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Gravitational Search Algorithm for Post-Outage Bus Voltage Magnitude Calculations

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***Abstract*—Branch outage problem is one of the key problems in power system security analysis. This paper solves branch outage problem using a bounded approach. Local constrained optimization problem in the bounded approach is solved by the gravitational search algorithm. Test results of IEEE 14, 30, and 118 bus systems are compared to those of ac load flow method in terms of both accuracy and speed.**

***Index Terms*—Branch outage, contingency studies, gravitational search algorithm, intelligent methods, modeling, optimization.**

I. INTRODUCTION

Electrical power systems are mainly composed of branches, transformers, and generators. Outage of any of these components could cause serious disturbances in a system. One of the main studies that an electricity management center operator should perform for post outage state estimation is the branch outage simulation.

AC power flow method can be used for branch outage simulation but it is not computationally effective even for moderate size power systems. To speed up the solution process, hence be able to study many scenarios, some linearized and approximated methods have been developed [1]–[3]. However, they suffer in handling bus voltage magnitudes and reactive power flows. A recent work simulates branch outage problem by a pair of fictitious reactive power sources formulating it as a local constrained optimization problem [4]. The model deals with a limited number of busses and corresponding reactive power mismatch equations included in a bounded region. It produces good results both in terms of accuracy and speed.

Optimization problems can either be solved analytically or numerically. Numerical methods can be classified into two types: gradient based methods and non-gradient based methods. Among the former are the steepest descent method, the conjugate gradient method, and the Lagrangian method. They use information of the derivatives with respect to the variables. On the other hand, the latters are genetic algorithms, particle swarm optimization method, simulated annealing method, differential evolution method, harmony search method etc. These methods first generate initial solution candidates, followed by generation of better solution candidates using evolutionary operators like mutation, crossover, and randomization.

One of the recently developed non-gradient based methods is the gravitational search algorithm [5], [6]. This algorithm is based on the Newtonian gravity principle, which states that every particle in the universe attracts every other particle with a force directly proportional to the product of its mass and inversely proportional to the square of the distance between the two. In the algorithm, solution candidates are considered as objects and their performances are measured by their masses. All the objects attract each other by the gravity force. The position of an object changes according to its acceleration. The algorithm first generates an initial population. In the second step, the fitness value of each agent is evaluated. Next, the algorithm finds the best and the worst elements of the population and calculates the mass and acceleration values for each object in order to update velocity and position. If a stopping criterion is met the algorithm terminates otherwise restarts from the second step.

This study presents gravitational search algorithm based solution of a branch outage problem defined in [4]. Up to now, this problem has been solved by genetic algorithms [7], particle swarm optimization [8], differential evolution, and harmony search [9]. The aim of the proposed algorithm is to speed up the solution without sacrificing the accuracy. Post outage system state is determined by solution of a local optimization problem with the gravitational search algorithm. Matlab based power system package, Matpower and an interacting software are used for the simulations. The proposed solution algorithm is tested with IEEE standard test cases (14, 30, and 118 bus systems). The results of the solutions are compared and discussed both from the point of accuracy and solution speed.

The rest of the paper is organized as follows: In the second part, a branch outage model and a local optimization method for solving branch outages are defined. In the third part, the basics of the gravitational search algorithm are introduced. Next, adoptation of the gravitational search algorithm for the branch outage problem is given and finally, test results for IEEE 14, 30, and 118 bus test systems are illustrated.

II. BRANCH OUTAGE MODELING

An interconnected power system transmission line's π equivalent, connecting two busses and the associated reactive power flows are given in Fig. 1.

Reactive power flowing through the line ij, transferred reactive power, and reactive power loss are represented by

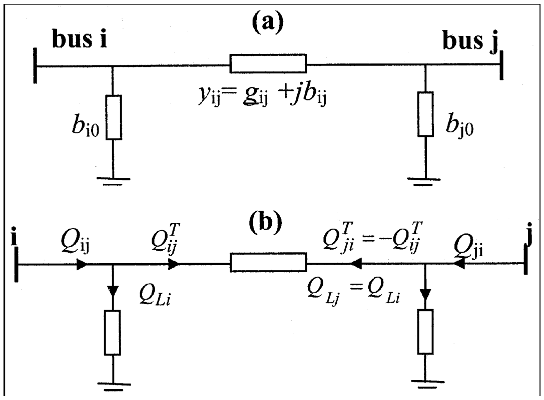

Fig. 1. Transmission line and reactive power flow model. a) π equivalent of a transmission line. b) reactive power flows.

Q_{ij}, Q_{ij}^T, and Q_{Li} respectively. These reactive powers can be expressed in terms of system variables as follows.

$$Q_{ij} = -[V_i^2 - V_iV_j cos\delta_{ji}]b_{ij} + V_iV_jg_{ij}\sin\delta_{ji} - V_i^2\frac{b_{i0}}{2} \quad (1)$$

$$Q_{ij}^T = -[V_i^2 - V_j^2]\frac{b_{ij}}{2} + V_iV_jg_{ij}sin\delta ji \quad (2)$$

$$Q_{Li} = -[V_i^2 + V_j^2 - 2V_iV_j\cos\delta ji]\frac{b_{ij}}{2} - (V_i^2 + V_j^2)\frac{b_{i0}}{4} \quad (3)$$

Pre-outage and actual outage states of a transmission line are shown in Figures 2.a and 2.b respectively. A line outage is simulated using fictitious sources as shown in Fig. 2.c [4].

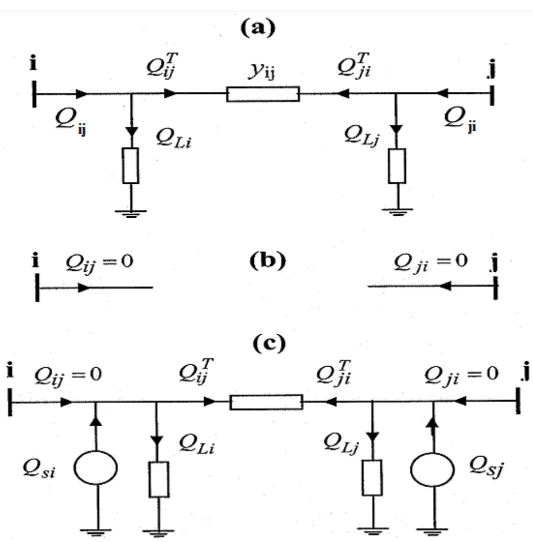

Fig. 2. Line outage modeling. a) pre-outage b) actual outage c) simulated post outage.

Local constrained optimization is solved in the bounded network which is composed of the first order neighbours of the outaged buses. Only load bus voltage magnitudes in this bounded region are taken into consideration during the computation process of the optimization problem.

The procedure for the existing method is as follows.

1) Select an outage of a branch, connected between busses i and j, and number it as k.
2) Calculate bus voltage phase angles using linearized MW flows (see [10] for details).

$$\delta_l = \delta_l - (X_{li} - X_{lj}) \triangle P_k, \quad l = 2, 3, \cdots, \text{NB} \quad (4)$$

$$\triangle P_k = \frac{P_{ij}}{\frac{1-(X_{ii}+X_{jj}-2X_{ij})}{x_k}} \quad (5)$$

where, X represents the inverse of the bus susceptance matrix, P_{ij} is the pre-outage active power flow through the line, and x_k represents the reactance of the line at hand. If the voltage magnitudes are calculated, then the calculation of the busses included in the bounded network would suffice.

3) Calculate the reactive power transfer $\overline{Q}_{ij}^T$between the busses. This power includes the increment due to the change in bus voltage phase angles.
4) Minimize the reactive power mismatches of busses i and j. This process is mathematically equivalent to the following constrained optimization problem.

$$\min_{wrt\ Q_{si},Q_{sj}} \| Q_i - (\overline{Q}_{ij} + \overline{Q}_{Li}) + Q_{Di} \quad Q_j - (-\overline{Q}_{ij} + \overline{Q}_{Li}) + Q_{Dj} \| \quad (6)$$
$$\text{subject to} \quad g_q(V_b) = \triangle Q_b - B_b \triangle V_b = 0$$

where, $\| \cdot \|$ is the Euclidean norm of a vector. Equation (6) is linear reactive power equation for load busses, $\triangle Q$ is reactive power mismatch vector, V is bus voltage magnitude vector and B is bus susceptance matrix. It should be stated that only two elements of $\triangle Q$ vector are nonzero and they are represented as shown below.

$$[\triangle Q] : [\triangle Q]_i = -[\triangle Q]_j = Q_{si} - Q_{ij} \quad (7)$$

On the other hand, we use subscript b to denote the bounded region where the optimization process is done. For the case of a transformer with tap t, these values are given as follows.

$$[\triangle Q]_k = \begin{cases} [\triangle Q]_i & = Q_{si} - Q_{ij}^T \\ [\triangle Q]_j & = \frac{tV_j}{-2V_i + tV_j}[\triangle Q]_i \\ [\triangle Q]_k & = 0 \quad \text{for} \quad k \neq i, j \end{cases} \quad (8)$$

III. GRAVITATIONAL SEARCH ALGORITHM

Gravitational Search Algorithm (GSA) is one of the recently developed evolutionary algorithms which is based on the law of gravity. Since it is a new algortihm, up to now it has not been applied to the traditional power system problems besides, but it has been applied to the allocation of static VAR compansators [11] and vertex covering problem [12].

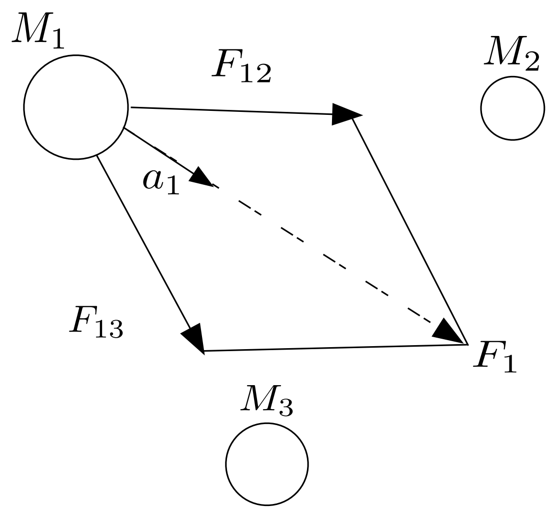

Fig. 3. Total force acting on an object and its acceleration.

In the algorithm agents are considered as objects and their performances are measured by their masses [6]. Each mass represents a solution and every object attracts the other objects by the gravity force. Hence, with respect to the law of motion, objects try to move towards the heavier objects. The heavier masses represent good solutions and they move slower than the lighter ones representing worse solutions.

Two well known equations are used in GSA. The first one is the gravitational force equation between the two particles, which is directly proportional to their masses and inversely proportional to the square of distance between them [6]:

$$F = G\frac{M_1 M_2}{R^2} \tag{9}$$

In the algorithm R is used instead of R^2 since the authors of [6] reported that this case provided better results.

The second one is the equation of accelaration of a particle when a force is applied to it:

$$a = \frac{F}{M} \tag{10}$$

Gravitational constant value is proportional to the ratio of the initial time with respect to the actual time as shown in (11). $G(t_0)$ in (11) is the value of the gravitational constant at the initial time.

$$G(t) = G(t_0) \times (\frac{t_0}{t})^{\beta} \tag{11}$$

Active gravitational mass, passive gravitational mass, and inertial mass are defined in physics. Active gravitational force is a measure of the strength of the gravitational field due to a particular object [6]. Passive gravitational force is a measure of the strength of an object's interaction with the gravitational field [6].

Steps of GSA can be given as follows.

1) Initialize gravitational constant G. Decide the number of agents, N, to use in GSA.
2) Initialize the positions of a system with N masses as follows.
$$X_i = (x_i^1, \cdots, x_i^d, \cdots, x_i^n) \quad \text{for} \quad i = 1, 2, \cdots, N \tag{12}$$
where, x_i^d represents the initial position of the ith mass in the dth dimension.
3) Decrease gravitational constant according to the following equaiton.
$$G(t) = G_0 e^{-\alpha \frac{t}{T}} \tag{13}$$
where α is a user specified constant, T is the total number of iterations (time steps), and t is the current iteration.
4) Evaluate the fitness of each object. Calculate the gravitational and inertial masses by using the following equations.
$$\begin{aligned} M_{ai} &= M_{pi} = M_{ii} = M_i, \qquad i = 1, 2, \cdots, N \\ m_i(t) &= \frac{\text{fitness}_i(t) - worst(t)}{\text{best}(t) - \text{worst}(t)} \\ M_i(t) &= \frac{m_i(t)}{\sum_{j=1}^{N} m_j(t)} \end{aligned} \tag{14}$$
In (14) $fitness_i(t)$ represents the fitness value of the ith mass at time t, $worst(t)$ and $best(t)$ are the minimum value of the all fitness values and maximum value of the all fitness values for a minimization problem respectively. For maximization problems the opposite of this expression applies.
5) Compute the force acting on mass i from mass j at time t as shown below.
$$F_{ij}^d(t) = G(t)\frac{M_{pi}(t) \times M_{aj}(t)}{R_{ij}(t) + \epsilon}(x_j^d(t) - x_i^d(t)) \tag{15}$$
where, M_{aj} is the active gravitational mass of object j, M_{pi} is the passive gravitational mass of object i, $G(t)$ is the gravitational constant at time t, ϵ is a small constant, R_{ij} is the Euclidean distance between two objects i and j.
6) Compute the total force that acts on object i in dimension d as follows.
$$F_i^d(t) = \sum_{j \in Kbest, j \neq i}^{N} rand_j F_{ij}^d(t) \tag{16}$$
where $rand_j$ is a random number between 0 and 1, and $Kbest$ is the set of first K objects with the best fitness value and biggest mass.
7) Find the acceleration of object i in dth dimension.
$$a_i^d(t) = \frac{F_i^d}{M_{ii}(t)} \tag{17}$$
8) Compute the velocity and the position of the object for time $t+1$ by using the following equations.
$$v_i^d(t+1) = rand_i \times v_i^d(t) + a_i^d \tag{18}$$
$$x_i^d(t+1) = x_i^d(t) + v_i^d(t+1) \tag{19}$$

where $rand_i$ is a random number between 0 and 1.

9) If the norm of two consecutive best values of x_i is smaller than a specified tolerance value, or the best values don't change for a spacified number of iterations stop, otherwise goto step 3.

A. Gravitational Algorithm For Branch Outage Problem

Application of gravitational search algorithm for solving line outage problem can be summarized as follows.

1) Run a base case load flow to obtain bus voltage magnitudes of the load buses in the bounded region.
2) Initialize gravitational constant $G = G_0 \geq 0$ and decide the number of agents N.
3) Create an initial Q_{si} solution candidate vector, composed of N elements.
4) Decrease G according to (13).
5) Determine $\triangle Q_b$ vector, either using (7) or (8), then solve the equations given below and update voltage magnitudes.
   $(B_b)^{-1} \triangle Q_b = \triangle V_b$ and $V_{new} = V_{old} + \Delta V$
6) After evaluating the objective function in (6) for all objects, and finding the best and the worst fitness values, calculate the gravitational and inertial masses by using (14).
7) Compute the force acting on mass i from mass j using (15).
8) Compute the total force on object i using (16) and the acceleration of object i using (17).
9) Compute the velocity and the position of all the objects for the next time step using (18) and (19).
10) If the norm of the difference between the two consecutive best values of Q_{si} is smaller than a specified tolerance value, or the best values don't change for a specified number of iterations stop, otherwise goto step 3.

IV. Test Results

GSA application of branch outage problem is tested on IEEE 14, 30, and 118 bus test systems. Matlab based opensource software Matpower [13] is used as a tool. Programs for simulating the outages are written in Matlab.

In all simulations, program parameters are chosen as follows:

$G_0 = 1$,
population size $= 15$,
$\epsilon = 0.0001$,
maximum iteration number $= 60$.

The program stops if the variables providing the best solution in the generation do not change for 10 consecutive iterations.

Tables 1-5 illustrate the simulation results for several test systems. In the tables V_{AC} repesents the post-outage voltage magnitude of a specific bus calculated by using full AC load flow where, V_{GSA} symbolizes the post-outage voltage magnitude of a specific bus calculated by using GSA method. $Err\%$ represents the percentage error of the specific bus voltage magnitude, and is computed as follows.

$$\text{Err}\% = 100 \times \frac{abs(V_{AC} - V_{GSA})}{V_{AC}} \tag{20}$$

Among several simulations, two representative outage results will be illustrated for IEEE 14 bus system. Table I shows post outage voltage magnitudes for the outage of a line connected between buses 7 and 9 and for the outage of a transformer connected between busses 5 and 6. Maximum percentage voltage magnitude errors for the outage of the line and for the outage of the transformer are 0.61 and 0.86 respectively.

TABLE I
TWO REPRESENTATIVE POST-OUTAGE VOLTAGE MAGNITUDE CALCULATIONS FOR IEEE 14 BUS TEST SYSTEM

	Outage of Line 7-9			Outage of Transformer 5-6		
Bus No	V_{AC}	V_{GSA}	Err(%)	V_{AC}	V_{GSA}	Err(%)
1	1.0600	1.0600	0.00	1.0600	1.0600	0.00
2	1.0450	1.0450	0.00	1.0450	1.0450	0.00
3	1.0100	1.0100	0.00	1.0100	1.0100	0.00
4	1.0169	1.0178	0.08	1.0181	1.0268	0.86
5	1.0174	1.0196	0.21	1.0272	1.0348	0.74
6	1.0700	1.0700	0.00	1.0700	1.0700	0.00
7	1.0671	1.0698	0.26	1.0656	1.0657	0.01
8	1.0900	1.0900	0.00	1.0900	1.0900	0.00
9	1.0291	1.0353	0.61	1.0682	1.0601	0.77
10	1.0282	1.0338	0.55	1.0614	1.0544	0.66
11	1.0446	1.0480	0.32	1.0623	1.0587	0.34
12	1.0535	1.0539	0.04	1.0543	1.0555	0.11
13	1.0459	1.0472	0.13	1.0525	1.0510	0.14
14	1.0179	1.0225	0.45	1.0422	1.0381	0.39

In table II maximum percantage errors and corresponding buses are given for all possible outages in IEEE 14 bus system.

TABLE II
MAXIMUM PERCENTAGE ERRORS FOR IEEE 14 BUS TEST SYSTEM

Branch	Max. Error (%)	Bus No.
1-5	0.16	4
2-4	0.32	5
2-5	0.10	4
3-4	0.10	5
4-5	0.73	9
4-7	0.79	9
4-9	0.13	9
5-6	0.86	4
6-11	0.41	11
6-12	0.87	12
6-13	1.07	13
7-9	0.60	9
9-10	0.15	10
9-14	0.07	9
10-11	0.15	10
12-13	0.08	12
13-14	0.36	14

Table III gives the post bus outage voltage magnitudes and their corresponding percentage errors for the two representative outages (outage of line connected between busses 4 and 6 and outage of a transformer connected between busses 4 and 12) in IEEE 30 bus test system. Maximum percentage errors for these two cases are 0.52 and 0.64, respectively.

TABLE III
TWO REPRESENTATIVE POST-OUTAGE VOLTAGE MAGNITUDE CALCULATIONS FOR IEEE 30 BUS TEST SYSTEM

	Outage of Line 4-6			Outage of Transformer 4-12		
Bus No	V_{AC}	V_{GSA}	Err(%)	V_{AC}	V_{GSA}	Err(%)
1	1.0500	1.0500	0.00	1.0500	1.0500	0.00
2	1.0500	1.0500	0.00	1.0500	1.0500	0.00
3	1.0233	1.0223	0.10	1.0419	1.0455	0.34
4	1.0165	1.0154	0.10	1.0396	1.0439	0.42
5	1.0500	1.0500	0.00	1.0500	1.0500	0.00
6	1.0399	1.0411	0.12	1.0381	1.0408	0.26
7	1.0364	1.0373	0.09	1.0355	1.0371	0.16
8	1.0500	1.0500	0.00	1.0500	1.0500	0.00
9	1.0507	1.0532	0.24	1.0498	1.0482	0.15
10	1.0449	1.0499	0.48	1.0462	1.0397	0.62
11	1.0500	1.0500	0.00	1.0500	1.0500	0.00
12	1.0526	1.0507	0.18	1.0252	1.0219	0.32
13	1.0500	1.0500	0.00	1.0500	1.0500	0.00
14	1.0392	1.0376	0.15	1.0131	1.0109	0.22
15	1.0339	1.0346	0.07	1.0150	1.0108	0.41
16	1.0397	1.0427	0.29	1.0288	1.0222	0.64
17	1.0387	1.0427	0.39	1.0348	1.0292	0.54
18	1.0253	1.0279	0.25	1.0136	1.0089	0.47
19	1.0235	1.0270	0.34	1.0161	1.0109	0.51
20	1.0280	1.0319	0.38	1.0229	1.0173	0.54
21	1.0329	1.0379	0.48	1.0334	1.0272	0.60
22	1.0336	1.0385	0.47	1.0338	1.0277	0.60
23	1.0260	1.0282	0.22	1.0133	1.0090	0.43
24	1.0243	1.0280	0.37	1.0198	1.0150	0.47
25	1.0285	1.0334	0.47	1.0306	1.0251	0.53
26	1.0110	1.0160	0.49	1.0132	1.0077	0.55
27	1.0400	1.0452	0.50	1.0457	1.0399	0.56
28	1.0376	1.0386	0.10	1.0357	1.0378	0.20
29	1.0205	1.0258	0.52	1.0264	1.0205	0.57
30	1.0092	1.0145	0.52	1.0152	1.0093	0.58

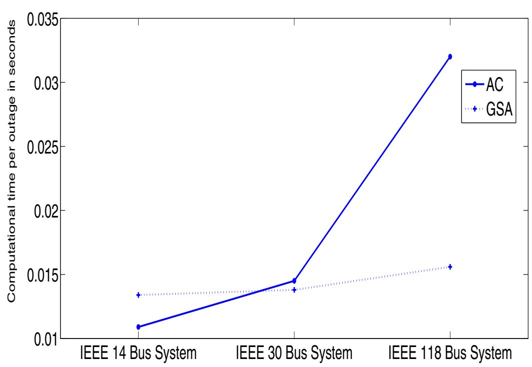

Fig. 4. Computation Time Graphic for Several Test Systems.

Table IV gives the results for the outage of line connected between busses 52 and 53 in IEEE 118 bus system. Table V gives the results for the outage of the transformer line between buses 30 and 17 in IEEE 118 bus system. Because of the limited space, Table IV comprises only the buses which have percentage errors greater then 0.05 and similarly Table V comprises only the buses which have percentage errors greater then 0.10.

TABLE IV
POST-OUTAGE VOLTAGE MAGNITUDES FOR THE OUTAGE OF THE BRANCH $52-53$ IN IEEE 118 BUS TEST SYSTEM

Bus	V_{AC}	V_{GSA}	Error (%)
51	0.9719	0.9726	0.07
52	0.9654	0.9669	0.15
53	0.9356	0.9371	0.17
58	0.9619	0.9623	0.05

TABLE V
POST-OUTAGE VOLTAGE MAGNITUDES FOR THE OUTAGE OF THE TRANSFORMER $30-17$ IN IEEE 118 BUS TEST SYSTEM

Bus	V_{AC}	V_{GSA}	Error (%)
13	0.9668	0.9683	0.16
20	0.9538	0.9569	0.33
21	0.9531	0.9577	0.49
22	0.9639	0.9690	0.53
23	0.9970	0.9995	0.24
30	1.0131	1.0183	0.51
33	0.9704	0.9719	0.16
38	0.9683	0.9727	0.45

Table VI gives the computation times required for post outage calculations per outage for IEEE 14, IEEE 30 and IEEE 118 bus systems using AC load flow and GSA method. Computation time versus system size is also shown in fig. (4).

Computation results have showed that the computation times required for the solution of the branch outage problem was not affected from the system size for GSA method. On the other hand, load flow based computations get very slow as the system size increases.

TABLE VI
COMPUTATION TIMES FOR SEVERAL TEST SYSTEMS

Test System	Simulation #	Computation time per outage		
		GS	σ_{GS}	AC
IEEE 14 Bus	17	0.0134	0.0025	0.0109
IEEE 30 Bus	36	0.0138	0.0013	0.0145
IEEE 118 Bus	126	0.0156	0.0010	0.0320

V. CONCLUSIONS

In this study, the constrained optimization problem representing the line outage phenomena in an electric power system has been solved by using the gravitational search algorithm.

Simulation results have shown that the problem at hand can be easily solved by GSA and reasonable accuracies for post outage voltage magnitude values can be obtained. Solution of the local constrained optimization problem is independent from the size of the system under test. This is confirmed by the speed test results of the all possible scenarios for IEEE 14, 30, and 118 test systems.

REFERENCES

[1] C. Y. Lee, and N. Chen, “Distribution factors of reactive power flow in transmission line and transformer outage studies,” *IEEE Trans. Power Syst.*, vol. 17, pp. 194–200, Feb. 1992.

[2] M. Ilic, and A. Phadke, “Redistribution of reactive power flow in contingency analysis,” *IEEE Trans. Power Syst.*, vol. 1, pp. 266–275, Aug. 1986.

[3] K. R. C. Mamandur, and G. J. Berg “Efficient simulation of line and transformer outages in power systems”, *IEEE Trans. Power App. Syst.*, vol. 101, pp. 3733–3741, Oct. 1982.

[4] A. Ozdemir, Y. J. Lim, and C. Singh, “Branch outage simulation for MWAR flows: bounded network solution”, *IEEE Trans. Power Syst.*, vol. 18, pp. 1523–1528, Nov. 2003.

[5] E. Rashedi, H. Nezamabadi-pour, and S. Saryazdi, “BGSA: A Gravitational Search Algorithm ”, *Natural Computing*, Dec. 2009.

[6] E. Rashedi, H. Nezamabadi-pour, and S. Saryazdi, “GSA: A Gravitational Search Algorithm ”, *Information Sciences, Special Section on High Order Fuzzy Sets*, vol. 179, pp. 2232–2248, June 2009.

[7] A. Ozdemir, Y. J. Lim, and C. Singh, “Post-outage reactive power flow calculations by genetic algorithm: constrained optimization approach”, *IEEE Trans. Power Syst.*, vol. 20, pp. 1266–1272, Aug. 2005.

[8] O. Ceylan, A. Ozdemir, and H. Dag, “Branch outage solution using particle swarm optimization”, *Australasian Universities Power Eng. Conf.*, Dec. 2008.

[9] O. Ceylan, A. Ozdemir, and H. Dag, “Comparison of post outage bus voltage magnitudes estimated by harmony search and differential evolution methods”, *The 15th International Conference on Intelligence System Applications to Power Systems*, Nov. 2009.

[10] A. J. Wood and B. F. Wollenberg, *Power Generation Operation and Control*, 2nd ed. New York: Wiley, 1996, pp. 410–450. bibitemWood A. J. Wood and B. F. Wollenberg, *Power Generation Operation and Control*, 2nd ed. New York: Wiley, 1996, pp. 410–450.

[11] E. Rashedi, H. Nezamabadi-pour, and S. Saryazdi, “Allocation of Static Var Compansator Using Gravitational Search Algorithm ”, *First Joint Congress on Fuzzy and Intelligent Systems, Ferdowsi University of Mashad, Iran*, August 2007.

[12] S. R. Balachandar, and K. Kannan, “A Meta-Heuristic Algorithm for Vertex covering problem Based on Gravity”, *International Journal of Mathematical and Statistical Sciences*, vol. 1, pp. 130–136, 2009.

[13] R. Zimmermann, and D. Gan, “Matpower Manual ”, *USA: PSERC*, Cornell Univ. 1997.

[14] “The mathworks distributed computing toolbox and matlab distributed computing engine” 2.0.1
http:www.mathworks.com/products/distribtb