Frequenz
58 (2004)
7-8

EM Based Stochastic Maximum Likelihood Approach for Localization of Near-field Sources in 3-D

Abstract
The goal of this paper is to estimate the locations of unknown sources in 3-D space from the data collected by a 2-D rectangular array. Various studies employing different estimation methods under near-field and far-field assumptions were presented in the past. In most of the previous studies, location estimations of sources at the same plane with the antenna array were carried out by using algorithms having constraints for various situations indeed. In this study, location estimations of sources that are placed at a different plane from the antenna array is given. In other words, locations of sources in 3-D space is estimated by using a 2-D rectangular array. Maximum likelihood (ML) method is chosen as the estimator since it has a better resolution performance than the conventional methods in the presence of less number and highly correlated source signal samples and low signal to noise ratio. Besides these superiorities, stability, asymptotic unbiasedness, asymptotic minimum variance properties as well as no restrictions on the antenna array are motivated the application of ML approach. Despite these advantages, ML estimator has computational complexity. However, this problem is tackled by the application of Expectation/Maximization (EM) iterative algorithm which converts the multidimensional search problem to one dimensional parallel search problems in order to prevent computational complexity. EM iterative algorithm is therefore adapted to the localization problem by the data (complete data) assumed to arrive to the sensors separately instead of observed data (incomplete data). Furthermore, performance of the proposed algorithm is tested by deriving Cramér-Rao bounds based on the concentrated likelihood approach. Finally, the applicability and effectiveness of the proposed algorithm is illustrated by some numerical simulations

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Index Terms
Maximum Likelihood estimation, Expectation, Maximization algorithm, Antenna arrays, Localization of near-field sources in 3-D space

1. Introduction

In the last decades, the basic interest in the array processing research turned towards the developing algorithms for localization of sources by using passive sensor array. These algorithms were applied to radar, sonar, radio astronomy, geophysics, seismology, robotic and biomedicine [1-4]. In the last, many studies were presented for solution to the problem of direction of arrival estimations of narrow-band far-field sources [5-21]. In far-field scenarios, each source location could be characterized by only the azimuth and/or elevation since waves impinging to antenna array were considered as plane waves. When the sources are located close to the array (i.e., near-field), the inherent curvature of the waveforms can no longer be neglected. Therefore, the spherical wavefronts in the near-field scenario must be considered and the location of each source has to be parametrized in terms of the direction of arrival (azimuth and/or elevation; DOA) together with range [22-24]. However, in [22-24], it was assumed that all sources were at the same plane with antenna array which introduces some restrictions to some applications in the real world.

Localization of sources at the different plane with antenna array is more applicable to the real world array processing problems. Primary research results presented under this assumption were for localization of narrow-band far-field source signals [25-27]. Moreover, recent research results on localization of near-field narrow-band sources in 3-D space were also presented [28, 29]. Faced with inability to completely evaluate performances of optimal 3-D near-field localization approaches from [28, 29], it is reasonable to resort to a asymptotically optimal estimators.

The localization of the near-field sources in 3-D space is in general nontrivial, since localization of near-field sources requires estimation of the azimuth and elevation together with the range parameters. Recently, an algorithm using 3-D Music with polynomial rooting has been developed [28]. High-order subspace based algorithms were introduced in [29]. In contrast to suboptimal approaches proposed in [28, 29], we now investigate an alternative estimator that is asymptotically efficient. Due to many attractive properties of maximum likelihood (ML) estimation methods such as consistency, asymptotic unbiasednes and asymptotic minimum variance, we concentrate on ML method for localization of near-field sources in 3-D space. Furthermore it has a better resolution performance than the other methods in the presence of less number and highly correlated source signal samples and low signal to noise ratios. Besides these superiorities, it also brings no restrictions on the antenna array structure. Regarding the assumption on the narrow-band source signals, there are two different types of models. These two models lead corresponding ML solutions. The models are

i. Deterministic Model (DM) which assumes the signals to be unknown but deterministic (i.e., the same in all realizations)
ii. Stochastic Model (SM) which assumes the signals to be random.

ML methods corresponding to the signal models (i) and (ii) are termed deterministic ML and stochastic ML respectively. Expectation/Maximization (EM) based deterministic ML (signal model (i)) near-field location estimator have been studied in [30]. The goal of the present paper is to provide a stochastic ML approach for the estimation of the DOA and range parameters of near-field sources. However, calculation of ML estimation from corresponding likelihood function for the stochastic case results in difficult nonlinear constrained optimization problem, which must be solved iteratively. We therefore employed the EM iterative method for obtaining ML estimator. The most important feature of the algorithm is that it decomposes the observed data into its signal components and then estimates the parameters of each signal component separately.

The asymptotical performance of an stochastic ML 3-D near-field localization technique is analyzed via the derivation of Cramér Rao Bounds (*CRB*) which provides benchmarks for evaluating the performance of actual estimators. The technique for the derivation of *CRB*s used here in relies as modulators of the log-likelihood function by replacing the nuisance parameters with their ML estimates. It therefore avoids the process of explicitly calculating and inverting the entire Fisher Information Matrix (FIM). We substitute the ML estimates of the array observation covariance matrix to obtain concentrated covariance matrix. We then calculated the *CRB*s for location parameters of near-field sources in 3-D space by modifying the bounds presented in [31].

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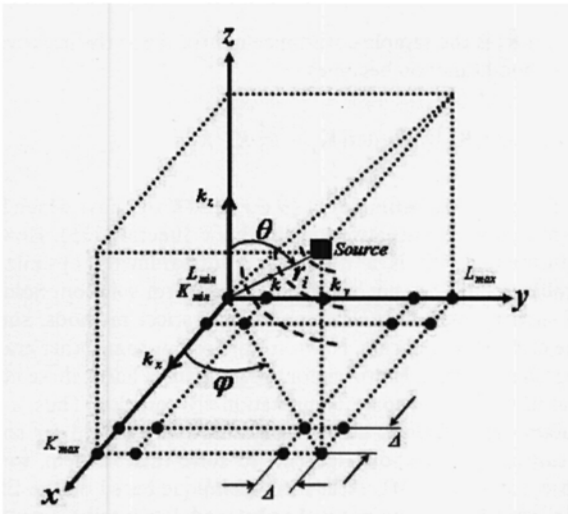

Fig. 1: Near-field scenario with a 2-D linear array

The remainder of the paper is organized as follows. The signal model for 3-D near-field scenario is presented in section 2. In section 3, the stochastic ML estimator and the corresponding assumptions on the signal model are outlined. Moreover, the iterative EM algorithm for obtaining ML estimates from corresponding stochastic cost function is introduced. While in section 4, the performance of the stochastic ML method is evaluated based on the concentrated ML approach to obtain *CRB*s. In section 5, simulation results are presented. Finally, some conclusions of the work are reported.

2. Signal Model

Consider a near-field scenario in which narrowband signals from d sources received by an $K \times L$ element antenna array. Let the array center be the phase reference point with index '(0,0)' as depicted in **Figure 1**.

Assuming 2-D rectangular uniform linear array consisting of omnidirectional sensors with interelement spacing Δ along each axes, we write the output of the $(k,l)^{th}$ sensor with narrowband, co-channel signal at time t_n as,

$$x_{k,l}(t_n)=\sum_{i=1}^{d} s_i(t_n)e^{j\tau_{kl}(i)}+n_{k,l}(t_n), \qquad 1\le t_n \le N \tag{1}$$

where $s_i(t_n)$ denotes the complex envelope of the i^{th} source signal, $n_{k,l}(t_n)$ is an additive complex Gaussian sensor noise and $\tau_{kl}(i)$ is the phase difference of the i^{th} signal collected at sensor (k,l) with respect to the i^{th} signal collected at reference sensor '(0,0)'. Due to our narrowband assumption, the phase difference is given by

$$\tau_{kl}(i)=\frac{2\pi}{\lambda}\left(r_{kl}(i)-r_i\right) \tag{2}$$

where $\lambda = c/2\pi f_0$ is the wavelength corresponding to the center frequency f_0 of the signals travelling at a velocity c. The distance between i^{th} source and the $(k,l)^{th}$ sensor equals

$$r_{kl}(i)=r_i\sqrt{1+\frac{(k^2+l^2)\Delta^2}{r_i^2}-\frac{2\Delta}{r_i}\sin\theta_i\left(k\cos\varphi_i+l\sin\varphi_i\right)} \tag{3}$$

The plane wave approximation of far-field sources is obtained by retaining only term up to the first power of Δ/r_i and its multiplier in the binomial expansion of (3). Since the near field sources are of interest in this paper, we should include an extra term to approximate the effect of spherical waves. By retaining terms up to the second power of Δ/r_i and its multiplier in the binomial expansion of (3), such an approximation can therefore be obtained. We then arrive at the *Fresnel* approximation of the distance:

$$\begin{aligned} r_{kl}(i)\approx r_i-k\Delta\sin\theta_i\cos\varphi_i+\frac{(k^2)\Delta^2}{2r_i}\left(1-\sin^2\theta_i\cos^2\varphi_i\right) \\ -l\Delta\sin\theta_i\sin\varphi_i+\frac{l^2\Delta^2}{2r_i}\left(1-\sin^2\theta_i\sin^2\varphi_i\right) \\ -\frac{kl\Delta^2}{2r_i}\left(\sin^2\theta_i\sin2\varphi_i\right). \end{aligned} \tag{4}$$

If we substitute (4) in (2), we obtain the phase difference $\tau_{kl}(i) = [\omega_{xi}k + \phi_{xi}k^2 + \omega_{yi}l + \phi_{yi}l^2 + \beta_i kl]$ as

$$\begin{aligned} \tau_{kl}(i)\approx\left(\frac{2\pi\Delta}{\lambda}\sin\theta_i\cos\varphi_i\right)k-\left(\frac{\pi\Delta^2}{\lambda r_i}\left(1-\sin^2\theta_i\cos^2\varphi_i\right)\right)k^2+ \\ +\left(\frac{2\pi\Delta}{\lambda}\sin\theta_i\sin\varphi_i\right)l-\left(\frac{\pi\Delta^2}{\lambda r_i}\left(1-\sin^2\theta_i\sin^2\varphi_i\right)\right)l^2+ \\ +\left(\frac{\pi\Delta^2}{\lambda r_i}\sin^2\theta_i\sin2\varphi_i\right)kl \end{aligned} \tag{5}$$

where

$$\begin{aligned} &\omega_{xi}=-\frac{2\pi\Delta}{\lambda}\sin\theta_i\cos\varphi_i, \quad \phi_{xi}=\frac{\pi\Delta^2}{\lambda r_i}\left(1-\sin^2\theta_i\cos^2\varphi_i\right), \\ &\omega_{yi}=-\frac{2\pi\Delta}{\lambda}\sin\theta_i\sin\varphi_i, \quad \phi_{yi}=\frac{\pi\Delta^2}{\lambda r_i}\left(1-\sin^2\theta_i\sin^2\varphi_i\right), \\ &\beta_i=\frac{\pi\Delta^2}{\lambda r_i}\sin^2\theta_i\sin2\varphi_i. \end{aligned} \tag{6}$$

Then, the noise corrupted array measurements at the $(k,l)^{th}$ sensor can be approximately expressed as:

$$x_{k,l}(t_n)=\sum_{i=1}^{d} s_i(t_n)e^{j\left[\omega_{xi}k+\phi_{xi}k^2+\omega_{yi}l+\phi_{yi}l^2+\beta_i kl\right]}+n_{k,l}(t_n), \quad 1\le t_n\le N. \tag{7}$$

For a collection of observed outputs of $K \times L$ sensors in 2-D array $\mathbf{x}(t_n) = [\mathbf{x}^T_{L_{min}}(t_n), ..., \mathbf{x}^T_{L_{max}}(t_n)]^T$, the model (7) is written more compactly in matrix notation as

$$\mathbf{x}(t_n)=\mathbf{A}(\boldsymbol{\theta},\boldsymbol{\varphi},\boldsymbol{r})\mathbf{s}(t_n)+\mathbf{n}(t_n), \qquad 1\le t_n\le N \tag{8}$$

where the super vector $\mathbf{x}(t_n)$ consists of $\mathbf{x}_l(t_n) = [x_{K_{min},l}(t_n) \; ... \; x_{K_{max},l}(t_n)]^T$ which is output of only one column sub-array of the 2-D rectangular array, $\mathbf{s}(t_n) = [s_1(t_n) \; ... \; s_d(t_n)]^T$ is the collection of d source signals impinging to 2-D array, $\mathbf{n}(t_n) = [\mathbf{n}^T_{L_{min}}(t_n) \; ... \; \mathbf{n}^T_{L_{max}}(t_n)]^T$ is super Gaussian complex vector with zero-mean and known spatial covariance $\sigma^2\mathbf{I}$, which consists of sub-array noise vectors one forming as $\mathbf{n}_\ell(t_n) = [n_{K_{min},l}(t_n) \; ... \; n_{K_{max},l}(t_n)]^T$, $\mathbf{A}(\boldsymbol{\theta},\boldsymbol{\varphi},\boldsymbol{r}) = [\mathcal{A}_1(\theta,\varphi,r) ... \mathcal{A}_d(\theta,\varphi,r)]$ is the arrays steering matrix in the near-field scenario which is known as a function of unknown set of parameters $\boldsymbol{\theta} = [\theta_1 \; ... \; \theta_d]^T$, $\boldsymbol{\varphi} = [\varphi_1 \; ... \; \varphi_d]^T$, $\boldsymbol{r} = [r_1 \; ... \; r_d]^T$, consisting of sub-array steering vectors one forming as $\mathcal{A}_i(\theta,\varphi,r) = [\mathbf{a}^T_{L_{min}}(\theta,\varphi,r) \; ... \; \mathbf{a}^T_{L_{max}}(\theta,\varphi,r)]^T$ and $\mathbf{a}_\ell(\theta,\varphi,r)$ is l^{th} sub-array steering vector for i^{th} source, in the $\mathbf{a}_{\ell,i}(\theta,\varphi,r) = [e^{j\tau_{K_{min}l}(i)}, ..., 1, e^{j\tau_{1l}(i)}, e^{j\tau_{2l}(i)}, ..., e^{j\tau_{K_{max}l}(i)}]^T$ form.

We are interested in stochastic ML approach in this paper for the estimation of 3-D near-field source location parameters $[\boldsymbol{\theta},\boldsymbol{\varphi},\boldsymbol{r}] = [(\theta_1,\varphi_1,r_1), ..., (\theta_d,\varphi_d,r_d)]$ from n observations $\boldsymbol{x} = [\mathbf{x}^T(1), ..., \mathbf{x}^T(N)]^T$ made from (8). The data for this problem consists of a set of discrete samples $\{\mathbf{x}(k);\; 1 \le k \le N\}$ of the process $\mathbf{x}(t_n)$. Our approach is to derive an iterative stochastic ML estimator based on the EM algorithm, that performs joint sample covariance and location parameters estimation in alternating steps.

3. Stochastic ML Estimator

In this section we derive the stochastic ML estimator for the problem defined above. To describe stochastic ML estimator's derivation, we made the following assumptions on the signal model (7):

AS1: The source signal $\mathbf{s}(k)$ is temporally and spatially uncorrelated circular complex Gaussian random process with zero-mean and nonsingular unknown covariance matrix $\mathbf{K}_s$,

$$E\left[\mathbf{s}(k_1)\mathbf{s}^H(k_2)\right]=\mathbf{K}_s\delta_{k_1,k_2}$$
$$E\left[\mathbf{s}(k_1)\mathbf{s}^T(k_2)\right]=\mathbf{0} \qquad \text{for all } k_1 \text{ and } k_2. \tag{9}$$

where δ_{k_1,k_2} is the Kronecker delta ($\delta_{k_1,k_2}=1$ if $k_1=k_2$ and 0 otherwise), $(\cdot)^H$ is the conjugate transpose and $(\cdot)^T$ is the transpose of a matrix.

AS2: The additive noise vector $\mathbf{n}(k)$ is temporally and spatially uncorrelated circular complex Gaussian process with zero-mean and standardt derivative σ^2 as

$$E\left[\mathbf{n}(k_1)\mathbf{n}^H(k_2)\right]=\sigma^2\mathbf{I}\delta_{k_1,k_2} \tag{10}$$

$$E\left[\mathbf{n}(k_1)\mathbf{n}^T(k_2)\right]=\mathbf{0} \qquad \text{for all } k_1 \text{ and } k_2. \tag{11}$$

AS3: The source signal $\mathbf{s}(k_l)$ and the noise $\mathbf{n}(k_2)$ are uncorrelated for all k_l and k_2.

Based on the assumptions AS2 and AS3, the array observations $\mathbf{x}$ are Gaussian distributed with zero-mean and covariance $\mathbf{K}_x(\boldsymbol{\theta},\boldsymbol{\varphi},\boldsymbol{r},\mathbf{K}_s)$, i.e.,

$$\mathbf{K}_x(\boldsymbol{\theta},\boldsymbol{\varphi},\boldsymbol{r},\mathbf{K}_s)=E\left[\mathbf{x}(k_1)\mathbf{x}^H(k_2)\right]$$
$$=\mathbf{A}(\boldsymbol{\theta},\boldsymbol{\varphi},\boldsymbol{r})\mathbf{K}_s\mathbf{A}^H(\boldsymbol{\theta},\boldsymbol{\varphi},\boldsymbol{r})+\sigma^2\mathbf{I}. \tag{12}$$

Then joint probability density function of the observation $\boldsymbol{x}=\mathbf{x}(k)$, $k=1,\ldots,N\}$ given $\{\boldsymbol{\theta},\boldsymbol{\varphi},\boldsymbol{r},\mathbf{K}_s\}$ can be written as follows:

$$f(\mathbf{x};\boldsymbol{\theta},\boldsymbol{\varphi},\boldsymbol{r},\mathbf{K}_s)=\prod_{k=1}^{N}2\pi^{-KL/2}\left(\det\mathbf{K}_\mathbf{x}\right)^{-1/2}$$
$$\exp\left(-\frac{1}{2}\mathbf{x}^H(k)\mathbf{K}_\mathbf{x}^{-1}\mathbf{x}(k)\right). \tag{13}$$

The joint probability function (13) can also be written as

$$f(\mathbf{x};\boldsymbol{\theta},\boldsymbol{\varphi},\boldsymbol{r},\mathbf{K}_s)=2\pi^{-NKL/2}\left(\det\mathbf{K}_\mathbf{x}\right)^{-N/2}$$
$$\exp\left(-\frac{1}{2}tr\left[\mathbf{K}_\mathbf{x}^{-1}\sum_{k=1}^{N}\mathbf{x}(k)\mathbf{x}^H(k)\right]\right). \tag{14}$$

where tr is the trace. The negative log-likelihood function (after neglecting unnecessary terms) is

$$\mathcal{L}(\mathbf{x};\boldsymbol{\theta},\boldsymbol{\varphi},\boldsymbol{r},\mathbf{K}_s)=-\ln\det\mathbf{K}_\mathbf{x}-\frac{1}{N}tr\left[\mathbf{K}_\mathbf{x}^{-1}\sum_{k=1}^{N}\mathbf{x}(k)\mathbf{x}^H(k)\right]. \tag{15}$$

AS2 implies that, by the law of large numbers $\mathbf{x}(k)$ is second-order ergodic, i.e.,

$$\mathbf{K}_\mathbf{x}=\lim_{N\to\infty}\hat{\mathbf{K}}_\mathbf{x}=\lim_{N\to\infty}\frac{1}{N}\sum_{k=1}^{N}\mathbf{x}(k)\mathbf{x}^H(k) \tag{16}$$

where $\hat{\mathbf{K}}_\mathbf{x}$ is the sample covariance matrix. Then the negative log-likelihood function becomes

$$\mathcal{L}(\mathbf{x};\boldsymbol{\theta},\boldsymbol{\varphi},\boldsymbol{r},\mathbf{K}_s)=-\ln\det(\mathbf{K}_\mathbf{x})-tr\left[K_\mathbf{x}^{-1}\hat{K}_\mathbf{x}\right]. \tag{17}$$

Then, the ML estimates of $\{\hat{\boldsymbol{\theta}},\hat{\boldsymbol{\varphi}},\hat{\boldsymbol{r}}\}$ and $\hat{\mathbf{s}}$ are those which locally minimize the negative log-likelihood function (15). However, minimizing (15) is a nonlinear multiparameter optimization problem and does not yield to a closed-form solution. Solutions of such problems usually requires numerical methods, such as the methods of Scoring, Newton-Raphson or some other gradient search algorithm. However, for the problem at hand, these numerical methods tend to be computationally complex. Thus, a computationally efficient iterative algorithm is required for solving resulting optimization problem. To solve this problem, we propose a stochastic ML estimation technique based on the EM algorithm which decomposes the observed data into its signal components and then estimates the parameters of each signal components separately. Thus, multidimensional maximization has been reduced to separate one dimensional maximizations yielding an easier problem in general. Moreover, the EM algorithm iterates as the parameter updates in a manner which guarantees an increase in the likelihood function.

3.1 EM Algorithm

The formulation of the estimation problem at hand in terms of the actual data sample (incomplete-data) and a hypothetical data set (complete-data) allows us to apply the EM algorithm. To be able to easily apply the EM algorithm the complete data must be chosen in such a way that:

- the complete data log-likelihood function is easily maximized and
- the complete data log-likelihood function can be easily estimated from the incomplete data. The choice for the complete data vector is obtained from hypothetical independent observations of each incident wave as

$$\boldsymbol{y}_i(k)=\mathcal{A}_i(\theta,\varphi,r)s_i(k)+\boldsymbol{n}_i(k), \qquad 1\le i\le d \tag{18}$$

where $\mathbf{n}_i(k)$ is the Gaussian noise vector belongs to i^{th} signal. The complete-data vector $\mathbf{y}_i(k)$ is the set of N samples of d independent Gaussian vectors with the i^{th} vector $\mathbf{y}_i(k)$ having mean $\mathcal{A}_i(\theta,\varphi,r)s_i(k)$ and with identical covariance $\sigma^2\mathbf{I}/d$. Thus, the complete-data $\mathbf{y}_i(k)$ and the incomplete data $\mathbf{x}(k)$ is can be related by $\mathbf{x}(k)=\sum_{i=1}^{d}\mathbf{y}_i(k)$.

Under AS1, the covariance matrix $\mathbf{K}_s$ is a diagonal matrix $\mathbf{K}_s=\text{diag}[\alpha_1,\ldots,\alpha_d]$, then the complete data $\mathbf{y}_i(t_n)$ is the Gaussian process with mean zero and covariance

$$\mathbf{K}_{y_i}(\theta_i,\varphi_i,r_i,\alpha_i)=\alpha_i\mathcal{A}_i(\theta,\varphi,r)\mathcal{A}_i^H(\theta,\varphi,r)+\frac{\sigma^2}{d}\mathbf{I}. \tag{19}$$

Then the log-likelihood function of the complete data $\mathbf{y}_i(k)$ is

$$\mathcal{L}_c(\mathbf{y}_i;\boldsymbol{\theta},\boldsymbol{\varphi},\boldsymbol{r},\mathbf{K}_s)=-\ln\det\mathbf{K}_{\mathbf{y}_i}-\frac{1}{N}tr\left[\mathbf{K}_{\mathbf{y}_i}^{-1}\sum_{k=1}^{N}\mathbf{y}_i(k)\mathbf{y}_i^H(k)\right]. \tag{20}$$

At the $(p+1)^{\text{th}}$ iteration, two step EM algorithm for our problem has the following steps:

Expectation Step: Compute conditional expectation of the sufficient statistics for the complete data log-likelihood. The sufficient statistics is the sample covariance of the complete-data,

$$\hat{\mathbf{K}}_{\mathbf{y}_i}=\frac{1}{N}\sum_{k=1}^{N}\mathbf{y}_i(k)\mathbf{y}_i^H(k). \tag{21}$$

At the $(p+1)^{\text{th}}$ iteration, expected value of $\hat{\mathbf{K}}_{\mathbf{y}_l}^{p+1}$ given $\mathbf{K}_{\mathbf{x}}^{p}$ and $\mathbf{K}_{\mathbf{y}_l}^{p}$ is

$$\hat{\mathbf{K}}_{\mathbf{y}_l}^{p+1} = E\left\{\hat{\mathbf{K}}_{\mathbf{y}_l} \middle| \mathbf{K}_{\mathbf{y}_l}^{p}, \mathbf{K}_{\mathbf{x}}^{p}, \hat{\mathbf{K}}_{\mathbf{x}}\right\}$$
$$= \mathbf{K}_{\mathbf{y}_l}^{p}\left(\mathbf{K}_{\mathbf{x}}^{p}\right)^{-1}\hat{\mathbf{K}}_{\mathbf{x}}\left(\mathbf{K}_{\mathbf{x}}^{p}\right)^{-1}\mathbf{K}_{\mathbf{y}_l}^{p} + \mathbf{K}_{\mathbf{y}_l}^{p} - \mathbf{K}_{\mathbf{y}_l}^{p}\left(\mathbf{K}_{\mathbf{x}}^{p}\right)^{-1}\mathbf{K}_{\mathbf{y}_l}^{p} \tag{22}$$

In (22), $\mathbf{K}_{\mathbf{y}_l}^{p}$ and $\mathbf{K}_{\mathbf{x}}^{p}$ can be obtained from the estimates of near-field parameters $\{\theta^p, \varphi^p, r^p\}$ at iteration p,

$$\mathbf{K}_{\mathbf{x}}^{p} = \mathbf{A}\left(\theta^p, \varphi^p, r^p\right)\mathbf{K}_{\mathbf{s}}^{p}\mathbf{A}^H\left(\theta^p, \varphi^p, r^p\right) + \sigma^2\mathbf{I}$$
$$\mathbf{K}_{\mathbf{y}_l}^{p} = \alpha_l^p \mathcal{A}_l\left(\theta^p, \varphi^p, r^p\right)\mathcal{A}_l^H\left(\theta^p, \varphi^p, r^p\right) + \frac{\sigma^2}{d}\mathbf{I}. \tag{23}$$

Maximization Step: The conditional expectation of the sufficient statistics obtained in Expectation Step is substituted in (20). Then the complete-data likelihood function is maximized with respect to the parameters to be estimated $\{\theta_l, \varphi_l, r_l, \mathbf{K}_{\mathbf{y}_l}\}$:

$$\left\{\theta_l^{p+1}, \varphi_l^{p+1}, r_l^{p+1}, \mathbf{K}_{\mathbf{y}_l}^{p+1}\right\} = \underset{\{\theta,\varphi,r,\alpha\}}{\arg\max}\left\{-\ln\det\mathbf{K}_{\mathbf{y}_l} - tr\left[\hat{\mathbf{K}}_{\mathbf{y}_l}^{p}\mathbf{K}_{\mathbf{y}_l}^{-1}\right]\right\} \tag{24}$$

The determinant of $\mathbf{K}_{\mathbf{y}_l}$ can be obtained by using the spectral decomposition. One eigenvector of $\mathbf{K}_{\mathbf{y}_l}$ is $\mathcal{A}_l(\theta,\varphi,r)/|\mathcal{A}_l(\theta,\varphi,r)|$, $K \times L - 1$ other mutually orthogonal eigenvector can be chosen from orthogonal complement of $\mathcal{A}_l(\theta,\varphi,r)$ and are equal to σ^2/d. Then the eigenvalue corresponding to the distinct eigenvector is $\alpha_l|\mathcal{A}_l(\theta,\varphi,r)|^2 + \sigma^2/d$.

Then the determinant of $\mathbf{K}_{\mathbf{y}_l}$ can be written as

$$\det\mathbf{K}_{\mathbf{y}_l} = \left(\alpha_l\left|\mathcal{A}_l(\theta,\varphi,r)\right|^2 + \frac{\sigma^2}{d}\right)\left(\frac{\sigma^2}{d}\right)^{KL-1}. \tag{25}$$

Since the inverse of $\mathbf{K}_{\mathbf{y}_l}$ is required in (24), it could be determined by employing matrix inverse lemma as

$$\mathbf{K}_{\mathbf{y}_l}^{-1} = \frac{d}{\sigma^2}\mathbf{I} - \frac{\mathcal{A}_l\left(\theta^p, \varphi^p, r^p\right)\mathcal{A}_l^H(\theta,\varphi,r)}{\left|\mathcal{A}_l\left(\theta^p, \varphi^p, r^p\right)\right|}\left(\frac{d}{\sigma^2} - \frac{1}{\frac{\sigma^2}{d} + \alpha_l\left|\mathcal{A}_l(\theta,\varphi,r)\right|^2}\right). \tag{26}$$

If we substitute the eigenvalues and the inverse of $\mathbf{K}_{\mathbf{y}_l}$ into (24), and maximizing (24) for $\alpha_l > 0$, the estimates of near-field parameters become

$$\left(\theta^{p+1}, \varphi^{p+1}, r^{p+1}\right) = \underset{\{\theta_l,\phi_l,r_l\}}{\arg\max}\frac{\mathcal{A}_l^H(\theta,\varphi,r)\hat{\mathbf{K}}_{\mathbf{y}_l}^{p+1}\mathcal{A}_l(\theta,\varphi,r)}{\left|\mathcal{A}_l(\theta,\varphi,r)\right|^2},$$
$$\alpha_l^{p+1} > 0 \tag{27}$$

$$\alpha_l^{p+1} = \frac{1}{\left|\mathcal{A}_l\left(\theta^{p+1}, \varphi^{p+1}, r^{p+1}\right)\right|^2}\left(\frac{\mathcal{A}_l^H\left(\theta^{p+1}, \varphi^{p+1}, r^{p+1}\right)\hat{\mathbf{K}}_{\mathbf{y}_l}^{p+1}\mathcal{A}_l\left(\theta^{p+1}, \varphi^{p+1}, r^{p+1}\right)}{\left|\mathcal{A}_l\left(\theta^{p+1}, \varphi^{p+1}, r^{p+1}\right)\right|^2} - \frac{\sigma^2}{d}\right). \tag{28}$$

Based on this results, the steps of the proposed unconditional ML algorithm are summarized as follows:

Repeat steps 1–3 for $i = 1, \ldots, d$

1. Given $\{\theta_l^0, \varphi_l^0, r_l^0\ \alpha_l^0\}$, $p = 0$,
2. $p = p + 1$,
   - Obtain $\hat{\mathbf{K}}_{\mathbf{y}_l}^{p+1}$ from (22),
   - Substitute $\hat{\mathbf{K}}_{\mathbf{y}_l}^{p+1}$ in (27), and then solve (27) for $\{\theta_l^{p+1}, \varphi_l^{p+1}, r_l^{p+1}\}$,
   - Substitute the estimates $\{\theta_l^{p+1}, \varphi_l^{p+1}, r_l^{p+1}\}$ in (28), then compute α_l^{p+1},
3. Continue this process until $\{\theta_l, \varphi_l, r_l\}$ and α_l converges.

For a sufficiently good initialization, this algorithm converges rapidly to the ML estimate of the near-field parameters $\{\hat{\theta}, \hat{\varphi}, \hat{r}\}$ and the source signals $\hat{\mathbf{K}}_s$.

4. Cramér Rao Bounds

The *CRB* provides a lower bound on the error variance of any unbiased estimators. In particular, it provides an asymptotic near-field source location estimator. The parameter of interest is $\tau = [\theta^T \varphi^T r^T]^T$. To focus on the parameters of interest, we shall use a concentrated likelihood approach to obtain the *CRB* [31]. Then the *ij*th element of the Fisher Information Matrix is given by

$$\mathbf{J}_{ij}(\tau) = N\,\mathrm{tr}\left[\mathbf{K}_{\mathbf{x}}^{-1}\left[\frac{\partial\tilde{\mathbf{K}}_{\mathbf{x}}}{\partial\tau_i}\right]_\infty \mathbf{K}_{\mathbf{x}}^{-1}\left[\frac{\partial\tilde{\mathbf{K}}_{\mathbf{x}}}{\partial\tau_j}\right]_\infty\right] \tag{29}$$

where $[\cdot]_\infty$ is defined as the almost sure (a.s.) limit of $[\cdot]$, and $\tilde{\mathbf{K}}_{\mathbf{x}}$ is the concentrated covariance after substituting the ML estimates $\hat{\mathbf{K}}_{\mathbf{x}}$ for $\mathbf{K}_{\mathbf{x}}$.

The ML estimate of $\mathbf{K}_s$ can be obtained as

$$\hat{\mathbf{K}}_s = \left[\mathbf{A}^H\mathbf{A}\right]^{-1}\mathbf{A}^H\hat{\mathbf{K}}_{\mathbf{x}}\mathbf{A}\left[\mathbf{A}^H\mathbf{A}\right]^{-1} - \sigma^2\left[\mathbf{A}^H\mathbf{A}\right]^{-1} \tag{30}$$

where we have suppressed the dependence of $\mathbf{A}$ on (θ,φ,r). It is well known that $\hat{\mathbf{K}}_s \rightarrow \mathbf{K}_s$ almost surely under mild conditions. Concentrating the covariance $\mathbf{K}_{\mathbf{x}}$ with respect to $\mathbf{K}_s$ yields

$$\tilde{\mathbf{K}}_{\mathbf{x}} = \mathbf{A}\hat{\mathbf{K}}_s\mathbf{A}^H + \sigma^2 I = \mathbf{P}\hat{\mathbf{K}}_{\mathbf{x}}P + \sigma^2\mathbf{P}^c \tag{31}$$

where

$$\mathbf{P} \triangleq \mathbf{A}\left[\mathbf{A}^H\mathbf{A}\right]^{-1}\mathbf{A}^H \quad \text{and} \quad \mathbf{P}^c \triangleq \mathbf{I} - \mathbf{P}. \tag{32}$$

We are now ready to evaluate CRB. Taking the first derivative $\partial\tilde{\mathbf{K}}_{\mathbf{x}}/\partial\tau_i$, the following can be obtained

$$\left(\frac{\partial\tilde{\mathbf{K}}_{\mathbf{x}}}{\partial\tau_i}\right) = \mathbf{P}_i\hat{\mathbf{K}}_{\mathbf{x}}\mathbf{P} + \mathbf{P}\hat{\mathbf{K}}_{\mathbf{x}}\mathbf{P}_i + \sigma^2\mathbf{P}_i^c. \tag{33}$$

where $\mathbf{P}_i = \partial\mathbf{P}/\partial\tau_i$.

We will make use of the properties of projection matrix to obtain,

$$\mathbf{P}^c\mathbf{P} = 0 \tag{34}$$
$$\mathbf{K}_{\mathbf{x}}\mathbf{P} = \mathbf{P}\mathbf{K}_{\mathbf{x}} \tag{35}$$
$$\mathrm{tr}\left[\mathbf{P}_i^c\mathbf{P}^c\right] = \frac{1}{2}\mathrm{tr}\left[\left(\mathbf{P}^c\right)_i^2\right] = \frac{1}{2}\left(\mathrm{tr}\left[\mathbf{P}^c\right]\right)_i = 0. \tag{36}$$

Thus, taking the limit $N \rightarrow \infty$ of (33)

$$\left(\frac{\partial\tilde{\mathbf{K}}_{x}}{\partial\tau_i}\right)_\infty = \mathbf{P}_i\mathbf{K}_{\mathbf{x}}\mathbf{P} + \mathbf{P}\mathbf{K}_{\mathbf{x}}\mathbf{P}_i + \sigma^2\mathbf{P}_i^c \tag{37}$$

Using (29) and the last result, we have

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$$\mathbf{J}_{ij}(\tau)=N\,\mathrm{tr}\Big[\mathbf{K}_x^{-1}\big(\mathbf{P}_i\mathbf{K}_x\mathbf{P}+\mathbf{P}\mathbf{K}_x\mathbf{P}_i+\sigma^2\mathbf{P}_i^c\big)\,\mathbf{K}_x^{-1}\big(\mathbf{P}_j\mathbf{K}_x\mathbf{P}+\mathbf{P}\mathbf{K}_x\mathbf{P}_j+\sigma^2\mathbf{P}_j^c\big)\Big]. \quad (38)$$

Next, we note that

$$\mathbf{P}_i=\mathbf{P}^c\mathbf{A}_i\mathbf{A}^\dagger+\big(\mathbf{A}^H\big)^\dagger\mathbf{A}_i^H\mathbf{P}^c \quad (39)$$

where $(\cdot)^\dagger$ is the pseudo inverse of $(\cdot)$.

To evaluate (38) explicitly, we consider its nine terms:

[1.]

$$\mathrm{tr}\big[\mathbf{K}_x^{-1}\mathbf{P}_i\mathbf{K}_x\mathbf{P}\mathbf{K}_x^{-1}\mathbf{P}_j\mathbf{K}_x\mathbf{P}\big]=\mathrm{tr}\big[\mathbf{P}_i\mathbf{P}\mathbf{P}_j\mathbf{P}\big]=0 \quad (40)$$

[2.]

$$\mathrm{tr}\big[\mathbf{K}_x^{-1}\mathbf{P}\mathbf{K}_x\mathbf{P}_i\mathbf{K}_x^{-1}\mathbf{P}_j\mathbf{K}_x\mathbf{P}\big]=\mathrm{tr}\big[\mathbf{P}\mathbf{P}_i\mathbf{K}_x^{-1}\mathbf{P}_j\mathbf{K}_x\big]=\frac{1}{\sigma^2}\mathrm{tr}\Big[\mathbf{A}_i^H\mathbf{P}^c\mathbf{A}_j\Big(\mathbf{K}_s+\sigma^2\big(\mathbf{A}^H\mathbf{A}\big)^{-1}\Big)\Big] \quad (41)$$

[3.]

$$-\sigma^2\mathrm{tr}\big[\mathbf{K}_x^{-1}\mathbf{P}_i^c\mathbf{K}_x^{-1}\mathbf{P}_j\mathbf{K}_x\mathbf{P}\big]=-\sigma^2\mathrm{tr}\big[\mathbf{P}\mathbf{P}_i\mathbf{K}_x^{-1}\mathbf{P}_j\big]=-\mathrm{tr}\Big[\mathbf{A}_i^H\mathbf{P}^c\mathbf{A}_j\big(\mathbf{A}^H\mathbf{A}\big)^{-1}\Big] \quad (42)$$

[4.]

$$\mathrm{tr}\big[\mathbf{K}_x^{-1}\mathbf{P}_i\mathbf{K}_x\mathbf{P}\mathbf{K}_x^{-1}\mathbf{P}\mathbf{K}_x\mathbf{P}_j\big]=-\sigma^2\mathrm{tr}\big[\mathbf{K}_x^{-1}\mathbf{P}_i\mathbf{K}_x\mathbf{P}_j\big]=\frac{1}{\sigma^2}\mathrm{tr}\Big[\mathbf{A}_i^H\mathbf{P}^c\mathbf{A}_i\Big(\mathbf{K}_s+\sigma^2\big(\mathbf{A}^H\mathbf{A}\big)^{-1}\Big)\Big] \quad (43)$$

[5.]

$$\mathrm{tr}\big[\mathbf{K}_x^{-1}\mathbf{P}\mathbf{K}_x\mathbf{P}_i\mathbf{K}_x^{-1}\mathbf{P}\mathbf{K}_x\mathbf{P}_j\big]=\mathrm{tr}\big[\mathbf{P}\mathbf{P}_i\mathbf{P}\mathbf{P}_j\big]=0 \quad (44)$$

[6.]

$$-\sigma^2\mathrm{tr}\big[\mathbf{K}_x^{-1}\mathbf{P}_i^c\mathbf{K}_x^{-1}\mathbf{P}\mathbf{K}_x\mathbf{P}_j\big]=-\sigma^2\mathrm{tr}\big[\mathbf{K}_x^{-1}\mathbf{P}_i\mathbf{P}\mathbf{P}_j\big]=-\mathrm{tr}\Big[\mathbf{A}_j^H\mathbf{P}^c\mathbf{A}_i\big(\mathbf{A}^H\mathbf{A}\big)^{-1}\Big] \quad (45)$$

[7.]

$$-\sigma^2\mathrm{tr}\big[\mathbf{K}_x^{-1}\mathbf{P}_i\mathbf{K}_x\mathbf{P}\mathbf{K}_x^{-1}\mathbf{P}_j^c\big]=-\sigma^2\mathrm{tr}\big[\mathbf{K}_x^{-1}\mathbf{P}_i\mathbf{P}\mathbf{P}_j\big]=-\mathrm{tr}\Big[\mathbf{A}_j^H\mathbf{P}^c\mathbf{A}_i\big(\mathbf{A}^H\mathbf{A}\big)^{-1}\Big] \quad (46)$$

[8.]

$$-\sigma^2\mathrm{tr}\big[\mathbf{K}_x^{-1}\mathbf{P}\mathbf{K}_x\mathbf{P}_i\mathbf{K}_x^{-1}\mathbf{P}_j^c\big]=-\sigma^2\mathrm{tr}\big[\mathbf{P}\mathbf{P}_i\mathbf{K}_x^{-1}\mathbf{P}_j\big]=-\mathrm{tr}\Big[\mathbf{A}_j^H\mathbf{P}^c\mathbf{A}_j\big(\mathbf{A}^H\mathbf{A}\big)^{-1}\Big] \quad (47)$$

[9.]

$$\sigma^4\mathrm{tr}\big[\mathbf{K}_x^{-1}\mathbf{P}_i^c\mathbf{K}_x^{-1}\mathbf{P}_j^c\big]=\sigma^4\mathrm{tr}\Big[\big(\mathbf{A}_j^H\mathbf{P}^c\mathbf{A}_i+\mathbf{A}_i^H\mathbf{P}^c\mathbf{A}_j\big)\mathbf{A}^\dagger\mathbf{K}_x^{-1}\big(\mathbf{A}^\dagger\big)^H\Big] \quad (48)$$

Plugging these results in (38), we have

$$\mathrm{tr}\left[\mathbf{K}_x^{-1}\left[\frac{\partial\tilde{\mathbf{K}}_x}{\partial\tau_i}\right]_{\infty}\mathbf{K}_x^{-1}\left[\frac{\partial\tilde{\mathbf{K}}_x}{\partial\tau_j}\right]_{\infty}\right]=2\,\mathrm{Re}\Big\{tr\Big\{\mathbf{A}_i^H\mathbf{P}^c\mathbf{A}_j\times\Big(\frac{\mathbf{K}_s}{\sigma^2}-\big(\mathbf{A}^H\mathbf{A}\big)^{-1}+\sigma^2\mathbf{A}^\dagger\mathbf{K}_x^{-1}\big(\mathbf{A}^\dagger\big)^H\Big)\Big\}\Big\} \quad (49)$$

Here,

$$\begin{aligned}\frac{\mathbf{K}_s}{\sigma^2}-\big(\mathbf{A}^H\mathbf{A}\big)^{-1}+\sigma^2\mathbf{A}^\dagger\mathbf{K}_x^{-1}\big(\mathbf{A}^\dagger\big)^H
&=\frac{1}{\sigma^2}\mathbf{A}^\dagger\big(\mathbf{A}\mathbf{K}_s\mathbf{A}^H-\sigma^2\mathbf{I}+\sigma^4\mathbf{K}_x\big)\big(\mathbf{A}^\dagger\big)^H\\
&=\frac{1}{\sigma^2}\mathbf{A}^\dagger\left(\mathbf{A}\mathbf{K}_s\mathbf{A}^H-\mathbf{A}\mathbf{K}_s\Big(\frac{1}{\sigma^2}\mathbf{A}^H\mathbf{A}\mathbf{K}_s+\mathbf{I}\Big)^{-1}\mathbf{A}^H\right)\big(\mathbf{A}^\dagger\big)^H\\
&=\frac{1}{\sigma^2}\Big\{\mathbf{K}_s-\mathbf{K}_s\Big(\mathbf{I}-\mathbf{A}^H\big(\mathbf{A}\mathbf{K}_s\mathbf{A}^H+\sigma^2\mathbf{I}\big)^{-1}\mathbf{A}\mathbf{K}_s\Big)\Big\}\\
&=\frac{1}{\sigma^2}\mathbf{K}_s\mathbf{A}^H\mathbf{K}_x^{-1}\mathbf{A}\mathbf{K}_s.\end{aligned} \quad (50)$$

Using (50) in (49), produces

$$\begin{aligned}J_{ij}&=\frac{2N}{\sigma^2}\mathrm{Re}\Big\{tr\big\{\mathbf{A}_i^H P^c\mathbf{A}_j\mathbf{K}_s\mathbf{A}^H\mathbf{K}_x^{-1}\mathbf{A}\mathbf{K}_s\big\}\Big\}\\
&=\frac{2N}{\sigma^2}\mathrm{Re}\Big\{\big(\mathbf{D}^H\mathbf{P}^c\mathbf{D}\big)_{ij}\big(1\otimes\mathbf{K}_s\mathbf{A}^H\mathbf{K}_x\mathbf{A}\mathbf{K}_s\big)_{ij}\Big\}\end{aligned} \quad (51)$$

Finally, using (51), we have CRB upon inversion

$$CRB=\frac{\sigma^2}{2N}\mathrm{Re}\Big\{\mathbf{D}^H\mathbf{P}^c\mathbf{D}\odot\big(1\otimes\mathbf{K}_s\mathbf{A}^H\mathbf{K}_x\mathbf{A}\mathbf{K}_s\big)^T\Big\}^{-} \quad (52)$$

where $\odot$ denotes the element-wise matrix products

5. Simulations

To illustrate the the effectiveness and applicability of the proposed method, we consider the following scenarios.

Case 1: A Uniform rectangular linear array of $K = L = 3$ totally 9 sensors with inter-element spacing $\Delta = \lambda/2$ was used to estimate the locations of two sources located at $\{\theta_1,\varphi_1,r_1\} = \{24^0, 80^0, 2\lambda\}$ and $\{\theta_2,\varphi_2,r_2\} = \{34^0, 5^0, 1.6\lambda\}$. The number of the snapshots (N) set to 80 and the SNR was varied from 0 to 30 dB. The proposed method was tested for $M = 100$ independent trials. The resulting $RMSE$ of the estimated DOAs (azimuths and elevations in degrees) and ranges in wavelength are shown in **Fig. 2, Fig. 3** and **Fig. 4**. The results were compared with the Cramer-Rao Bounds.

Case 2: To illustrate bias and convergence rate of the estimator, we consider a new scenerio. A Uniform rectangular linear array of $K = L = 5$ totally 25 sensors with inter-element spacing $\Delta = \lambda/2$ was used to estimate the locations of two sources located at $\{\theta_1,\varphi_1,r_1\} = \{70^0, 24^0, 4.5\lambda\}$ and $\{\theta_2,\varphi_2,r_2\} = \{70^0, -24^0, 7\lambda\}$.

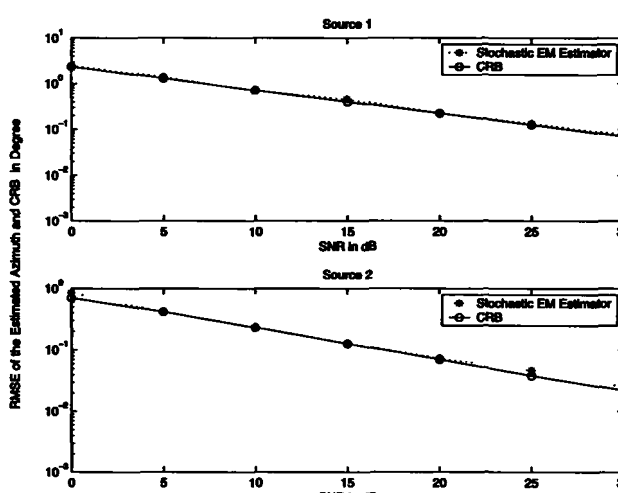

Fig. 2: RMSE of the estimated azimuths

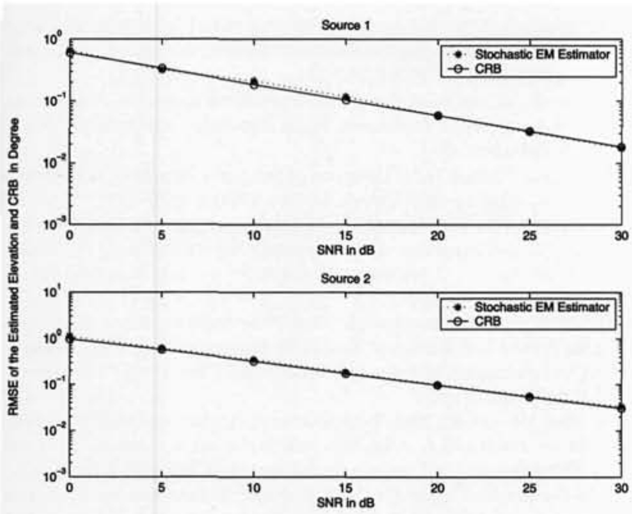

Fig. 3: RMSE of the estimated elevations

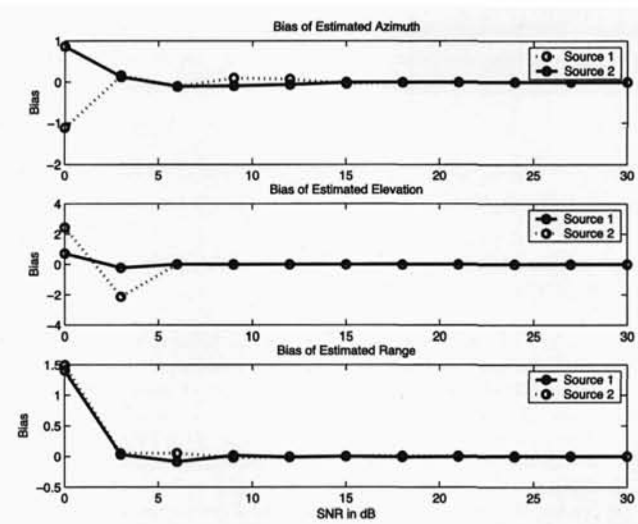

Fig. 5: Bias of the estimated azimuths,elevations and ranges versus SNR

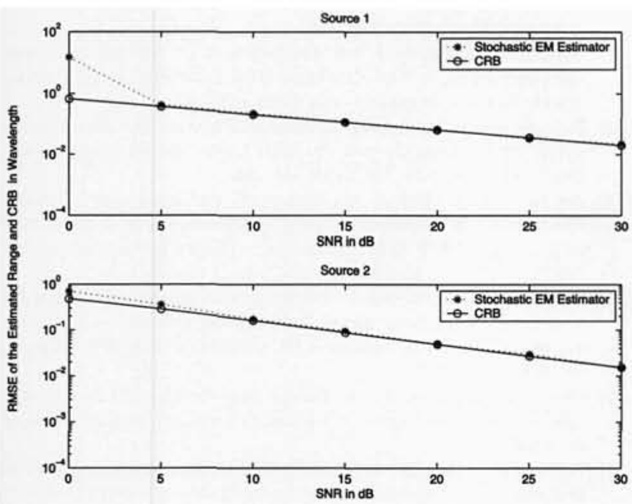

Fig. 4: RMSE of the estimated ranges

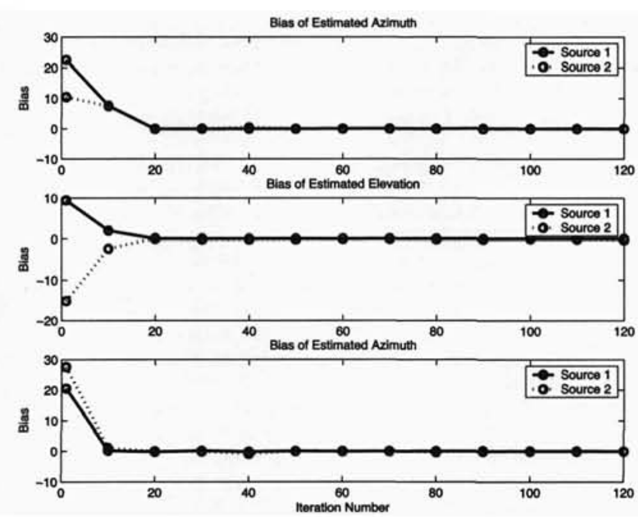

Fig. 6: Bias of the estimated azimuths,elevations and ranges versus iteration number

The number of the snapshots (*N*) set to 1000 and the *SNR* was varied from 0 to 30 dB and The proposed method was tested for $M = 500$ independent trials. Bias of the estimated azimuth, elevation and range versus *SNR* and iterations number are shown in **Fig. 5** and **Fig. 6** respectively. Also, the convergence rates of the estimated source parameters are shown in **Fig. 7**.

Based on the simulation results we made the following observations:

- For a sufficiently good initialization, proposed algorithm converges rapidly to the ML estimate of $\{\hat{\theta},\hat{\varphi},\hat{r}\}$ and $\hat{\mathbf{K}}_s$. Since the spàtial structure of the array matrix is known, then the good initial estimates of the steering matrix can be obtained from MUSIC and ESPRIT algorithms.
- For high *SNRs* the *RMSEs* obtained from simulations becomes almost identical to the *CRB* results derived by modifying the results in [31].
- As expected, for a high enough *SNR*, the estimator is essentially unbiased. Moreover, for larger iterations, it is verified that the mean equals to true value.
- In the near-field scenario, it is observed that the proposed algorithm converges in around 10 to 20 steps.

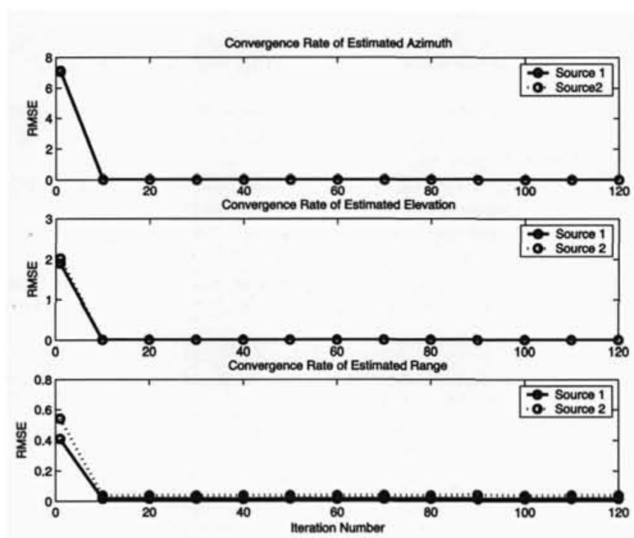

Fig. 7: Convergence of the proposed algorithm of estimated azimuths,elevations and ranges

6. Conclusions

In this paper, we have described a stochastic approach to estimate the parameters of near-field sources in 3-D space. We derived an EM based iterative method for obtaining ML estimator. Furthermore the performance of the proposed method is studied based on the derivation of Cramér-Rao bounds from concentrated likelihood function. We also presented Monte Carlo simulations to verify the theoretically predicted estimator's performance. The examples demonstrated that the proposed stochastic ML achieve the concentrated *CRB* for moderate and high *SNR* values. Furthermore, they also demonstrated that proposed stochastic ML has nearly zero bias, and it converges rapidly for all source location parameters.

This work was supported by The Research Fund of The University of Istanbul, Project Numb: 220/29042004.

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(Received on May 24, 2004)