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NONUNIFORM SAMPLING FOR DETECTION OF ABRUPT CHANGES*

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Abstract. In this work, detection of abrupt changes in continuous-time linear stochastic systems and selection of the sampling interval to improve the detection performance are considered. Cost functions are proposed to optimize both uniform and nonuniform sampling intervals for the well-known cumulative sum algorithm. Some iterative techniques are presented to make online optimization computationally feasible. It is shown that considerable improvement in the detection performance can be obtained by using nonuniform sampling intervals.
Key words: Change detection, cumulative sum test, nonuniform sampling.

1. Introduction

In most change detection and isolation applications, a statistical test forms the basic part of the detection mechanism. This simply arises because the data used for detection purposes is corrupted in noise or other disturbances, which can be modeled by statistical tools. An exhaustive treatment of the statistical change detection algorithms has been presented in [4]. Two basic goals are generally to be sought by any reasonable change detection technique: to detect the change as quickly as possible, and to avoid false alarms as much as possible. Because these two criteria are typically in conflict with each other, the choice of a threshold or

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other parameters for a statistical test is based on optimizing one, while keeping the other at a tolerable level.

One of the likelihood-ratio-based methods in this context is the well-known cumulative sum (CUSUM) test [4], [6], which can be used to detect a change from one known operating mode to another one. The CUSUM test is a discrete-time algorithm. In applications where continuous-time systems or signals are involved, the detection of a change should be based on data that is generated by sampling. Obviously, a suitably chosen sampling strategy is expected to increase the test performance. For independently distributed data, fast sampling may seem advantageous from the detection-delay point of view, simply because it reduces the duration between each data sample. However, this will also shorten the mean time between false alarms, which is undesirable. However, in detecting changes in the dynamics of signals or systems, the sampling interval will certainly affect the correlation structure of the discrete-time data. In other words, statistical properties of the sampled data will depend on the sampling strategy applied in collecting this data. From this latter point of view, the selection of a suitable sampling rate can be seen as a *hypothesis generation* problem. In this case, the relationships between the sampling rate and the detection performance and the trade-off between the detection delay and the false alarm rate are more complex.

Although there are some works on the selection of various design parameters such as auxiliary signals [6], [8], [10] and sensor locations [3] to improve the detection performance of statistical tests, reports on sampling rate selection in change detection are very sparse. Çağdaş and Kerestecioğlu [5] discussed offline selection of an optimal sampling rate and solved the resulting constrained optimization problem by numerical methods. Nevertheless, they presented strong simulation evidence in favor of their hypothesis that a wisely selected sampling rate can considerably improve the detection performance.

The main goal of this work is to improve the performance of statistical tests by using a non-uniform sampling strategy. By using the data available online, one can obtain sampling intervals that yield better performance as compared to the fixed-sampling-rate case. The problem is analyzed for the simple two-hypotheses case, where the statistical hypotheses before and after the change (denoted as $\mathcal{H}_0$ and $\mathcal{H}_1$, respectively) are known. In the next section, the CUSUM test, its performance characteristics, and its application to the detection of abrupt changes in state-space models are described. Then, selection of sampling intervals for change detection is investigated in Section 3. Our objective is to incorporate both performance characteristics in a single cost function, which can be used for an online selection of the sampling interval. We also present some iterative techniques for online minimization of this cost function. Finally, some simulation results are presented in Section 4 for both uniform and nonuniform sampling strategies. Section 5 includes some conclusions and comments on possible further extensions of the ideas presented in this article.

2. Cumulative sum test

Let us consider the following mth-order single-output discrete-time time-varying model:

$$\begin{aligned} \mathbf{x}(k+1) &= \mathbf{A}_i(k)\,\mathbf{x}(k) + \mathbf{v}_i(k) \\ y(k) &= \mathbf{C}_i(k)\,\mathbf{x}(k). \end{aligned} \tag{1}$$

In (1), $\mathbf{A}_i(k)$ and $\mathbf{C}_i(k)$ denote the $m \times m$ system and $1 \times m$ output matrices, respectively, under the hypothesis $\mathcal{H}_i$ $(i = 0, 1)$. On the other hand, $\mathbf{v}_i(k)$ is an $m \times 1$ white Gaussian noise vector with zero mean and covariance matrix $\mathbf{Q}_i(k)$ under either hypothesis. Note that the model in (1) could have been extended to include any control inputs or known disturbances as well. Further, the dimension of the state vector could have been assumed to be different under either hypothesis. Nevertheless, for clarity of the analysis in the sequel, we shall restrict ourselves to (1), without losing any generality. We are interested in detecting a change from the $\mathcal{H}_0$ mode to $\mathcal{H}_1$, which might occur at an unknown instant, by using the CUSUM test.

The CUSUM test is conducted by computing the statistics

$$S(k) = \max[0, S(k-1) + z(k)],$$

starting with $S(k) = 0$ [4], [6]. The cumulative sum $S(k)$ is compared to a predetermined positive threshold β at each sampling instant, and if $S(k)$ exceeds β, the test is terminated and a change is declared. In other words, the alarm time is given as

$$n = \inf\{k \ni S(k) \geq \beta\}.$$

The increments of the test statistics are given as [6]

$$z(k) = \ln \frac{f_1(y(k) \mid y(k-1), \ldots, y(1))}{f_0(y(k) \mid y(k-1), \ldots, y(1))}, \tag{2}$$

where f_i denotes the conditional density of the observations, $y(k)$, under $\mathcal{H}_i$ $(i = 0, 1)$. It follows from (2) and the Gaussianity of $\mathbf{v}_i(k)$ that [10]

$$z(k) = \ln \frac{\sigma_0(k)}{\sigma_1(k)} + \frac{1}{2\sigma_0^2(k)} e_0^2(k) - \frac{1}{2\sigma_1^2(k)} e_1^2(k). \tag{3}$$

Here, $e_i(k)$ is the error in the one-step-ahead output prediction obtained by the model corresponding to the ith hypothesis. That is,

$$\begin{aligned} e_i(k) &= y(k) - \hat{y}_i(k \mid k-1) \\ &= y(k) - \mathbf{C}_i(k)\hat{\mathbf{x}}_i(k \mid k-1) \end{aligned}$$

where $\sigma_i^2(k)$ is its variance and $\hat{\mathbf{x}}_i(k \mid k-1)$ is the best linear predicted state estimate based on the data available up to time $k-1$. Note that $\hat{\mathbf{x}}_i(k \mid k-1)$ and $\sigma_i^2(k)$ can be obtained using a Kalman filter based on the $\mathcal{H}_i$ hypothesis as

$$\begin{aligned} \hat{\mathbf{x}}_i(k+1 \mid k) &= \mathbf{A}_i(k)\hat{\mathbf{x}}_i(k \mid k) \\ \hat{\mathbf{x}}_i(k+1 \mid k+1) &= \hat{\mathbf{x}}_i(k+1 \mid k) + \mathbf{K}_i(k+1)e_i(k+1), \end{aligned} \tag{4}$$

where $\mathbf{K}_i(k)$ is the Kalman gain matrix and generated by recursive computations

$$
\begin{aligned}
\mathbf{P}_i(k+1 \mid k) &= \mathbf{A}_i(k)\mathbf{P}_i(k \mid k)\mathbf{A}_i{}^{\mathrm{T}}(k) + \mathbf{Q}_i(k+1) \\
\sigma_i^2(k+1) &= \mathbf{C}_i(k+1)\mathbf{P}_i(k+1 \mid k)\mathbf{C}_i{}^{\mathrm{T}}(k+1) \\
\mathbf{K}_i(k+1) &= \frac{\mathbf{P}_i(k+1 \mid k)\mathbf{C}_i{}^{\mathrm{T}}(k+1)}{\sigma_i^2(k+1)} \\
\mathbf{P}_i(k+1 \mid k+1) &= \mathbf{P}_i(k+1 \mid k) - \mathbf{K}_i(k+1)\mathbf{C}_i(k+1)\mathbf{P}_i(k+1 \mid k).
\end{aligned}
\tag{5}
$$

Also note that, in (5), $\mathbf{P}_i(k+1 \mid k)$ and $\mathbf{P}_i(k \mid k)$ denote the covariance matrices of the predicted and filtered state estimates, respectively.

The performance of the CUSUM test has been examined extensively in [4], [6]. Two important characteristics of performance, namely the average detection delay (ADD) and the mean time between false alarms (MTBFA), can be obtained by the average run length of the test under the hypotheses $\mathcal{H}_1$ and $\mathcal{H}_0$, respectively. An exact calculation of the ADD as well as the MTBFA of the test is difficult in most cases. Therefore, generally, approximations for ADD and MTBFA are used. An approximate formula for the ADD is given [9], [6] as

$$
E\{n \mid \mathcal{H}_1\} \approx \frac{\beta - 1 + e^{-\beta}}{E\{z(k) \mid \mathcal{H}_1\}}. \tag{6}
$$

On the other hand, the MTBFA can be approximated [9], [6] by

$$
E\{n \mid \mathcal{H}_0\} \approx \frac{\beta + 1 - e^{\beta}}{E\{z(k) \mid \mathcal{H}_0\}}. \tag{7}
$$

Note that both of these approximations are originally established for a case in which the observations are identically and independently distributed. Nevertheless, it can also be shown [6] that these approximations are also valid for changes in the dynamics of the data generating mechanisms, which can be modeled as autoregressive moving average processes. Hence, the selection of the sampling interval described in the next section will be based on these approximations.

3. Sampling interval selection

3.1. The cost function

In many practical cases, the discrete-time data used by the CUSUM test might be obtained by sampling continuous-time signals. The dynamics of the sampled system and, hence, the hypotheses and the test performance will depend on the sampling interval. Moreover, if a nonuniform sampling strategy is employed, the corresponding discrete-time description of the observations will be given by a time-varying model. Let us consider a continuous-time autoregressive moving average model of a stationary stochastic system with white noise input. That is,

$$\begin{aligned}&\frac{d^m}{dt^m}y(t) + a_1\frac{d^{m-1}}{dt^{m-1}}y(t) + \cdots + a_m y(t)\\&\quad = \frac{d^m}{dt^m}w(t) + c_1\frac{d^{m-1}}{dt^{m-1}}w(t) + \cdots + c_{m-1}w(t)\end{aligned} \tag{8}$$

where $w(t)$ is Wiener process with incremental variance s^2. Further, let an mth-order state-space realization of (8) be

$$\begin{aligned}d\mathbf{x}(t) &= \mathbf{A}_c\,\mathbf{x}(t) + \mathbf{B}dw(t)\\ y(t) &= \mathbf{C}\,\mathbf{x}(t).\end{aligned} \tag{9}$$

After sampling the output, $y(t)$, of the continuous-time system in (9), the sampled output, $y(k)$, can be described so as to be generated by the discrete-time system

$$\begin{aligned}\mathbf{x}(k+1) &= \mathbf{A}(k)\,\mathbf{x}(k) + \mathbf{v}(k)\\ y(k) &= \mathbf{C}\,\mathbf{x}(k),\end{aligned} \tag{10}$$

where

$$\mathbf{A}(k) = \exp\{\mathbf{A}_c T(k)\}.$$

Here, $T(k)$ is the sampling period used to obtain the data sample $y(k)$. Also, the white noise $\mathbf{v}(k)$ is an $m \times 1$ vector with zero mean. Its covariance matrix $\mathbf{Q}(k)$ can be obtained by solving the Lyapunov equation [5]

$$\mathbf{A}_c\mathbf{Q}(k) + \mathbf{Q}(k)\mathbf{A}_c{}^{\mathrm{T}} = \exp(\mathbf{A}_c T(k))\mathbf{B}s^2\mathbf{B}^{\mathrm{T}}\exp(\mathbf{A}_c{}^{\mathrm{T}}T(k)) - \mathbf{B}s^2\mathbf{B}^{\mathrm{T}}. \tag{11}$$

For a fixed sampling rate, i.e. $T(k) = T$, it follows from (6) and (7) that the ADD and MTBFA can be approximately expressed as

$$E\{t_n \mid \mathcal{H}_1\} \approx \frac{T(\beta - 1 + e^{-\beta})}{E\{z(k) \mid \mathcal{H}_1\}} \tag{12}$$

and

$$E\{t_n \mid \mathcal{H}_0\} \approx \frac{T(\beta + 1 - e^{\beta})}{E\{z(k) \mid \mathcal{H}_0\}}, \tag{13}$$

respectively, where t_n is the test duration in continuous time. In selecting a suitable sampling interval, one has to take into consideration that the detection delay should be shortened but not the MTBFA. A cost function that will be minimized can be proposed as

$$E\{t_n \mid \mathcal{H}_1\} - \bar{g}E\{t_n \mid \mathcal{H}_0\},$$

where $\bar{g}$ is a positive constant expressing a relative weight between the detection delay and false alarm rate from the designer's point of view. On the other hand,

$$\frac{1}{E\{t_n \mid \mathcal{H}_1\}} - \frac{1}{\bar{g}E\{t_n \mid \mathcal{H}_0\}} \tag{14}$$

can be maximized toward the same goal. In view of (12) and (13), it follows that the maximization of (14) with respect to the sampling interval as well as any other design variable is equivalent to maximizing

$$\frac{E\{z(k) \mid \mathcal{H}_1\} + gE\{z(k) \mid \mathcal{H}_0\}}{T}$$

or minimizing

$$J_u(T) = \frac{T}{E\{z(k) \mid \mathcal{H}_1\} + gE\{z(k) \mid \mathcal{H}_0\}}, \tag{15}$$

where

$$g = -\frac{\beta - 1 + e^{-\beta}}{\bar{g}(\beta + 1 - e^{\beta})},$$

which depends only on $\bar{g}$ and β.

Note that the cost function $J_u(T)$ in (15) does not make use of any information available online and, hence, is suitable for obtaining a fixed sampling period, by an offline minimization. In choosing a nonuniform sampling interval online, one can also make use of the data collected up to then. For this case, we propose a cost function of the same form as in (15), save that the expectations are replaced by conditional ones, namely,

$$J_n(k, T(k)) = \frac{T(k)}{E\{z(k) \mid k-1, \mathcal{H}_1\} + gE\{z(k) \mid k-1, \mathcal{H}_0\}}. \tag{16}$$

Here, $E\{\cdot \mid k-1, \mathcal{H}_i\}$ denotes the conditional expectation given the data up to and including the sampling instant $k-1$ and given that the system is operating under $\mathcal{H}_i$.

A strategy employing the cost in (16) benefits from the extra information available online which would be absent in the minimization of (15). Therefore, the test performance is expected to be improved, even compared to that under the sampling strategy which aims to minimize (15). Also, by using the cost function in (16), both ADD and MTBFA objectives are taken into consideration so that a trade-off can be reached between them. On the other side, in this strategy, the next sampling instant is determined by using the expected value of the increments $z(k)$. However, note that considering $z(k)$ at each step is, in fact, a myopic policy and may not always mean an improvement in the whole trajectory of the test statistics. Beyond all these considerations, it should also be kept in mind that the formulas in (6) and (7) are just approximations.

For the nonuniform sampling case, to compute the cost function given in (16), conditional expectations of the increments have to be obtained. Under $\mathcal{H}_0$, an error of the output prediction based on $\mathcal{H}_1$ is given as

$$e_1^2(k) = y(k) - \hat{y}_1(k \mid k-1), \tag{17}$$

and $e_0(k)$ is a zero-mean white noise with the variance

$$E\{e_0^2(k)\} = \sigma_0^2. \tag{18}$$

By substituting (17) and (18) into (3) and taking expectations, we get

$$E\{z(k) \mid k-1, \mathcal{H}_0\} = \frac{1}{2}\left[\ln\frac{\sigma_0^2}{\sigma_1^2} - \frac{1}{\sigma_1^2}E\left\{[y(k) - \hat{y}_1(k \mid k-1)]^2\right\} + 1\right]. \quad (19)$$

Because

$$y(k) = \hat{y}_0(k \mid k-1) + e_0(k),$$

whenever the system is operating under $\mathcal{H}_0$, (19) can be written as

$$E\{z(k) \mid k-1, \mathcal{H}_0\} = \frac{1}{2}\left[\ln\frac{\sigma_0^2}{\sigma_1^2} - \frac{\sigma_0^2}{\sigma_1^2} + 1 - \frac{1}{\sigma_1^2}[\hat{y}_0(k \mid k-1) - \hat{y}_1(k \mid k-1)]^2\right]. \quad (20)$$

Similarly, the expected value of the increments $z(k)$ under $\mathcal{H}_1$ turns out to be

$$E\{z(k) \mid k-1, \mathcal{H}_1\} = \frac{1}{2}\left[\ln\frac{\sigma_0^2}{\sigma_1^2} + \frac{\sigma_1^2}{\sigma_0^2} - 1 + \frac{1}{\sigma_0^2}[\hat{y}_0(k \mid k-1) - \hat{y}_1(k \mid k-1)]^2\right]. \quad (21)$$

The expectations given in (20) and (21) are used in (16) to evaluate the cost function to obtain the next sampling instant. The estimates $\hat{y}_0(k \mid k-1)$ and $\hat{y}_1(k \mid k-1)$ in these equations are obtained by using the Kalman filter structure described in (4),(5).

3.2. *Iterative techniques*

At each step of the CUSUM test, the cost function given in (16) is minimized over the sampling interval that is going to be used in the next step of the test. To adjust the sampling interval, different techniques can be used. A straightforward way is a direct search [1] in a given interval determined by possible constraints that may be imposed due to the hardware or software requirements of the case at hand. However, direct search techniques can present difficulties from the point of view of the computational burden involved, if the search algorithm is to be executed for every sampling interval.

To overcome such problems, suboptimal approaches can be used in which the optimization is done in an iterative manner. A gradient approach to adjust the sampling period iteratively can be obtained by the well-known MIT rule [2], which is originally proposed for updating controller parameters in adaptive controllers. The rationale is changing the sampling period in the direction of the negative gradient of the cost function with respect to the sampling interval. By using a discrete-time counterpart of the MIT rule, an iterative adjustment rule for the sampling interval can be written as

Author: Does MIT need to be defined, or is it just MIT?

$$T(k+1) = T(k) - \gamma\frac{\partial J_n(k, T(k))}{\partial T(k)}, \quad (22)$$

where γ is a positive constant determining the adaptation rate. However, it is not possible to obtain $\partial J_n(k, T(k))/\partial T(k)$ in (22) from (16) in a closed form. Thus, one can use an estimate of it obtained from the recently used sampling intervals. That is,

$$\frac{\partial J_n(k, T(k))}{\partial T(k)} \approx \frac{J_n(k, T(k)) - J_n(k, T(k-1))}{T(k) - T(k-1)}. \tag{23}$$

In (23), $J_n(k, T(k-1))$ is the value of the cost function at step k, obtained by using the sampling period at $k-1$. To further simplify the evaluation of $J_n(k, T(k-1))$, we can use the approximation

$$J_n(k, T(k-1)) \approx J_n(k-1, T(k-1)). \tag{24}$$

Note that $J_n(k-1, T(k-1))$ is the value of J_n attained by the sampling interval used to obtain the previous sample. In this way, J_n has to be evaluated only once for every sampling interval.

Hence, using (24), the adjustment rule of the sampling period can be written as

$$T(k+1) = T(k) - \gamma \frac{J_n(k, T(k)) - J_n(k-1, T(k-1))}{T(k) - T(k-1)}. \tag{25}$$

An even simpler implementation can be obtained if the sampling period is changed by a fixed amount at each step. This technique is known as the sign-sign rule [2] and can be formalized in our case as

$$T(k+1) = T(k) - \lambda\, \text{sign}[T(k) - T(k-1)]\, \text{sign}[J_n(k, T(k)) - J_n(k-1, T(k-1))], \tag{26}$$

where λ is a positive constant determining the change in the sampling period at each step.

4. Simulation example

In this section, a simulation example that compares the techniques that we have discussed is presented. Monte Carlo simulations have been used to estimate the ADD and MTBFA by taking the mean values obtained from 200 runs for each case. Four methods are applied to detect the same change in a continuous-time system. The first one is the uniform sampling strategy, in which the sampling period is determined by an offline minimization of (15) and kept fixed throughout the simulation. For nonuniform sampling, three different sampling interval adjustment mechanisms are used: the direct search technique, the MIT rule, and the sign-sign rule. To estimate the ADD, the data is generated according to the $\mathcal{H}_0$ hypothesis up to $t = 40$ s, which is the instant when the change occurs. It has been assumed that the sampling interval is constrained in an interval between 0.1 and 10 s.

Table 1. Detection performances of different sampling strategies

	β	ADD (s)	MTBFA (s)
Uniform sampling ($g = 1$)	0.93	27.2	159
Direct search ($g = 1$)	4.10	27.9	1447
MIT rule ($\gamma = 10^{-5}$)	3.00	29.0	592
Sign-sign rule ($\lambda = 0.2$)	2.20	27.7	174

We consider the following two operation modes for the system:

$$\mathcal{H}_0: \frac{d^2}{dt^2}y(t) + \frac{d}{dt}y(t) + 2y(t) = 2\frac{d}{dt}w(t)$$

$$\mathcal{H}_1: \frac{d^2}{dt^2}y(t) + \frac{d}{dt}y(t) + 3y(t) = 3\frac{d}{dt}w(t),$$

and the incremental variance of $w(t)$ is taken as unity.

To make a fair comparison between different methods, different thresholds are used for each case to obtain approximately the same ADD for all cases. Therefore, various sampling techniques can be compared on the basis of their false alarm performances.

The proposed methods are presented in Table 1. The parameters of the cost functions for each case (g, γ, λ) have been determined by extensive simulations so as to yield the best possible performance. It is seen in Table 1 that all three nonuniform sampling strategies result in a better false alarm behavior compared to the uniform-sampling case. Obviously, the test performance is considerably improved by using the online data available in the selection of the current sampling interval. The direct search method gives the best performance. However, keep in mind that such a direct search at each sampling instant may not be computationally feasible. On the other hand, the worst performance among the iterative methods is exhibited by the sign-sign method, which is the simplest one. Nevertheless, even this technique yields a better performance than that obtained using a fixed sampling rate.

5. Conclusions

Our aim in this paper has been to discuss the effect of the sampling interval on the performance of discrete-time statistical change detection methods. To achieve this goal, different schemes for selecting uniform and nonuniform sampling intervals are proposed. The conflicting aims of reducing both the detection delay and the false alarm rate are combined in choosing a suitable cost function. Simulation studies suggest that an improvement can be obtained in the test performance by using nonuniform sampling techniques, which make use of online available data, rather than fixed sampling periods. It is also shown that, for the small-change cases, simplified iterative adjustment methods (MIT rule, sign-sign rule) are inferior to the direct search methods; nevertheless, they can still result in better performances than that obtained with the fixed sampling rate.

Obviously, the choice of a suitable sampling strategy should be made by considering not only the improvement achieved in the test performance, but also the computational complexity that can be tolerated by the detection problem at hand. Also, the weighting or step-size coefficients in the relevant cost functions should be seen as design parameters, rather than a priori requirements. Hence, extensive simulations may be necessary to determine them.

In this work, we have considered a relatively simple change detection problem; namely, detecting a change from a known hypothesis to another known one. Nevertheless, it is also possible to extend the ideas presented here to other types of detection problems. For example, in [7] it is suggested that, for extensions of the CUSUM test to cases where the hypothesis after the change is completely or partially unknown, cost functions based on the Kullback information or Fisher information matrix can be employed for design of optimal inputs for change detection. Similar cost functions can also be applied to the sampling interval selection for such extended CUSUM tests.

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