$. For the rest $2^M - 1$ combinations even if a single primary system is active, then the CR is not allowed to communicate. The hypotheses corresponding to having at least one active system can be represented by $\mathbf{H}_{1,i}$, $1 \leq i \leq 2^M - 1$, where the active and passive systems in each hypothesis can be determined by the relation

$$(i)_{10} = (x_M \cdots x_2\, x_1)_2 \quad (6)$$

with $(\cdot)_n$ representing the logarithmic base n. Hence, the probability of false alarm and probability of detection for multiple systems can be expressed as

$$P_f = \Pr\left[P_{det} | \mathbf{H}_0\right] \quad (7)$$

$$P_d = \sum_{i=1}^{2^M-1} \Pr\left[P_{det} \middle| \mathbf{H}_{1,i}\right] \Pr[\mathbf{H}_{1,i} | \mathbf{H}_1] \quad (8)$$

where $P_{det} = 1 - \bigcap_{m=1}^{M} \Pr[d_m < \lambda_m]$ and $\mathbf{H}_1 = \bigcup_{i=1}^{2^M-1} \mathbf{H}_{1,i}$.

The probability of detection expression given in (8) is different from the conventional expression mainly due to the P_{det} term being conditioned on different hypotheses. Hence, the probability of these hypotheses is important in determining (8). Accordingly, the probability that all the primary systems are passive is $p_0 = \Pr[\mathbf{H}_0]$, whereas $p_i = \Pr[\mathbf{H}_{1,i}]$,$1 \leq i \leq 2^M - 1$, is the probability that $\mathbf{H}_{1,i}$ holds, where $\sum_{i=0}^{2^M-1} p_i = 1$. For example, if there are $M = 4$ interdependent systems and p_7 is close to unity, that means the first three systems are jointly active most of the time while the fourth system is not active (i.e., $(7)_{10} = (0\,1\,1\,1)_2$). To note, the probabilities $\{p_i\}$ can also be referred to as joint system activity values.

3. Detection Methods

In the following, we consider the implementation of two detection methods for $M > 2$ primary systems that are interdependent. For both methods, it is assumed that the systems' joint activity values $\{p_i\}$ and the probability density functions (pdfs) of the decision variables $\{d_m\}$ are known a priori. This is a reasonable assumption as the traffic information of the primary systems may be available to secondary users, and the SNR of the primary signals can be estimated at the receiver. In practice, the traffic information can be obtained from either licensed service providers or research groups conducting measurement campaigns. Accordingly, this information can be used to model the percentage of time the corresponding frequency band is occupied or not. For multiple bands, the statistics for the bands being jointly active or not can also be determined. As for the χ^2 distribution, the only unknown is the SNR. Hence, knowing the noise power level in an available frequency band, the SNR can be measured if a primary user becomes active. Based on the availability of the conditions above, we explain the detection methods next.

3.1 Joint Detection

Knowing $\{p_i\}$ and the pdfs of $\{d_m\}$, the MAP decision rule serves as an optimal decision rule. The hypothesis can be estimated by finding the maximum of the MAP decision metrics as

$$\hat{\imath} = \arg \max_{i \in \{0,1,\ldots,2^M-1\}} PM_i$$
$$\hat{\mathbf{H}} = \mathbf{H}_0 \text{ if } \hat{\imath} = 0; \quad \hat{\mathbf{H}} = \mathbf{H}_1 \text{ if } \hat{\imath} = \{1, 2, \ldots, 2^M - 1\} \tag{9}$$

where the decision metrics are $PM_0 = b_0\, p_0\, f_{D_1,D_2,\ldots,D_M|\mathbf{H}_0}(d_1, d_2, \ldots, d_M)$ and $PM_i = b_i\, p_i\, f_{D_1,D_2,\ldots,D_M|\mathbf{H}_{1,i}}(d_1, d_2, \ldots, d_M)$, $\{i = 1, 2, \ldots, 2^M - 1\}$. The bias terms $\{b_i \,|\, i = 0, 1, 2, \ldots, 2^M - 1\}$ are the intentionally introduced terms to achieve a desired trade-off between the probabilities of false alarm and detection, and $f_{D_1,D_2,\ldots,D_M|\mathbf{H}_x}(d_1, d_2, \ldots, d_M)$ are the joint pdfs conditioned on the hypothesis $\mathbf{H}_x$. The decision metrics can be simplified as

$$PM_i = b_i\, p_i\, C \prod_{m=1}^{M} \frac{\exp\left(\frac{-d_m}{2(\gamma_m+1)^{x_m}}\right)}{(\gamma_m + 1)^{x_m N_m/2}}, \quad \{i = 0, 1, 2, \ldots, 2^M - 1\} \tag{10}$$

where $C = \prod_{m=1}^{M} \frac{d_m^{N_m/2-1}}{2^{N_m/2}\Gamma(N_m/2)}$ is a common term for all PM_i and $\{x_m\}$ can be obtained from the index i using (6). Considering (7)–(9), the probabilities of false alarm and detection can be defined as

$$P_f = 1 - \Pr\left[\bigcap_{i=1}^{2^M-1} (PM_0 > PM_i)|\mathbf{H}_0\right] \tag{11}$$

$$P_d = 1 - \sum_{i=1}^{2^M-1} \frac{p_i}{1-p_0} \Pr\left[\bigcap_{j=1}^{2^M-1} (PM_0 > PM_j)|\mathbf{H}_{1,i}\right]. \tag{12}$$

By substituting (10) into the comparison term $\{PM_0 > PM_i\}$, (11) and (12) can be simplified to

$$P_f \;=\; 1 - P_{cond,\mathbf{H}_0}\left[\prod_{m=1}^{M} (1 - P_{f,m})\right] \tag{13}$$

$$P_d \;=\; 1-\sum_{i=1}^{2^M-1}\frac{p_i}{1-p_0}\,P_{cond,\mathbf{H}_{1,i}}\left[\prod_{m=1}^{M}(1{-}P_{d,m})^{x_m}(1{-}P_{f,m})^{(1-x_m)}\right] \tag{14}$$

where $P_{cond,\mathbf{H}_x}$, $\{x=0\}$ or $\{x=1,i\}$ is the conditional probability term obtained as

$$P_{cond,\mathbf{H}_x} = \prod_{i=1}^{2^M-1} P_{\lambda_i\,|\,(\lambda_1)^{x_1},(\lambda_2)^{x_2},\ldots,(\lambda_{2^{m-1}})^{x_m},\ldots,(\lambda_{2^{M-1}})^{x_M},\mathbf{H}_x} \quad \text{with} \tag{15}$$

$$P_{\lambda_i|(\lambda_1)^{x_1},\ldots,(\lambda_{2^{M-1}})^{x_M},\mathbf{H}_x}=\Pr\left[\sum_{m=1}^{M}x_m a_m d_m<\lambda_i\middle|x_1a_1d_1<\lambda_1,\ldots,x_Ma_Md_M<\lambda_{2^{M-1}},\mathbf{H}_x\right] \tag{16}$$

and $a_m = \frac{\gamma_m}{2(\gamma_m+1)}$. The resulting threshold values are

$$\lambda_i = \left[\sum_{m=1}^{M}\frac{x_m N_m}{2}\ln\Big(\gamma_m+1\Big)+\ln\Big(\frac{p_0}{p_i}\Big)+\ln\Big(\frac{b_0}{b_i}\Big)\right] \text{ for } i=\{1,2,\ldots,2^M-1\} \tag{17}$$

where $\{x_m\}$ are obtained from (6). It should be noted that $\{\lambda_{2^{m-1}}|m=1,2,\ldots,M\}$ correspond to independent threshold values[1] for each band, m, whereas the rest of the $\{\lambda_i\}$ values (i.e., 2^M-M-1 values) correspond to the joint bands. For example, λ_5 corresponds to bands 1 and 3. We calculate the probabilities of false alarm and detection using (13)–(14), where (16) can be calculated numerically. By letting $b = b_1 = b_2 = \cdots = b_{2^M-1}$ in (17) and varying it, a tradeoff between (P_f, P_d)-pairs can be obtained with a close-to-optimal performance [10].

3.2 Independent Detection

The probabilities of false alarm and detection for multiple bands can be expressed as

$$P_f \;=\; 1-\prod_{m=1}^{M}(1-P_{f,m}) \tag{18}$$

$$P_d \;=\; 1-\sum_{i=1}^{2^M-1}\frac{p_i}{1-p_0}\prod_{m=1}^{M}(1{-}P_{d,m})^{x_m}(1{-}P_{f,m})^{(1-x_m)} \tag{19}$$

if the bands are independently processed. These equations can also be obtained by letting $P_{cond,\mathbf{H}_x}=1$ in (13) and (14). In order to obtain the best detection performance the NP test can be employed, which optimizes the threshold values in order to maximize P_d for a given target $P_f=\alpha$:

$$\max_{\{\lambda_{2^{m-1}}|m=1,2,\ldots,M\}} \quad P_d \qquad s.t.\;\; P_f=\alpha. \tag{20}$$

Here, $\{\lambda_{2^{m-1}}|m=1,2,\ldots,M\}$ are the independent threshold values to be optimized. This is equivalent to maximizing P_d over an M-dimensional search space.

Alternatively, we use the optimized threshold values obtained from MAP detection instead of the NP test. These values are easier to compute compared to the NP test. Accordingly, the threshold values that will be used in (18) and (19) can be simplified as

$$\lambda_{2^{m-1}} = \left[\frac{N_m}{2}\ln\Big(\gamma_m+1\Big)+\ln\Big(\frac{p_0}{p_{2^{m-1}}}\Big)+\ln\Big(\frac{b_0}{b_{2^{m-1}}}\Big)\right]\Big/a_m \text{ for } m=\{1,2,\ldots,M\}. \tag{21}$$

[1] Note that these values resulting from MAP detection and corresponding to the mth band are different from the conventional threshold values $\{\lambda_m|m=1,2,\ldots,M\}$ given after (8).

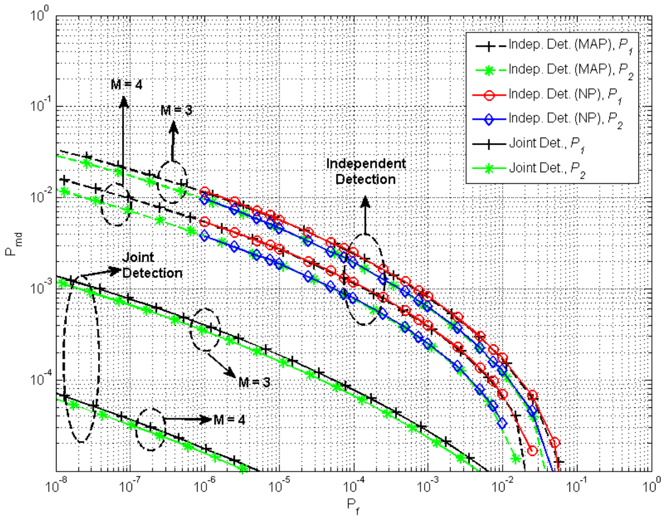

Figure 1: Complementary ROC curves for various M and system activity values.

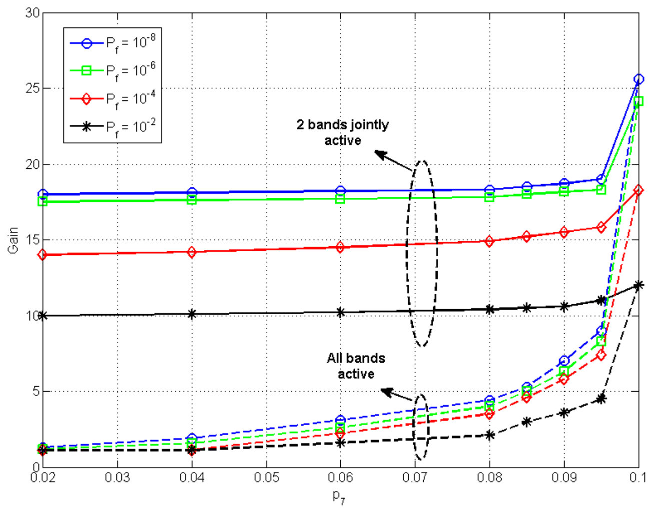

Figure 2: The effect of system activity values on the detection performance when $M = 3$.

Similar to joint detection, by letting $b = b_1 = b_2 = \cdots = b_{2^M-1}$ in (21) and varying it, a tradeoff between (P_f, P_d)-pairs can be obtained.

4. Results

In this section, we evaluate the effects of number of interdependent systems, joint system activity values, and SNR on the joint and independent detection performances. For all scenarios, it is assumed that $\Pr[\mathbf{H}_0] = 0.90$ and $\Pr[\mathbf{H}_1] = 0.10$. Also, SNR and N values for each band are fixed to 10dB and 8, respectively, unless otherwise indicated.

In Fig. 1, we consider two sets of system activity values, $P_1 : \{p_{2^M-1} = 0.07, p_1 = p_2 = \ldots = p_{2^{M-1}} = 0, p_i\}$ and $P_2 : \{p_{2^M-1} = 0.08, p_1 = p_2 = \ldots = p_{2^{M-1}} = 0, p_i\}$ for $M = 3$ and $M = 4$, where p_i represent the probability of jointly active bands and have equal values, $p_i = (1 - p_{2^M-1})/(2^M - M - 2)$, $\forall i$. For performance comparison, complementary receiver operating characteristic (ROC) curves (i.e., P_f vs. $P_{md} = 1 - P_d$) are plotted. It can be observed that the joint detection performs better than

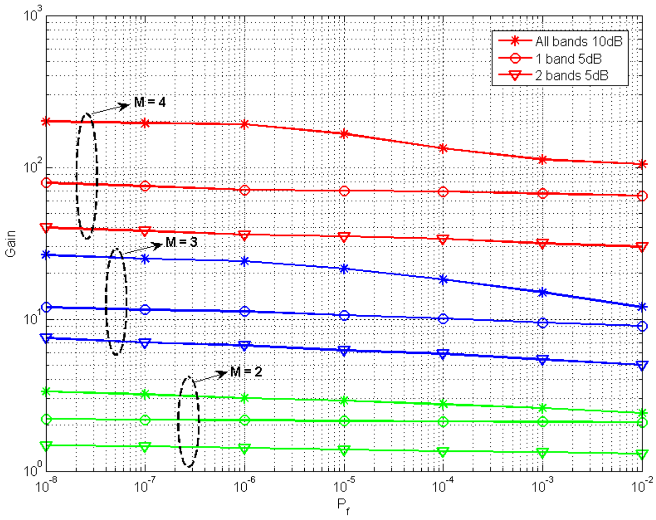

Figure 3: The effect of SNR values on the detection performance for various M values.

the independent detection with M increasing and for P_2 (all M bands are active more frequently) as expected. Also, it is important to note that the threshold values obtained by MAP detection and used in independent detection achieve the same performance as the NP test results. Hence, it can be used with an easier implementation to replace the NP test.

In Fig. 2, missdetection performance gains are compared at various P_f values for 2-band (i.e., only $\{p_3, p_5, p_6\}$ are non-zero) and no-band (i.e., only $\{p_1, p_2, p_4\}$ are non-zero) jointly active cases when $M = 3$ and p_7 varying. The probability value $(1 - p_7)$ is equally distributed among the non-zero probability values. The gains are defined as the P_{md} ratios of joint and independent detection at fixed P_f values. It can be observed that the gains increase with P_f decreasing and p_7 increasing. When $p_7 = 0.1$, the performances for both cases merge at the best gain value.

Finally, in Fig. 3, the effect of SNR degradation on the detection performance is investigated for various M values. Again, the missdetection gain values are calculated at various P_f values. For each case, we assume that all systems are jointly active all the time, i.e., $p_{2^M-1} = 0.1$, for $M = \{2, 3, 4\}$. When $M = 2$, gains similar to the ones reported in [10] are obtained. When M is increased, there is a significant increase in gain due to jointly processing the bands. When the SNR decreases in a band, the $M = 4$ case can compensate better due to other active bands having significant SNR values.

5. Conclusion

In this paper, we studied the primary system detection performance of a wideband CR assuming that there are multiple coexisting primary systems and that these primary systems may be jointly active. Accordingly, we considered the implementation of MAP based joint detection and the optimum NP test based independent detection methods. We initially showed that the optimum independent detection results can be obtained with a simpler implementation if the MAP detection based thresholds are used. We then quantified the detection gain that can be achieved with joint detection for practical scenarios. The results show that the detection gain increases with M, i.e., the number of jointly active systems, and the value of p_{2^M-1}, i.e., the case when all systems are jointly active, for lower false alarm values. The study presented is important for researchers working on CRs and UWB systems with a special focus on wideband spectrum sensing.

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