KADİR HAS UNIVERSITY
GRADUATE SCHOOL OF SCIENCE AND ENGINEERING
PROGRAM OF INDUSTRIAL ENGINEERING

MULTI-OBJECTIVE DISASTER RELIEF LOGISTICS

MAHDI M. SAMARAH

MASTER'S THESIS

ISTANBUL, AUGUST, 2018

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DECLARATION OF RESEARCH ETHICS /

METHODS OF DISSEMINATION

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This work entitled **MULTI-OBJECTIVE DISASTER RELIEF LOGISTICS** prepared by **MAHDI M. SAMARAH** has been judged to be successful at the defense exam held on **15/08/2018** and accepted by our jury as **MASTER'S THESIS**.

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DATE OF APPROVAL: 15/08/2018

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ABSTRACT

Disaster relief logistics is one of the major fields of operations research. Deciding the locations of depots before the disaster by minimizing total costs and total distances between nodes of demands and these depots is the main purpose of this study. The efficiency of disaster relief logistics is expressed in terms of the total transportation cost. The other objective function is considering minimizing total accumulated time to represent efficacy to supply different number of pallets which include basic materials and necessary types of foods. Equity is represented by minimizing the percentage of unsatisfied demand achieved by balancing the capability to serve demand nodes and the ability to diminish number of pallets that would not reach the nodes. Dealing with uncertainty in both demands and distances create different scenarios for our study, and the results explain how each objective function affects the logistics decisions for each scenario.

Keywords: Disaster Management, Humanitarian Relief Logistics, Location Selection, Multi-Objective Programming, Efficacy, Equity, Demand and Distance Uncertainty.

ÇOK AMAÇLI AFET YARDIMI LOJİSTİĞİ

ÖZET

Afet yardım lojistiği, yöneylem araştırmasının başlıca alanlarından biridir. Bu çalışmanın temel amacı, toplam maliyetlerin en aza indirilmesi ve talep noktaları ile depolar arasındaki toplam mesafelerin en aza indirilmesi yoluyla, felaket öncesinde depoların bulunduğu yerlere karar verilmesidir. Afet yardım lojistik planının verimliliği, toplam ulaşım maliyeti şeklinde ifade edilmiştir. Diğer amaç fonksiyonu, çözümün verimliliğini temsil etmek için, temel malzeme ve gerekli gıda türlerini içeren farklı sayıda paletleri tedarik etmek için gereken toplam sürenin en aza indirilmesidir. Eşitlik, talep düğümlerine hizmet verebilme kapasitesinin dengelenmesi ve düğümlere ulaşmayacak palet sayısının azaltılması ile elde edilen tatminsiz talebin yüzdesinin en aza indirilmesiyle temsil edilmektedir. Hem talepler hem de mesafelerdeki belirsizliği ifade etmek için farklı senaryolar oluşturulmuş ve sonuçlara göre her bir hedefin her bir senaryo için lojistik kararları nasıl etkilediği açıklanmıştır.

Anahtar Sözcükler: Afet Yönetimi, İnsani Yardım Lojistiği, Lokasyon Seçimi, Çok Amaçlı Programlama, Verimlilik, Eşitlik, Talep ve Mesafe Belirsizliği.

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Last but not the least, there is no word can express my feelings toward my mother, my father and my family who offered everything for me to reach this level.

To my family

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LIST OF ABBREVIATIONS

WFP	World Food Program
NGO	Non-Governmental Organization
DOM	Disaster of Operation Management
DSS	Decision Support System
IFRC	International Federation of Red Cross
OR/MS	Operation Research and Management Science
MS	Management Science
MC	Management Consulting
ME	Management Engineering
CA	Cluster Approach
VRP	Vehicle Routing Problem
SDVRP	Split Delivery Vehicle Routing Problem
LMDP	Last Mile Delivery Problem
FDP	Final Destination Problem
EVPI	Expected Value of Perfect Information
VSS	Value of Stochastic Model
LMRD	Last Mile Relief Distribution
LDC	Local Distribution Center
POD	Point Distribution Center
GIS	Geological Information System
OCHA	Office for the Collaboration of Humanitarian Affairs
SMIP	Stochastic Mixed Integer Programming
SCAP	Single Commodity Allocation Problem
RCC	Rescue Command Center
RRA	Relief Resource Allocation
HL	Humanitarian Logistics

1. INTRODUCTION

One of the most important difficulties that humanity faces is the different types of disasters, including two major groups. The first one is natural disasters like floods, hurricanes, earthquakes, and cyclones, and the other group is caused by human beings, like wars, famines, and epidemics. Earthquakes are a common natural disasters group that cause huge damage and a high number of life losses. For example, Shanxi earthquake, the deadliest earthquake in history, stroke China in 1556 and killed 830,000 people (Tzeng et al., 2007), Tsunami attacked Indonesia in 2004 and killed more than 165,000 people.

Recently another earthquake stroke Haiti in 2010 which is assumed to be the worst earthquake encountered by United Nations (UN). The problem of facing the earthquakes is related to the uncertainty of the time and the area that the earthquake will strike. Even the advanced technology, computers and seismographs cannot help the scientists to determine where and when the earthquake will attack.

Table 1.1 shows the top 5 natural disasters within the period 1980-2010, and statistics of lives lost, the number of affected people, and damage in million dollars.

Table 1.1 Top five disasters within 1980-2010 (De La Torre et al., 2012)

Disaster Type	Number of disasters 1980-2010	Year	Country	Lost lives	Affected population	Economic damage (million $)
Flood	3120	2004	Haiti	2665	31283	--
		1999	Venezuela	30000	483185	160
		1998	China	3536	238973000	30000
		1996	China	2775	154634000	12600
		1980	China	6200	67000	160
Cyclone	2516	2008	Myanmar	138366	2420000	1780
		1999	India	9843	12628312	2500
		1998	Honduras	14600	2112000	3794
		1991	Bangladesh	138866	15438849	1780
		1985	Bangladesh	15000	1810000	50
Earthquake	786	2010	Haiti	222570	3700000	8000
		2008	China	87476	45976596	85000
		2005	Pakistan	73338	5128000	5200
		2004	Indonesia	165708	532898	4452
		1990	Iran	40000	710000	8000

Earthquake scientists expect a disastrous earthquake that could strike Istanbul within the coming years, the magnitude on Richter scale is expected to be between 7 and 7.4, depending on the seismic site. Figure 1.1 shows the north and east Anatolia fault and specifically the epicenter of the earthquake in the red line. The north Anatolia fault line is a highly active tectonic fault line. This site shows hard subterranean movements, these movements create interlocking between earth plates, and that is increasing the tension in the ground and releasing this tension leads to new earthquakes.

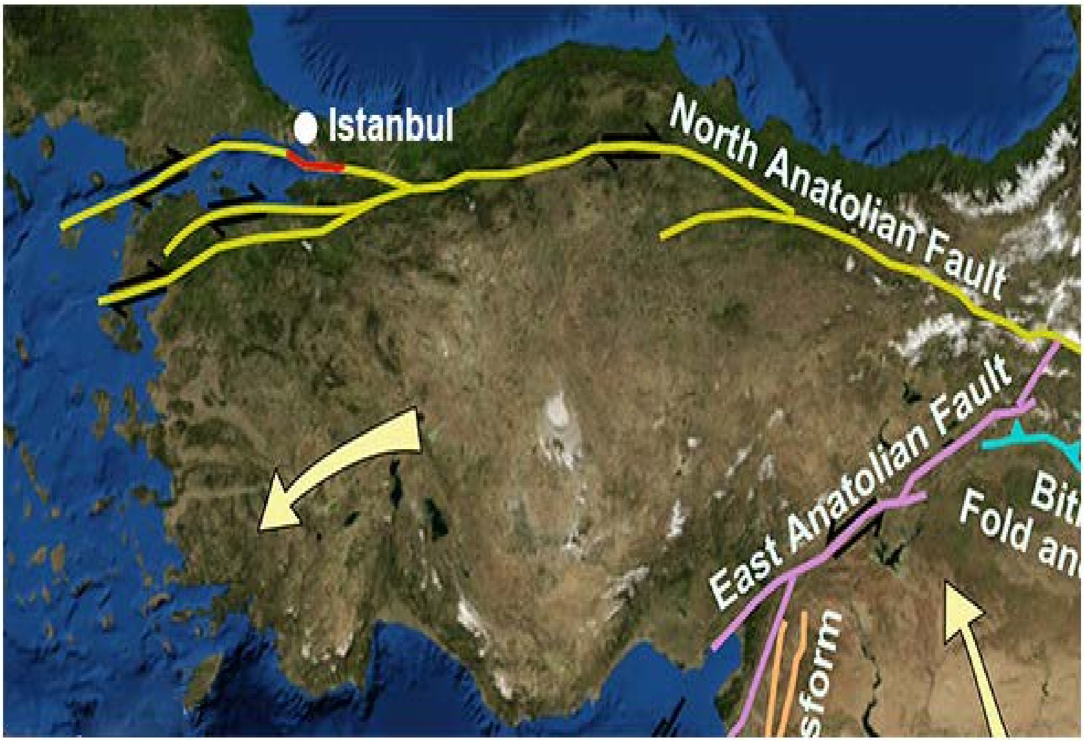

Figure 1.1 North and East Anatolia fault (Weston, 2017)

According to the scientists, the earthquake will occur in the eastern part of Marmara Sea, twenty kilometers south of Istanbul, where there has not been an earthquake since 1776. Figure 1.2 appoints the region from the Marmara Sea that will be affected by the earthquake and the plates that faced earthquake before. UN estimated the number of affected people to be between seventy thousand to ninety thousand and the economic losses to be around five hundred million dollar.

The best way to decrease the consequences of the earthquake by preparing the community for a sudden attack and investment of sufficient budget in multiple kinds of research, including operations research and management sciences (OR/MS) in disaster operation management (DOM) to increase the efficiency of actions needed during the disaster. This area of research deals with decisions regarding the numbers of depots, vehicles, and shelters, the capacity of vehicles, relief demand, transportation cost and mode, in addition to the cost of materials.

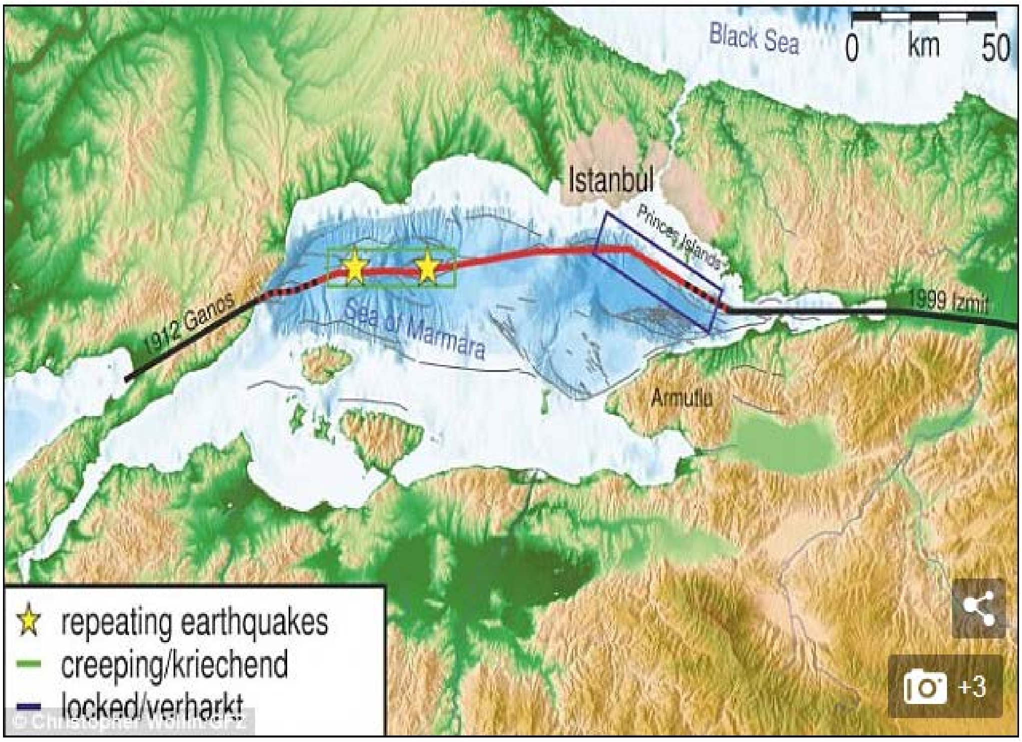

Figure 1.2 Marmara Sea and historical earthquake in the region (Weston, 2017)

DOM is a scientific approach to perform activities before, during, and after the disaster in order to diminish the losses in human life and economy and to revive the community to return to its normal situation.

DOM has four main stages, starting with the mitigation stage. At this stage, the performance of all activities within the period of the disaster are measured. Then, there is the preparedness stage and response stage, ending with recovery stage. Using operations research in disaster management as a method to incorporate the braces of OR/MS (MS, MC, ME) to increase the effectivity of stages (mitigation, preparedness, response, recovery).

In our research, we will consider uncertainty as an attempt to simulate the reality with the model applied within the work, since the deterministic case may result in infeasibilities if any condition changes unexpectedly.

The stochastic model is the most familiar model used to deal with disaster operation management and the reason behind that is related to the ability of a stochastic model to cope the uncertainty during the disaster.

In general, the stochastic model is established in two stages, the first one depends on preparedness which is the second stage of disaster operation management, this stage considering any type of outfitting the society and environment before the occurrence of the disaster to increase the efficiency of response actions during the disaster, the second stage of the stochastic model relying on the response and recovery stages (third and fourth stages in disaster operation management).

Response is defined as the actions applied during the disaster by the government and other non-governmental organizations (NGOs) to help affected population and to utilize the available resources in a suitable way, while the recovery is the short and long-term activities helped to revive the community and its functioning to the normal situation.

In this model, the preparedness stage focusing on the locations that would be chosen to build depots and storages, where the uncertainty will be beneficiaries demand which changed because of moving between shelters and depots, in a trial to find a bigger relief and disease epidemics.

The reaction stage considering the performance parameters efficiency, efficacy, and equality, the first parameter in terms of minimizing total cost, efficiency highlighted the quick and sufficient distribution, and equality comparing the variety of service stage through different sites.

The thesis is organized as follows: In Chapter 2, the related literature is reviewed. In Chapter 3, the mathematical model development is presented. In Chapter 4, the methodology used for solving multi-objective disaster relief problem is explained. In Chapter 5, computational results of the mathematical model based on a case study is presented and analyzed. Finally, in Chapter 6, concluding remarks and future research directions are provided.

2. REVIEW OF RELATED LITERATURE

The challenges that create the barriers and difficulties for disaster operation field were built by destabilized infrastructure, duration and capacity needed to distribute relief materials and uncertainty in demand, these challenges have been discussed by De La Torre et al. (2012). The article explained how United Nations (UN) established Sphere handbook; the standard of minimum humanitarian relief materials like daily calories of food (2100 calorie per day per person) and amount of water (2.5 liters per day per person). This handbook expresses the collaborative effort between a large number of local and international non-governmental organizations. The article also used special software to generate damage scenarios for infrastructure modeling the Federal Emergency Management Agency's (FEMA) HAZUS program, and defined the recovery stage as equality in delivery.

Another important article comprises multi-objective optimal planning for relief system is Tzeng et al. (2007), where the key to reducing human losses and damage is the distribution of relief materials. The model has three objectives starting with minimizing total cost and total travel time (efficiency goals), and increasing the minimal satisfaction as the last objective function for fairness target.

One of the popular and traditional locations problem is P-median location problem, P-median has several properties which makes this kind of problems capable and useable even in the present time. P-median is focusing on minimizing total distances between candidate nodes to choose best locations for different facilities, another important property that P-median has the ability to deal with un-capacitated storages, and the primary principle for P-median is trying to get nearest nodes to chosen locations as a final result of the model.

Advanced approach was derived from P-median is P-center, this approach uses minimax objective function in order to decrease the maximum amount of distances between located facilities and nodes of demand. The difference between P-median and P-center is minimizing average distances for P-median problem, while minimizing maximum distances for P-center.

In the model we used uncertainty in distances and demand as a basic concept, because of that for each scenario the located depots will change, and for demand we suggest upper and lower bounds, and P-median works with un-capacitated storages while we are limits our depots with six hundred pallets for each. The other properties of P-median has been used like the objective function for the first phase considers minimizing total distances between nodes of demand and depots.

We can say that we used P-median and its properties with new practical application includes stochastic approach, and P-center gave us the way to define and solve the fourth objective function in our model by using minimax methodology to get the percentages of unmet demand for each demand node.

Stochastic model has two different stages, preparedness stage and reaction stage, both of them considering uncertainty in multiple ways. While using OR/MS in DOM rises the possibility to incorporate the model with the reality. The different characteristics of assumptions will be available for the model, first type reasonable assumptions do not compromise the applicability of the study (finding space for improvement), second type is limited assumptions which specified to model inappropriate to others, non-realistic is the last type of assumptions, it doesn’t work with general settings but useful to diminish the complexity of the model.

By Table 2.1 many classifications was created to compare between different studies that have been made between 1978 and 2018, these classifications related to the type of the objective function like minimizing total distances, total costs, total unsatisfied demand and total waiting time. Another characteristic have been taken which is the stochasticity of demand and distances, in addition to use multi commodity, multi depot, heterogeneous vehicles, data of real disaster and review of the past works as additional criteria to discover more details about these articles. The arrangement of the table based on the arrangement of the articles through literature review. The flow of our literature review: 2.1 General reviews for different articles in OR/MS in DOM, 2.2 Review for relief routing articles, and 2.3 Review for articles related to DOM levels: 2.3.1 mitigation, 2.3.2 preparedness, 2.3.3 response, 2.3.4 recovery, and 2.4 Review different aspects of disaster management.

Table 2.1 Comparing different characteristics of disaster relief articles

	Features										
Articles	Min. distances	Min. cost	Min. unmet demand	Min. waiting time	Stochastic demand	Multi-commodity	Multi-depot	Heterogeneous vehicles	Stochastic distances	Data from real disaster	Literature review
Allahverdi et al. 2018				×	×		×	×	×		
Battara et al. 2018					×					×	
Besiou et al. 2018		×									
Carracsco et al. 2018				×	×		×	×	×		
Ferrer et al. 2018	×	×	×		×		×				
Franco et al. 2018	×									×	
Kirac and Milburn 2018		×								×	
Moshtagh et al. 2018		×		×							
Yucel et al. 2018										×	×
Bonmee et al. 2017	×	×				×	×	×	×		
Pradhananga et al. 2016		×	×	×		×	×			×	

Table 2.1 Comparing different characteristics of disaster relief articles (continued)

	Features										
Articles	Min. distances	Min. cost	Min. unmet demand	Min. waiting time	Stochastic demand	Multi-commodity	Multi-depot	Heterogeneous vehicles	Stochastic distances	Data from real disaster	Literature review
Das and Okumura 2015			×	×	×	×					
Huang et al. 2015	×	×	×	×	×	×		×			
Noyan et al. 2015		×			×		×		×		
Rodriguez-Espindola et al. 2015	×	×				×	×				
Zhan et al. 2014		×		×		×					
Balliue 2013										×	×
Davis et al. 2013				×				×		×	
Galdino and Batta 2013										×	×
Milburn and Rainwater 2013		×					×			×	
Ortunu et al. 2013	×									×	
Rekik 2013							×				
Anaya-Arenas et al. 2012										×	×
Roy et al. 2012	×	×		×	×	×	×	×		×	
De La Torre et al. 2012		×		×	×		×				

Table 2.1 Comparing different characteristics of disaster relief articles (continued)

Articles	Features										
	Min. distances	Min. cost	Min. unmet demand	Min. waiting time	Stochastic demand	Multi-commodity	Multi-depot	Heterogeneous vehicles	Stochastic distances	Data from real disaster	Literature review
Han et al. 2011		×		×	×						
Leiras at el. 2010		×	×	×	×		×				
Mete and Zabinsky 2010		×	×		×	×		×	×	×	
Rawls and Turnquist 2010		×	×		×	×			×	×	
Beamon and Balcik 2008		×			×		×				
Tzeng et al. 2007	×	×	×			×	×	×		×	
Yi and Ozdamar 2007			×			×	×	×		×	
Altay and Green 2006										×	×
Fiorucci et al. 2005					×					×	
Klose and Drexl 2005										×	×
Barbarosoğlu and Arda 2004		×			×	×				×	
Denzel et al. 2003										×	×
Rand 1976		×			×		×				
Our thesis	×	×	×	×	×		×		×		×

2.1 Review Articles on OR/MS in DOM

Galindo and Batta (2013) reviewed main studies for operation research and management science in disaster operation management between 2005-2010, and through 155 article 28.4% talked about preparedness stage, 33.5% focused on response stage, and 3.2% considered recovery stage, which means 36.7% for reaction stage in our stochastic model.

Altay and Green (2006) reviewed OR/MS in DOM, between 1980 and 2004, out of 109 articles, 21.1% related to preparedness stage, 23.9% for response stage, and 11% recovery. According to these reviews in Table 2.2, the proportion of articles and searches for the response and preparedness stages increased, because in these stages the successful of the models will be examined, if the decisions of preparedness stage has a high percentage of inaccurateness, the result will effect on the response stage then all stages of the model, at the end will lead to disastrous stochastic situation, on the other side the scientists concentrated on increasing the effectivity for these stages to catch higher possibility of success.

Table 2.2 Percent of research articles in 4 DOM stages (Galindo and Batta, 2013)

	Altay and Green	Galindo and Batta
No. of articles	109	155
Mitigation	48	37
Preparedness	23	44
Response	26	52
Recovery	12	5
Multi-stages	0	17

Other aspects of Altay and Green (2006) the most important review in the last decades, provide starting points for researches and scientists who interested in OR/MS in DOM field, highlighted the role of DOM stages within the disaster, providing different sectors for future work, surviving the OR/MS which is done until 2004, focusing on sociological and psychological impact on the community, and finally the review was not limited to specific kind of disaster management, it took all of the societies that related to international federation of red cross (IFRC). Through 109 article Figure 2.1 shows the percentages and numbers of articles relying on DOM stages and classification frameworks, management engineering (ME), management consulting (MC), management science (MS).

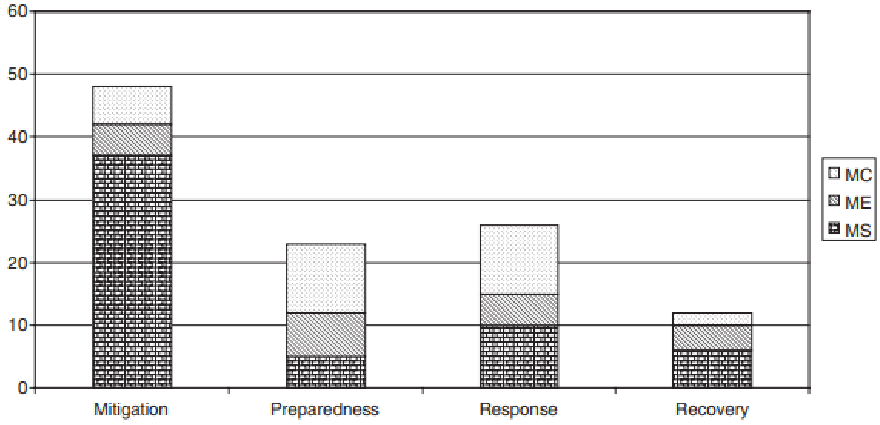

Figure 2.1 Distribution of DOM research type, ME, MS, MC. (Altay and Green, 2006)

Denizel et al. (2003) discussed the limitations of operation research and management science these days, it is suffering from the low number of researchers developing conjunction with humanitarian organizations, low technology magnifies the gap between what we have and the demand, questions with answers of how service allocation for affected people models will contain critical factors. Research percentages decreased with time for recovery stage, the design of the infrastructure and community business has low number of articles and searches, lack of researchers considering socio-economic situation for displaced and affected families.

Papers that highlighted response stage in disaster operation management with relief distribution network reviewed by Anaya-Arenas et al. (2012). The article examined different available studies in multiple criteria to specify the degree of advancement for actions needed within DOM and emphasize the most effective approaches and contributions in the literature, in addition to defining each stage of DOM (pre-disaster stages: mitigation and preparedness, post-disaster stages: response and recovery) and which articles presenting the suitable model and solutions for each stage.

The study that reviewed models with decision support system (DSS) is Ortunu et al. (2013). This paper declared human-made and natural hazard effects as a threatening event with a probability of a potential damage in specific region and specific period, the damage could be in health, different properties, infrastructure, human being, animal and plants life, the main objective for the article is incorporating humanitarian logistics models from different agencies into decision support system. This search try to define the fairness as the effective efforts to ensure and deliver each demand point with relief materials, the physical delivery consist three levels supply, transportation and demand, so the Ideal model driving for both effectiveness and fairness not for profit in business, which leads to find combinations of variables to decrease total travel time, size of vehicle fleet, fixed and variable costs.

2.2 Relief Routing Literature

An optimization stochastic model applied in Ethiopia-Africa by Leiras et al. (2010) for humanitarian supply to distribute of World food program (WFP). The model relies on two linear programming stages, the first one is the preparedness stage, and the second stage depends on recovery stage, uncertainty will appear through limited accessibility and warehouse capacities and will be calculated using two methodologies expected value of the perfect information (EVPI) and value of stochastic solution (VSS). Different kind of transportation affects distribution models, were characterized in primary transportation (PT) like air (high cost), railway (suffering from poor infrastructure and maintenance) and road (most used), secondary transportation is the transportation within two points from the same country.

One of the researches that focused on modeling relief routing with the principles of efficiency in terms of transportations cost, efficacy as fast and accurate response and equity represented by the measurement of fairness between deviated recipients is Huang et al. (2010). This article avoided ad-hoc distribution decisions that create problems in response time, amount of deliveries and usefulness of resources, the study explains how the Split delivery vehicle routing problem (SDVRP) model that split the demand through the fleet to minimize fleet cost and travel cost can incorporate within last-mile delivery problem (LMDP).

Figure 2.2 below shows multiple solutions (different roads) produced by using SDVRP in last mile delivery problem:

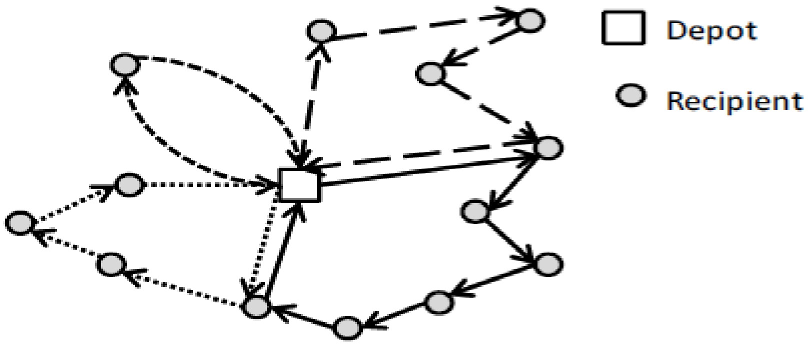

Figure 2.2 SDVRP solutions for LMPD (Huang et al., 2010)

The important role of OR within the last decades were studied by Besiou et al. (2018) included how OR can effect on different aspects of humanitarian logistics. In the same field Ferrer et al. (2018) produced a new model with multi-objective function to schedule many trips for relief vehicles where the performance has measured by security, and Pakistani flood 2010 were the case study for the model.

Ray and Albores (2012) develop logistical framework using the ability of last mile relief distribution (LMRD) then implementing the framework in optimization and humanitarian logistics model. Four main factors affects final decisions of the model are facility location (the location of most effectively inventory in relief network) , transportation mode(the most suitable mode of transportation with minimum travel cost and transportation cost), distribution decisions(fast and equity distribution between affected population) and inventory management(arrange and design incoming and left relief materials).

Allahverdi et al. (2018) have introduced a new algorithm with the name of 'AA', where the purpose from this algorithm to minimize total lateness of specific number of machines without wait flow-shop, and considering a maximum value of makespan as a limit for all machines. Another article has improved a new algorithm to build a specific schedule depending on total resource cost by Carrasco et al. (2018), the objective function of total costs and total completion time were minimized in order to test the addition heuristics of the new algorithm an to know if we can apply it on real instances or not.

Stochastic optimization model used for designing the last mile relief network, this study was applied by Noyan et al. (2015). The article established the model by determining capacities and locations for each relief distribution point through last mile network as a first step, the second one designing the distribution network, third step building stochastic model with two stages, focusing at the same time on equity and achieving high levels of accessibility. This model used Branch-and-cut algorithm based on Benders decomposition, the last step explaining the effectiveness of uncertainty method to build stochastic model demand and transportation stages.

Figure 2.3 explains how local distribution center (LDC) deliver relief materials to points of Distribution (POD) which send the relief materials for demand points, these points serve different beneficiaries (like villages and neighborhoods) in the affected region:

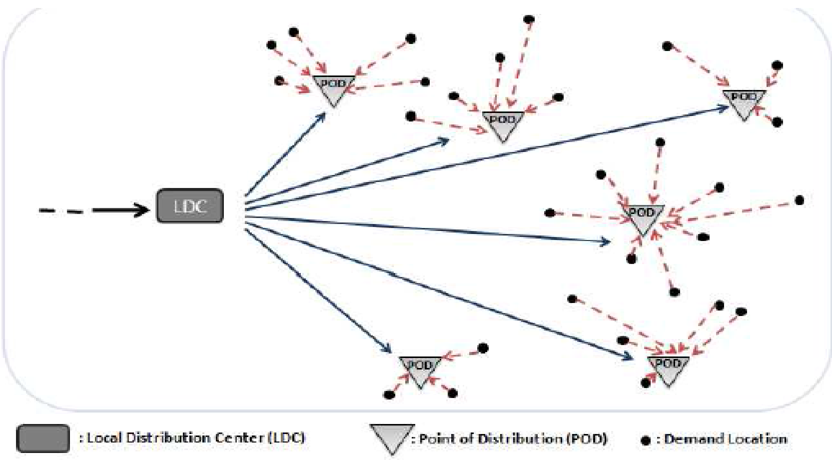

Figure 2.3 Last mile relief distribution (LMRD) (Noyan et al., 2015)

Das and Okumura (2015) studied perishable kind of relief materials and how it effects on both declining urgency and distribution of the demand, this study used optimization dynamic programming model to determine the amount and the time for ordering perishable relief materials against demand and urgency, perishability influences service level of the system, so decision makers should balance between the efficiency of the system and wastage resulted from perishability.

Figure 2.4 declares trend of relief materials (bread and clothes) and a number of requests for the earthquake that stroke Japan 2011. We noticed here, the natural declining in demand with time, Das and Okumura (2015) tried to reach usefulness stage of perishability before the demand decreases until zero demand.

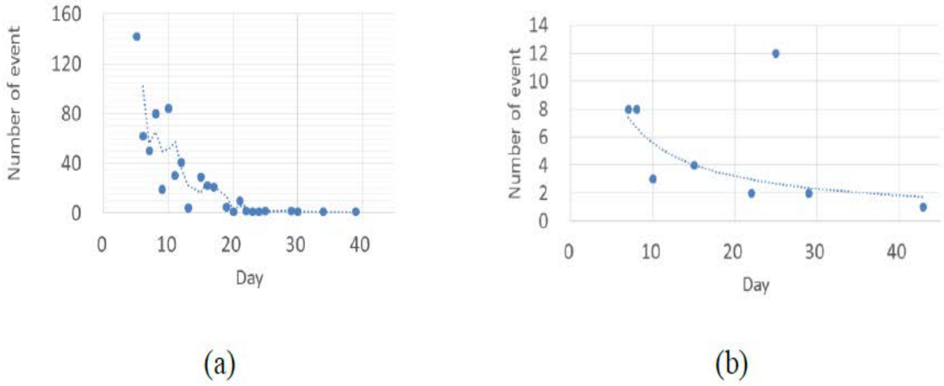

Figure 2.4 Number of requests from the shelter in Japan (a) Bread requests, (b) Clothes requests (Das and Okumura, 2015)

Another article that used relief resources allocation (RRA) principles to establish multi-objective function model in order to improve the relation between equity and efficiency to reach most suitable decisions of fleet routing and allocation of primary relief materials.

Zhan et al. (2014) tried to get most appropriate period before making critical decisions to response the consequences of the disaster. This article has defined RRA as an activities based on critical logistic orders which supplies different transportations with high security to deliver multiple relief materials (water, medications, different types of primary food and tents) from wide range resources (by sea, land and air) to act with the occurrence of any kind of disasters either natural or human-made, then the optimization model correlated OR with Bayesian analysis to get periodic results.

A systematic review has been applied to disaster relief routing by Anaya-Arenas et al. (2012). The review has targeted three aspects at the beginning searching for latest articles where the model dealing with supplying of disaster materials by improving and optimizing it's logistics, the second aspect is pursuing the number of challenges for these kinds of models, and the mathematical methods to solve it, the third aspect shows the fields and the scientific gaps into the current models in order to create new paths and to discover advanced approaches for logistics and routing challenges. This article has ensured that the articles which the dynamic models were built on uncertainty and solved by stochastic principles still have very small number because it is very hard models and sophisticated, on the other hand, keep using of deterministic models lead to low performances and inaccurate results.

2.3 Articles Related to DOM Stages

Two types of decisions related to disaster management, long-term decision and short-term decision, Rekik (2013), applied short-term decision type for deterministic problem to build optimization model helping decision makers in the early moments after the disaster, and determining number of warehouses in addition to planning for relief materials sent towards affected population. This study explains decision support system (DSS) that relying on expert's discussions in crisis management and resulted observations, then DSS should specify the most suitable approach form operation research field for each stage through the model.

United Nations (UN) worked on a paper for sharing information in humanitarian emergencies, this paper recommended building systems instead of depending on information flow of individual within the disaster, the system will establish by collaboration between international humanitarian organizations.

Its work starts from what happened through the disaster, what kind of actions had done to decrease the effect of the disaster, what kind of actions was necessary but none of the organizations applied it, what we can extract for better collaboration between governmental and non-governmental organization (office of the collaboration of humanitarian affairs OCHA), the scenario is chosen in the article was from the earthquake of Haiti 2010.

Gate (1, q) an algorithm was created by Franco et al. (2018), where the algorithm has tested a new approach to measure the distance between two nodes in tow separated areas, using straight line between the areas, than the algorithm compared with a famous algorithm to determine whether the performances have improved or not.

2.3.1 Mitigation Stage

Mitigation stage in DOM shares the principles of performance within different types of disaster and the necessity of modern approaches to improving actions performance, Beamon and Balcik (2008), framed new methodology to increase the efficiency of system performance in the relief aid field by comparing performance measurements in relief aid chains with performance measurements for commercial supply chains. New performance metrics will be produced to help relief professionals to achieve more targets in the models of mitigation stage and system performance. The paper also provide new ideas about how the performance of the system will get rid of negative and bottlenecks points if the collaboration between multiple chains reach maximum, limit and that will lead to saving more lives with decreasing amount of losses in different economy sectors.

Integrated methodology with two main columns, optimal appointing of resources and modelling systems are built to evaluate decision support system (DSS) in mitigation stage and providing tools to ease forecasting procedures to decision makers, this study was presented by Fiorucci et al. (2005). The article worked on formalizing and describing real time risk management in two phases, real time phase, and pre-operative phase. The first one refers to the horizon time intervals which belong to uncertainty transit time with dummy nodes (null demand), the other phase used expected daily information for the disaster as a base input data to allocate available resources. Three types of frameworks effect on second phase daily information, static hazard assessment represented by topographic and climate data, while dynamic hazard assessment based on model outputs and active hazard assessment related to semi-physical propagation model.
Figure 2.5 declared how daily information can be generated and the conditions that play an important role to determine data kinds:

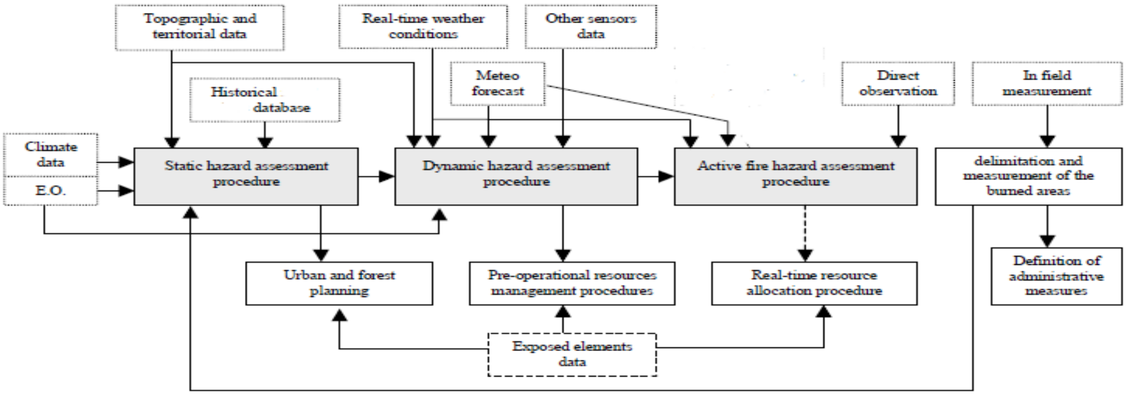

Figure 2.5 Representation of the various functions of the information flows (Fiorucci et al., 2005)

Different types of models focusing on choosing depots locations, these types of studies have been introduced by Klose and Drexl (2005). The article has explained eight kinds of models considering optimization functions to discuss most appropriate locations for storages in relief operations. First kind of the models worked on finding depots and its locations based on geological maps and available roads, second kind helps decision makers to minimize distances between affected people and relief storages, and third kind deals with incapacitated storages to capture any size of demand under different scenarios. fourth type looking after supplying and logistics system for only a single stage model, fifth one has assumed all of the capacity, cost and demand for relief materials as a single aggregated unit, the sixth kind performing a special assumption under fixed and independent demand, seventh kind improves performance metrics within a single period, the last kind isolates every pair of depots and collection points to rising up the quality of relief materials allocation.

Bonmee et al. (2017) have studied how decision makers can appoint different locations for relief storages relying on multiple kinds of problems with various types of models, at the same time concentrating on some factors according to location characteristics which play the main role in determining where will be collection points, medical centers, relief storages, and other important facilities. This paper also defined humanitarian logistics (HL) as an operational activities established to evacuate effected people from the affected region to secure shelters, and performing specific actions to control the capacity of depots and calculates total costs for performed actions, while accurate information about the disaster, stricken region, affected people and accessible ways have been gathered by expert team, (HL) assumed distances as a constant (actual distances between pair of points) not as a function (Euclidean, square Euclidean and rectilinear). By HL definition many types of location problems can be solved such as minisum problem, where there is an ability to use as much as needed of depots and storages to minimize total distances between these depots and demand nodes, another type of problems is covering problem, in this problem the time is limited and demand should be satisfied within this time, Third type of problem is maximal problem, the target is maximizing total demand with limited number of kilometers that can be covered, minimax problem focusing on developing performance metrics for the whole supply chain, while dynamic problem searching for compromising between total costs and times spent, last type is robust problem where uncertainty factors probabilities determine the shape of the model with specific uniform scenarios.

2.3.2 Preparedness Stage

Preparedness stage in disaster operation management taking higher importance within the last two decades, Rodriguez-Espindola et al. (2015) create a combination between multi-objective optimization which considers relief materials allocation and prepositioning of stocks. The other side of combination is geographical information system (GIS) to deal with appointing emergency facilities locations (demand points, distribution centers, and shelters), achieving preparedness depending on the coordination between different non-governmental organizations, governmental organizations, and international agencies.

One of the most important relief materials is dedication materials, medical supply needs unique conditions to avoid any kind of spoilage, Mete and Zabinsky (2010) suggests stochastic optimization model in preparedness and response stage to assign warehouses points with inventory available capacity for each type of medications and drugs. The paper tried to balance between various percentages risks and uncertainty of events, also gives procedures to reach best ways to deliver medications, taking into account particular medical supplies as a priority for the hospitals. The model based on two stages, the first stage to determine warehouses areas with its stages, second one focusing on the amount supplied to hospitals, the amount for each scenario equals aggregated stage which changes to number of vehicles, its capacity and the suitable routing, the different earthquake scenarios applied to Seattle state in the United States of America.

Yucel et al. (2018) have discussed the methodology of strengthening the roads before the disaster, for the first step they built a random model (randomness in roads), then they have calculated each road probability by Bayesian, after that they have worked on strengthen the weak links using stochastic model programming, so as a result the weak road's probabilities has been diminished.

Social cost was the objective function of a three-echelon model introduced by Pradhananga et al. (2016) to build a critical planes for preparedness stage and response stage. Social cost was calculated using both of logistics cost and deprivation cost, where these costs increase exponentially with deprivation time. This study also shows how propositioning and purchasing decisions with multiple resources can reduce the shortage in the amount of relief materials and decrease the total cost. The network has a set of supply points (a large facilities near from the disaster) at the highest echelon to serve a large region which stroke by a disaster, assuming the type of shipping in this model is direct and flexible from these points to the victims and the affected areas.

Prepositioning of various supplies has a probability to rise the efficiency of preparedness stage, the results reflects on the actions of response stage after the disaster, finding the preposition stages for different kinds of equipment's and emergency relief materials was the target of Rawls and Turnquist (2010) using two stages mixed integer program (SMIP), uncertainty in the model appears through demand and transportation availability, the study solved the model by decomposing the problem into series of sub-problems to make it easier to calculate, then finding total expected total cost as a first stochastic level and specifying suitable prepositioning level for the second stochastic level.

Using earthquake sciences and variety approaches for forecasting was made by Battara et al. (2018) to specify the amount of needs before the disaster and they have used in the article different scenarios of demands reflect with uncertainty, this study was suggested to use upon preparedness stage to judge the level of prepositioning level of the relief materials.

Discussing different characteristics and criteria's to determine most suitable choices of relief storages location were performed by Rand (1976), the study suggested seven main aspects that should be satisfied in order to take right decisions for depots and storages locations. The first aspect is questioning about which is more important between minimizing total costs or applying performance metrics principles, second aspect wondering if the number of locations is limited or not, third one about the type of methodology which used in the model is optimizing or in heuristic way, the fourth point considering the periods is it in months, years or decades, in single period or not, fifth aspect is wondering if the current sites are included in the model, sixth aspect about limit capacity storages or incapacitated relief storages, seventh aspect comprise total supplying costs if it will be calculated related to fleet scheduling or information analysis, finally this paper suggest an algorithm to build a model by:

1) Determine the number of storages and depots
2) Allocate them randomly to suggest regions
3) Serve effected people by these regions
4) By minimizing total distances choose best regions
5) Do not continue if there are no better regions
6) Otherwise go back to step 3

2.3.3 Response Stage

The evaluation of first two stages of disaster operation management, mitigation, and preparedness stages, occurred in response stage, Milburn and Rainwater (2013), presented how we can build disaster response model, starting with determining distribution points after the disaster occurrence, ending with the advancement of the infrastructure, and preparing actions to face the consequences of the disaster, the scenario of the disaster in this paper had taken from New Madrid Seismic Zone (NMSZ) to test the results of the response model.

With eighty percent of total disaster relief operations, logistics considered as the most important stage for relief aid, Balliue (2013) characterized logistics as on-going process and explained its role in saving lives within and after the disaster, also the importance of logistics in emergency plans for response stage and how collaboration between supply chains provide the most effective possible aid for shelters, demand and distribution points, and finally affected population.

Two stages model for logistical relief aid to effected region in the response stage was established by Barbarosoğlu and Arda (2004), they explained the flow of materials over modern types of transportation through multi-objective model network, uncertainty in the model represented by variety of supply and capacities amounts which resulted from affected infrastructure. To solve uncertainty and variety in the linear programming model they used different scenarios for demand, supply flow, and capacities, and dealing with two stages needs a model with multi-commodity and network flow, then first stochastic stage is a structural component with free uncertainty and fixed variables, while control components the second stage is affected by uncertainty in input data, the article studied Avcilar region in Istanbul with different earthquakes scenario to find the most effective stochastic model.

Multi-objective function model was developed by Huang et al. (2015), in this model three objective functions have been integrated and improved under two important terms emergency distribution and resource allocation. The paper suggests multiple time horizons to consider latest updates of information and to modify the wrong logistics decisions. Lifesaving utility is the first objective function which has the highest importance between other objective functions, delay cost was assumed as the second objective function and it was offered to show how postponing relief materials maximize the suffering of the victims, and to show the importance of time urgency and how to deal with it, last objective function is fairness, it was defined by the degree of the equality and the priority through the affected region.

Highly important operations applied through disaster response stage, evacuation and logistical support, Yi and Ozdamar (2007) built a model with integrated distribution-location, the target of the model is reaching accurate results for logistical plans, depots, and distribution locations, establishing emergency and shelters regions within affected zone and relief materials delivery, like medications, food, water, tents, and rescue equipment's, then to obtain vehicles numbers, capacities, and routs. The article will consider mixed-integer network flow multi-commodity model, finally the study characterizes relief materials and affected population into hierarchy priorities, that means the vehicles will serve higher priorities locations, this approach achieving minimum delay in supplying time.

2.3.4 Recovery Stage

Recovery stage did not take the same importance of other stages, related to low number of articles and studied performed at this stage, but one of the recent papers explained recovery stage in stochastic model which is Van Hentenryck et al. (2010). The paper focused on Single Commodity Allocation Problem (SCAP) as the main stage for stochastic optimization model to obtain series of depots and fleet routing, also to minimize inefficiency in resource allocation and supplying time. The scenario generated by Los Alamos Library for expected hurricane that will strike USA, and uncertainty in this paper is the outcome of hurricane effected people lives and infrastructure

2.4 Articles Reviewed for Different Aspects of Disaster Management

Volunteer management one of the aspects in humanitarian relief has not a high concentration, this aspect could improve the actions in response level but it is depending on the way that the organizations and agencies will manage the volunteers and the field experiences. Falasca and Zobel (2012) examined a model with multi-criteria to find a systematic approach to assign volunteers and different tasks in addition to labor scheduling this assignment related to volunteers experience and field skills for each of them. This approach will help decision makers to reach the best choices in the early period after occurrence of disaster consequences.

An interesting paper shows the effect of the social media on response stage was discussed by Kirac and Milburn (2018). The writers have viewed how awareness level will be changed significantly after using social media through the disaster, and the way that affected people can use mobile app to finally determine total needs into the affected area, and providing valuable information’s about the disaster.

Health sector were studied in term of performance metrics, many of these studies explained different methods to improve delivery services. Davis et al. (2013) built a model depending on equity, efficiency, and effectiveness to rank 35 general service hospitals in New Zealand between 2001 and 2009. The study applied for more than 500 admissions per year, where the total number of admissions in that period around 4 million admission, each performance metric relied on two measurements. For equity as a first metric both of level of ethnic and the variation through socio-economic for the hospitals considered as variables to calculate equity, the correlation between pooled data variation and inter hospital variation for first metric is 0.41 which is moderate correlation. The second performance the efficiency measured by patient stay and number of surgeries for each day, the correlation of variances for this metric is 0.2 that means low correlation. Effectiveness as a last metric related to morality of period before disaster and unplanned readmission, for the last metric the correlation is low and equals 0.2.

To highlight the importance of queue problems, Moshtagh et al. (2018) have been published to illustrate the different situations of the roads through the disaster and which one of them has a huge effect on the evacuation operations, and developed a model with travel time and total costs as an objective functions.

Our model has common aspects shared with relevant articles, but the distinguishable points in our research are:

- Uncertainty in demand and supply.
- Simultaneous optimization of performance parameters.
- Multi-objective function model, the first objective function is determining depots locations, while other objective functions related to performance parameters (efficiency, efficacy, and equity).

The closest parameters for our research are Huang et al. (2010) who worked on optimizing performance parameters, and Klose and Drexl (2005), they built a model to obtain best possible locations for depots and storages.

3. MATHEMATICAL MODEL DEVELOPMENT

Disaster operations management has four stages, each stage considers a specific time (before or after the disaster) that will effect on the final results of any model. To incorporate the science with the reality in terms of the existence of inaccurate data and uncertainty in details, the stochastic model of two stages can create the perfect environment to solve the uncertainty problem. Two stage stochastic model are applied with different cases through different scenarios, starting with the first stage that deal with preparedness (first stage of DOM), where the target is selecting the locations of different depots, in the model it was represented as a minimization function for total distance to demand nodes and it takes (Z_1) symbol, the decisions of this stage are applied before the disaster.

Response (second stage of DOM) represented by performance metrics as a second stage of the stochastic model, in this stage the decisions are made after the disaster. Performance metrics starts with efficiency (Z_2), it is focusing on minimizing total transportation cost by minimizing both amount of pallets delivered and the distance between depot and demand node. Second performance metric is the efficacy, this metric can be evaluated by using total accumulated waiting time (Z_3). Last performance metric related to equity the one with many approaches to be calculated, and it does not have a direct way to solve like other performance metrics but in our study unmet demand and it's percentage is the key to find the value of equity metric as (Z_4) function, the following Figure 3.1 shows how each stage connect to different objective functions.

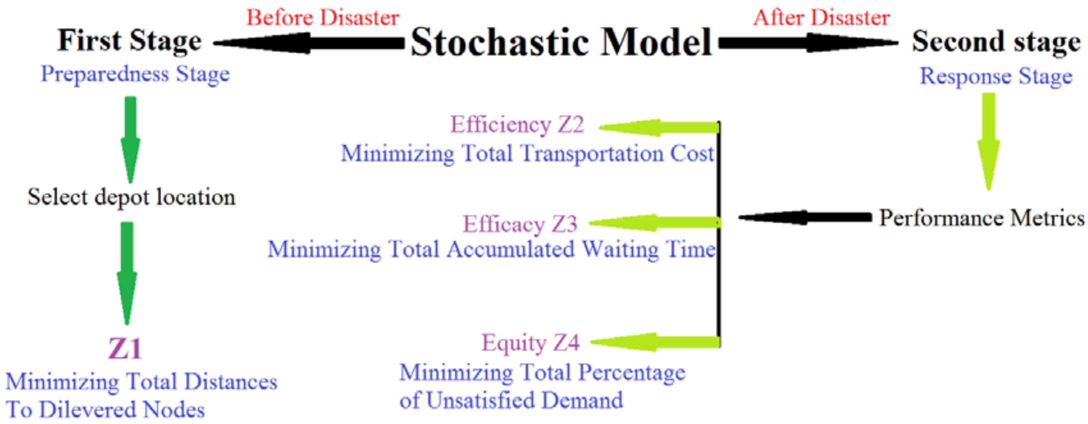

Figure 3.1 Model framework

In this study the model has some similarities and differences with P-median, as one of the famous approaches to solve locations selection problems. P-median works with un-capacitated storages while we are limits our depots with a specific number of pallets, and we have used the principle of uncertainty in the distances between demand nodes and the multiple location of depots. The other properties of P-median has been used within the model like the common objective function which considers minimizing total distances between nodes of demand and depots. We can say that we used P-median and its properties with new practical application includes stochastic approach, and P-center gave us the way to define and solve the fourth objective function in our model by using minimax methodology to get the percentages of unmet demand for each demand node.

In Figure 3.2 which refers to the sample network that the case study in Chapter 5 is based on, we can find the following:

- Demand nodes in twelve blue points,
- 7 different alternative locations depot in orange color to choose 4 or less from,
- The distances between all depots and demand nodes in kilometers,
- Two different collection points, shown as black squares.

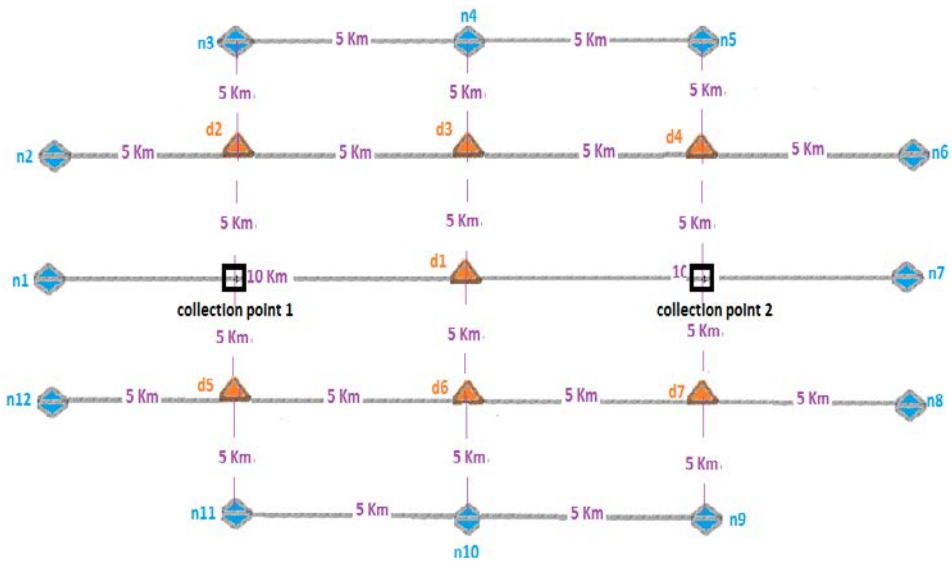

Figure 3.1 Sample network

The assumptions of the proposed model are as follows:

1. The total storage capacity of the depots is greater than or equal to the total demand.
2. A specific number of locations will be chosen out of the total number of available locations.
3. A specific average speed of trucks and a predetermined unit cost have been assigned for different objective functions.
4. Each depot can send relief aid to a specific number of demand nodes based on the number of available vehicles for that depot.
5. The unit transportation cost is a constant that does not depend on the depot-demand node pair.
6. Percent of unmet demand for each demand node is assumed as the equity measure.
7. Loading time and unloading time for a truck are assumed to be equal.
8. There is no vehicle restriction, i.e., there are enough number of trucks available at time zero such that all demand can be loaded starting at the same time and delivered immediately.

The notation used in the model is defined as follows.

Sets:

$\boldsymbol{S}$: set of alternative depot nodes, $i = 1, 2, \ldots, S$

$\boldsymbol{D}$: set of demand nodes, $j = 1, 2, \ldots, D$

$\boldsymbol{K_i}$: set of available vehicles (trucks) at depot i, $k = 1, 2, \ldots, Nv_i$

$\boldsymbol{\Omega}$: set of disaster scenarios, $\omega = 1, 2, \ldots, \Omega$

Parameters:

cap_i : Capacity of depot i in number of pallets

dem_j : Demand of node j in number of pallets

d_{ij} : Distance between depot i and demand node j in kilometers

tc : Unit transportation cost per kilometer in dollars

MD : Maximum number of depots that can be opened

Nv_i : Number of vehicles available at each depot at time zero

$vcap$: Capacity of a truck in number of pallets

LT : Loading time for a truck in minutes

UT : Unloading time for a truck in minutes

SP : Average speed of trucks (km/hour)

f_{ij} : Fuel cost for the trip between i and j in dollars

$Dist_{ij}$: Distribution cost between demand nodes and depots in dollars

Wr_{ij} : Worker's cost in dollars

$Accu_{ij}$: Accumulated waiting time in minutes

Decision variables:

x_{ij} : Number of pallets delivered from depot i to demand node j

$$w_{ij} = \begin{cases} 1, & \text{depot } i \text{ serves node } j \\ 0, & \text{otherwise} \end{cases}$$

$$y_i = \begin{cases} 1, & \text{if depot location } i \text{ is selected} \\ 0, & \text{otherwise} \end{cases}$$

Before applying the model we should calculate the parameters $Accu_{ij}$, f_{ij}, $Dist_{ij}$, and Wr_{ij}. Accumulated waiting time parameter $Accu_{ij}$ is the sum of loading time, unloading time, and the time needed to reach demand nodes. Fuel cost parameter f_{ij} equals the product of the unit fuel cost ($ per km) and the distance between depots and demand nodes. The distribution cost parameter $Dist_{ij}$ has different levels based on the quality of service of each depot. In our case study, based on sample network in Figure 3.2, we will define three levels of distribution cost such that depot 1 has the lowest cost, depots 2, 3 and 4 have an average cost, and depots 5, 6 and 7 have the highest cost. Worker cost parameter Wr_{ij} is the product of the number of workers at a depot and the wage of each worker for that depot.

In our study, we define certain scenarios based on the level of demand and the impact of the disaster on the transportation network. We will be using scenario-dependent parameter values for demand, where dem_j^s is the demand of node j in number of pallets under scenario $s \in \Omega$, and distances, where d_{ij}^s is the distance between depot i and demand node j under scenario s. Accordingly, the decision variables in each scenario will be defined as x_{ij}^s, w_{ij}^s, and y_i^s. The description of the demand and distance scenarios and the corresponding numerical results will be provided in Chapter 5.

The mathematical model for the preparedness stage problem is given as Model 1 below.

Model 1:

$$\text{Minimize} \quad Z_1 = \sum_{i \in S} \sum_{j \in D} (x_{ij} \times d_{ij}) \tag{3.1}$$

Subject to:

$$\sum_{j \in D} x_{ij} \leq Cap_i \times y_i \qquad \forall i \in S \tag{3.2}$$

$$\sum_{i \in S} x_{ij} \geq dem_j \qquad \forall j \in D \tag{3.3}$$

$$\sum_{i \in S} y_i \leq MD \tag{3.4}$$

$$x_{ij} \geq 0, \text{integer} \qquad \forall i \in S, \forall j \in D \tag{3.5}$$

$$y_i \in \{0,1\} \qquad \forall i \in S \tag{3.6}$$

The objective function Z_1 in (3.1) minimizes the total distance between the demand nodes and the depot locations selected. Constraint (3.2) declares that for each delivery, the storage capacity of depots should not be exceeded. Constraint (3.3) ensures the amount of relief materials will satisfy the demand for each node. Constraint (3.4) ensures that the number of selected depot locations does not exceed the maximum number of depots required. Constraint (3.5) is the non-negativity constraint for the integer x_{ij} variables and constraint (3.6) defines the binary y_i variables.

The mathematical models for the response stage problems are given as Models 2, 3, and 4 below.

Model 2:

$$\text{Minimize} \quad Z_2 = \sum_{i \in S} \sum_{j \in D} \left(x_{ij} \times \left(f_{ij} + Dist_{ij} + Wr_{ij} \right) \right) \tag{3.7}$$

Subject to: (3.2)-(3.6)

The objective function Z_2 in (3.7) minimizes the total transportation cost that consists of fuel cost, distribution cost, and worker cost.

Model 3:

$$\text{Minimize} \quad Z_3 = \sum_{i \in S} \sum_{j \in D} \left(x_{ij} \times Accu_{ij} \right) \tag{3.8}$$

Subject to: (3.2)-(3.6)

$$\sum_{j \in D} x_{ij} \leq vcap \times Nv_i \qquad \forall i \in S \tag{3.9}$$

$$x_{ij} \leq vcap \times w_{ij} \qquad \forall i \in S, \forall j \in D \tag{3.10}$$

$$\sum_{j \in D} w_{ij} \leq Nv_i \times y_i \qquad \forall i \in S \tag{3.11}$$

$$w_{ij} \in \{0,1\} \qquad \forall i \in S, \forall j \in D \tag{3.12}$$

The objective function Z_3 in (3.8) minimizes the total accumulated waiting time for the demand nodes to receive relief items. In addition to constraints (3.2)-(3.6), constraint (3.9) limits delivered pallets with the number of vehicles. Constraint (3.10) ensures the number of delivered pallets is at most as much as the capacity of vehicles for each trip. Constraint (3.11) limits the number of demand nodes that can be served by a depot with the number of available vehicles at that depot. Constraint (3.12) defines the binary w_{ij} variables.

Model 4:

Minimize Z_4 (3.13)

Subject to: (3.2), (3.4)-(3.6), (3.9)-(3.12)

$$Z_4 \geq \frac{dem_j - \sum_{i \in S} x_{ij}}{dem_j} \qquad \forall j \in D \tag{3.14}$$

$$\sum_{i \in S} x_{ij} \leq dem_j \qquad \forall j \in D \tag{3.15}$$

$$Z_w \leq \sum_{f=1}^{3} c_f Z_f^* \tag{3.16}$$

$$Z_w \geq 0 \tag{3.17}$$

The objective function Z_4 in (3.13) minimizes the maximum percent of unmet demand defined by constraint (3.14). In order to allow for unmet demand, constraint (3.3) is modified as constraint (3.15). Also, a target performance measure is defined as the weighted sum of the optimal objective functions of Models 1-3 (Z_w) in constraint (3.16), where c_f is the weight of objective function of Model f and Z_f^* is the optimal objective function value for Model f. The weighted sum variable is defined to be non-negative in constraint (3.17).

The disaster relief logistics optimization models defined above are related to decision making in the preparedness stage (Z_1) and the response stage (Z_2, Z_3, Z_4). These decisions are based on different performance metrics, which requires a multi-objective decision making approach. In the next chapter, the methodology used for solving the multi-objective optimization problem is described.

4. MULTI-OBJECTIVE OPTIMIZATION METHODOLOGY

The relief distribution problem introduced in the previous chapter has multiple objective functions. When multi-objective optimization problems are solved, there usually is a trade-off between various objectives. Different studies have offered many approaches to model the trade-off between multiple objective functions from the decision maker's perspective. Chiandussi et al. (2012) characterized the most popular techniques within three major groups as shown in Figure 4.1: *a priori* preference articulation (decide → search), *a posteriori* preference articulation (search → decide), and progressive preference articulation (decide ↔ search). This paper explains and compares four multi-objective optimization techniques: global criterion method, linear combination of weights, *ε*-contraint method, and multi-objective genetic algorithm (MOGA).

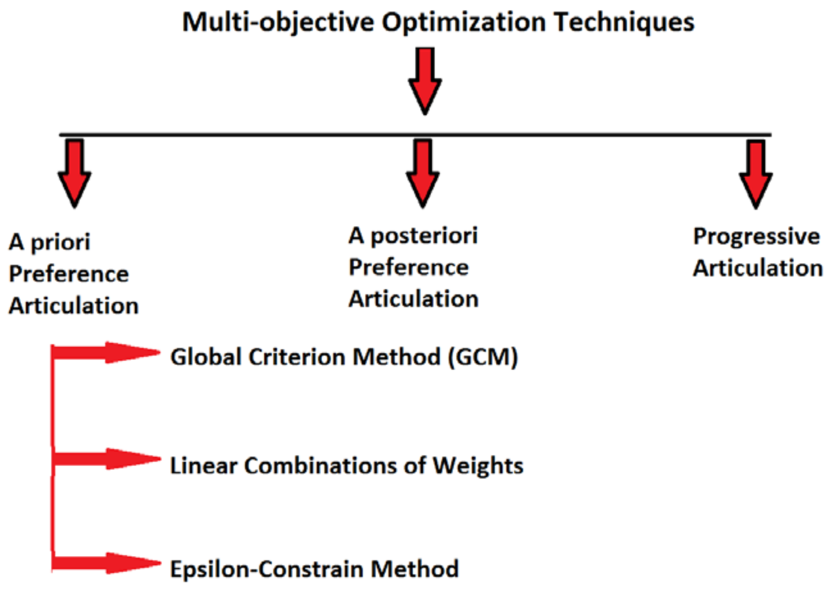

Figure 4.1 Multi-objective optimization methods (based on Chiandussi et al. (2012) and Cui et al. (2017))

According to Chiandussi et al. (2012), *a priori* preference articulation assumes that the decision maker can pre-order the objectives before making the search for the solution. The Global Criterion Method (GCM) is one of the *a priori* preference articulation methods. The target of GCM is to know how close the model is to the ideal solution or ideal vector (the vector of optimal solutions for every objective function separately, while achieving all of the objective functions at the same point). The GCM is applied using equation (4.1):

$$L_p = \left[\sum_{z=1}^{Z} W_z^p \times \left(\frac{f_z - f_z^0}{f_z^{max} - f_z^0}\right)^p\right]^{1/p} \tag{4.1}$$

where W_z^p is the weight of objective function z, f_z is the implemented function value, f_z^0 is the ideal function value, f_z^{max} is the maximum value of objective function z, and L_p is the closeness percentage. The choice of exponent p affects the results significantly and Boychuk and Ovichinnikov (1973) assumed a p value of 1, because the GCM became more complicated when p takes higher values. Table 4.1 points the most important advantages and disadvantages of GCM.

Table 4.1 Advantages and disadvantages of GCM

Advantages	Disadvantages
Simple to apply	Difficulty in definition of optimized functions
Effectiveness in results	
Don not need any ranking procedures	Need extra computational effort

The second methodology, linear combination of weights, is of the *a posteriori* preference articulation type according to Chiandussi et al. (2012). However, Cui et al. (2017) classify this method as an *a priori* method. We observe that linear combination of weights or the weighted sum method is an *a priori* method, because the objective functions in this technique also have predetermined weights which are linearly combined based on equation (4.2):

$$\min \sum_{z=1}^{Z} \alpha_z \times f_z \tag{4.2}$$

where α_z is the weight of objective function z and f_z is the implemented function value. Table 4.2 shows the advantages and disadvantages of using linear combinations of weights.

Table 4.2 Advantages and disadvantages of linear combination of weights

Advantages	Disadvantages
Simplicity in implementation	Some applications are very difficult to determine the weights of functions
Effectiveness in computation	

The third methodology is ε-constraint method, another *a posteriori* preference articulation according to Chiandussi et al. (2012). However, Cui et al. (2017) classify this method as an *a priori* method. We observe that the ε-constraint method is an *a priori* method, because the upper bounds on the objective functions are determined before the search for the solution. The ε-constraint method is introduced by Haimes et al. (1971). This technique optimizes one objective function subject to other objective functions defined as constrains with specific limits, ε_z, as shown in equations (4.3)-(4.4):

$$\text{Min } f_u \tag{4.3}$$

$$\text{Subject to: } f_z \leq \varepsilon_z, z = 1, \ldots, Z, z \neq u \tag{4.4}$$

where f_u is the optimized function and f_z's are the other functions that are represented as constraints. Table 4.3 explains advantages and disadvantages of ε-constraint method.

Table 4.3 Advantages and disadvantages of ε-constraint method

Advantages	Disadvantages
No mixing among objectives and they keep their identity	High computational cost
Guarantees ideal values of the objective function with consideration of $z-1$ objectives	Encoding objective functions is extremely difficult in some industrial applications

The fourth method is the evolutionary algorithm which is based on Darwin's theory (survival of the fittest) with three steps to apply:

1. Starting with random population
2. Sequence of generation
3. Choosing best solutions (lowest values of the objective functions)

Examples of evolutionary algorithms are MOGA and SOGA (single-objective genetic algorithm). The main advantage of this method is being able to obtain multiple solutions in a single application. Its main disadvantage is related to high computation cost.

Another important study reviews all the available methods that are used in multi-objective optimization (Cui et al., 2017). This review demonstrates 13 different techniques in four major groups, which are *a priori* methods, interactive methods, Pareto-dominated methods, and new dominance methods.

Through this study both the linear combination of weights and GCM are used to choose the best option of weights out of possible sets of weights for the objective functions as presented in the next chapter.

5. COMPUTATIONAL RESULTS

In this chapter, we first define the case study parameters used in computational experiments, followed by the results of models with different objective functions introduced in Chapter 3.

5.1 Case Study Parameters

The case study is defined on a sample network that is already provided in Figure 3.2. In this network, there are 12 demand nodes (blue nodes) and 7 alternative depot locations (orange triangular nodes). In this case study, at most 4 depot locations can be selected out of 7 to open depots at ($MD = 4$). The capacity of each depot is 600 pallets ($cap_i = 600, \forall i \in S$).

The vehicles used for transportation are trucks with a capacity of 160 pallets ($vcap = 160$). The unit transportation cost is \$2.5 per kilometer ($tc = \$2.5$). The average truck speed is assumed to be 45 kilometers per hour ($SP = 45\ km/hr$). The total loading and unloading time is assumed to be 40 minutes ($LT + UT = 40\ min$). It is assumed that 5 workers and 5 vehicles are available at each depot ($Nv_i = 5$). Since the number of vehicles available at each depot is 5 and we assume that each vehicle is sent to one demand node, each depot can serve at most 5 demand nodes.

There are two collection points (black squares) in the network that will be used when the direct access from depots to demand nodes is limited because of the disaster impact. In those scenarios, the relief items will be sent from depots to the available collection point and then sent to the demand nodes, which will increase the distances to be travelled. These scenarios will be explained in detail below.

The relief items are stored on pallets at depots. The types of items that should be stored on a pallet are determined based on Tzeng et al. (2006) and the quantities of items per pallet are determined so as to provide relief aid for four people. The value or cost of items on a pallet are estimated by online search for these types of relief aid items. The number of items of each type and their costs are shown in Table 5.1.

Table 5.1 Pallet contents

Relief Materials	Amount	Volume (cm^3)	Volume (unit)	Price ($)
Sleeping bag	4	45×25×11= 12375	1	7.5
Tent	1	27300	2.21	50
Box of mineral water	1	28080	2.27	18
Rice (5kg)	2	5225	0.42	10
Box of instant noodles	1	21199	1.71	12
Box of dry food	2	18468	1.49	15
Box of canned food	2	3532	0.29	36

Total transportation cost is defined as the summation of fuel cost, distribution cost, and worker cost, where fuel cost f_{ij} is calculated by multiplying the distances between demand nodes and depots by $2.5. Distribution cost $Dist_{ij}$ is related to shipping taxes, truck driver cost, and maintenance cost. We assume that there are three levels of distribution costs (low, medium, and high) that depends on the location of depot as shown in Table 5.2. Worker cost Wr_{ij} is equal to the number of workers multiplied by the cost of each worker which also depends on the type of depot. So for the fourth and seventh depots have the highest cost (Grade A) with $40 paid for each worker, while depots 2 and 5 with moderate cost (Grade B) as $30 paid for each worker, the others 1, 3 and 6 have the lowest cost (Grade C) with $20 paid to workers. These worker costs are also given in Table 5.2. The distribution and worker cost parameters are not assumed to be dependent on the demand nodes in this case study, but this assumption can be easily changed.

Table 5.2 Distribution and worker costs

	Depots (i)	Distribution Cost ($Dist_{ij}$) ($)		Depots (i)	Worker Cost (Wr_{ij}) ($)
Low	1	120	Grade A	4, 7	40
Medium	2, 3, 4	140	Grade B	2, 5	30
High	5, 6, 7	160	Grade C	1, 3, 6	20

In this study, we consider the demand and distances to be the random parameters. We use a set of scenarios, Ω, to represent uncertainty regarding demand and distance in the model. The probability of each scenario is $P^s, s \in \Omega$, where $P^s \in [0, 1]$ and $\sum_{s\in\Omega} P^s = 1$. In this case study, the demand is random between 100 and 150 pallets for each demand node. As shown in Table 5.3, we define three demand scenarios: high demand scenario with 30% probability, medium demand scenario with 45% probability, and low demand scenario with 25% probability. A set of demand values are generated from the Uniform distribution between 100 and 150 pallets for the medium demand case. Then, 125% of the medium demand is taken as the high demand case and 75% of the medium demand is taken as the low demand case. The fractional values are rounded up to obtain integer numbers. The randomly generated demand values are shown in Table 5.4.

Table 5.3 Scenario probabilities

		Demand		
		Low (25%)	**Medium (45%)**	**High (30%)**
Distances	**Direct transportation (40%)**	10% S1	18% S2	12% S3
	Limited accessibility (collection point 1) (35%)	8.75% S4	15.75% S5	10.5% S6
	Highly affected network (collection point 2) (25%)	6.25% S7	11.25% S8	7.5% S9

The other random factor is the condition of the transportation network. The distances to be traveled depend on how much the roads are affected by the disaster encountered. We define three distance scenarios: normal transportation conditions scenario with 40%

probability (transportation is possible directly from depots to demand nodes), limited accessibility of demand nodes with 35% probability (direct transportation is not possible, collection point 1 in Figure 3.2 must be used), and highly affected transportation rate scenario with 25% probability (direct transportation or using collection point 1 is not possible, collection point 2 in Figure 3.2 must be used). The scenario numbers are also provided in each cell of Table 5.3, therefore, S1 is the best-case scenario in terms of low demand and short distances to be travelled, whereas S9 is the worst-case scenario with high demand and long distances to be travelled.

Table 5.4 Randomly generated demands (in pallets) according to $Uniform[100, 150]$ distribution

Demand Scenario	**Demand Nodes (j)**												**Total Demand**
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	
Low	81	76	81	85	80	90	79	75	87	87	80	86	987
Medium	107	101	108	113	106	120	105	100	116	116	106	114	1312
High	134	127	135	142	133	150	132	125	145	148	133	143	1647

Total transportation cost ($f_{ij} + Dist_{ij} + Wr_{ij}$) and total accumulated time ($Accu_{ij}$) depend on the distances between depot locations and demand nodes. In this case study, three different matrices for distances, total transportation cost, and total accumulated time is pre-calculated for each of the three distance scenarios as shown in Tables 5.5-5.13.

Table 5.5 Distance matrix for scenarios S1, S2, S3 (in km)

Depot (i)	**Demand Node (j)**											
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
1	10	11	11	10	11	11	10	18	11	10	11	11
2	7	5	5	7	11	15	16	18	18	16	15	11
3	11	10	7	5	7	10	11	14	16	15	16	14
4	16	15	11	7	5	5	7	11	15	16	18	18
5	7	11	15	16	18	18	16	15	11	7	5	5
6	11	14	16	15	16	14	11	10	7	5	7	10
7	16	18	18	16	15	11	7	5	5	7	11	15

Table 5.6 Total transportation cost matrix for scenarios S1, S2, S3 (in $)

Depot (*i*)	Demand Node (*j*)											
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
1	245	248	248	245	248	248	245	265	248	245	248	248
2	308	303	303	308	318	328	330	335	335	330	328	318
3	268	265	258	253	258	265	268	275	280	278	280	275
4	380	378	368	358	353	353	358	368	378	380	385	385
5	328	338	348	350	355	355	350	348	338	328	323	323
6	288	295	300	298	300	295	288	285	278	273	278	285
7	400	405	405	400	398	388	378	373	373	378	388	398

Table 5.7 Total accumulated waiting time matrix for scenarios S1, S2, S3 (in minutes)

Depot (*i*)	Demand Node (*j*)											
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
1	54	55	55	54	55	55	54	64	55	54	55	55
2	50	47	47	50	55	60	62	64	64	62	60	55
3	55	54	50	47	50	54	55	59	62	60	62	59
4	62	60	55	50	47	47	50	55	60	62	64	64
5	50	55	60	62	64	64	62	60	55	50	47	47
6	55	59	62	60	62	59	55	54	50	47	50	54
7	62	64	64	62	60	55	50	47	47	50	55	60

Table 5.8 Distance matrix for scenarios S4, S5, S6 (in km)

Depot (*i*)	Demand Node (*j*)											
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
1	10	12	15	16	19	21	20	21	19	16	15	12
2	10	12	15	16	19	21	20	21	19	16	15	12
3	12	14	17	18	21	23	22	23	21	18	17	14
4	16	18	21	22	25	27	26	27	25	22	21	18
5	10	12	15	16	19	21	20	21	19	16	15	12
6	12	14	17	18	21	23	22	23	21	18	17	14
7	16	18	21	22	25	27	26	27	25	22	21	18

Table 5.9 Total transportation cost matrix for scenarios S4, S5, S6 (in $)

Depot (i)	Demand Node (j)											
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
1	245	250	258	260	268	273	270	273	268	260	258	250
2	315	320	328	330	338	343	340	343	338	330	328	320
3	270	275	283	285	293	298	295	298	293	285	283	275
4	380	385	393	395	403	408	405	408	403	395	393	385
5	335	340	348	350	358	363	360	363	358	350	348	340
6	290	295	303	305	313	318	315	318	313	305	303	295
7	400	405	413	415	423	428	425	428	423	415	413	405

Table 5.10 Total accumulated waiting time matrix for scenarios S4, S5, S6 (in minutes)

Depot (i)	Demand Node (j)											
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
1	54	56	60	62	66	68	67	68	66	62	60	56
2	54	56	60	62	66	68	67	68	66	62	60	56
3	56	59	63	64	68	71	70	71	68	64	63	59
4	62	64	68	70	74	76	75	76	74	70	68	64
5	54	56	60	62	66	68	67	68	66	62	60	56
6	56	59	63	64	68	71	70	71	68	64	63	59
7	62	64	68	70	74	76	75	76	74	70	68	64

Table 5.11 Distance matrix for scenarios S7, S8, S9 (in km)

Depot (i)	Demand Node (j)											
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
1	20	21	19	16	15	12	10	12	15	16	19	21
2	26	27	25	22	21	18	16	18	21	22	25	27
3	22	23	21	18	17	14	12	14	17	18	21	23
4	20	21	19	16	15	12	10	12	15	16	19	21
5	26	27	25	22	21	18	16	18	21	22	25	27
6	22	23	21	18	17	14	12	14	17	18	21	23
7	20	21	19	16	15	12	10	12	15	16	19	21

Table 5.12 Total transportation cost matrix for scenarios S7, S8, S9 (in $)

Depot (i)	Demand Node (j)											
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
1	270	273	268	260	258	250	245	250	258	260	268	273
2	355	358	353	345	343	335	330	335	343	345	353	358
3	295	298	293	285	283	275	270	275	283	285	293	298
4	390	393	388	380	378	370	365	370	378	380	388	393
5	375	378	373	365	363	355	350	355	363	365	373	378
6	315	318	313	305	303	295	290	295	303	305	313	318
7	410	413	408	400	398	390	385	390	398	400	408	413

Table 5.13 Total accumulated waiting time matrix for scenarios S7, S8, S9 (in minutes)

Depot (i)	Demand Node (j)											
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
1	67	68	66	62	60	56	54	56	60	62	66	68
2	75	76	74	70	68	64	62	64	68	70	74	76
3	70	71	68	64	63	59	56	59	63	64	68	71
4	67	68	66	62	60	56	54	56	60	62	66	68
5	75	76	74	70	68	64	62	64	68	70	74	76
6	70	71	68	64	63	59	56	59	63	64	68	71
7	67	68	66	62	60	56	54	56	60	62	66	68

5.2 Computational Results for Demand-Distance Scenarios

The case study problem is solved for Model 1-4 under scenarios S1-S9. In the following nine subsections, the computational results for the nine demand-distance scenarios are shown. First, Model 1-4 are separately solved for each scenario and the optimal solutions are discussed. Then, in order to find a solution that combines the four objective functions, a linear combination of weights method is used. For the multi-objective optimization using linear combination of weights, the weights of objective functions must be determined. Let c_1, c_2, c_3, and $c_4$be the weights of the four objective functions Z_1, Z_2, Z_3, and Z_4. To decide which weights will be given for every objective function, we have experimented with all the possible combinations of weights between 0.1 and 0.7. Appendix B.1 shows how the combinations and the results were used, and how the objective functions react through these linear combinations of weights in addition to using the global criterion method (GCM) to measure which one of these combinations is the closest to the ideal model. It can be noticed from the Appendix B.1 that the most suitable weights for objective functions are $c_2 = 0.1$ and c_1=c_3= c_4= 0.3, because of the summation of the total percentages of differences for these weights is the lowest among 84 combinations. GCM supports this result because it gives also the nearest value from the ideal model. The results shown here are obtained using GAMS 24.6.1 software with CPLEX 12.6.3 solver on a computer with 1.50 GHz CPU AMD processor and 4 GB RAM (64 bit Windows operating system).

5.2.1 Results for Scenario 1

Scenario 1 is the best-case scenario with low demand and short distances. The optimal solutions for Model 1, Model 2, Model 3, and Model 4 under Scenario 1 are provided in Table 5.14 below. When we compare Model 1 and Model 2, we can notice the change in selected depots and x_{ij} values (number of delivered pallets), because the target of Model 1 is minimizing the total distance whereas the target of Model 2 is minimizing the total transportation cost.

In Model 4, the percent of unmet demand ranges from 5.6 to 6.7 and there is an equitable distribution of pallets to demand nodes as we aimed. We note that Model 4, minimizing the maximum unmet demand, gives the same optimal solution in scenarios S1, S4, and S7, because we assume there is no vehicle restriction, i.e., there are enough number of trucks available at time zero such that all demand can be loaded starting at the same time and delivered immediately. Therefore, we report the different values just for scenarios with low, medium, and high demand at S1, S2, and S3.

The optimal solution for minimizing the weighted sum of the four objective functions is given in Table 5.15, where the weights are $c_2 = 0.1$ and c_1=c_3= c_4= 0.3 (c = [0.3, 0.1, 0.3, 0.3]) as explained above. The target of the weighted sum model is to minimize total distances between depots and demand nodes while maintaining service level by minimizing transportation cost, accumulated waiting time, and percent of unsatisfied demand.

We also compare Model 1 and Model 2 that differ only in the objective function coefficients. Table 5.16 shows the differences in results between assigning the total distances as the primary objective function (Model 1) and assigning total transportation cost with giving higher weights to distribution cost and worker cost (Model 2). It is clear from results that the weights of Wr_{ij} and $Dist_{ij}$ are higher than the weight of fuel cost through calculating total transportation cost, and if fuel cost gets higher, then the supply decisions would be the same with the first objective function of minimizing the total distance. When we compare the objective function values for the two models, Z_1 value is 71.72% more than its optimal value when Z_2 is minimized (5,599 versus 9,615), whereas Z_2 value is 32.24% more than its optimal value when Z_1 is minimized (249,220 versus 329,571). This is due to the fact that the transportation cost parameters are not directly proportional to the distances and significantly longer distances may be travelled while minimizing the transportation cost which is a combination of fuel cost, distribution cost, and worker cost. The summary of the results for Scenario 1 is given in Table 5.17.

Table 5.14 Optimal solutions (x_{ij}^*) of four models separately for Scenario 1

S1	**Model 1**				**Model 2**		**Model 3**				**Model 4**		
Z_1	Z_1^* = 5,599				9,615		8,875				7,778		
Z_2	329,571				Z_2^* = 249,220		347,402				240,821		
Z_3	47,385				52,787		Z_3^* = 47,385				47,934		
Z_4	0%				0%		0%				Z_4^* = 6.7%		
$\boldsymbol{x_{ij}^*}$	**Selected Depots ($\boldsymbol{i}$)**												
$\boldsymbol{j}$	2	4	5	7	1	3	2	4	5	7	1	3	6
1	81	0	0	0	81	0	0	0	81	0	71	5	0
2	76	0	0	0	10	66	76	0	0	0	71	0	0
3	81	0	0	0	0	81	81	0	0	0	0	76	0
4	85	0	0	0	0	85	0	85	0	0	0	80	0
5	0	80	0	0	0	80	0	80	0	0	0	75	0
6	0	90	0	0	90	0	0	90	0	0	85	0	0
7	0	79	0	0	79	0	0	0	0	79	74	0	0
8	0	0	0	75	0	75	0	0	0	75	0	0	70
9	0	0	0	87	87	0	0	0	0	87	0	0	82
10	0	0	87	0	87	0	0	0	87	0	0	0	82
11	0	0	80	0	80	0	0	0	80	0	0	0	75
12	0	0	86	0	86	0	0	0	86	0	81	0	0

Table 5.15 Optimal solutions (x^*_{ij}) of $Z_{weighted}$ for Scenario 1

Selected Depots (i)	$Z_{weighted}$ = 44,135 (c = [0.3, 0.1, 0.3, 0.3])											
	Demand Node (j)											
	1	2	3	4	5	6	7	8	9	10	11	12
1	81	76	81	85	80	90	79	0	0	0	0	28
6	0	0	0	0	0	0	0	75	87	87	80	58

Table 5.16 Comparison of Model 1 and Model 2 results for Scenario 1

S1 x^*_{ij}	**Model 1**				**Model 2**	
	Z^*_1 = 5,599				Z_1 = 9,615	
	Z_2 = 329,571				Z^*_2 = 249,220	
	Selected Depots (i)					
Demand Node (j)	2	4	5	7	1	3
1	81	0	0	0	81	0
2	76	0	0	0	10	66
3	81	0	0	0	0	81
4	85	0	0	0	0	85
5	0	80	0	0	0	80
6	0	90	0	0	90	0
7	0	79	0	0	79	0
8	0	0	0	75	0	75
9	0	0	0	87	87	0
10	0	0	87	0	87	0
11	0	0	80	0	80	0
12	0	0	86	0	86	0
Total	323	249	253	162	600	387

Table 5.17 Scenario 1 summary

Objective Function	**Value**	**Selected Depots (i)**
Z_1	5,599	2, 4, 5, 7
Z_2	249,220	1, 3
Z_3	47,385	2, 4, 5, 7
Z_4	6.7%	1, 3, 6
$Z_{weighted}$	44,135	1, 6

5.2.2 Results for Scenario 2

In Scenario 2, demand is medium-level and distances are short. The optimal solutions for Model 1, Model 2, Model 3, and Model 4 under Scenario 2 are provided in Table 5.18 below. We note that Model 4, minimizing the maximum unmet demand, gives the same optimal solution as in Scenario 1. The weighted sum minimization solution is presented in Table 5.19. The summary of the results for Scenario 2 is given in Table 5.20.

Table 5.18 Optimal solutions (x_{ij}^*) of four models separately for Scenario 2

S2	**Model 1**				**Model 2**			**Model 3**				**Model 4**			
Z_1	Z_1^* = 7,442				12,137			7,442				10,797			
Z_2	438,131				Z_2^* = 335,830			440,230				321,230			
Z_3	62,987				69,349			Z_3^* = 62,987				64,439			
Z_4	0%				0%			0%				Z_4^* = 6.2%			
$\boldsymbol{x_{ij}^*}$	**Selected Depots ($\boldsymbol{i}$)**														
$\boldsymbol{j}$	2	4	5	7	1	3	6	2	4	5	7	1	2	3	6
1	107	0	0	0	107	0	0	107	0	0	0	100	1	0	0
2	101	0	0	0	0	101	0	101	0	0	0	95	0	0	0
3	108	0	0	0	0	108	0	108	0	0	0	0	2	100	0
4	113	0	0	0	0	113	0	113	0	0	0	0	6	100	0
5	0	106	0	0	0	106	0	0	106	0	0	0	0	100	0
6	0	120	0	0	0	120	0	0	120	0	0	0	13	100	0
7	0	105	0	0	105	0	0	0	0	0	105	99	0	0	0
8	0	0	0	100	0	52	48	0	0	0	100	0	0	0	94
9	0	0	0	116	116	0	0	0	0	0	116	100	0	0	9
10	0	0	116	0	52	0	64	0	0	116	0	0	0	0	100
11	0	0	106	0	106	0	0	0	0	106	0	0	0	0	100
12	0	0	114	0	114	0	0	0	0	114	0	100	0	0	7

Table 5.19 Optimal solutions (x_{ij}^*) of $Z_{weighted}$ for Scenario 2

Selected Depots ($\boldsymbol{i}$)	$Z_{weighted}$ = 57,728 (c = [0.3, 0.1, 0.3, 0.3])											
	Demand Node ($\boldsymbol{j}$)											
	1	2	3	4	5	6	7	8	9	10	11	12
1	107	101	0	0	0	120	105	0	0	0	0	0
3	0	0	108	113	106	0	0	0	0	0	53	114
6	0	0	0	0	0	0	0	100	116	116	53	0

Table 5.20 Scenario 2 summary

Objective Function	**Value**	**Selected Depots (i)**
Z_1	7,442	2, 4, 5, 7
Z_2	335,830	1, 3, 6
Z_3	62,987	2, 4, 5, 7
Z_4	6.2%	1, 2, 3, 6
$Z_{weighted}$	57,728	1, 3, 6

5.2.3 Results of Scenario 3

In Scenario 3, demand is high and distances are short. The optimal solutions for Model 1, Model 2, Model 3, and Model 4 under Scenario 3 are provided in Table 5.21 below. We note that Model 4, minimizing the maximum unmet demand, gives the same optimal solution as in Scenario 1. The weighted sum minimization solution is presented in Table 5.22.

Table 5.21 Optimal solutions (x_{ij}^*) of four models separately for Scenario 3

S3	**Model 1**				**Model 2**			**Model 3**				**Model 4**			
Z_1	Z_1^* = 9,347				14,028			Z_1^* = 9,347				13,875			
Z_2	549,950				Z_2^* = 427,823			559,730				415,841			
Z_3	Z_3^* = 79,077				85,474			Z_3^* = 79,077				78,409			
Z_4	0%				0%			0%				Z_4^* = 24.8%			
x_{ij}^*	**Selected Depots (i)**														
j	2	4	5	7	1	3	6	2	4	5	7	1	3	4	6
1	134	0	0	0	134	0	0	0	0	134	0	100	0	34	0
2	127	0	0	0	87	40	0	127	0	0	0	100	0	0	0
3	135	0	0	0	0	135	0	135	0	0	0	2	100	0	0
4	142	0	0	0	0	142	0	0	142	0	0	0	100	42	0
5	0	133	0	0	0	133	0	0	133	0	0	0	100	0	0
6	0	150	0	0	0	150	0	0	150	0	0	100	0	50	0
7	0	132	0	0	132	0	0	0	132	0	0	100	0	0	0
8	0	0	0	125	0	0	125	0	0	0	125	0	0	0	95
9	0	0	0	145	0	0	145	0	0	0	145	0	0	100	45
10	0	0	148	0	0	0	148	0	0	148	0	0	48	0	100
11	0	0	133	0	104	0	29	0	0	133	0	0	0	21	100
12	0	0	143	0	143	0	0	0	0	143	0	0	43	0	100

Table 5.22 Optimal solutions (x_{ij}^*) of $Z_{weighted}$ for Scenario 3

Selected Depots (i)	$Z_{weighted}$ = 72,571 (c = [0.3, 0.1, 0.3, 0.3])											
	Demand Node (j)											
	1	2	3	4	5	6	7	8	9	10	11	12
1	134	41	0	0	0	150	132	0	0	0	0	143
3	0	86	135	142	133	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	125	145	148	133	0

The summary of the results for the third scenario is given in Table 5.23.

Table 5.23 Scenario 3 summary

Objective Function	**Value**	**Selected Depots (i)**
Z_1	9,347	2, 4, 5, 7
Z_2	427,820	1, 3, 6
Z_3	86,313	4, 5, 6, 7
Z_4	24.8%	1, 3, 4, 6
$Z_{weighted}$	72,571	1, 3, 6

5.2.4 Results of Scenario 4

In Scenario 4, demand is low and distances are medium length. The optimal solutions for Model 1, Model 2, Model 3, and Model 4 under Scenario 4 are provided in Table 5.24 below. The weighted sum minimization solution is presented in Table 5.25. The summary of the results for Scenario 4 is given in Table 5.26.

Table 5.24 Optimal solutions (x_{ij}^*) of four models separately for Scenario 4

S4	Model 1		Model 2		Model 3			Model 4		
Z_1	Z_1^* = 16,139		16,913		16,139			16,745		
Z_2	334,560		Z_2^* = 267,410		323,740			266,622		
Z_3	61,305		62,236		Z_3^* = 61,305			60,885		
Z_4	0%		0%		0%			Z_4^* = 3.5%		
x_{ij}^*	**Selected Depots (i)**									
j	2	5	1	3	1	2	5	1	3	6
1	0	81	0	81	0	0	81	0	79	0
2	0	76	0	76	0	76	0	0	0	74
3	0	81	0	81	0	0	81	79	0	0
4	0	85	0	85	0	0	85	0	82	0
5	16	64	16	64	0	0	80	0	78	0
6	90	0	90	0	0	90	0	87	0	0
7	79	0	79	0	0	79	0	0	77	0
8	75	0	75	0	75	0	0	0	0	73
9	87	0	87	0	87	0	0	84	0	0
10	87	0	87	0	0	87	0	84	0	0
11	80	0	80	0	0	80	0	0	78	0
12	86	0	86	0	0	0	86	83	0	0

Table 5.25 Optimal solutions (x_{ij}^*) of $Z_{weighted}$ for Scenario 4

Selected Depots (i)	$Z_{weighted}$ = 53,624 (c = [0.3, 0.1, 0.3, 0.3])											
	Demand Node (j)											
	1	2	3	4	5	6	7	8	9	10	11	12
3	0	0	0	0	16	90	79	75	87	87	80	86
6	81	76	81	85	64	0	0	0	0	0	0	0

Table 5.26 Scenario 4 summary

Objective Function	Value	Selected Depots (i)
Z_1	16,139	2, 5
Z_2	267,410	1, 3
Z_3	61,305	1, 2, 5
Z_4	3.5%	1, 3, 6
$Z_{weighted}$	53,624	3, 6

5.2.5 Results of Scenario 5

In Scenario 5, demand is medium-level and distances are medium length. The optimal solutions for Model 1, Model 2, Model 3, and Model 4 under Scenario 5 are provided in Table 5.27 below. The weighted sum minimization solution is presented in Table 5.28. The summary of the results for Scenario 5 is given in Table 5.29.

Table 5.27 Optimal solutions (x_{ij}^*) of four models separately for Scenario 5

S5	Model 1			Model 2			Model 3			Model 4			
Z_1	Z_1^* = 21,462			22,886			21,462			22,322			
Z_2	394,700			Z_2^* = 362,660			440,230			360,640			
Z_3	81,503			83,313			Z_3^* = 81,503			80,940			
Z_4	0%			0%			0%			Z_4^* = 5.7%			
x_{ij}^*	**Selected Depots (i)**												
j	2	4	5	1	3	6	1	2	5	1	3	5	6
1	0	0	107	0	0	107	0	0	107	0	100	5	0
2	0	96	5	0	96	5	0	101	0	0	0	0	96
3	0	108	0	0	108	0	0	0	108	100	0	0	8
4	0	113	0	0	113	0	0	0	113	0	0	13	100
5	0	106	0	0	106	0	0	0	106	0	100	0	0
6	0	120	0	0	120	0	0	120	0	100	0	0	17
7	48	57	0	48	57	0	0	105	0	100	0	0	0
8	100	0	0	100	0	0	100	0	0	0	92	0	8
9	116	0	0	116	0	0	116	0	0	0	100	13	0
10	116	0	0	116	0	0	0	116	0	100	0	0	0
11	106	0	0	106	0	0	0	106	0	100	0	0	0
12	114	0	0	114	0	0	0	0	114	100	0	0	11

Table 5.28 Optimal solutions (x_{ij}^*) of $Z_{weighted}$ for Scenario 5

Selected Depots (i)	$Z_{weighted}$ = 68,078 (c = [0.3, 0.1, 0.3, 0.3])											
	Demand Node (j)											
	1	2	3	4	5	6	7	8	9	10	11	12
1	0	0	55	0	0	120	105	100	0	0	106	114
3	0	96	53	113	106	0	0	0	116	116	0	0
6	107	5	0	0	0	0	0	0	0	0	0	0

Table 5.29 Scenario 5 summary

Objective Function	Value	Selected Depots ($\boldsymbol{i}$)
Z_1	21,462	2, 4, 5
Z_2	362,660	1, 3, 6
Z_3	81,503	1, 2, 5
Z_4	5.7%	1, 3, 5, 6
$Z_{weighted}$	68,078	1, 3, 6

5.2.6 Results of Scenario 6

In Scenario 6, demand is high and distances are medium length. The optimal solutions for Model 1, Model 2, and Model 3 under Scenario 6 are provided in Table 5.30 below. The weighted sum minimization solution is presented in Table 5.31. The summary of the results for Scenario 6 is given in Table 5.32.

Table 5.30 Optimal solutions (x_{ij}^*) of four models separately for Scenario 6

S6	Model 1			Model 2			Model 3			Model 4			
Z_1	Z_1^* = 26,937			29,031			26,937			25,572			
Z_2	512,320			Z_2^* = 465,210			526,090			423,890			
Z_3	102,310			105,040			Z_3^* = 102,308			93,501			
Z_4	0%			0%			0%			Z_4^* = 24.8%			
$\boldsymbol{x_{ij}^*}$	**Selected Depots ($\boldsymbol{i}$)**												
$\boldsymbol{j}$	1	2	5	1	3	6	1	2	5	1	2	3	6
1	0	0	134	0	0	134	0	0	134	100	34	0	0
2	0	0	127	0	0	127	0	71	56	100	0	0	0
3	0	0	135	0	0	135	0	0	135	100	0	0	35
4	0	91	51	0	91	51	0	0	142	0	28	100	10
5	0	133	0	0	133	0	0	0	133	0	0	0	100
6	0	150	0	0	150	0	0	150	0	0	0	100	46
7	0	132	0	0	132	0	132	0	0	100	0	0	0
8	31	94	0	31	94	0	125	0	0	100	41	100	0
9	145	0	0	145	0	0	145	0	0	0	44	100	0
10	148	0	0	148	0	0	0	148	0	0	0	100	0
11	133	0	0	133	0	0	45	88	0	0	39	0	0
12	143	0	0	143	0	0	0	143	0	0	0	0	100

Table 5.31 Optimal solutions (x^*_{ij}) of $Z_{weighted}$ for Scenario 6

Selected Depots (i)	$Z_{weighted}$ = 86,678 (c = [0.3, 0.1, 0.3, 0.3])											
	Demand Node (j)											
	1	2	3	4	5	6	7	8	9	10	11	12
1	0	0	0	0	0	67	132	125	0	0	133	143
3	0	0	0	91	133	83	0	0	145	148	0	0
6	134	127	135	51	0	0	0	0	0	0	0	0

Table 5.32 Scenario 6 summary

Objective Function	**Value**	**Selected Depots (i)**
Z_1	26,937	1, 2, 5
Z_2	465,210	1, 3, 6
Z_3	102,308	1, 2, 5
Z_4	24.8%	1, 2, 3, 6
$Z_{weighted}$	86,678	1, 3, 6

5.2.7 Results of Scenario 7

In Scenario 7, demand is low and distances are high. The optimal solutions for Model 1, Model 2, Model 3, and Model 4 under Scenario 7 are provided in Table 5.33 below. The weighted sum minimization solution is presented in Table 5.34. The summary of the results for Scenario 7 is given in Table 5.35.

Table 5.33 Optimal solutions (x_{ij}^*) of four models separately for Scenario 7

S7	Model 1		Model 2		Model 3			Model 4		
Z_1	Z_1^* = 16,108		16,882		16,108			16,718		
Z_2	383,830		Z_2^* = 267,330		364,910			266,553		
Z_3	61,259		62,254		Z_3^* = 61,259			60,768		
Z_4	0%		0%		0%			Z_4^* =3.5%		
$\boldsymbol{x_{ij}^*}$	**Selected Depots ($\boldsymbol{i}$)**									
$\boldsymbol{j}$	4	7	1	3	1	4	7	1	3	6
1	0	81	0	81	0	0	81	79	0	0
2	0	76	0	76	0	76	0	0	0	74
3	0	81	0	81	0	0	81	0	79	0
4	0	85	0	85	0	0	85	0	82	0
5	16	64	16	64	0	0	80	0	78	0
6	90	0	90	0	0	90	0	87	0	0
7	79	0	79	0	0	79	0	0	77	0
8	75	0	75	0	75	0	0	9	0	73
9	87	0	87	0	87	0	0	84	0	0
10	87	0	87	0	0	87	0	84	0	0
11	80	0	80	0	0	80	0	0	78	0
12	86	0	86	0	0	0	86	83	0	0

Table 5.34 Optimal solutions (x_{ij}^*) of $Z_{weighted}$ for Scenario 7

Selected Depots ($\boldsymbol{i}$)	$Z_{weighted}$ = 50,429 (c = [0.3, 0.1, 0.3, 0.3])											
	Demand Node ($\boldsymbol{j}$)											
	1	2	3	4	5	6	7	8	9	10	11	12
1	81	76	0	0	80	90	0	75	87	0	25	0
3	0	0	81	85	0	0	79	0	0	87	55	86

Table 5.35 Scenario 7 summary

Objective Function	**Value**	**Selected Depots ($\boldsymbol{i}$)**
Z_1	16,108	4, 7
Z_2	267,330	1, 3
Z_3	61,259	1, 4, 7
Z_4	3.5%	1, 3 ,6
$Z_{weighted}$	50,429	1, 3

5.2.8 Results of Scenario 8

In Scenario 8, demand is medium-level and distances are high. The optimal solutions for Model 1, Model 2, Model 3, and Model 4 under Scenario 8 are provided in Table 5.36 below. The weighted sum minimization solution is presented in Table 5.37. The summary of the results for Scenario 8 is given in Table 5.38.

Table 5.36 Optimal solutions (x_{ij}^*) of four models separately for Scenario 8

S8	**Model 1**			**Model 2**			**Model 3**			**Model 4**			
Z_1	Z_1^* = 21,405			22,829			21,405			21,869			
Z_2	430,160			Z_2^* = 362,520			484,960			366,600			
Z_3	81,421			83,279			Z_3^* = 81,421			79,496			
Z_4	0%			0%			0%			Z_4^* =5.7%			
x_{ij}^*	**Selected Depots (i)**												
j	1	4	7	1	3	6	1	4	7	1	3	4	6
1	0	0	107	0	0	107	0	0	107	4	0	97	0
2	0	96	5	0	96	5	0	101	0	96	0	0	0
3	0	108	0	0	108	0	0	0	108	0	100	0	2
4	0	113	0	0	113	0	0	0	113	0	99	8	0
5	0	106	0	0	106	0	0	0	106	100	0	0	0
6	0	120	0	0	120	0	0	120	0	0	0	14	100
7	48	57	0	48	57	0	0	105	0	0	100	0	0
8	100	0	0	100	0	0	100	0	0	0	0	0	95
9	116	0	0	116	0	0	116	0	0	0	99	0	11
10	116	0	0	116	0	0	0	116	0	99	0	16	0
11	106	0	0	106	0	0	0	106	0	0	100	0	1
12	114	0	0	114	0	0	0	0	114	100	0	12	0

Table 5.37 Optimal solutions (x_{ij}^*) of $Z_{weighted}$ for Scenario 8

Selected Depots (i)	$Z_{weighted}$ = 68,024 (c = [0.3, 0.1, 0.3, 0.3])											
	Demand Node (j)											
	1	2	3	4	5	6	7	8	9	10	11	12
1	0	0	0	0	0	0	48	100	116	116	106	114
3	0	96	108	113	106	120	57	0	0	0	0	0
6	107	5	0	0	0	0	0	0	0	0	0	0

Table 5.38 Scenario 8 summary

Objective Function	**Value**	**Selected Depots (i)**
Z_1	21,405	1, 4, 7
Z_2	362,520	1, 3, 6
Z_3	81,421	1, 4, 7
Z_4	5.7%	1, 3, 4, 6
$Z_{weighted}$	68,024	1, 3, 6

5.2.9 Results of Scenario 9

In Scenario 9, demand is high and distances are high. The optimal solutions for Model 1, Model 2, Model 3, and Model 4 under Scenario 9 are provided in Table 5.39 below. The weighted sum minimization solution is presented in Table 5.40. The summary of the results for Scenario 9 is given in Table 5.41.

Table 5.39 Optimal solutions (x_{ij}^*) of four models separately for Scenario 9

S9	**Model 1**			**Model 2**			**Model 3**			**Model 4**			
Z_1	Z_1^* = 26,872			28,966			26,872			25,820			
Z_2	564,510			Z_2^* = 465,040			585,930			461,020			
Z_3	102,210			104,950			Z_3^* = 102,210			93,739			
Z_4	0%			0%			0%			Z_4^* =24.8%			
x_{ij}^*	**Selected Depots (i)**												
j	1	4	7	1	3	6	1	4	7	1	3	4	6
1	0	0	134	0	0	134	0	0	134	95	0	0	39
2	0	0	127	0	0	127	0	71	56	100	0	20	0
3	0	0	135	0	0	135	0	0	135	0	0	99	36
4	0	91	51	0	91	51	0	0	142	98	44	0	0
5	0	133	0	0	133	0	0	0	133	0	0	0	100
6	0	150	0	0	150	0	0	150	0	0	50	100	0
7	0	132	0	0	132	0	132	0	0	0	100	0	0
8	31	94	0	31	94	0	125	0	0	0	0	94	0
9	145	0	0	145	0	0	145	0	0	10	100	0	0
10	148	0	0	148	0	0	0	148	0	0	48	0	100
11	133	0	0	133	0	0	45	88	0	0	0	0	100
12	143	0	0	143	0	0	0	143	0	43	0	100	0

Table 5.40 Optimal solutions (x^*_{ij}) of $Z_{weighted}$ for Scenario 9

Selected Depots (i)	$Z_{weighted}$ = 86,615 (c = [0.3, 0.1, 0.3, 0.3])											
	Demand Node (j)											
	1	2	3	4	5	6	7	8	9	10	11	12
1	0	0	0	0	37	150	0	125	145	0	0	143
3	0	0	0	91	96	0	132	0	0	148	133	0
6	134	127	135	51	0	0	0	0	0	0	0	0

Table 5.41 Scenario 9 summary

Objective Function	**Value**	**Selected Depots (i)**
Z_1	26,872	1, 4, 7
Z_2	465,040	1, 3, 6
Z_3	102,210	1, 4, 7
Z_4	24.8%	1, 3, 4, 6
$Z_{weighted}$	86,615	1, 3, 6

5.3 Comparison of Scenario Results

We provide summary tables of optimal objective function values and selected depot locations for Models 1-4 in Table 5.42 and Table 5.43 to analyze the solutions for the case study under nine demand-distance scenarios.

Table 5.42 Summary of optimal objective function values

Scenario	Z_1^*	Z_2^*	Z_3^*	Z_4^*
S1	5,599	249,220	47,385	0.067
S2	7,442	335,830	62,987	0.062
S3	9,347	427,823	79,077	0.248
S4	16,139	267,410	61,305	0.035
S5	21,462	362,660	81,503	0.057
S6	26,937	465,210	102,308	0.248
S7	16,108	267,330	61,259	0.035
S8	21,405	362,520	81,421	0.057
S9	26,872	465,040	102,210	0.248

Table 5.43 Summary of optimal depot locations

Scenario	Model 1	Model 2	Model 3	Model 4
S1	2, 4, 5, 7	1, 3	2, 4, 5, 7	1, 3, 6
S2	2, 4, 5, 7	1, 3, 6	2, 4, 5, 7	1, 2, 3, 6
S3	2, 4, 5, 7	1, 3, 6	2, 4, 5, 7	1, 3, 4, 6
S4	2, 5	1, 3	1, 2, 5	1, 3, 6
S5	1, 2, 5	1, 3, 6	1, 2, 5	1, 3, 5, 6
S6	1, 2, 5	1, 3, 6	1, 2, 5	1, 2, 3, 6
S7	4, 7	1, 3	1, 4, 7	1, 3, 6
S8	1, 4, 7	1, 3, 6	1, 4, 7	1, 3, 4, 6
S9	1, 4, 7	1, 3, 6	1, 4, 7	1, 3, 4, 6

We have deduced the following observations from these results.

1. Given a certain distance scenario, Z_1^*, Z_2^*, and Z_3^* values decrease as the demand is decreased. These objective function values are at their lowest level (best value) for the low demand scenarios (S1, S4, S7).
2. Given a certain demand scenario, Z_1^*, Z_2^*, and Z_3^* values are at their lowest level (best value) for the low demand scenarios (S1, S4, S7) and at their highest level (worst value) for the medium demand scenarios (S2, S5, S8). These objective functions have slightly better (lower) values for the high demand scenarios (S3, S6, S9) than for the medium demand scenarios.
3. In all the models, depot location decisions do not change significantly depending on the level of demand. There are exceptions only in the form of selecting a subset of the depots when the demand is lower, such as in S4 and S7 for Model 1 and S1, S4, and S7 in Model 2.
4. Depot location decisions in Model 1, Model 3, and Model 4 change as the matrices of distance, total transportation cost, and total accumulated waiting time are changed, i.e., as the distances change. We can see this behavior for Model 1: In S4 and S7 two depot locations are selected as opposed to four locations in S1. Also, in S5 (or S6) and S8 (or S9) three depot locations are selected as opposed to four locations in S2 (or S3). Model 3 results also show a similar pattern. However, Model 2 depot location decisions are not affected by the changing distances and this is due to the fact that the objective function cost coefficients are

proportionally increasing as the distances increase. Therefore, the optimal solutions for S4 and S7 are the same for Model 2 and the optimal solution for S1 is slightly different in terms of only 6 x_{ij}^* values.

5. The optimal Model 4 objective function values (Z_4^*) are the same in S3, S6, S9, in which cases the demand is high. So, no matter what the distance scenario is, the best possible maximum percent of unmet demand is 24.8% for this case study. The Z_4^* values for S4 and S7 are equal, the Z_4^* values of S5 and S8 are equal, as well.
6. We can also see from the results of Model 4 that the equity of percent of unmet demand gets worse as the demand gets higher. The percent of unmet demand for each demand node in each scenario are provided in Table 5.44. The standard deviation of percent of unmet demand among demand nodes in S3, S6, and S9 are significantly higher than other scenarios.
7. It is clear from the results that the first model can also present the optimal values not just for Z_1^* also it gives the optimal vales for Z_3^*.

Figures 5.1-5.4 below display the optimal values for each objective function in each scenario.

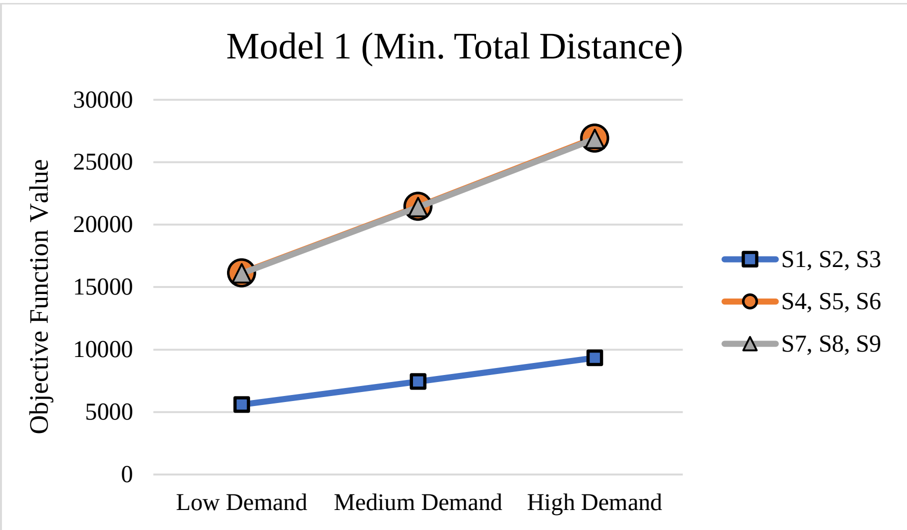

Figure 5.1 Model 1 optimal objective function values for all scenarios

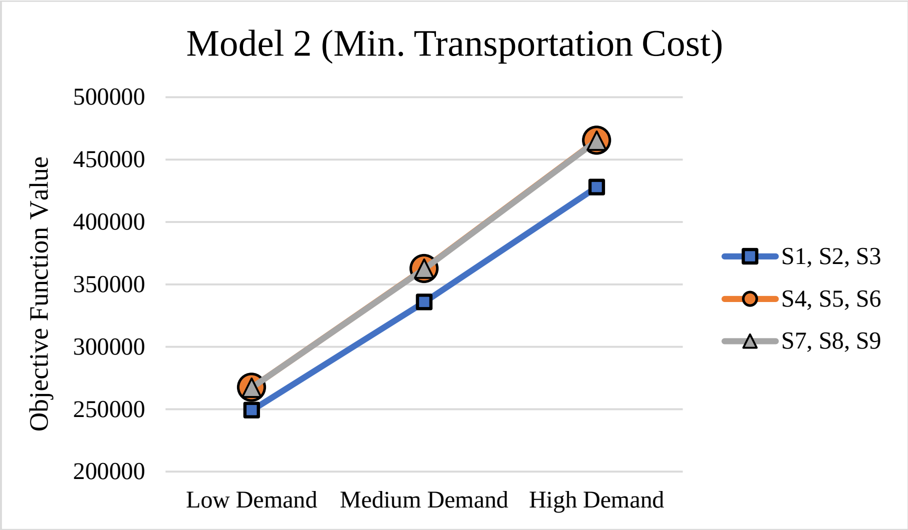

Figure 5.2 Model 2 optimal objective function values for all scenarios

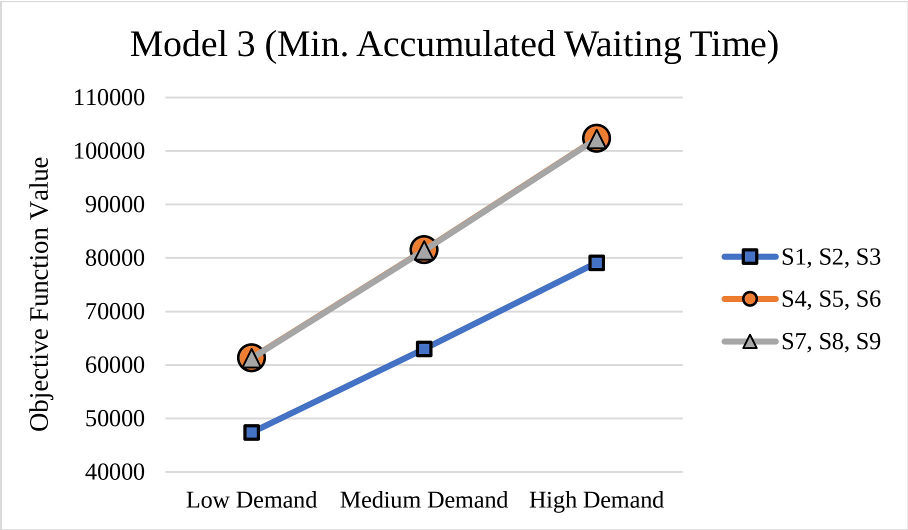

Figure 5.3 Model 3 optimal objective function values for all scenarios

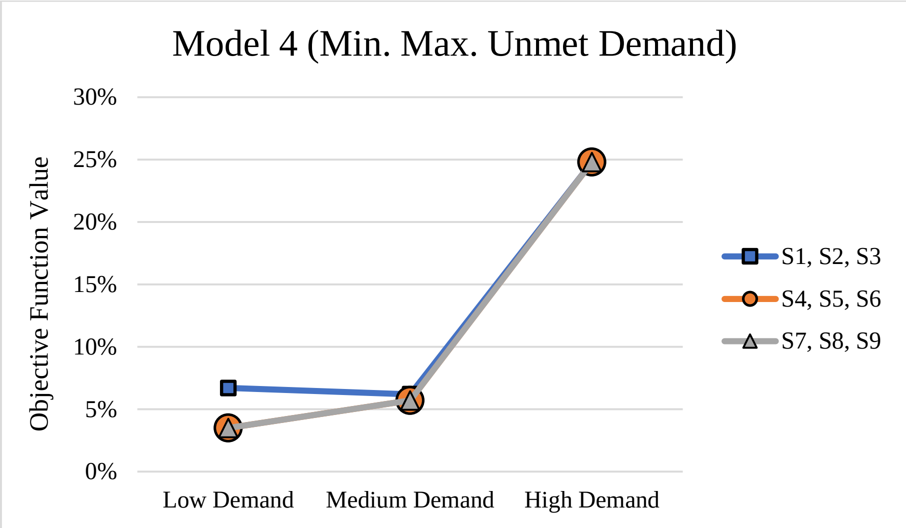

Figure 5.4 Model 4 optimal objective function values for all scenarios

Table 5.44 Percent of unmet demand for each demand node based on Model 4 results in each scenario

j	S1	S2	S3	S4	S5	S6	S7	S8	S9
1	6.7	5.6	0.0	2.5	1.9	0.0	2.5	5.6	0.0
2	6.2	5.9	21.3	2.6	5.0	21.3	2.6	5.0	5.5
3	6.6	5.6	24.4	2.5	0.0	0.0	2.5	5.6	0.0
4	6.2	6.2	0.0	3.5	0.0	0.0	3.5	5.3	0.0
5	5.9	5.7	24.8	2.5	5.7	24.8	2.5	5.7	24.8
6	6.2	5.8	0.0	3.3	2.5	0.0	3.3	5.0	0.0
7	5.6	5.7	24.2	2.5	4.8	24.2	2.5	4.8	24.2
8	6.3	6.0	24.0	2.7	0.0	20.0	2.7	5.0	24.8
9	6.7	6.0	0.0	3.4	2.6	0.0	3.4	5.2	24.1
10	5.7	6.0	0.0	3.4	2.6	0.0	3.4	0.9	0.0
11	6.2	5.7	9.0	2.5	5.7	24.8	2.5	4.7	24.8
12	5.8	6.1	0.0	3.5	2.6	0.0	3.5	1.8	0.0
Standard Deviation	0.372	0.202	11.862	0.458	2.136	11.929	0.458	1.548	12.340

We also compare the results of minimizing the linear combination of weights for the four objective functions in Figure 5.5. The weights of c = [0.3, 0.1, 0.3, 0.3] are used in computations after comparing the performance of various weight combinations. The weighted sum of four objective functions increases as the demand gets higher given any distance scenario. The $Z_{weighted}$ value also increases in the second and third distance scenarios compared to the normal traffic conditions scenario.

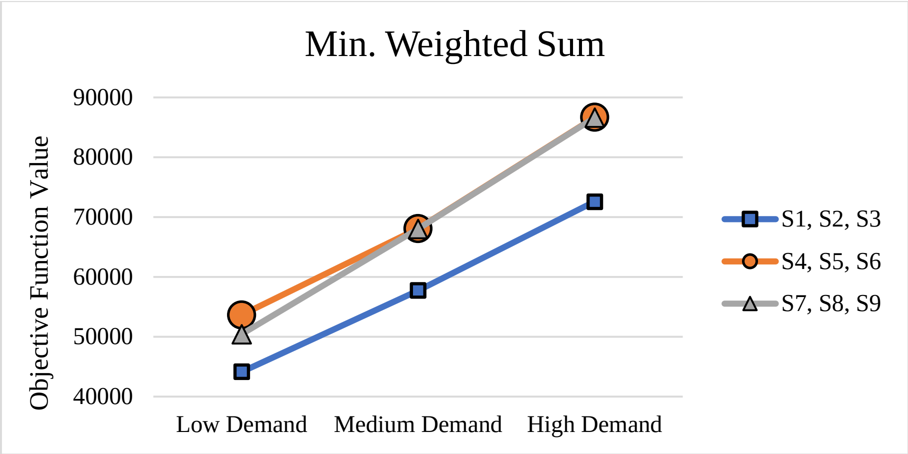

Figure 5.5 Optimal Z_{weight} values for all scenarios

5.4 Computational Results for Stochastic Outcomes

The demand and distance scenarios are defined to have certain probabilities and the scenario probabilities are provided in Table 5.3. Based on these probabilities, we compute the expected objective function values ($E[Z] = \sum_{s\in\Omega} z^* P^s$, where P^s is the probability of scenario s) as shown in Table 5.45 below. These expected costs can help decision makers in planning for disaster relief logistics operations depending on the expected scenarios.

Table 5.45 Expected values of the objective values

	Model 1	**Model 2**	**Model 3**	**Model 4**	**min $z_{weighted}$**
Scenarios	$P^s z_1^*$	$P^s z_2^*$	$P^s z_3^*$	$P^s z_4^*$	$P^s z_w^*$
S1	559.900	24,922.000	4,738.500	0.0067	4,413.500
S2	1,339.560	60,449.400	11,337.660	0.0112	10,391.040
S3	1,121.640	51,338.760	9,489.240	0.0298	8,708.520
S4	1,412.162	23,398.375	5,364.187	0.0031	4,692.100
S5	3,380.265	57,118.950	12,836.722	0.0090	10,722.285
S6	2,828.385	48,847.050	10,742.340	0.0260	9,101.190
S7	1,006.750	16,708.125	3,828.687	0.0022	3,151.812
S8	2,408.062	40,783.500	9,159.862	0.0064	7,652.700
S9	2,015.400	34,878.000	7,665.750	0.0186	6,496.125
$E[Z]$	16,072.125	358,444.160	75,162.950	0.1129	65,329.273

6. CONCLUSIONS

This study focuses on disaster relief logistics where the locations for depots are determined and the distribution of relief items is planned. Different performance measures are defined as the objective functions for determining the number of pallets that should be supplied from each depot to demand nodes. These objective functions helped in choosing suitable depots among several options.

Based on the comparison of results for different objective functions and different scenarios, we can identify the characteristics of the decisions made in each case. We have deduced the following observations from the computational results for the case study. Given a certain distance scenario, total distance (Z_1^*), total transportation cost (Z_2^*), and accumulated waiting time (Z_3^*) values decrease as the demand is decreased. These objective function values are at their lowest level (best value) for the low demand scenarios (S1, S4, S7). Given a certain demand scenario, these three objective function values are at their lowest level (best value) for the low demand scenarios (S1, S4, S7) and at their highest level (worst value) for the medium demand scenarios (S2, S5, S8). We also observed that depot location decisions do not change significantly depending on the level of demand. There are exceptions only in the form of selecting a subset of the depots when the demand is lower. Depot location decisions in Model 1, Model 3, and Model 4 change as the matrices of distance, total transportation cost, and total accumulated waiting time are changed, i.e., as the distances change. However, Model 2 depot location decisions are not affected by the changing distances and this is due to the fact that the objective function cost coefficients are proportionally increasing as the distances increase. The optimal Model 4 objective function values (Z_4^*) are the same in S3, S6, S9, in which cases the demand is high. So, no matter what the distance scenario is, the best possible maximum percent of unmet demand is the same. We can also see from the results of Model 4 that the equity of percent of unmet demand gets worse as the demand gets higher. The standard deviation of percent of unmet demand among demand nodes in S3, S6, and S9 are significantly higher than other scenarios. It is clear from the results that Model 1 can produce the optimal values not only for Z_1 but also for Z_3.

As a future research direction, the assumption that there are enough vehicles to deliver relief aid can be modified such that not all the demanded pallets can be loaded starting at time zero. In this case, either additional vehicles must be waited or the initial vehicles must be waited to return from the demand nodes after delivery. Then, the model for minimizing the accumulated waiting time would be a more realistic representation of the actual problem. Another future research direction would be application of the proposed models based on real data for a central region of a city where the population that can be affected by a disaster is dense.

Uncertainty in distances and demand creates different scenarios for disaster relief logistics. Every scenario represents a unique situation where the different disaster conditions are reflected on the parameter values. These types of stochastic scenarios give us a good opportunity to connect with the reality and to ensure that there is a chance to improve these kinds of models in the future in order to protect the lives of the people and to decrease the huge amount of losses due to different types of disasters.

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APPENDIX A

A.1 GAMS file for Model 1

```
sets
J demand nodes /j1*j12/
I depots /i1*i7/;
parameters
cap(i) capacity of depot i
/ i1=   600
  i2=   600
  i3=   600
  i4=   600
  i5=   600
  i6=   600
  i7=   600 /
dem(j) demand in pallets at demand node j
/ j1 =       81
  j2 =       76
  j3 =       81
  j4 =       85
  j5 =       80
  j6 =       90
  j7 =       79
  j8 =       75
  j9 =       87
  j10=        87
  j11=        80
  j12=        86 /
table d(i,j) distance in km
    j1   j2   j3   j4   j5   j6   j7   j8   j9   j10   j11   j12
i1  10   11   11   10   11   11   10   18   11   10    11    11
i2   7    5    5    7   11   15   16   18   18   16    15    11
i3  11   10    7    5    7   10   11   14   16   15    16    14
i4  16   15   11    7    5    5    7   11   15   16    18    18
i5   7   11   15   16   18   18   16   15   11    7     5     5
i6  11   14   16   15   16   14   11   10    7    5     7    10
i7  16   18   18   16   15   11    7    5    5    7    11    15
```

```
variables
x(i,j) number of pallets delivered to demand nodes
z1 total distance in km;
integer variable x;
binary variable y(i)  if depot i were chosen or not ;

equations
obj1 minimizing total distances to demand nodes
capacity(i) observe capacity limit at depot i
limitation (j) limits the demand with the capacity
numberofdepot(j) limits number of depots to 4;

obj1.. z1 =e= sum((i,j),x(i,j)*d(i,j)  )    ;
capacity(i).. sum(j,x(i,j)) =l= cap(i)*y(i);
limitation(j)..  sum (i,x(i,j)) =g= dem(j);
numberofdepot(j).. sum (i,y(i)) =l= 4;

model location /all/;
Option optcr = 0.00;
solve location using mip minimizing z1;
display  x.l;
```

A.2 GAMS file for Model 2

```
sets
J demand nodes /j1*j12/
I depots /i1*i7/;

parameters
cap(i) capacity of depot i
/ i1=   600
  i2=   600
  i3=   600
  i4=   600
  i5=   600
  i6=   600
  i7=   600 /
dem(j) demand in pallets at demand node j
/ j1=        81
  j2=        76
  j3=        81
  j4=        85
  j5=        80
  j6=        90
  j7=        79
  j8=        75
  j9=        87
  j10=        87
  j11=        80
  j12=        86 /
table tot(i,j) percentage of distribution cost of each pair of depots
and nodes
    j1   j2   j3   j4   j5   j6   j7   j8   j9   j10   j11   j12
i1  245  248  248  245  248  248  245  265  248  245   248   248
i2  308  303  303  308  318  328  330  335  335  330   328   318
i3  268  265  258  253  258  265  268  275  280  278   280   275
i4  380  378  368  358  353  353  358  368  378  380   385   385
i5  328  338  348  350  355  355  350  348  338  328   323   323
i6  288  295  300  298  300  295  288  285  278  273   278   285
i7  400  405  405  400  398  388  378  373  373  378   388   398
```

```
variables
x(i,j) number of pallets delivered to demand nodes
z2 total transportation cost in $;
integer variable x;
binary variable y(i)  if depot i were chosen or not ;

equations
obj2 minimizing total distribution cost
capacity(i) observe capacity limit at depot i
limitation(j) limits the demand with the capacity
numberofdepot(j) limits number of depots to 4;

obj2.. z2 =e= sum ((i,j),(x(i,j)*(tot(i,j)))) ;
capacity(i).. sum(j,x(i,j)) =l= cap(i)*y(i);
limitation(j)..  sum (i,x(i,j)) =g= dem(j);
numberofdepot(j).. sum (i,y(i)) =l= 4;

model location /all/;
Option optcr = 0.00;
solve location using mip minimizing z2;
display  x.l ;
```

A.3 GAMS file for Model 3

```
sets
J demand nodes /j1*j12/
I depots /i1*i7/;

parameters
nv(i) num of vehicles at each depot
/ i1=        5
  i2=        5
  i3=        5
  i4=        5
  i5=        5
  i6=        5
  i7=        5 /
vcap vehicle capacity
/160/
cap(i) capacity of depot i
/ i1=   600
  i2=   600
  i3=   600
  i4=   600
  i5=   600
  i6=   600
  i7=   600 /
dem(j) demand in pallets at demand node j
/ j1=        81
  j2=        76
  j3=        81
  j4=        85
  j5=        80
  j6=        90
  j7=        79
  j8=        75
  j9=        87
  j10=        87
  j11=        80
  j12=        86 /
```

```
table acc(i,j) accumulated waiting time (transportation
time+loading+unloading)
    j1   j2   j3   j4   j5   j6   j7   j8   j9   j10   j11   j12
i1  54   55   55   54   55   55   54   64   55   54    55    55
i2  50   47   47   50   55   60   62   64   64   62    60    55
i3  55   54   50   47   50   54   55   59   62   60    62    59
i4  62   60   55   50   47   47   50   55   60   62    64    64
i5  50   55   60   62   64   64   62   60   55   50    47    47
i6  55   59   62   60   62   59   55   54   50   47    50    54
i7  62   64   64   62   60   55   50   47   47   50    55    60
variables
x(i,j) number of pallets delivered to demand nodes
z3 total accumulated waiting time;
integer variable x;
binary variable
w(i,j) depot i services demand node j
y(i)  if depot i were chosen or not ;
equations
obj3 minimizing total accumulated waiting time
capacity(i) observe capacity limit at depot i
capa(i) limits supplied pallets to nv * vc
cap2(i,j) limits supplied pallets to # of vehicles which is equal # of
served nodes
limitation (j) limits the demand with the capacity
numberofdepot(j) limits number of depots to 4
serving(i) every depot will serve just 5 nodes;
obj3.. z3 =e= sum((i,j), acc(i,j)*x(i,j));
capacity(i).. sum(j,x(i,j)) =l= cap(i)*y(i);
capa(i).. sum(j,x(i,j)) =l= vcap*nv(i);
cap2(i,j)..  x(i,j) =l= vcap*w(i,j)  ;
limitation(j)..  sum(i,x(i,j)) =g= dem(j);
numberofdepot(j).. sum(i,y(i)) =l= 4;
serving(i).. sum (j, w(i,j)) =l= nv(i)*y(i) ;

model location /all/;
Option optcr = 0.00;
solve location using mip minimizing z3;
display  x.l , y.l , w.l;
```

A.4 GAMS file for Model 4

```
sets
J demand nodes /j1*j12/
I depots /i1*i7/;

parameters
nv(i) num of vehicles at each depot
/ i1=        5
  i2=        5
  i3=        5
  i4=        5
  i5=        5
  i6=        5
  i7=        5 /
vcap vehicle capacity
/160/
cap(i) capacity of depot i
/ i1=   600
  i2=   600
  i3=   600
  i4=   600
  i5=   600
  i6=   600
  i7=   600 /
dem(j) demand in pallets at demand node j
/ j1=        81
  j2=        76
  j3=        81
  j4=        85
  j5=        80
  j6=        90
  j7=        79
  j8=        75
  j9=        87
  j10=        87
  j11=        80
  j12=        86 /
```

```
table d(i,j) distance in km
    j1   j2   j3   j4   j5   j6   j7   j8   j9   j10   j11   j12
i1  10   11   11   10   11   11   10   18   11   10    11    11
i2   7    5    5    7   11   15   16   18   18   16    15    11
i3  11   10    7    5    7   10   11   14   16   15    16    14
i4  16   15   11    7    5    5    7   11   15   16    18    18
i5   7   11   15   16   18   18   16   15   11    7     5     5
i6  11   14   16   15   16   14   11   10    7    5     7    10
i7  16   18   18   16   15   11    7    5    5    7    11    15

table tot(i,j) percentage of distribution cost of each pair of depots
and nodes
    j1   j2   j3   j4   j5   j6   j7   j8   j9   j10   j11   j12
i1  245  248  248  245  248  248  245  265  248  245   248   248
i2  308  303  303  308  318  328  330  335  335  330   328   318
i3  268  265  258  253  258  265  268  275  280  278   280   275
i4  380  378  368  358  353  353  358  368  378  380   385   385
i5  328  338  348  350  355  355  350  348  338  328   323   323
i6  288  295  300  298  300  295  288  285  278  273   278   285
i7  400  405  405  400  398  388  378  373  373  378   388   398

table acc(i,j) accumulated waiting time (transportation
time+loading+unloading)
    j1   j2   j3   j4   j5   j6   j7   j8   j9   j10   j11   j12
i1  54   55   55   54   55   55   54   64   55   54    55    55
i2  50   47   47   50   55   60   62   64   64   62    60    55
i3  55   54   50   47   50   54   55   59   62   60    62    59
i4  62   60   55   50   47   47   50   55   60   62    64    64
i5  50   55   60   62   64   64   62   60   55   50    47    47
i6  55   59   62   60   62   59   55   54   50   47    50    54
i7  62   64   64   62   60   55   50   47   47   50    55    60

variables
x(i,j) number of pallets delivered to demand nodes
z4,z percent of unsatisfied demand;
integer variable x;
binary variable
w(i,j) depot i services demand node j
y(i)  if depot i were chosen or not
```

```
zw;
equations
obj  minimizing max unsatisfied demand
obj4(j) max unsatisfied demand
capacity(i) observe capacity limit at depot i
capa(i) limits supplied pallets to nv * vc
cap2(i,j) limits supplied pallets to # of vehicles which is equal # of
served nodes
limitation(j) limits the demand with the capacity
numberofdepot(j) limits number of depots to 4
serving(i) every depot will serve just 5 nodes
zwvalue
zwupper;

obj.. z =e= z4;
obj4(j).. z4 =g= (dem(j)- sum(i,x(i,j)))/dem(j);
capacity(i).. sum(j,x(i,j)) =l= cap(i)*y(i);
capa(i).. sum(j,x(i,j)) =l= vcap * nv(i);
cap2(i,j)..  x(i,j) =l=  vcap* w(i,j)  ;
limitation(j)..  sum(i,x(i,j)) =l= dem(j);
numberofdepot(j).. sum(i,y(i)) =e= 4;
serving(i).. sum (j, w(i,j)) =l= nv(i)*y(i);
zwvalue.. zw =e= (0.3* sum((i,j), x(i,j)*d(i,j))) + (0.1*sum((i,j),
x(i,j)*tot(i,j))) + (0.3*sum((i,j), acc(i,j)*x(i,j)));
zwupper.. zw =l= 40817;

model location /all/;
Option optcr = 0.00;
solve location using minlp minimizing z;
display  x.l , y.l , w.l;
```

APPENDIX B

B.1 Linear Combinations of Weights with GCM

Weights Combinations				Values										
Z1	Z2	Z3	Z4	Z1	Z2	Z3	Z4	Location Depots	ΔZ1%	ΔZ2%	ΔZ3%	ΔZ4%	Total Δ %	GCM %
0.1	0.3	0.3	0.3	979	66482	12943	0.09	1,3,4,6	14.5%	200.0%	200.0%	200%	614.5%	25.679%
0.3	0.1	0.3	0.3	2936	22161	12943	0.03	1,3,4,6	243.4%	0.0%	200.0%	0%	443.4%	25.960%
0.2	0.3	0.2	0.3	1957	66284	8629	0.09	1,3,4,6	128.9%	199.1%	100.0%	200%	628.0%	26.915%
0.2	0.2	0.3	0.3	1957	44321	17257	0.09	1,3,4,6	128.9%	100.0%	300.0%	200%	728.9%	30.272%
0.1	0.2	0.3	0.4	979	44321	12943	0.12	1,3,4,6	14.5%	100.0%	200.0%	300%	614.5%	30.392%
0.1	0.3	0.2	0.4	979	66482	8629	0.12	1,3,4,6	14.5%	200.0%	100.0%	300%	614.5%	30.760%
0.4	0.2	0.2	0.2	2408	71695	10251	0.06	2,4,5,7	181.6%	223.5%	137.6%	100%	642.8%	31.115%
0.4	0.1	0.3	0.2	2408	35847	15377	0.06	2,4,5,7	181.6%	61.8%	256.4%	100%	599.8%	32.099%
0.4	0.1	0.2	0.3	2408	35847	10251	0.09	2,4,5,7	181.6%	61.8%	137.6%	200%	581.0%	32.689%
0.3	0.2	0.2	0.3	2936	44321	8629	0.09	1,3,4,6	243.4%	100.0%	100.0%	200%	643.4%	33.684%
0.4	0.2	0.3	0.1	2408	71695	15377	0.03	2,4,5,7	181.6%	223.5%	256.4%	0%	661.6%	33.858%
0.2	0.2	0.2	0.4	1957	44321	12943	0.12	1,3,4,6	128.9%	100.0%	200.0%	300%	728.9%	34.123%
0.2	0.3	0.1	0.4	1957	66284	4314	0.12	1,3,4,6	128.9%	199.1%	0.0%	300%	628.0%	34.455%
0.4	0.2	0.1	0.3	2408	71695	5126	0.09	2,4,5,7	181.6%	223.5%	18.8%	200%	624.0%	34.629%
0.1	0.1	0.4	0.4	979	22161	17257	0.12	1,3,4,6	14.5%	0.0%	300.0%	300%	614.5%	35.127%
0.1	0.3	0.4	0.2	855	102231	16885	0.09	1,2,5	0.0%	361.3%	291.4%	200%	852.7%	35.329%
0.1	0.4	0.1	0.4	979	88643	4314	0.12	1,3,4,6	14.5%	300.0%	0.0%	300%	614.5%	36.231%
0.3	0.3	0.1	0.3	2936	66482	4314	0.09	1,3,4,6	243.4%	200.0%	0.0%	200%	643.4%	36.511%
0.2	0.3	0.3	0.2	1710	102231	16885	0.09	1,2,5	100.0%	361.3%	291.4%	200%	952.7%	36.835%
0.3	0.3	0.2	0.2	2213	81348	11588	0.12	1.2	158.8%	267.1%	168.6%	300%	894.5%	36.863%
0.4	0.3	0.2	0.1	2408	107543	10251	0.03	2,4,5,7	181.6%	385.3%	137.6%	0%	704.5%	37.151%
0.1	0.2	0.4	0.3	855	68154	16885	0.135	1,2,5	0.0%	207.5%	291.4%	350%	848.9%	37.321%
0.4	0.3	0.1	0.2	2408	107543	5126	0.06	2,4,5,7	181.6%	385.3%	18.8%	100%	685.7%	37.331%
0.2	0.1	0.3	0.4	1957	22161	17257	0.12	1,3,4,6	128.9%	0.0%	300.0%	300%	728.9%	37.628%
0.2	0.1	0.4	0.3	1710	34077	16885	0.135	1,2,5	100.0%	53.8%	291.4%	350%	795.2%	37.634%
0.2	0.2	0.4	0.2	1710	68154	22514	0.09	1,2,5	100.0%	207.5%	421.9%	200%	929.4%	37.995%
0.3	0.2	0.3	0.2	2565	68154	16885	0.09	1,2,5	200.0%	207.5%	291.4%	200%	898.9%	38.172%
0.3	0.4	0.2	0.1	2213	108464	11588	0.06	1.2	158.8%	389.4%	168.6%	100%	816.9%	38.530%
0.1	0.2	0.2	0.5	979	44321	8629	0.15	1,3,4,6	14.5%	100.0%	100.0%	400%	614.5%	38.806%
0.5	0.1	0.2	0.2	3010	35847	10251	0.06	2,4,5,7	252.0%	61.8%	137.6%	100%	551.4%	39.603%
0.2	0.3	0.4	0.1	1710	102231	22514	0.045	1,2,5	100.0%	361.3%	421.9%	50%	933.2%	41.003%
0.3	0.1	0.2	0.4	2936	22161	8629	0.12	1,3,4,6	243.4%	0.0%	100.0%	300%	643.4%	41.040%
0.1	0.1	0.3	0.5	979	22161	12943	0.15	1,3,4,6	14.5%	0.0%	200.0%	400%	614.5%	41.082%
0.3	0.3	0.3	0.1	2565	102231	16885	0.045	1,2,5	200.0%	361.3%	291.4%	50%	902.7%	41.180%

Linear Combinations of Weights with GCM (continued)

0.3	0.2	0.1	0.4	2936	44321	4314	0.12	1,3,4,6	243.4%	100.0%	0.0%	300%	643.4%	41.224%
0.5	0.2	0.2	0.1	3010	71695	10251	0.03	2,4,5,7	252.0%	223.5%	137.6%	0%	613.2%	41.362%
0.2	0.4	0.3	0.1	2213	108464	17382	0.06	1.2	158.8%	389.4%	302.9%	100%	951.2%	41.517%
0.5	0.2	0.1	0.2	3010	71695	5126	0.06	2,4,5,7	252.0%	223.5%	18.8%	100%	594.4%	41.542%
0.1	0.3	0.1	0.5	979	66482	4314	0.15	1,3,4,6	14.5%	200.0%	0.0%	400%	614.5%	41.633%
0.5	0.1	0.3	0.1	3010	35847	15377	0.03	2,4,5,7	252.0%	61.8%	256.4%	0%	570.2%	42.346%
0.3	0.2	0.4	0.1	2565	68514	22514	0.045	1,2,5	200.0%	209.2%	421.9%	50%	881.0%	42.383%
0.1	0.4	0.4	0.1	855	136308	16885	0.045	1,2,5	0.0%	515.1%	291.4%	50%	856.5%	42.402%
0.1	0.4	0.3	0.2	1107	108464	17382	0.12	1.2	29.5%	389.4%	302.9%	300%	1021.8%	42.517%
0.2	0.4	0.2	0.2	2213	108464	11588	0.12	1.2	158.8%	389.4%	168.6%	300%	1016.9%	42.822%
0.5	0.1	0.1	0.3	3010	35847	5126	0.09	2,4,5,7	252.0%	61.8%	18.8%	200%	532.6%	43.116%
0.3	0.4	0.1	0.2	2213	108464	5794	0.12	1.2	158.8%	389.4%	34.3%	300%	882.6%	43.138%
0.3	0.1	0.4	0.2	2565	34077	22514	0.09	1,2,5	200.0%	53.8%	421.9%	200%	875.7%	43.398%
0.2	0.2	0.1	0.5	1957	44321	8629	0.15	1,3,4,6	128.9%	100.0%	100.0%	400%	728.9%	43.766%
0.2	0.1	0.2	0.5	1957	22161	12943	0.15	1,3,4,6	128.9%	0.0%	200.0%	400%	728.9%	44.812%
0.2	0.2	0.5	0.1	1710	68154	28142	0.045	1,2,5	100.0%	207.5%	552.3%	50%	909.9%	45.372%
0.1	0.2	0.5	0.2	855	68154	28142	0.09	1,2,5	0.0%	207.5%	552.3%	200%	959.9%	46.116%
0.2	0.1	0.5	0.2	1710	34077	28142	0.09	1,2,5	100.0%	53.8%	552.3%	200%	906.1%	46.429%
0.1	0.5	0.3	0.1	1107	135580	17382	0.06	1.2	29.5%	511.8%	302.9%	100%	944.2%	47.418%
0.5	0.3	0.1	0.1	3010	107543	5126	0.03	2,4,5,7	252.0%	385.3%	18.8%	0%	656.1%	47.578%
0.2	0.5	0.2	0.1	2213	135580	11588	0.06	1.2	158.8%	511.8%	168.6%	100%	939.2%	47.723%
0.3	0.5	0.1	0.1	2213	135580	5974	0.06	1.2	158.8%	511.8%	38.5%	100%	809.1%	48.091%
0.1	0.5	0.2	0.2	1107	135580	11588	0.12	1.2	29.5%	511.8%	168.6%	300%	1009.9%	48.723%
0.1	0.3	0.5	0.1	855	102231	28142	0.045	1,2,5	0.0%	361.3%	552.3%	50%	963.7%	49.124%
0.1	0.4	0.2	0.3	1107	108464	11588	0.18	1.2	29.5%	389.4%	168.6%	500%	1087.5%	50.488%
0.3	0.1	0.5	0.1	2565	34077	28142	0.045	1,2,5	200.0%	53.8%	552.3%	50%	856.1%	50.774%
0.3	0.1	0.1	0.5	2936	22161	4314	0.15	1,3,4,6	243.4%	0.0%	0.0%	400%	643.4%	51.914%
0.1	0.1	0.5	0.3	855	34077	28142	0.135	1,2,5	0.0%	53.8%	552.3%	350%	956.1%	52.173%
0.2	0.5	0.1	0.2	2213	135580	5794	0.12	1.2	158.8%	511.8%	34.3%	300%	1004.9%	52.331%
0.1	0.1	0.2	0.6	979	22161	8629	0.18	1,3,4,6	14.5%	0.0%	100.0%	500%	614.5%	52.829%
0.4	0.1	0.4	0.1	3420	34077	22514	0.045	1,2,5	300.0%	53.8%	421.9%	50%	825.7%	52.832%
0.1	0.2	0.1	0.6	979	44321	4314	0.18	1,3,4,6	14.5%	100.0%	0.0%	500%	614.5%	53.013%
0.6	0.1	0.2	0.1	3613	35847	10251	0.03	2,4,5,7	322.6%	61.8%	137.6%	0%	522.0%	53.451%
0.6	0.1	0.1	0.2	3613	35847	5126	0.06	2,4,5,7	322.6%	61.8%	18.8%	100%	503.2%	53.631%
0.2	0.4	0.1	0.3	2213	108464	5794	0.18	1.2	158.8%	389.4%	34.3%	500%	1082.6%	54.096%
0.4	0.4	0.1	0.1	3010	143391	5126	0.03	2,4,5,7	252.0%	547.0%	18.8%	0%	817.9%	54.811%
0.6	0.2	0.1	0.1	3613	71695	5126	0.03	2,4,5,7	322.6%	223.5%	18.8%	0%	564.9%	55.391%
0.4	0.1	0.1	0.4	3915	22161	4314	0.12	1,3,4,6	357.9%	0.0%	0.0%	300%	657.9%	56.429%
0.1	0.2	0.6	0.1	855	68154	33770	0.045	1,2,5	0.0%	207.5%	682.8%	50%	940.3%	56.701%
0.1	0.6	0.2	0.1	1107	162692	11588	0.06	1.2	29.5%	634.1%	168.6%	100%	932.2%	56.858%
0.2	0.1	0.6	0.1	1710	34077	33770	0.045	1,2,5	100.0%	53.8%	682.8%	50%	886.6%	57.014%

Linear Combinations of Weights with GCM (continued)

0.1	0.1	0.6	0.2	855	34077	33770	0.09	1,2,5	0.0%	53.8%	682.8%	200%	936.6%	57.758%
0.2	0.1	0.1	0.6	1957	22161	8629	0.18	1,3,4,6	128.9%	0.0%	100.0%	500%	728.9%	57.790%
0.1	0.5	0.1	0.3	1107	135580	5794	0.18	1.2	29.5%	511.8%	34.3%	500%	1075.6%	59.998%
0.2	0.6	0.1	0.1	2213	162692	5794	0.06	1.2	158.8%	634.1%	34.3%	100%	927.3%	60.466%
0.1	0.6	0.1	0.2	1107	162692	5794	0.12	1.2	29.5%	634.1%	34.3%	300%	997.9%	61.466%
0.1	0.1	0.1	0.7	979	22161	4314	0.21	1,3,4,6	14.5%	0.0%	0.0%	600%	614.5%	70.369%
0.7	0.1	0.1	0.1	4215	35847	5126	0.03	2,4,5,7	393.0%	61.8%	18.8%	0%	473.6%	71.048%
0.1	0.1	0.7	0.1	855	34077	39399	0.045	1,2,5	0.0%	53.8%	813.3%	50%	917.1%	71.554%
0.1	0.7	0.1	0.1	1107	189812	5794	0.06	1.2	29.5%	756.5%	34.3%	100%	920.3%	72.839%