Unconditional Maximum Likelihood Approach for Localization of Near-Field Sources in 3-D Space

Nihat Kabaoğlu
Technical Vocational School
Kadir Has University
Fatih, 34230, Istanbul,Turkey
nihat@khas.edu.tr

Hakan A. Çırpan
Department of Electrical Engineering
Istanbul University
Avcilar, 34850, Istanbul, Turkey
hcirpan@istanbul.edu.tr

Selçuk Paker
Electronics and Communication Engineering Department
Istanbul Technical University
Maslak, 80626, Istanbul, Turkey
spaker@ehb.itu.edu.tr

Abstract

Since maximum likelihood (ML) approaches have better resolution performance than the conventional localization methods in the presence of less number and highly correlated source signal samples and low signal to noise ratios, we propose unconditional ML (UML) method for estimating azimuth, elevation and range parameters of near-field sources in 3-D space in this paper. Besides these superiorities, stability, asymptotic unbiasedness, asymptotic minimum variance properties are motivated the application of ML approach. Despite these advantages, ML estimator has computational complexity. Fortunately, this problem can be tackled by the application of Expectation/Maximization (EM) iterative algorithm which converts the multidimensional search problem to one dimensional parallel search problems in order to prevent computational complexity.

1. Introduction

Localization of sources at the different plane with antenna array is more applicable to the real world array processing problems. Primary research results presented under this assumption were for localization of narrow-band far-field source signals [1], [2], [3], [4]. Moreover, recent research results on localization of near-field narrow-band sources in 3-D space were also presented [5], [6]. Faced with inability to completely evaluate performances of optimal 3-D near-field localization approaches from [5], [6], it is reasonable to resort to a asymptotically optimal estimators.

The localization of the near-field sources in 3-D space is in general nontrivial, since localization of near-field sources requires estimation of the azimuth and elevation together with the range parameters. Recently, an algorithm using 3-D Music with polynomial rooting have been developed [5]. High-order subspace based algorithms was introduced in [6]. In contrast to suboptimal approaches proposed in [5], [6], we now investigate an alternative estimator that is asymptotically efficient. Due to many attractive properties of maximum likelihood (ML) estimation methods such as consistency, asymptotic unbiasedness and asymptotic minimum variance, we concentrate on ML method for localization of near-field sources in 3-D space. Furthermore it has a better resolution performance than the other methods in the presence of less number and highly correlated source signal samples and low signal to noise ratios. Besides these superiorities, bring no restrictions on the antenna array are the additional reasons for the decision of this method. Regarding the assumption on the narrow-band source signals, there are two different types of models. These two models lead corresponding ML solutions. The models are: i. Deterministic Model which assumes the signals to be unknown but deterministic (i.e., the same in all realizations) and ii. Stochastic Model (SM) which assumes the signals to be random. ML methods corresponding to the signal models (i) and (ii) are termed conditional ML (CML) and unconditional ML (UML) respectively. Expectation/Maximization (EM) based deterministic ML (signal model (i)) near-field location estimator have been studied in [7]. The goal of the present paper is to provide an UML approach for the estimation of the DOA and range parameters of near-field sources. However, calculation of ML estimation from corresponding likelihood function for the unconditional case results in further difficult nonlinear constrained optimization problem, which must be solved iteratively. We therefore employed the EM iterative method for obtaining ML estimator. The most important feature of the algorithm is that it decomposes the observed data into its signal components and then estimates the parameters of each signal component separately.

2. Signal Model

Consider a near-field scenario in which narrowband signals from d sources received by an $K \times L$ element antenna array. Let the array center be the phase reference point with index $'(0,0)'$. Assuming 2-D rectangular uniform linear array consisting of omnidirectional sensors with interelement spacing Δ along each axes, we write the output of the $(k,l)^{th}$ sensor with narrowband, co-channel signal at time t_n as,

$$x_{k,l}(t_n) = \sum_{i=1}^{d} s_i(t_n) e^{j\tau_{kl}(i)} + n_{k,l}(t_n), \quad 1 \le t_n \le N \tag{1}$$

where $s_i(t_n)$ denotes the complex envelope of the i^{th} source signal, $n_{k,l}(t_n)$ is an additive complex Gaussian sensor noise and $\tau_{kl}(i)$ is the phase difference of the i^{th} signal collected at sensor (k,l) with respect to the i^{th} signal collected at reference sensor '$(0,0)$'. The phase difference is $\tau_{kl}(i) = [\omega_{xi}k + \phi_{xi}k^2 + \omega_{yi}l + \phi_{yi}l^2 + \beta_i kl]$ where $\omega_{xi} = -\frac{2\pi\Delta}{\lambda}\sin\theta_i\cos\varphi_i$, $\phi_{xi} = \frac{\pi\Delta^2}{\lambda r_i}(1-\sin^2\theta_i\cos^2\varphi_i)$, $\omega_{yi} = -\frac{2\pi\Delta}{\lambda}\sin\theta_i\sin\varphi_i$, $\phi_{yi} = \frac{\pi\Delta^2}{\lambda r_i}(1-\sin^2\theta_i\sin^2\varphi_i)$ and $\beta i = \frac{\pi\Delta^2}{\lambda r_i}\sin^2\theta_i\sin 2\varphi_i$.

For a collection of observed outputs of $K \times L$ sensors in $2-D$ array $\mathbf{x}(t_n) = [\mathbf{x}^T{}_{L_{min}}(t_n), \cdots, \mathbf{x}^T{}_{L_{max}}(t_n)]^T$, the model (1) is written more compactly in matrix notation as

$$\mathbf{x}(t_n) = \mathbf{A}(\boldsymbol{\theta},\boldsymbol{\varphi},\mathbf{r})\mathbf{s}(t_n) + \mathbf{n}(t_n), \quad 1 \le t_n \le N \tag{2}$$

where the super vector $\mathbf{x}(t_n)$ consists of $\mathbf{x}_l(t_n) = [x_{K_{min},l}(t_n) \cdots x_{K_{max},l}(t_n)]^T$ which is output of only one column sub-array of the 2-D rectangular array, $\mathbf{s}(t_n) = [s_1(t_n)\cdots s_d(t_n)]^T$ is the collection of d source signals impinging to $2-D$ array, $\mathbf{n}(t_n) = [\mathbf{n}^T_{L_{min}}(t_n)\cdots\mathbf{n}^T_{L_{max}}(t_n)]^T$ is super Gaussian complex vector with zero-mean and known spatial covariance $\sigma^2\mathbf{I}$, which consists of sub-array noise vectors one forming as $\mathbf{n}_l(t_n) = [n_{K_{min},l}(t_n)\cdots n_{K_{max},l}(t_n)]^T$, $\mathbf{A}(\boldsymbol{\theta},\boldsymbol{\varphi},\mathbf{r}) = [\mathcal{A}_1(\theta,\varphi,r)\cdots\mathcal{A}_d(\theta,\varphi,r)]$ is the arrays steering matrix in the near-field scenario which is known as a function of unknown set of parameters $\boldsymbol{\theta} = [\theta_1\cdots\theta_d]^T$, $\boldsymbol{\varphi} = [\varphi_1\cdots\varphi_d]^T$, $\mathbf{r} = [r_1\cdots r_d]^T$, consisting of sub-array steering vectors one forming as $\mathcal{A}_i(\theta,\varphi,r) = [\mathbf{a}^T_{L_{min}}(\theta,\varphi,r)\cdots\ \mathbf{a}^T_{L_{max}}(\theta,\varphi,r)]^T$ and $\mathbf{a}_l(\theta,\varphi,r)$ is l^{th} sub-array steering vector for i^{th} source, in the $\mathbf{a}_{l,i}(\theta,\varphi,r) = [e^{j\tau_{K_{min}l}(i)},\cdots,1,e^{j\tau_{1l}(i)},e^{j\tau_{2l}(i)},\cdots, e^{j\tau_{K_{max}l}(i)}]^T$ form.

We are interested in UML approach for the estimation of 3-D near-field source location parameters $\{\boldsymbol{\theta},\boldsymbol{\varphi},\mathbf{r}\} = \{(\theta_1,\varphi_1,r_1),\cdots,(\theta_d,\varphi_d,r_d)\}$ from N observations $\boldsymbol{x} = [\mathbf{x}^T(1),\cdots,\mathbf{x}^T(N)]^T$ made from (2). The data for this problem consists of a set of discrete samples $\{\mathbf{x}(k); 1 \le k \le N\}$ of the process $\mathbf{x}(t_n)$. Our approach is to derive an iterative UML estimator based on the EM algorithm, that performs joint sample covariance and location parameters estimation in alternating steps.

3. UML Estimator

In this section we derive the UML estimator for the problem defined above. To describe stochastic ML estimator's derivation, we made the following assumptions on the signal model (1):

AS1: The source signal $\mathbf{s}(k)$ is temporally and spatially uncorrelated circular complex Gaussian random process with zero-mean and nonsingular unknown covariance matrix $\mathbf{K_s}$,

$$\begin{aligned} E\left[\mathbf{s}(k_1)\mathbf{s}^H(k_2)\right] &= \mathbf{K_s}\delta_{k_1,k_2} \\ E\left[\mathbf{s}(k_1)\mathbf{s}^T(k_2)\right] &= \mathbf{0} \quad \text{for all } k_1 \text{ and } k_2\,. \end{aligned} \tag{3}$$

where δ_{k_1,k_2} is the Kronecker delta ($\delta_{k_1,k_2} = 1$ if $k_1 = k_2$ and 0 otherwise), $(\cdot)^H$ is the conjugate transpose and $(\cdot)^T$ is the transpose of a matrix.

AS2: The additive noise vector $\mathbf{n}(k)$ is temporally and spatially uncorrelated circular complex Gaussian process with zero-mean and standard derivative σ^2 as

$$E\left[\mathbf{n}(k_1)\mathbf{n}^H(k_2)\right] = \sigma^2\mathbf{I}\delta_{k_1,k_2} \tag{4}$$

$$E\left[\mathbf{n}(k_1)\mathbf{n}^T(k_2)\right] = \mathbf{0} \quad \text{for all } k_1 \text{ and } k_2\,. \tag{5}$$

AS3: The source signal $\mathbf{s}(k_1)$ and the noise $\mathbf{n}(k_2)$ are uncorrelated for all k_1 and k_2.

Based on the assumptions **AS2** and **AS3** the array observations $\mathbf{x}$ are Gaussian distributed with zero-mean and covariance $\mathbf{K_x}(\boldsymbol{\theta},\boldsymbol{\varphi},\mathbf{r},\mathbf{K_s})$, where $\mathbf{K_x}(\boldsymbol{\theta},\boldsymbol{\varphi},\mathbf{r},\mathbf{K_s}) = E[\mathbf{x}(k_1)\mathbf{x}^H(k_2)] = \mathbf{A}(\boldsymbol{\theta},\boldsymbol{\varphi},\mathbf{r})\mathbf{K_s}\mathbf{A}^H(\boldsymbol{\theta},\boldsymbol{\varphi},\mathbf{r}) + \sigma^2\mathbf{I}$. Then joint probability density function of the observation $\boldsymbol{x} = \{\mathbf{x}(k), \quad k = 1,\cdots,N\}$ given $\{\boldsymbol{\theta},\boldsymbol{\varphi},\mathbf{r},\mathbf{K_s}\}$ can be written as follows:

$$\begin{aligned} f(\boldsymbol{x};\boldsymbol{\theta},\boldsymbol{\varphi},\mathbf{r},\mathbf{K_s}) &= \prod_{k=1}^{N} 2\pi^{-KL/2}(\det\mathbf{K_x})^{-1/2} \\ &\times \exp\left(-\frac{1}{2}\mathbf{x}^H(k)\mathbf{K_x}^{-1}\mathbf{x}(k)\right) \end{aligned} \tag{6}$$

The joint probability function (6) can also be written as

$$f(\boldsymbol{x};\boldsymbol{\theta},\boldsymbol{\varphi},\mathbf{r},\mathbf{K_s}) = 2\pi^{-NKL/2}(\det \mathbf{K_x})^{-N/2} \times \exp\left(-\frac{1}{2}\mathrm{tr}\left[\mathbf{K_x}^{-1}\sum_{k=1}^{N}\mathbf{x}(k)\mathbf{x}^H(k)\right]\right). \quad (7)$$

and the negative log-likelihood function (after discarding unnecessary terms) is

$$\mathcal{L}(\boldsymbol{x};\boldsymbol{\theta},\boldsymbol{\varphi},\mathbf{r},\mathbf{K_s}) = -\ln\det(\mathbf{K_x}) - \frac{1}{N}\mathrm{tr}\left[\mathbf{K_x}^{-1}\sum_{k=1}^{N}\mathbf{x}(k)\mathbf{x}^H(k)\right]. \quad (8)$$

AS2 implies that, by the law of large numbers $\mathbf{x}(k)$ is second-order ergodic, i.e.,

$$\mathbf{K_x} = \lim_{N\to\infty}\widehat{\mathbf{K}}_\mathbf{x} = \lim_{N\to\infty}\frac{1}{N}\sum_{k=1}^{N}\mathbf{x}(k)\mathbf{x}^H(k) \quad (9)$$

where $\widehat{\mathbf{K}}_\mathbf{x}$ is the sample covariance matrix. Then the negative log-likelihood function becomes

$$\mathcal{L}(\boldsymbol{x};\boldsymbol{\theta},\boldsymbol{\varphi},\mathbf{r},\mathbf{K_s}) = -\ln\det(\mathbf{K_x}) - \mathrm{tr}\left[\mathbf{K_x}^{-1}\widehat{\mathbf{K}}_\mathbf{x}\right]. \quad (10)$$

The ML estimates of $\{\hat{\boldsymbol{\theta}},\hat{\boldsymbol{\varphi}},\hat{\mathbf{r}}\}$ and $\hat{\boldsymbol{s}}$ are those which locally minimizes the negative log-likelihood function (8). However, minimizing (8) is a difficult nonlinear constraint optimization problem, and does not yield to a closed-form solution. Thus, a computationally efficient iterative algorithm is required for solving resulting optimization problem. To solve this problem, we propose an UML estimation technique based on the EM algorithm which decomposes the observed data into its signal components and then estimates the parameters of each signal components separately. The EM algorithm iterates as the parameter updates in a manner which guarantees an increase in the likelihood function. The EM algorithm requires the definition of the complete data and its associated log-likelihood function. The choice for the complete data vector is obtained from hypothetical independent observations of each incident wave as

$$\mathbf{y}_i(k) = \mathcal{A}_i(\theta,\varphi,r)s_i(k) + \mathbf{n}_i(k), \quad 1 \le i \le d \quad (11)$$

where $\mathbf{n}_i(k)$ is the Gaussian noise vector belongs to i^{th} signal. Motivation behind this choice is that, if one could somehow observe each of the incident waves separately, the estimation of its near-field parameters would be straightforward by performing d parallel maximizations. The incomplete data is the set of observations themselves.

Under **AS1**, the covariance matrix $\mathbf{K_s}$ is a diagonal matrix $\mathbf{K_s} = \mathrm{diag}[\alpha_1,\cdots,\alpha_d]$, then the complete data $\mathbf{y}_i(k)$ is the Gaussian process with mean zero and covariance

$$\mathbf{K}_{\mathbf{y}_i}(\theta_i,\varphi_i,r_i,\alpha_i) = \alpha_i\mathcal{A}_i(\theta,\varphi,r)\mathcal{A}_i^H(\theta,\varphi,r) + \frac{\sigma^2}{d}\mathbf{I}. \quad (12)$$

Then the log-likelihood function of the complete data $\mathbf{y}_i(k)$ is

$$\mathcal{L}_c(\mathbf{y}_i;\boldsymbol{\theta},\boldsymbol{\varphi},\mathbf{r},\mathbf{K_s}) = -\ln\det\mathbf{K}_{\mathbf{y}_i} - \frac{1}{N}\mathrm{tr}\left[\mathbf{K}_{\mathbf{y}_i}^{-1}\sum_{k=1}^{N}\mathbf{y}_i(k)\mathbf{y}_i^H(k)\right]. \quad (13)$$

At the $(p+1)^{th}$ iteration, two step EM algorithm for our problem has the following steps:

Expectation Step: Compute conditional expectation of the sufficient statistics for the complete data log-likelihood. The sufficient statistics is the sample covariance of the complete-data,

$$\widehat{\mathbf{K}}_{\mathbf{y}_i} = \frac{1}{N}\sum_{k=1}^{N}\mathbf{y}_i(k)\mathbf{y}_i^H(k). \quad (14)$$

At the $(p+1)^{th}$ iteration, expected value of $\widehat{\mathbf{K}}_{\mathbf{y}_i}^{p+1}$ given $\mathbf{K}_\mathbf{x}^p$ and $\mathbf{K}_{\mathbf{y}_i}^p$ is

$$\begin{aligned}\widehat{\mathbf{K}}_{\mathbf{y}_i}^{p+1} &= E\{\widehat{\mathbf{K}}_{\mathbf{y}_i} \mid \mathbf{K}_{\mathbf{y}_i}^p, \mathbf{K}_\mathbf{x}^p, \widehat{\mathbf{K}}_\mathbf{x}\} \\ &= \mathbf{K}_{\mathbf{y}_i}^p(\mathbf{K}_\mathbf{x}^p)^{-1}\widehat{\mathbf{K}}_\mathbf{x}(\mathbf{K}_\mathbf{x}^p)^{-1}\mathbf{K}_{\mathbf{y}_i}^p \\ &\quad + \mathbf{K}_{\mathbf{y}_i}^p - \mathbf{K}_{\mathbf{y}_i}^p(\mathbf{K}_\mathbf{x}^p)^{-1}\mathbf{K}_{\mathbf{y}_i}^p. \end{aligned} \quad (15)$$

In (15), $\mathbf{K}_{\mathbf{y}_i}^p$ and $\mathbf{K}_\mathbf{x}^p$ can be obtained from the estimates of near-field parameters $\{\boldsymbol{\theta}^p,\boldsymbol{\varphi}^p,\mathbf{r}^p\}$ at iteration p,

$$\mathbf{K}_\mathbf{x}^p = \mathbf{A}(\boldsymbol{\theta}^p,\boldsymbol{\varphi}^p,\mathbf{r}^p)\mathbf{K}_\mathbf{s}^p\mathbf{A}^H(\boldsymbol{\theta}^p,\boldsymbol{\varphi}^p,\mathbf{r}^p) + \sigma^2\mathbf{I}$$

$$\mathbf{K}_{\mathbf{y}_i}^p = \alpha_i^p\mathcal{A}_i(\theta^p,\varphi^p,r^p)\mathcal{A}_i^H(\theta^p,\varphi^p,r^p) + \frac{\sigma^2}{d}\mathbf{I} \quad (16)$$

Maximization Step: The conditional expectation of the sufficient statistics obtained in Expectation Step is substituted in (13). Then the complete-data likelihood function is maximized with respect to the parameters to be estimated $\{\theta_i,\varphi_i,r_i,\mathbf{K_y}\}$:

$$\{\theta_i^{p+1},\varphi_i^{p+1},r_i^{p+1},\mathbf{K}_{\mathbf{y}_i}^{p+1}\} = \arg\max_{\{\theta,\varphi,r,\alpha\}}\left\{-\ln\det\mathbf{K}_{\mathbf{y}_i} - \mathrm{tr}\left[\widehat{\mathbf{K}}_{\mathbf{y}_i}^p\mathbf{K}_{\mathbf{y}_i}^{-1}\right]\right\} \quad (17)$$

The determinant of $\mathbf{K}_{\mathbf{y}_i}$ can be obtained by using the spectral decomposition. One eigenvector of $\mathbf{K}_{\mathbf{y}_i}$ is $\mathcal{A}_i(\theta,\varphi,r)/|\mathcal{A}_i(\theta,\varphi,r)|$, $KxL-1$ other mutually orthogonal eigenvector can be chosen from orthogonal complement of $\mathcal{A}_i(\theta,\varphi,r)$ and are equal to σ^2/d. Then the eigenvalue corresponding to the distinct eigenvector is $\alpha_i\,|\mathcal{A}_i(\theta,\varphi,r)|^2 + \sigma^2/d$.

Then the determinant of $\mathbf{K}_{\mathbf{y}_i}$ can be written as

$$\det\mathbf{K}_{\mathbf{y}_i} = \left(\alpha_i\,|\mathcal{A}_i(\theta,\varphi,r)|^2 + \frac{\sigma^2}{d}\right)\left(\frac{\sigma^2}{d}\right)^{KL-1}. \quad (18)$$

Since the inverse of $\mathbf{K}_{\mathbf{y}_i}$ is required in (17), it could be determined by employing matrix inverse lemma as

$$\mathbf{K}_{\mathbf{y}_i}^{-1} = \frac{d}{\sigma^2}\mathbf{I} - \frac{\mathcal{A}_i(\theta,\varphi,r)\mathcal{A}_i{}^H(\theta,\varphi,r)}{|\mathcal{A}_i(\theta,\varphi,r)|^2} \times \left(\frac{d}{\sigma^2} - \frac{1}{\frac{\sigma^2}{d} + \alpha_i\,|\mathcal{A}_i(\theta,\varphi,r)|^2}\right). \quad (19)$$

If we substitute the eigenvalues and the inverse of $\mathbf{K}_{\mathbf{y}_i}$ into (17), and maximizing (17) for $\alpha_i > 0$, the estimates of near-field parameters become

$$\{\theta_i^{p+1},\varphi_i^{p+1},r_i^{p+1}\} = \arg\max_{\{\theta_i,\varphi_i,r_i\}} \frac{\mathcal{A}_i^H(\theta,\varphi,r)\widehat{\mathbf{K}}_{\mathbf{y}_i}^{p+1}\mathcal{A}_i(\theta,\varphi,r)}{|\mathcal{A}_i(\theta,\varphi,r)|^2} \quad (20)$$

where $\alpha_i^{p+1} > 0$,

$$\alpha_i^{p+1} = \frac{1}{|\mathcal{A}_i(\theta^{p+1},\varphi^{p+1},r^{p+1})|^2} \times \left(\frac{\mathcal{A}_i^H(\theta^{p+1},\varphi^{p+1},r^{p+1})\widehat{\mathbf{K}}_{\mathbf{y}_i}^{p+1}\mathcal{A}_i(\theta^{p+1},\varphi^{p+1},r^{p+1})}{|\mathcal{A}_i(\theta^{p+1},\varphi^{p+1},r^{p+1})|^2} - \frac{\sigma^2}{d}\right). \quad (21)$$

Based on this results, the steps of the proposed UML algorithm are summarized as follows:

Repeat steps **1-3** for $i = 1,\cdots,d$

1. Given $\{\theta_i^0,\varphi_i^0,r_i^0\alpha_i^0\}, p = 0,$
2. $p = p+1,$
   - Obtain $\widehat{\mathbf{K}}_{\mathbf{y}_i}^{p+1}$ from (15),
   - Substitute $\widehat{\mathbf{K}}_{\mathbf{y}_i}^{p+1}$ in (20), and then solve (20) for $\{\boldsymbol{\theta}^{p+1}, \boldsymbol{\varphi}^{p+1}, \mathbf{r}^{p+1}\}$,
   - Substitute the estimates $\{\boldsymbol{\theta}^{p+1}, \boldsymbol{\varphi}^{p+1}, \mathbf{r}^{p+1}\}$ in (21), then compute α_i^{p+1},
3. Continue this process until $\{\theta_i,\varphi_i,r_i\}$ and α_i converges.

4. Simulation

To illustrate the the effectiveness and applicability of the proposed method, we consider the following scenario. A Uniform rectangular linear array of $K = L = 3$ totaly 9 sensors with inter-element spacing $\Delta = \frac{\lambda}{2}$ was used to estimate the locations of two sources located at $\{\theta_1,\varphi_1,r_1\} = \{24^0, 80^0, 2\lambda\}$ and $\{\theta_2,\varphi_2,r_2\} = \{34^0, 5^0, 1.6\lambda\}$. The number of the snapshots (N) set to 80 and the SNR was varied from 0 to $30dB$. The proposed method was tested for $M = 100$ independent trials. The resulting $RMSE$ of the estimated DOAs (azimuths and elevations) in degrees and ranges in wavelengths are shown in Fig.1., Fig.2. and Fig.3. respectively. The results were compared with the Cramer-Rao Bounds.

5. Conclusions

Based on the simulation results we made the following observations:

-For a sufficiently good initialization, proposed algorithm converges rapidly to the ML estimate of $\{\hat{\boldsymbol{\theta}}, \hat{\boldsymbol{\varphi}}, \hat{\mathbf{r}}\}$ and $\widehat{\mathbf{K}}_{\mathbf{s}}$. Since the spatial structure of the array matrix is known, then the good initial estimates of the steering matrix can be obtained from MUSIC and ESPRIT algorithms.

-For high SNRs the $RMSE$s obtained from simulations becomes almost identical to the CRB results derived by modifying the results in [8].

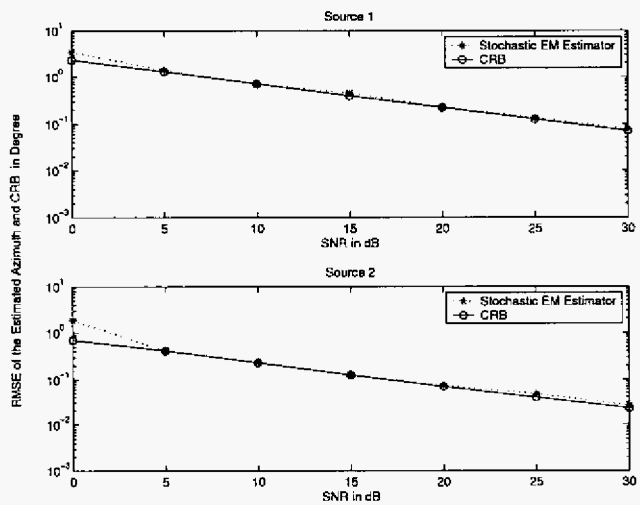

Fig. 1. RMSE of the estimated azimuths

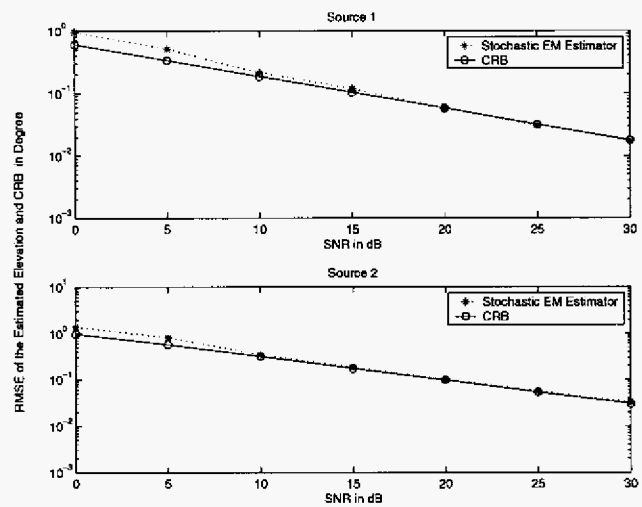

Fig. 2. RMSE of the estimated elevations

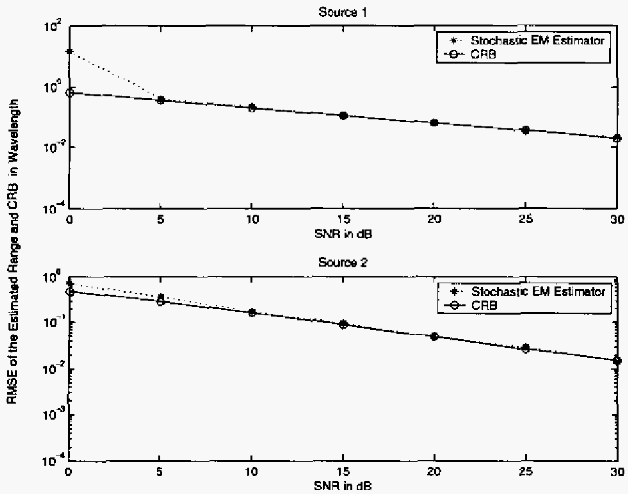

Fig. 3. RMSE of the estimated ranges

References

[1] Van der Veen, A. J., Ober, P. B. and Deprettere, E. D., "Azimuth and Elevation Computation in High Resolution DOA Estimation", IEEE Transaction on Signal Processing, vol.40, July, pp.1828-1832, 1992.

[2] Swindlehurst, A. L. and Kailath, T., "Azimuth/Elevation Direction Finding Using Regular Array Geometries", IEEE Transaction on Aerospace and Electronic Systems, 29, January, pp.145-156, 1993.

[3] Zoltowski, M. D., Haardt M. and Mathews C. P., "Closed-form 2-D Angle Estimation with Rectangular Arrays in Element Space or Beamspace via Unitary ESPRIT", IEEE Transaction on Signal Processing, February, vol.44, pp.316-328, 1996.

[4] M. Haardt, "Efficient One-, Two, and Multidimensional High-Resolution Array Signal Processing", Ph.D. dissertation, Technische Universität München, 1997.

[5] Hung H., Change S. and Wu C., "3-D Music with Polynomial Rooting for Near-Field Source Localization", International Conference on Acoustic, Speech, and Signal Processing (ICASSP-96), Atlanta, Georgia, vol.6, May, pp.3065-3069, 1996.

[6] Challa, R. N. and Shamsunder, S., "Passive Near-Field Localization of Multiple Non-Gaussian Sources in 3-D using Cumulants", Signal Processing 65, pp.39-53, 1998.

[7] Kabaoglu, N., Cirpan, H., Cekli, E. and Paker, S., Deterministic Maximum Likelihood Approach for 3-D Near-Field Source Localization, AEU, International Journal of Electronics and Communication, 57(5) 345-350, 2003.

[8] B. Hochwald, A. Nehorai, "Concentrated Cramer Rao Bound Expression", *IEEE Trans. on Information Theory*, vol.40, pp.363-371 March 1994.