Realization of Ideal Filter Characteristics via Genetic Algorithm

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Abstract

In this paper, realization of ideal filter characteristics via genetic algorithm has been studied. The filter is defined as a lossless two-port terminated normalized source and load resistances, and the coefficients of its describing scattering polynomials have been calculated via genetic algorithm. An element value table has been given for a low-pass normalized prototype filter containing only one element to ten elements.

1. Introduction

A filter is a two-port network used to control the frequency response at a certain region in a system by providing transmission within the pass-band and attenuation in the stop-band. The ideal filter would have zero insertion loss in the pass-band, infinite attenuation in the stop-band, and a linear phase response in the pass-band.

By using the image parameter method, filters can be designed, and they consist of a cascade of simpler two-port filter sections to provide the desired cutoff frequencies and attenuation characteristics, but do not allow the specification of a frequency response over the complete operating band. Thus although the procedure is relatively simple, the design of filters by the image parameter method often must be iterated many times to achieve the desired response.

A more modern procedure is the insertion loss method. It uses network synthesis techniques to design filters with a completely specified frequency response. The design is simplified by forming low-pass normalized filter prototypes. Then frequency and impedance transformations are applied to convert the prototype filters to the desired frequency range and impedance level.

In this paper, low-pass normalized prototype filter element values are obtained via a genetic algorithm based approach. In the optimization part, ideal low-pass filter characteristic has been utilized.

In the next section, description of a lossless two-port is summarized shortly, and then genetic algorithm is explained in a brief manner. Then after describing the proposed procedure, element value table for low-pass normalized prototype filter is given.

2. Scattering Description of Lossless Two-ports

For a lumped-element lossless two-port like the one depicted in Fig. 1, the scattering matrix can be written as [1]:

$$S(p)=\begin{bmatrix} S_{11}(p) & S_{12}(p) \\ S_{21}(p) & S_{22}(p) \end{bmatrix}=\frac{1}{g(p)}\begin{bmatrix} h(p) & \mu f(-p) \\ f(p) & -\mu h(-p) \end{bmatrix} \tag{1}$$

where $g(p)$, $h(p)$ and $f(p)$ are real polynomials in complex frequency $p=\sigma+j\omega$, μ is a constant and $g(p)$ is a strictly Hurwitz polynomial. The three polynomials $g(p)$, $h(p)$, $f(p)$ are related by the Feldtkeller equation as $g(p)g(-p)=h(p)h(-p)+f(p)f(-p)$.

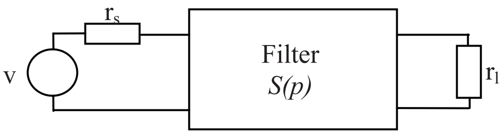

Fig. 1. Filter as a lossless two-port.

If the two-port is reciprocal, then the polynomial $f(p)$ is either even or odd. In this case, $\mu=+1$ if $f(p)$ is even, and $\mu=-1$ if $f(p)$ is odd. As a result, for a lossless reciprocal two-port

$$\mu=\frac{f(-p)}{f(p)}=\pm 1 \tag{2a}$$

and the Feldtkeller equation can be modified as

$$g(p)g(-p)=h(p)h(-p)+\mu f(p)^2 . \tag{2b}$$

3. Genetic Algorithm

Genetic algorithm is a heuristic search algorithm that inspired by the biological evolution process and used to find the solution of the optimization problems. Algorithm is started with a set of possible solutions. This set of solutions is called as population and represented by chromosomes.

One common application of genetic algorithm is function optimization, where the goal is to find a set of parameter values that maximize a multi-parameter function (fitness function).

3.1. Genetic Algorithm Operators

The basic form of genetic algorithm consists of three types of operators: selection, crossover and mutation [2]. Selection operator selects chromosomes in the population for reproduction. Crossover decomposes two distinct solutions and then randomly mixes their parts to form new solutions. Mutation randomly alters some of gene values in a chromosome from its initial state. This operation results in new gene values and better solutions values can be obtained from this new gene values.

A simple genetic algorithm works as follows [2,3]:

1- Start with a randomly picked population. (Produce candidate solutions to a problem.)

2- Calculate the fitness values of each chromosome in the population.

3- Create new solutions (offspring) by repeating the following steps:

- Using fitness probabilities select a pair of parent chromosomes from the current population.
- Cross over the pair at a randomly chosen point to form two offspring.
- Mutate the two offspring and place the obtained chromosomes in the new population.
- Replace the current population with the new population.
- Repeat process by starting from fitness value calculation step.

4. Design Procedure via Genetic Algorithm

The desired filter response is assumed to be the ideal low-pass filter characteristic shown in Fig. 2. Since it is low-pass type, the polynomial $f(p)$ is chosen as $f(p)=1$, namely all the transmission zeros are put to infinity. To be able to obtain the normalized termination resistor $r_l = 1$, the lowest ordered coefficient of polynomial $h(p)$ is set to zero. The other coefficients of polynomial $h(p)$ are chosen as the optimization parameters. Then by using (2b), the polynomial $g(p)$ is obtained.

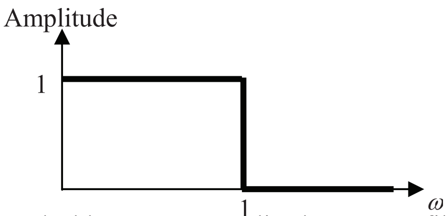

Fig. 2. Ideal low-pass normalized prototype filter characteristic.

Error is defined as the difference between the ideal characteristic seen in Fig. 2 and the characteristic calculated at each step as follows [6],

$$TPG(\omega) = \frac{1}{\left|g(j\omega)\right|^2} . \tag{3}$$

At the end of the optimization process, it is desired that TPG of the designed filter will have the same form as seen in Fig. 2. So it is desired to realize the ideal characteristic. But after performing the genetic algorithm based design procedure, it is seen that it is not possible to realize the ideal characteristic, and the obtained characteristic converges to Butterworth characteristic naturally.

Then an element value table similar to the table exist in literature has been formed, (Table 1). By using the table, low-pass normalized prototype filter containing one element to ten elements can be designed easily without any calculation. In this table, N represents the number of elements in the filter, the last element value is the normalized termination resistance ($r_l = 1$), and normalized source resistance is equal to $r_s = 1$.

From the low-pass normalized Butterworth prototype filter tables exist in literature (Table 2), it is seen that these filters are symmetric [4]. So to be able to design symmetric filters, we have restricted the coefficient of the polynomial $h(p)$, namely except the highest ordered coefficient, all the coefficients must be zero [5]. So the highest ordered coefficient is chosen as optimization parameter. At the end of the optimization process, it is seen that the obtained elements values are exactly the same as values given in Table 2.

Table 1. Obtained Element Values

N	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9	e_{10}
1	2.2654	1								
2	1.4883	1.4879	1							
3	1.8859	1.5111	1.131	1						
4	1.051	1.5782	2.023	1.3104	1					
5	0.82236	1.6185	1.9283	1.8504	1.2341	1				
6	1.3986	1.8857	1.8342	1.9507	1.5231	0.84717	1			
7	0.64229	1.6229	1.8953	2.0874	1.6641	1.6598	0.67206	1		
8	0.40624	1.6177	1.9586	1.8537	1.9847	1.6959	1.4478	0.52885	1	
9	0.2745	1.0876	1.7229	2.0032	2.0131	1.7684	1.8034	1.3289	0.49289	1

Table 2. Element Values for Low-pass Normalized Butterworth Prototype Filters [4]

N	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9	e_{10}
1	2.0000	1.0000								
2	1.4142	1.4142	1.0000							
3	1.0000	2.0000	1.0000	1.0000						
4	0.7654	1.8478	1.8478	0.7654	1.0000					
5	0.6180	1.6180	2.0000	1.6180	0.6180	1.0000				
6	0.5176	1.4142	1.9318	1.9318	1.4142	0.5176	1.0000			
7	0.4450	1.2470	1.8019	2.0000	1.8019	1.2470	0.4450	1.0000		
8	0.3902	1.1111	1.6629	1.9615	1.9615	1.6629	1.1111	0.3902	1.0000	
9	0.3473	1.0000	1.5321	1.8794	2.0000	1.8794	1.5311	1.0000	0.3473	1.0000

Now let us design a low-pass filter with cutoff frequency $1GHz$ and termination resistances 50Ω. From Table 1 and Table 2, normalized element values of the filter are as follows:

$e_1 = 0.82236, e_2 = 1.6185, e_3 = 1.9283, e_4 = 1.8504,$

$e_5 = 1.2341, r_l = 1$ (*from Table* 1),

$e_1 = 0.6180, e_2 = 1.6180, e_3 = 2, e_4 = 1.6180, e_5 = 0.6180, r_l = 1$

(from Table 2).

These normalized values must be denormalized by using frequency normalization number as $f_n = 1GHz$, and impedance normalization number as $R_n = 50\Omega$. Then actual values of the filter elements are calculated by using the following formulae:

$$C_i = \frac{e_i}{2\pi f_n R_n}, \quad L_i = \frac{e_i R_n}{2\pi f_n}, \quad R_i = r R_n,$$

where C_i, L_i and R_i represent the capacitor, inductor and resistor values, respectively.

For Table 1, the real elements values are:

$C_1 = 2.6177\,pF, C_2 = 6.1380\,pF, C_3 = 3.9283\,pF,$

$L_1 = 12.88nH, L_2 = 14.725nH, R_L = 50\Omega.$

For Table II, the real element values are:

$C_1 = 1.9672\,pF, C_2 = 6.3662\,pF, C_3 = 1.9672\,pF,$

$L_1 = 12.876nH, L_2 = 12.876nH, R_L = 50\Omega.$

The designed filter and its characteristic are given in Fig. 3 and 4, respectively.

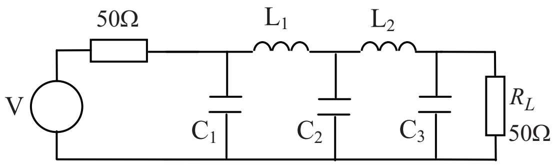

Fig. 3. Designed low-pass filter.

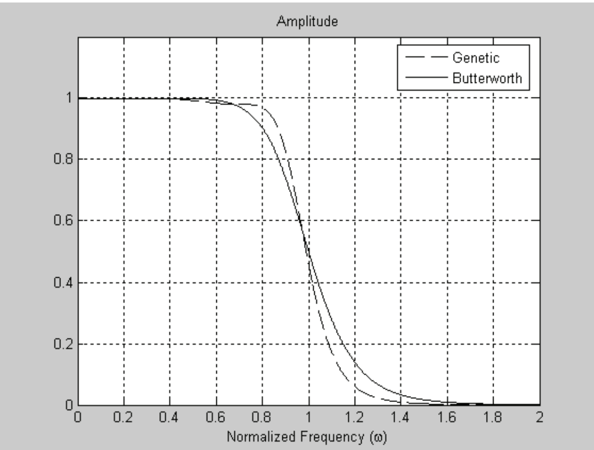

Fig. 4. Characteristics of the designed filter and the filter given in literature.

5. Conclusions

So it is concluded that ideal filter characteristic is not possible to realize. In this paper, after trying to realize the ideal characteristic, it is seen that the filter response naturally converges to Butterworth response. By using genetic algorithm, after restricting some coefficients, exactly the same component values are obtained as the values obtained in literature via insertion loss method analytically.

6. References

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