A PRECONDITIONER FOR TRANSIENT STABILITY TIME DOMAIN SIMULATION

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***Abstract*— Transient Stability analysis, which is one of the most important tasks of power system dynamic security analysis, determines the dynamic behaviour of the power system after a large disturbance. Differential and algebraic equations (DAEs) model the nonlinear dynamic power system. The conventional time domain solution process uses a Newton method to simultaneously solve the differential equations, discretized via an implicit integration method, alongside the non-linear algebraic network equations. Direct or iterative methods can be used to solve the resulting set of sparse linearized algebraic equations. A well-known and faster solution algorithm used in some popular transient stability packages is the Very Dishonest Newton Method (VDHN), which uses a current balance form and a very infrequent LU factorization in the solution of the algebraic equations. In order to prevent any trouble arising from VDHN in today's complex power system models, an exact Newton method with a Preconditioned Generalized Minimal Residual (GMRES) iterative method forms the basis of this paper. A new incomplete LU based preconditioner is proposed to achieve solution speeds comparable to that of the VDHN method. Results are given for a power system with 1169 buses, 392 generators, 2855 branches. Thanks to the proposed incomplete LU based preconditioner, a full Newton method approach with preconditioned GMRES can be used in the simulation of transient stability behavior with negligible impact on the solution speed.**

I. INTRODUCTION

The analysis of power system stability discovers the power system's response to any disturbance. Power system stability is classified into two main categories: rotor angle stability, which is an ability to maintain synchronism, and voltage stability, which is an ability to maintain steady acceptable voltage [1]. Angle stability is usually categorized according to the severity of a disturbance. It is called either small signal stability under small disturbances or transient stability under large disturbances. The objective of transient stability analysis is to determine the dynamic behaviour of an electric power system after a large disturbance. Transient stability analysis is indispensable in terms of power system planning and power system operations.

In this analysis, a differential algebraic model is used to describe the electric power system. That model consists of differential and algebraic equations (DAEs). They are shown as

$$\dot{X} = f(X,Y) \tag{1}$$

$$0 = g(X,Y) \tag{2}$$

Equation (1) represents the dynamic devices and equation (2) shows the algebraic equations of synchronous machines and power system network. So, X is called as a vector of state variables and Y is called as a vector of algebraic variables. In the solution of DAEs, firstly differential equations are discretized by implicit or explicit methods. Then non-linear algebraic equations are obtained. These equations and algebraic equations can be solved separately or simultaneously at each time step. Because of numerical stability, Simultaneous implicit method is preferred. Newton Raphson method can be used to solve non-linear algebraic equations due to its quadratic convergence. Sparse linear system in the Newton method is solved by LU factorization direct method in many production grade programs. Very Dishonest Newton Method (VDHN) has been used in these programs so as to obtain faster solution.

Iterative methods are alternative solution techniques for the solution of sparse linear system in transient stability analysis [2]. There are two categories in the iterative methods: stationary and non-stationary iterative methods [3]. Many researchers have applied iterative methods to the simulation of power system dynamics [4], [5], [6]. Nonstationary iterative methods cover Krylov subspace based methods, which have been widely discussed in the numerical analysis literature. The most robust and generic method in the family of Krylov subspace solvers is the Generalized Minimal Residual (GMRES) method. The GMRES was proposed in 1986 for nonsymmetric systems[7]. Preconditioning is a necessary step to accelerate the convergence process of the iterative solvers. Preconditioned GMRES with ILU preconditioner was applied to transient stability simulation of a power system [5]. Incomplete LU based dishonest preconditioner and Incomplete LU (ILU) preconditioner were compared in simulations by using 10-generator, 39-bus system. GMRES with dishonest preconditioner demonstrated better result than GMRES with ILU preconditioner. The variant of GMRES and some preconditioning techniques were used to simulate the transient stability of the system [4]. Techniques were tested on the 3-generator, 9-bus system and 16-generator, 68-bus system. The different iterative methods which can handle unsymmetric matrices are used by a dishonest preconditioner [6]. Tests were run on 10-generator, 39-bus system. GMRES was shown to be the most robust of all the method. Also dishonest preconditioner, ILU preconditioner with different strategies, produced better performance for all iterative methods used. In [5], [6], the preconditioner was recomputed depending on the threshold value of GMRES iterations and some parameters. In our case, it is recomputed only three times. The computation of preconditioner is not dependent to the size of power system.

In this paper, a new preconditioner, the incomplete LU based factorization, is constructed. VDHN approach with LU factorization and substitution schemes is replaced by a Newton method with preconditioned GMRES(m). The proposed preconditioner ensures the usage of a complete Newton approach. Large scale power system data was used to show the performance of the Newton method with preconditioned GMRES(m). The combination prevents any trouble arising from VDHN approach in today's large power system. According to the integration time step, the convergence problem might occur in the VDHN approach.

II. Transient Stability Analysis

Transient stability analysis is concerned with the effects on generator synchronism due to a large disturbance such as loss of an huge load or loss of generators. A differential algebraic model is used to describe the power system in this simulation method. Dynamic devices are modeled as a set of differential equations and the power system network is modeled as a set of algebraic equations. The differential-algebraic equations are

$$\dot{X} = f(X,V,u), \qquad X(0) = X_0 \tag{3}$$

$$I_1(X,V) = \mathbf{Y_N}V, \qquad V(0) = V_0 \tag{4}$$

Equation (3) can be discretized by trapezoidal integration method. Then, equations (5) and (6) are written.

$$\begin{aligned} F(X_{n+1},V_{n+1}) &\triangleq X_{n+1} - X_n - \\ &\tfrac{h}{2}[f(X_{n+1},V_{n+1}) + f(X_n,V_n)] = 0 \end{aligned} \tag{5}$$

$$G(X_{n+1},V_{n+1}) \triangleq \mathbf{Y_N}V_{n+1} - I_1(X_{n+1},V_{n+1}) = 0 \tag{6}$$

where h is an integration time step. Using the rectangular coordinate representation of I_1, V and $\mathbf{Y_N}$ as ${I_1}^e$, V^e and $\mathbf{Y_N^e}$ in equations (5) and (6), one can obtain

$$\begin{aligned} F(X_{n+1},V^e_{n+1}) &\triangleq X_{n+1} - X_n - \\ &\tfrac{h}{2}[f(X_{n+1},V^e_{n+1}) + f(X_n,V^e_n)] = 0 \end{aligned} \tag{7}$$

$$\begin{aligned} H(X_{n+1},V^e_{n+1}) &\triangleq \mathbf{Y_N^e}V^e_{n+1} \\ &-{I_1}^e(X_{n+1},V^e_{n+1}) = 0 \end{aligned} \tag{8}$$

where $\mathbf{Y_N}$, ${I_1}^e$, V^e, $\mathbf{Y_N^e}$ and H are defined as

$$\mathbf{Y_N} = \mathbf{G_N} + \mathbf{jB_N},\ V^e = \begin{bmatrix} V^r \\ V^i \end{bmatrix},\ {I_1}^e = \begin{bmatrix} I_1^r \\ I_1^i \end{bmatrix}$$

$$\mathbf{Y_N^e} = \begin{bmatrix} \mathbf{G_N} & -\mathbf{B_N} \\ \mathbf{B_N} & \mathbf{G_N} \end{bmatrix},\ H(X,V) = \begin{bmatrix} Re(G(X,V)) \\ Im(G(X,V)) \end{bmatrix},$$

The linear equations are

$$\mathbf{J}^{(k)}_{n+1}\Delta\mathcal{X}^{(k)}_{n+1} = -\mathcal{F}_{n+1}{}^{(k)} \tag{9}$$

the unknown variables updated are

$$\mathcal{X}^{(k+1)}_{n+1} = \mathcal{X}^{(k)}_{n+1} + \Delta\mathcal{X}^{(k)}_{n+1} \tag{10}$$

where

$$\mathcal{F} = \begin{bmatrix} F \\ H \end{bmatrix},\ \mathcal{X} = \begin{bmatrix} X \\ V \end{bmatrix}$$

$$\begin{bmatrix} F \\ H \end{bmatrix} = -\begin{bmatrix} \frac{\partial F}{\partial X} & \frac{\partial F}{\partial V^e} \\ \frac{\partial H}{\partial X} & \frac{\partial H}{\partial V^e} \end{bmatrix}\begin{bmatrix} \Delta X \\ \Delta V^e \end{bmatrix}$$

The Jacobian has the following structure:

$$\mathbf{J} = \begin{bmatrix} \frac{\partial F}{\partial X} & \frac{\partial F}{\partial V^e} \\ \frac{\partial H}{\partial X} & \frac{\partial G}{\partial V^e} \end{bmatrix} = \begin{bmatrix} \mathbf{J_A} & \mathbf{J_B} \\ \mathbf{J_C} & \mathbf{J_D} \end{bmatrix}$$

$\mathbf{J_D}$ is written

$$\mathbf{J_D} = \begin{bmatrix} \mathbf{G_N} & -\mathbf{B_N} \\ \mathbf{B_N} & \mathbf{G_N} \end{bmatrix} - \begin{bmatrix} \frac{\partial I_1^r}{\partial V^r} & \frac{\partial I_1^r}{\partial V^i} \\ \frac{\partial I_1^i}{\partial V^r} & \frac{\partial I_1^i}{\partial V^i} \end{bmatrix} \tag{11}$$

Equation (11) is composed of two parts denoted by $\mathbf{Y_N^e}$ and $\mathbf{Y_D^e}$ respectively.

$$\mathbf{Y_N^e} = \begin{bmatrix} \mathbf{G_N} & -\mathbf{B_N} \\ \mathbf{B_N} & \mathbf{G_N} \end{bmatrix} \quad \textit{and} \quad \mathbf{Y_D^e} = -\begin{bmatrix} \frac{\partial I_1^r}{\partial V^r} & \frac{\partial I_1^r}{\partial V^i} \\ \frac{\partial I_1^i}{\partial V^r} & \frac{\partial I_1^i}{\partial V^i} \end{bmatrix}$$

Then, equations (12) and (13) are obtained from equation (9)

$$\Delta X_{n+1} = -\mathbf{J_A}^{-1}\left[F_{n+1} + \mathbf{J_B}\Delta V^e_{n+1}\right]. \tag{12}$$

$$\begin{aligned} \mathbf{J_1}V^e_{n+1}{}^{(k+1)} &= I_1^{e(k)} + (\mathbf{Y_D^e} - \mathbf{J_C}\mathbf{J_A^{-1}}\mathbf{J_B})V^e_{n+1}{}^{(k)} \\ &+\mathbf{J_C}\mathbf{J_A^{-1}}F_{n+1}{}^{(k)} \end{aligned} \tag{13}$$

$\mathbf{J_1}$ is defined as:

$$\mathbf{J_1} = \mathbf{J_D} - \mathbf{J_C}\mathbf{J_A^{-1}}\mathbf{J_B}. \tag{14}$$

State variables and algebraic variables are obtained from equations (12) and (13). Equation (13) can be solved by using either direct method or iterative method.

There are two categories in the iterative methods: stationary and nonstationary iterative methods [3]. Nonstationary iterative methods cover Krylov subspace based methods, which have been widely discussed in the numerical analysis literature. The most robust and generic method in the family of Krylov subspace solvers is the Generalized Minimal Residual (GMRES) method. GMRES method converges in a finite number of iterations but its cost increases step by step. Therefore, it is usually restarted after m iterations to reduce the cost both computationally and storage wise. GMRES(m) method attempts to solve the $\mathbf{A}x = b$ linear system by minimizing the residual r defined by

$$r(x) = b - \mathbf{A}x. \tag{15}$$

Depending on the spectral property of $\mathbf{A}$ matrix, preconditioning is a required step for the solution process. The original system is transformed into a new system which has a more favorable eigenvalue distribution for the Krylov subspace solvers. Thus, the number of GMRES iterations can be decreased in the solution process. There are three different ways of implementing preconditioning: left preconditioning, right preconditioning and two-sided preconditioning [8]. If a matrix $\mathbf{M}$ approximates the coefficient matrix $\mathbf{A}$, they can be shown as below:

- Left preconditioning
  $\mathbf{M^{-1}A}x = \mathbf{M^{-1}}b$
  $\mathbf{A_L}x = b_L$
- Right preconditioning
  $(\mathbf{AM^{-1}})(\mathbf{M}x) = b$
  $\mathbf{A_R}x_R = b$
- Left and Right preconditioning ($\mathbf{M} = \mathbf{M_1M_2}$)
  $(\mathbf{M_1^{-1}AM_2^{-1}})(\mathbf{M_2}x) = \mathbf{M_1^{-1}}b$
  $\mathbf{A_{LR}}x_{LR} = b_{LR}$

Right preconditioning is a general approach in applying GMRES. The preconditioned GMRES(m) algorithm has been described in Figure 1

1) $r_0 = b - Ax_0$, $k = 0$, $\rho = \|r\|_2$, $v_1 = r_0/\rho$
   $g = \rho(1,0,0,...) \in \mathbb{C}^{m+1}$
2) While $\rho > errtol$ and $k < m$ do
   - $k = k+1$
   - $v_{k+1} = AM^{-1}v_k$
   - for $j = 1,...,k$
     - $h_{j,k} = v_j^H v_{k+1}$
     - $v_{k+1} = v_{k+1} - h_{j,k}v_j$
   - $h_{k+1,k} = \|v_{k+1}\|_2$
   - $v_{k+1} = v_{k+1}/h_{k+1,k}$
   - Apply and create Givens Rotations
     - If $k > 1$ apply Q_{k-1} to the kth column of H
     - $v = \sqrt{(|h_{k,k}|^2 + |h_{k+1,k}|^2)}$
     - $c_k = |h_{k,k}|/v$; $s_k = -\eta \overline{h}_{k+1,k}/v$; $\eta = h_{k,k}/|h_{k,k}|$
       $h_{k,k} = c_k h_{k,k} - s_k h_{k+1,k}$; $h_{k+1,k} = 0$
     - $g = G_k(c_k, s_k)g$
   - $\rho = |g_{k+1}|$
3) Set $r_{i,j} = h_{i,j}$ for $1 \le i,\ j \le k$
   Set $w_i = g_i$ for $1 \le i \le k$
   Solve the upper triangular system $Ry_k = w$
4) $x = x_0 + M^{-1}V_k y_k$

Fig. 1. The preconditioned GMRES(m) algorithm for $A \in \mathbb{C}^{nxn}$.

III. A NEW PRECONDITIONER

Due to slow convergence property of the iterative methods, preconditioning is a vital ingredient for the solution process. Any preconditioner should have the following fundamental features.

- The construction and the storage of preconditioner should not be expensive.
- The preconditioner should be a good approximation to inverse of $\mathbf{A}$.
- The preconditioned system should be easier than original linear system to solve.

Preconditioners can be categorized as implicit and explicit forms. $\mathbf{M}$ is defined as $\mathbf{M} = \overline{\mathbf{LU}}$ in implicit preconditioning method. $\overline{\mathbf{L}}$ and $\overline{\mathbf{U}}$ are approximation of the exact $\mathbf{L}$ and $\mathbf{U}$ factors. Incomplete factorization methods are represented as implicit preconditioners, which are suitable for right preconditioning.

A new incomplete LU based preconditioner is designed for GMRES(m) in the solution of equation (13). $\mathbf{Y_D^e}$, which is used in equation (13), can be decomposed into two parts: $\mathbf{Y_{Dn}^e}$ and $\mathbf{Y_{Ds}^e}$. Former is a constant part whereas $\mathbf{Y_{Ds}^e}$ is a inconstant part. If $\mathbf{J_1}$ is rewritten

$$\mathbf{J_1} = \mathbf{Y_N^e} + \mathbf{Y_D^e} - \mathbf{J_C J_A}^{-1}\mathbf{J_B} \tag{16}$$

then substituting $\mathbf{Y_D^e}$ as two parts into $\mathbf{J_1}$

$$\mathbf{J_1} = \mathbf{Y_N^e} + \mathbf{Y_{Dn}^e} + \mathbf{Y_{Ds}^e} - \mathbf{J_C J_A}^{-1}\mathbf{J_B} \tag{17}$$

Equation (17) consists of two parts termed as respectively $\mathbf{J_{11}}$ and $\mathbf{J_{12}}$.

$$\mathbf{J_{11}} = \mathbf{Y_N^e} + \mathbf{Y_{Dn}^e}$$

$$\mathbf{J_{12}} = \mathbf{Y_{Ds}^e} - \mathbf{J_C J_A^{-1} J_B}$$

By neglecting the $\mathbf{J_{12}}$ from $\mathbf{J_1}$, the remaining part, $\mathbf{J_{11}}$, is used as a new incomplete LU based preconditioner. The technique is to decompose $\mathbf{J_{11}}$ to a lower and an upper triangular factors and to use these factors as a preconditioner in GMRES(m) method.

IV. TEST RESULTS

A 392-generator, 624-load, 2855-branch, 1169-bus system was tested to study the performance of two different implementations that cover the solution of equation (13) with VDHN method by using LU direct method and Newton method by using with preconditioned GMRES(m) method. A three phase fault was used for the transient stability simulation. The fault occurred at 0.4 seconds and was cleared at 0.6 seconds. The total simulation time was 2 seconds. The integration time step was 0.01 seconds. The GMRES restart parameter m was 4 in the case of the 1169 bus system simulation. In table I, these methods are compared in terms of total number of Newton iterations and serial solution time. The total number of Newton iterations in the LU based solution is bigger than in the preconditioned GMRES(m) based solution. The usage of VDHN method causes a decrease in the simulation time but at the same time it leads to an increase in the number of Newton iterations. In today's large power systems, this approach can pose a risk in terms of power system control. If equation (13) is solved with an exact Newton method, the undesirable increase of the total number of Newton iterations will not happen. In this situation, Exact Newton with LU method brings a slower solution for transient stability simulation. To prevent a slowdown in the solution time, Newton method with preconditioned GMRES(m) method was used in the solution algorithm. Both the quality of the preconditioner and the initial

TABLE I

THE COMPARISON OF LU BASED AND PRECONDITIONED GMRES(M) BASED LINEAR SOLVER.

1169 bus system	LU based	preconditioned GMRES(m) based
Newton iterations	570	528
Simulation time (seconds)	6.56	6.77

guess for the GMRES(m) method determine the number of GMRES iterations for each Newton iteration at each time step in the time domain simulation. In table II, the maximum number of GMRES iterations taken in any one of the Newton steps is given for three different periods, which are pre-fault period, fault period and post-fault period.

TABLE II

THE MAXIMUM NUMBER OF GMRES ITERATIONS FOR THE 1169 BUS SYSTEM.

Period	GMRES maximum iterations
Pre-fault	1
Fault	3
Post-fault	3

V. CONCLUSION

In this paper, a simultaneous implicit method was used to simulate power system transient stability behavior. The core of this approach is the solution of the linearized equations via Newton's method. A VDHN method with LU factorization is popular in commercial simulation packages due to the typically faster solution process. However, convergence problems could cause the VDHN method to slow down. This paper presented an alternative approach with an exact Newton method and GMRES. The advantages over the VDHN method are that the simulation time does not increase significantly, while eliminating the potential for convergence problems. A new incomplete LU based preconditioner was designed to satisfy the simulation time constraint. For the 392-generator, 624-load, 2855-branch, 1169-bus system, it was observed that the proposed approach can be better than the VDHN method for large scale power systems.

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