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APPLICABILITY OF EVOLUTIONARY GAME THEORETIC MODELS TO DYNAMIC SOCIAL NETWORKS

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MELİS YILMAZ

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KADİR HAS UNIVERSITY

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To my family...

KADIR HAS UNIVERSITY

GRADUATE SCHOOL OF SCIENCE AND ENGINEERING

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“I, Melis Yılmaz, confirm that the work presented in this thesis is my own. Where information has been derived from other sources, I confirm that this has been indicated in the thesis.”

MELİS YILMAZ

ABSTRACT

APPLICABILITY OF EVOLUTIONARY GAME THEORETIC MODELS TO DYNAMIC SOCIAL NETWORKS

Melis YILMAZ

Master of Science in MIS

Advisor: Assoc.Prof.Dr. Mehmet N. AYDIN

JUNE, 2016

It is known that in social communities; humans display two main patterns of behavior: acting in their own utility or acting for the utility of the community in which they exist (cooperation). Mentioned two basic behaviors are addressed by the traditional game theory models which take as their basis the rational human. Therefore, these models accept that the behavior which would bring in the optimal outcome, is the one that maximizes the personal benefits of decision makers. On the other hand, evolutionary game-theoretic models hold that if one player does not know the next move of the other player, a dilemma is created and two basic behaviors of each one of the two players are simultaneously made possible to continue. Thus, it is possible to analyze the behaviors of humans with studies that combine network science, which allows examining dynamic social networks, and evolutionary game theory. In this context; the thesis study thereof explains a social network connection game, in light of the options of sending or not sending connections. Additionally, the study examines the results of relevant researches to investigate effects of network topologies on strategy choices of individuals. In conclusion, it is indicated that a social network connection game can be applicable in measuring decision-making skills of individuals.

Keywords: Cooperation, evolutionary game theory, dynamic social networks, social connectivity

ÖZET

YÜKSEK LİSANS TEZİ
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Sosyal topluluklarda insanların iki temel davranış sergiledikleri bilinmektedir: bireysel çıkarları doğrultusunda hareket etmeleri veya içinde bulundukları sosyal topluluğun çıkarları doğrultusunda hareket etmeleri (kooperasyon). Bu iki temel davranış rasyonel insan varsayımını baz alan geleneksel oyun teorisi modelleri ile incelenmektedir. Dolayısıyla bu modeller optimal sonuç getirecek olan davranışın karar alıcıların kişisel çıkarlarını maksimize edecek olan davranış olduğu kabul etmektedirler. Diğer taraftan evrimsel oyun teorisi modellerinde eğer bir oyuncu diğer oyuncunun gelecek hamlesini bilmiyorsa, çıkmaz yaratılarak oyuncuların iki temel davranışının da devam ettirilmesine imkan sağlanmaktadır. Dolayısıyla dinamik sosyal ağları araştırmaya olanak sağlayan ağ biliminin ve evrimsel oyun teorisinin beraber kullanıldığı çalışmalarda insan davranışlarının analiz edilmesi mümkün olmaktadır. Bu bağlamda bu tez çalışmasında sosyal ağlardaki bağlantı gönderme ve bağlantı göndermeme seçenekleri ele alınarak sosyal ağ bağlantı oyunu gösterilmiştir. Ayrıca sosyal ağların topolojik özelliklerinin bireylerin seçimlerine etkisini araştırmak için ilgili çalışmalar incelenmiştir. Sonuç olarak, sosyal ağ bağlantı oyununun bireylerin karar alma yeteneklerinin ölçülmesi için uygulanılabilir olduğu gösterilmiştir.

Anahtar Kelimeler: Kooperasyon, evrimsel oyun teorisi, dinamik sosyal ağlar, sosyal bağlantısallık

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Table of Contents

List of Tables

List of Figures

Chapter 1 Introduction

1.1 Introduction to the Research

Dynamic social networks have been subject to investigations in an emerging field so-called network science to understand the very nature of interactions among various people in social contexts such as online interactive platforms.

Among others, scholars (e.g., Barabasi, 2007) have emphasized the importance of network topologies, such as scale-free networks or random networks, and evolutionary game theoretical accounts have been employed to illuminate the underpinnings of human interactions in dynamic social networks.

In the context of social networks, both socioemotional and socioeconomic behaviors of people have been examined. Sociologists have argued that socioemotional behaviors of people might be affected by numerous factors. For example, homophily can shape social interactions (Christakis and Fowler, 2010).

Whereas, in socioeconomic context, generally game theorists, who examine strategic decision-makings of people, accept the rationality assumption in traditional game-theoretic models. Therefore, solution concepts of traditional game theory, such as Nash equilibrium, have focused on the best response that results in the highest outcome of a game.

However, there are studies (e.g., Zhang *et al.*, 2015) that focus on the effects of the network topologies (e.g. scale-free network topology) on the evolution of cooperative strategy, which does not yield the highest outcome of a game, in dynamic social networks.

Thus, a conceptual ground on which specific concepts of evolutionary game theory that are linked to certain characteristics of dynamic aspect of social networks might enable researchers to analyze effects of network topologies on evolution of cooperative strategy. In addition, strategic decision-making abilities can be examined with evolutionary game-theoretic models.

In this respect, a social network connection game is considered, along with connect and not connect strategies. Defective strategy (not connect) dominates cooperative strategy (connect), while mutual choice of cooperative strategy yields a higher outcome than mutual choice of defective strategy. Therefore, in a social network connection game, a dilemma occurs that can allow observation of cooperative strategy's evolution in dynamic social networks.

Consequently, one of the purposes of this thesis study is to discuss outcomes of relevant studies to find out effects of network topologies on the emergence and evolution of cooperation. Another aim of this study is to explain a social network connection game to demonstrate application of 2x2 symmetric games to dynamic social networks for measuring "decision-making abilities" of humans regarding risky situations.

1.2 Findings

By following a number of empirical works that combine evolutionary game theory with social networks (e.g., Gracia-Lázaro *et al*., 2012) it can be said that humans have been cooperating in social networks. However, the concept of evolutionary game theory depends on the Darwinian natural selection mechanism that eliminates cooperation from the network population sooner or later (Zhang *et al*., 2015; Nowak, 2006). Therefore, one might think that a 2x2 symmetric matrix can represent two main choices of humans in spatial social networks, but the complexity of social interactions may not be properly explained if strategy update rule depends on the accumulated payoff.

Furthermore, the results of studies (Poncela *et al*., 2009; Ifti, Killingback and Doebeli, 2004) showed that evolution of cooperation hinges upon network topologies (with respect to scale-free characteristics or characteristics of a lattice) in dynamic spatial networks.

On the other hand, Gracia-Lázaro *et al*. (2012) indicated that levels of cooperation are same in both scale-free network and square lattice, when it comes to behaviors of humans in heterogeneous networks (Gracia-Lázaro *et al*., 2012). Given this evidence, it can be inferred that artificial algorithms, which are generated to observe the emergence and evolution of cooperative strategy, differ from the human behaviors in real world. Additionally, contrary to results of other studies, such as (Zhang *et al*., 2015), it can be said that it is possible to measure “decision-making abilities” of humans with the application of continuous 2x2 symmetric games to dynamic social networks.

Therefore, besides examining the effects of social network topologies on levels of cooperation, comparing cooperation levels of people, such as individuals from different range of age groups or professions, might be possible by means of the application of continuous 2x2 symmetric games (e.g. continuous prisoner's dilemma games) to dynamic spatial social networks. This might allow analysis of decision-making abilities of individuals from different social groups, social classes, etc.

In this sense, a social network connection game, which might theoretically allow an empirical survey to compare decision-making skills of humans from different social groups, range of ages, etc., is considered. In this game, defective strategy (not connect) dominates cooperative strategy (connect), whereas mutual selection of cooperative strategy (connect) results in higher outcome than mutual selection of defective strategy (not connect). Thus, a dilemma occurs that makes possible to sustain two strategies (connect and not connect) if this game is continuously played.

As a result, the social interactions of individuals might not be adequately described with a symmetric 2x2 game because of the existence of numerous sociological underlying factors that affect human behaviors in the real world. Whereas, strategic situations (the choice between "connect" and "not connect" strategies) might be described with a 2x2 symmetric game (a social network connection game) to investigate and compare decision-making skills of humans in dynamic spatial social networks.

1.3 Thesis Structure

The thesis consists of four parts. Chapter two focuses on the basic concept of networks from the perspective of network science. The purpose is to supply information about network topological measures, typical network models, and real-world network types. Furthermore, properties of complex systems are clarified due to their importance in terms of understanding evolutionary processes of complex systems.

Chapter three gives an introduction to concept of game theory, and explains both assumptions and solution methods of game theory. Throughout the chapter, concept of evolutionary game theory is expounded, as well as the notions cooperation and defection are made clear.

The last chapter explains a social network connection game that might take place in any social network platform where users interact with each other by sending/receiving connections. Finally, to investigate the effects of network topologies on social cooperation, the results of relevant models are examined.

Chapter 2 Research Background of Networks

In this chapter real-world networks, types of networks and network topological measures are explained. Furthermore, properties of complex systems are clarified. This helps us to point out the importance of complex systems' behavior.

2.1 Network Representation

"A network can be modelled as a graph $G(N, E)$ where N is a finite set of nodes (vertices) and E is a finite set of edges (links) such that each edge is associated with a pair of nodes i and j" (Öbayashi, 2007, p. 84). If each edge of a graph G indicates a direction; then, a graph G is directed that is also known as digraph (Ore and Wilson, 1990).

Barabasi (2012) made clear that the difference between terminologies of the network and the graph is that networks often refer to real-world systems that consist of the combination of nodes and links, whereas vertex and edge are related to the mathematical representations of networks.

Many objects of contexts, such as social sciences (e.g., Centola, 2010) or semantic descriptions (e.g., Borge-Holthoefer and Arenas 2010), have been represented as networks (Baronchelli *et al*., 2013).

To investigate networks as complex systems, there is an emerging discipline called network science. According to The National Research Council, "In short, network science consists of the study of network representations of physical, biological, and social phenomena, leading to predictive models of these phenomena" (Strategy for an Army center for network science, technology, and experimentation, 2007, p. 3).

In this sense, before representing real-world phenomena in network data, a network representation concept is a requirement to differ networks from other phenomena (Bachmaier, Brandes and Schreiber, 2013). Initially, a phenomenon needs to be converted to a network concept; then, the network concept can be represented as network data (Bachmaier, Brandes and Schreiber, 2013).

2.2 Real-World Networks

There are various ways to categorize networks, but for the sake of understanding basic types, in the following, we explain four types real-world networks: information networks, technological networks, biological networks, and social networks. Moreover, main characteristics of social networks and perspective of social network analysis are clarified.

- Technological networks: These types of networks are men- made networks that are generally designed to spread commercial properties or resources, for instance, networks of electricity or information (Newman, 2003). The internet, which physically connects a large number of computers, is another important instance of technological networks (Easley and Kleinberg, 2010).
- Information networks: These networks are represented with vertices and citation patterns (edges). A network of citations between academic papers is a classical example of information networks (Egghe and Rousseau, 1990).

Vertices represent articles and directed edges from each vertex denote citations between the articles (Newman, 2003).

- Biological Networks: One of the most common types of biological networks is the protein-protein interaction network where proteins are represented as agents (Bachmaier, Brandes and Schreiber, 2010). Biological networks, where interactions of different biological organisms arise, can also be called as ecological networks that refer to greater space or timescales (Junker and Schreiber, 2008).

2.2.1 Social Networks

A social network can be represented as a graph where nodes refer to individuals and ties refer to interactions of individuals (Das, 2010). To analyze social interactions, individuals of social networks and their connections are represented as vertices and edges, respectively; it is then possible to explain social networks using graph-based representations (Hansen, Schneiderman and Smith, 2011). Social networking platforms can be networks of professionals and contacts, such as LinkedIn, or Facebook, or can be networks where individuals share contents such as Flickr or Youtube (Benevenuto *et al.*, 2009). Connections of online social networks can be messages, links, posts, documents, locations, or other objects that are created by users (Hansen, Schneirderman and Smith, 2011).

2.2.1.1 Social Network Analysis

In social networks, where individuals are connected by a particular pattern of ties, connections indicate who is connected to whom (Christakis and Fowler, 2010). Some of tie types are communication ties that denote who talks to whom, or who shares information with whom, formal ties that denote who reports to whom, or proximity ties that denote who is spatially close to whom (Katz *et al*., 2004).

Ties of social networks are classified into weak ties and strong ties. Acquaintances who are less likely to be communicated are called weak ties, and close friends who are more likely to be communicated are called strong ties (Granovetter, 1983).

Contrary to physical and biological hierarchies, observations depend on identifying who interacts with whom, but not who is physically close to whom to describe social hierarchies in spatial terms (Simon,1991). In this sense, in terms of social network analysis the primary concern is describing positions of individuals in relation to others to focus on social interactions (Hansen, Schneiderman and Smith, 2011).

Milgram (1967) introduced the small-world phenomenon that is one of the important features of social networks. Milgram indicated that a wide range of short paths exist in social networks and individuals can efficiently detect these short paths in a collective way (Milgram, 1967 as cited in Easley and Kleinberg, 2010). This phenomenon is also known as six degrees of separation which indicates that instead of the wide-spreading pattern of the diffusion of information or an emergent behavior, people tend to prefer a kind of focused search (Milgram, 1967 as cited in Easley and Kleinberg, 2010).

2.2.1.2 Social Capital

Putnam (1995) clarified notion of social capital."By "social capital," I mean features of social life-networks, norms, and trust-that enable participants to act together more effectively to pursue shared objectives. Whether or not their shared goals are praiseworthy is, of course, entirely another matter" (Putnam, 1995, pp. 664-665).

Putnam (2000) examined social networks as specific forms of the social capital (Scott and Carrington, 2011). Scott and Carrington (2011) stated that social networks might be relevant resources of social capital, but they are much more than to be only resources. Therefore, considering a social network as a source of social capital is too restrictive (Scott and Carrington, 2011).

2.3 Types of Network Topologies

The network topologies, such as random graphs, small world networks, or scale-free networks, have an influence on evolutionary dynamics of networks (Barabasi *et al*., 2002).

- Regular networks: Regular networks, which have a high clustering coefficient and characteristic path length, can be explained as nearest-neighbor coupled networks where nodes are connected by a few of their neighbors (Kirley, 2005).
- Random networks: Erdös and Renyi (1959) introduced random networks theory. In random networks, the probability of the connection of two nodes is

random, and the primary concern of random networks is achieving the optimal placement of links randomly between nodes (Barabasi, 2012).

2.3.1 Scale-Free and Small-world as Phenomena of Real-World Networks

The small-world phenomenon exists if any two individuals in the network are connected to their intermediate acquaintances in a short sequence (Kleinberg, 2000).

In the first place, there is a fixed number of vertices in both Erdős-Rényi networks and small world networks; then, the vertices are randomly connected each other in Erdős-Rényi networks, while the vertices are reconnected in small world networks without modifying the fixed number of vertices (Barabasi *et al*., 1999). However, real-world networks are not static, because they continuously evolve by the addition of new nodes to the existing nodes (Barabasi *et al*., 1999).

Furthermore, from the perspective of evolving network theory, instead of the abrupt emergence of the system, evolution of the system as the whole is the main interest (Papacharissi, 2011). Therefore, if an evolving network model is applied, many nodes then would be added, and one can observe that emergent network would be a scale-free network in which hubs would naturally arise (Papacharissi, 2011).

In a network, highly connected nodes are hubs whose removal have larger impact than removal of ones who have lower degrees (Albert *et al*., 2000; Baronchelli *et al*., 2013).

Scale-free network topology is explained in two steps: first step is the growth step which initially starts with a small number (m_0) of vertices, and a newcomer, with m $(\leq m_0)$ edges, is added at each time step; then, this newcomer attaches to a vertex that is already existed in the network (Barabasi *et al*., 1999). The second step is the probability of preferential attachment stage in which the probability π indicates that a newcomer is most likely to attach to a vertex with large number of connections (Barabasi *et al*., 1999).

2.4 Network Topological Measures

In this section quantitave topological network measures: degree distribution, average path length and diameter, and clustering coefficient are explained, as well as the concept of centrality is clarified.

- Degree distribution: Total number of links that are attached to a node is a general definition of degree (Kirley, 2005). Considering degree k_i of a node i, the probability is that a randomly chosen node has exactly k edges, and the distribution function $P(k)$ characterizes distribution of node degrees over the network (Kirley, 2005). In a network, the importance of nodes is determined by the sum of node's connected links, therefore, higher degree implies more importance (Kirley, 2005).
- Average path length and diameter: A path length refers to total number of a path's links (Barabasi, 2012). The shortest path is a path that has the shortest distance between any pair of nodes (Barabasi, 2012). Whereas, average path length measures average distance (number of edges along the shortest path that connects a pair of nodes) in a graph (Kirley, 2005). On the other hand, the diameter refers to the largest distance between any pair of nodes (Barabasi, 2012).
- Clustering coefficient: The local clustering coefficient and the global clustering coefficient are considerable to be aware of meanings of network structures

(Katzir and Hardiman, 2015). Explanation of the local clustering coefficient is "the local clustering coefficient of a node in a graph as the ratio of the number of edges between its neighbors to the maximal possible number of such edges" (Katzir and Hardiman, 2015, p. 540). The clustering coefficient as a local property involves the probability that two nearest neighbors of a node are also the nearest neighbors of each other (Kirley, 2005). Whereas, "The global clustering coefficient of a graph is the ratio of the number of triangles (ordered triples of different nodes in which are all nodes connected) to the number of connected triplets (ordered triples of different nodes in which consecutive nodes are connected)" (Katzir and Hardiman, 2015, p. 540).

- Centrality: One can review various measures used for centrality such as closeness centrality. Centrality, which is often considered as degree centrality, evaluates centrality of nodes by the number of their connected edges (Lerman, Ghosh and Kang, 2010). Centrality measure can only examine the importance of nodes depending on the structure of network in static network analysis (Lerman, Ghosh and Kang, 2010). Whereas, a novel centrality metric, which measures centrality of a node by the number of its paths, of any length, that is attached to this node, is for dynamic network analysis (Lerman, Ghosh and Kang, 2010).

2.5 Complex Systems

Whitesides (1999) stated that a complex system, in which the number of independent components are interacted with each other is large in degree, is very vulnerable to its first conditions or to small perturbations.

On the other hand, a general definition might not adequate to explain intricate characteristics of complex networks. To stress this inadequacy, Barabasi (2007) noted that "A complete theory for complexity does not yet exist, due to failure of the available tools for various reasons. First, most complex systems are not made of identical components, such as gases and magnets. Rather, each gene in a cell or each individual in society has its own characteristic behavior. Second, while the interactions among the components are manifestly nonlinear, truly chaotic behavior is more the exception than the rule. Third, and most important, molecules and people do not obey either the extreme disorder of gases, where any molecule can collide with any other molecule, or the extreme order of magnets, where spins interact only with their immediate neighbors in a periodic lattice. Rather, in complex systems, the interactions form networks, where each node interacts with only a small number of selected partners whose presence and effects might be felt by far away nodes" (Barabasi, 2007, p. 1).

2.5.1 Properties of Complex Systems

- Nonlinearity: According to the superposition principle of linear systems if A and B are both solutions; then, their sum would be $A + B$ (Rickles, Hawe and Shiell, 2007). This means that a linear system can be separate into its parts, and every part of linear systems can be solved one by one for the full solution

(Rickles, Hawe and Shiell, 2007). However, solution of the superposition principle cannot be possible for nonlinear systems due to existence of functions of variables such as $xy, sin(x), x^3$ (Rickles, Hawe and Shiell, 2007). Ladyman, Lambert and Wiesner (2012) pointed out that complex systems' nonlinearity property is not solely enough to represent the whole complexity, because even a simple system can display the nonlinearity property.

- Hierarchical organization: As Simon (1991) stated that organization of organs, tissues, and cells can be an instance of hierarchical organization in biological systems. Cells are organized into tissues, tissues are organized into organs, organs are organized into systems, and the nucleus, mitochondria, or microsomes as well-defined subsystems can be detected with the downward movement in animal cells (Simon, 1991). On the other hand, the hierarchical structure of physical systems generally contains two levels, for instance, planetary systems or galaxies are at a macro level, and elementary particles, atoms, or molecules are at micro level (Simon, 1991).
- Feedback: Rickles, Hawe and Shiell (2007) explained that feedback may arise between micro and macro levels of organisations in complex systems. The micro level interactions between the subunits create some pattern in the macro level interactions; then, the created patterns back-react onto the subunits of the system that leads to a formation of a new pattern which back-reacts again on and on (Rickles, Hawe and Shiell, 2007).
- Robustness: The notion of robustness refers how well the network stays connected when either vertices or edges have been removed (Van Steen, 2010). According to Zhang and Sundaram (2012), the connectivity is a primary interest of the studies of structural robustness that focus on how networks resist

the loss of nodes or edges owing to either unexpected failures or planned attacks. Whereas, the main concern of dynamic robustness analysis is how nodes carry out certain purposes when even some nodes display atypical behaviors (Zhang and Sundaram, 2012). Ladyman, Lambert and Wiesner (2012) concluded that "Robustness seems to be necessary but not sufficient for complexity because a random system can be said to be robust in the trivial sense that perturbations do not affect its order because it doesn't have any" (Ladyman, Lambert and Wiesner, 2012, p.7).

2.5.1.1 Emergence

In complex systems, interactions of agents often result in large-scale behaviors, which are known as emergent behaviors, that occur as a result of unexpected collective interactions which cannot be easily foreseen from knowledge of other agents' behaviors (Mitchell and Newman, 2001).

According to Heylighen (1989), one of the important principles of emergence is that the macro properties of complex systems cannot be separated into the lower order subsystems or its own parts.

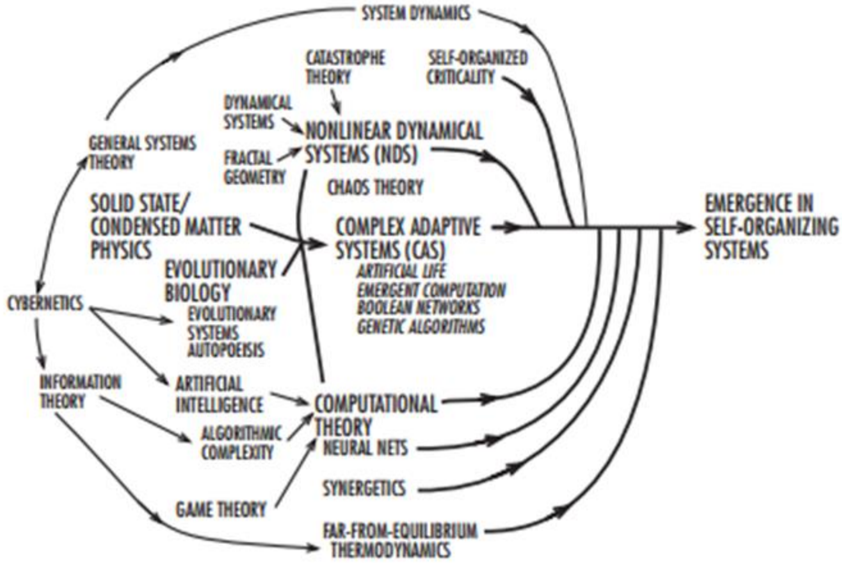

Figure 2.1. Mathematical and scientific roots of emergence (Goldstein, 1999)

It is the view of Mitchell and Newman (2001) that the evolution itself in ecosystems, where species of life forms interact with each other in a cooperative or competitive way, is one of the considerable examples of emergent behaviors. In the process of evolution, both evolution itself and ecosystem variety, which are the examples of emergent behaviors, are mutually responsible for existence of each other (Mitchell and Newman, 2001).

In the social context, the language variation and change, which is a result of the evolutionary change of vocabularies throughout centuries, is an example of emergent behaviors (Sawyer, 2005). Language variation and change often occur due to numberless routine speeches of small groups in societies (Sawyer, 2005).

Chapter 3 Research Background of Game Theory

In this chapter notions about traditional game theory and evolutionary game theory are clarified. Moreover, assumptions and solution concepts of both traditional game theory and evolutionary game theory are explained.

3.1 Concept of Game Theory

Myerson defined game theory as “the study of mathematical models of conflict and cooperation between rational decision makers” (Myerson, 1997, p. 1).

In a normal form (strategic form) game, players adopt strategies so as to make their payoff values as maximum as possible depending on their current information about the game (Rasmusen, 2001). A payoff, which represents profitability of the outcomes to players, is gained by each player at the end of a game (Turocy and Stengel, 2002).

The extensive form is the other representation form of games. In an extensive form game, which is represented with a decision tree or a game tree, nodes represent players who have to make decisions, branches of nodes represent choices that are available to every player, and vectors indicate payoffs of players (Romp, 1997).

3.2 Assumptions of Traditional Game Theory

Common knowledge is a prevalent assumption of traditional game theory. A strategic form (normal form) game consists of rational players who need common knowledge to maximize their utility (Varian, 1992). "Common knowledge of the rules goes even a few steps further: first it says yes, everybody knows that everybody knows that the constitution is available to all. Second, it says that everybody knows that the constitution is widely available. And third, that everybody knows that everybody knows, ad infinitum" (Dutta, 1999, p. 18).

Aumann (1995) explained the solution concept for conditions when common knowledge of rationality would associate with the backward induction method. Williams (2013) clarified the method of backwards induction as "backwards involves working backwards up the game to determine, first, what a rational player would do at the last node of the tree, what the player with the previous move would do given that the player with the last move is rational, and so on until the first node of the tree is reached" (Williams, 2013, p. 55).

However, although decision makers have been assumed to be rational who always move to increase their utilities in traditional game theoretic models, it has been argued that determination criteria of rationality are unclear due to the difficulties of rational decision-making. For instance, the opinion of Gintis (2005) is that rationality assumption can result in both informational and material limitations. Furthermore, expecting from humans to be objective or to be unmoved by both impulsive and predictable emotionality leads to restrictions (Gintis, 2005).

Perfect information is another assumption of traditional game theory. "A game has perfect information when at any point in time only one player makes a move, and knows all the actions that have been made until then" (Turocy and Stengel, 2002, p. 3).

3.3 Nash Equilibrium

Nash (1951) introduced the concept of Nash Equilibrium. Nash equilibrium is a solution concept of game theory that has been prevalently used to examine strategic choices of decision makers (e.g., Daskalakis and Papadimitriou, 2015).

Two types of Nash equilibrium have been defined. According to Darity (2008), "A pure-strategy Nash equilibrium is an action profile with the property that no single player can obtain a higher payoff by deviating unilaterally from this profile" (p. 540). Whereas, "A Nash equilibrium (mixed strategy) is a strategy profile with the property that no single player can, by deviating unilaterally to another strategy, induce a lottery that he or she finds strictly preferable" (Darity, 2008, p. 540).

In order to maximize utility, a natural consistency requirement is that a player's expectations from choices of the opponent have to coincide with each other (Varian, 1992). If expectations are consistent with the real results; then, expectations are considered to be rational, and a Nash equilibrium is related to these rational expectations (Varian, 1992).

Perfect equilibrium is another solution concept of traditional game theory. Selten (1975) proposed concept of a perfect equilibrium to explain conditions when games have many equilibria.

3.4 The Prisoner's Dilemma and The Snowdrift game

Concept of the Prisoner's Dilemma game and the Snowdrift game enable analysis of social dilemmas that occurs due to the ubiquitous emergence and survival of cooperation (Doebeli and Hauert, 2005).

3.4.1 The Prisoner's Dilemma Game

Tucker (1980) considered a fictional scenario that is the basis of the prisoner's dilemma game that describes a situation in which two prisoners have to choose either "confess" or "deny", although they cannot communicate with each other.

Table 3.1. The Prisoner's Dilemma game (Doebeli, M. and Hauert, C., 2005)

	C	D
C	b-c	-c
D	b	0

As shown in Table 3.1, if the mutual choice is C, then a player gains the reward $R = b - c$, but if she adopts D, while the opponent plays C, she obtains $T = b$ (Doebeli and Hauert, 2005). On the other hand, combination of C and D results in a payoff of $-c$, whereas mutual choice of D yields payoff of 0 (Doebeli and Hauert, 2005).

3.4.2 The Snowdrift Game

In the two-player Snowdrift game, players can either choose cooperation (C) or defection (D). Cooperation results in a benefit, b, to both the cooperator and the opponent (Doebelli and Hauert, 2004; Doebelli and Hauert, 2005). If the opponent defects, a cooperator pays cost, c, however, if the opponent cooperates as well, a cooperator then pays only a cost $c/2$ (Doebelli and Hauert, 2004; Doebelli and Hauert, 2005).

Table 3.2. The Snowdrift Game (Doebeli, M. and Hauert, C., 2005)

	C	D
C	b-c/2	b-c
D	b	0

For both cooperator and the opponent, mutual choice of cooperation results in a reward $R = b - c/2$, while mutual choice of defection results in $P = 0$ (Doebelli and Hauert, 2004; Doebelli and Hauert, 2005). When $b > c > 0$, cooperation is a better strategy than defection if the opponent chooses defection, however, if the opponent chooses cooperation, defection is still the best response as shown in Table 2.2. (Doebelli and Hauert, 2004; Doebelli and Hauert, 2005).

3.5 Continuous Prisoner's Dilemma Games

Axelrod (1984) generated algorithm to play continuously the prisoner's dilemma game and introduced the concept of Continuous Prisoner's Dilemma Games.

In spatial continuous prisoner's dilemma games, which is a common model to examine evolution of cooperation in spatial structures, people can either choose defection or cooperation (Ifti, Killingback and Doebeli, 2004). It is known that if individuals interact with their neighbors and learn to contrast their results with their neighbors; then, cooperative investments can evolve to noticeable levels in spatial networks (Ifti, Killingback and Doebeli, 2004).

3.6 Cooperation and Defection in Evolution

Darwin (1859) initiated theory of evolution and introduced the concept natural selection based on his observations about the evolution of biological species. "Evolution is the physical, genetic, or behavioral change in populations of biological organisms over time. Evolution's more interesting and significant manifestations result from natural selection, a process that engineers biological systems" (Vincent and Brown 2005, p. 24).

In the process of evolution, both cooperative and defective behaviors can occur."A cooperator is someone who pays a cost, c, for another individual to receive a benefit, b. A defector has no cost and does not deal out benefits" (Nowak, 2006, p. 1560). Natural selection needs contribution to constitute cooperation in mixed populations, since without any mechanism natural selection always acts to increase the relative abundance of defectors, then leads to disappearance of cooperators from the populations eventually (Nowak, 2006).

However, the presence of altruistic behaviors of biological organisms have been examined in studies of evolutionary biology. For instance, Wilson (2008) examined underlying factors of insects' altruistic behaviors.

Whereas, in the social context, studies of evolutionary game theory generally have focused on cultural evolution. For example, Mesoudi (2011) argued applicability of Darwin’s theory of evolution to cultural evolution such as evolution of the religions or customs.

3.7 Evolutionary Game Theory

Smith and Price (1973) applied Darwin’s theory of evolution to game theory and introduced the concept of evolutionary game theory in their seminal paper. It is known that in evolutionary game-theoretic models, fitness determines the success of organisms, therefore, biological organisms continuously update their strategies in the process of evolution (Aumann and Hart, 1992; McNamara and Weissing, 2010). “In the evolutionary game, players generally inherit their strategies and occasionally acquire a novel strategy as a mutation. The strategy set is determined by genetic, physical, and environmental constraints that may change with time” (Vincent and Brown, 2005, p. 17).

The Moran process designates strategy update rule of organisms in the biologic process of evolution. Cooney, Allen and Veller (2015) clarified the Moran process as “one member of the population is chosen to die; each member is equally likely to be chosen. A new individual is then born, taking the place of the one chosen to die. The new individual is of the same type as its parent, the member chosen for reproduction (which can be the same as the individual that was chosen to die)” (Cooney, Allen and Veller, 2015, p. 9). The indication can be therefore that the concept of evolutionary game theory is designed for biologic populations. However, evolutionary game-theoretic models have been applied to dynamic social networks. For instance, Roos *et al*. (2015) examined cultural distinctions in social norm strength and explain the roots of evolutionary cultural variation in strength of social norms with evolutionary game-theoretic models.

Chapter 4 Evolutionary Game Theoretical Approach to Social Networks

In this chapter relevant models that associate evolutionary game theoretical approach with dynamic social networks are examined in detail. In particular, we explain an application of 2x2 symmetric games to evolving social networks. Then, we introduce a game called social network connection game for measuring decision-making abilities of humans, and we calculate both accumulated payoff and evolutionary stable strategy of this game.

4.1 Relevant Models

As shown in Table 4.1., a number of relevant models investigate effects of network topologies, such as scale-free or square lattice, on emergence and evolution of cooperation in spatial evolving networks.

Ifti, Killingback and Doebeli (2004) aimed to investigate the effects of increasing neighborhood size and connectivity on cooperation in spatial continuous prisoner's dilemma games. The work of Iyer and Killingback (2016) examined social dilemmas (in the prisoner's dilemma, hawk-dove and coordination classes of game) to find out how cooperation evolves in spatial social networks.

Zhang *et al.* (2015) investigated scale-free networks' topological effects on evolution of cooperation in social networks. Similarly, Poncela *et al.* (2009)

examined the effects of the scale-network on evolution of cooperative strategy in a homogeneous network.

Alternatively, Gracia-Lázaro *et al.* (2012) tested human behaviors to find out effects of network topologies (scale-free network and square lattice) on cooperation in heterogeneous networks. In this respect, the results of the studies, which investigate effects of network topologies on evolution of cooperation in social networks, are reviewed to discuss applicability of continuous 2x2 symmetric games to dynamic spatial networks as indicated in Table 4.1.

Table 4.1. Relevant Models Examined

Authors	Models	Purpose
(Zhang, *et al.*, 2015)	Unfavorable Individuals in Social Gaming Networks	Investigate topological properties of less-connected individuals in scale-free networks
(Poncela *et al.*, 2009)	Evolutionary Game Dynamics in Growing Structured Populations	Examine emergence and evolution of cooperation in a growing scale-free network
(Ifti, Killingback and Doebeli, 2004)	Effects of neighbourhood size and connectivity on the spatial Continuous Prisoner's Dilemma	Investigate the effects of neighbourhood size and connectivity on the spatial Continuous Prisoner’s Dilemma Games
(Gómez-Gardeñes *et al.*, 2008)	Natural selection of cooperation and degree hierarchy in heterogeneous populations	Indicate how the dynamical partition correlates with classes of connectivity and typify the temporal fluctuations of the fluctuating set, revealing the mechanisms that hold cooperation stable in macroscopic scale-free structures.
(Iyer and Killingback, 2016)	Evolution of Cooperation in Social Dilemmas on Complex Networks	Study the evolution of cooperation in three exemplars of key social dilemmas in the prisoner’s dilemma, hawk-dove and coordination classes of game.
(Gracia-Lázaro *et al.*, 2012)	Heterogeneous networks do not promote cooperation when humans play a Prisoner’s Dilemma	Examine relevance of playing a spatial Prisoner’s Dilemma on a lattice and a scale-free network with population structure in a human experimental group

4.2 A Social Network Connection Game in Normal Form

In evolutionary game-theoretic models, 2x2 symmetric games have been applied to social networks. For instance, Zhang *et al*. (2015) considered a 2x2 symmetric game, in which $T > R > P > S$, and $2R > T + S$. $\Omega_{ij'}$ denotes the strategy of i against j that obtain vectors $(1,0)^{tr}$and $(0,1)^{tr}$ for the cooperative and defective strategies, respectively (Zhang *et al*., 2015). The payoff G_i represents strategy of i against strategy of j in one stage of the game (Zhang *et al*., 2015).

$$G_i = \Omega_{ij'}{}^{tr} \begin{pmatrix} R & S \\ T & P \end{pmatrix} \Omega_{ij'} \qquad (4.4)$$

On this basis, a scenario is considered to show how 2x2 symmetric games can be applied to social networks, where players interact with each other by sending and/or receiving connections, for measuring decision-making abilities of humans. According to this scenario, mutual not connect results in payoff of (0,0) because there is no interaction between players, and the optimal choice is combination of "not connect" and "connect" strategies (2,-1), since the best response of each player is being not connected while receiving a connection from other player.

On the other hand, mutual connect results in (1,1), because it is assumed that although players send connections to each other, they reciprocally receive connections. While, if a player chooses connect, whereas the opponent chooses not connect; then, she obtains the lowest payoff value of a social network connection game (-1,2).

In a social network connection game, players do not have certain information about the opponent's future actions, therefore, they might choose either connect or not connect strategies. As shown in Table 4.2., there are two players to gain payoffs. Let's explain all eight conditions:

- Player 1 chooses connect and player 2 chooses not connect. In this case payoff value is (-1, 2).
- Player 1 chooses not connect and player 2 chooses not connect. In this case payoff value is (0,0).
- Player 1 chooses connect and player 2 chooses connect. In this case payoff value is (1,1).
- Player 1 chooses not connect and player 2 chooses connect. In this case payoff value is (2,-1).
- Player 2 chooses connect and player 1 chooses not connect. In this case payoff value is (-1,2).
- Player 2 chooses not connect and player 1 chooses not connect. In his case payoff value is (0,0).
- Player 2 chooses connect and player 1 chooses connect. In this case payoff value is (1,1).
- Player 2 chooses not connect and player 1 chooses connect. In this case payoff value is (2,-1).

Table 4.2 A Social Network Connection Game

		Player 2	
		Connect	Not Connect
Player 1	Connect	(1,1)	(-1,2)
	Not Connect	(2,-1)	(0,0)

As a consequence, if players do not know future choices of opponents; then, a dilemma occurs, even though not connect strategy dominates connect strategy (2>-1), because mutual choice of connect strategy yields higher payoff value than mutual choice of not connect strategy (1>0).

4.3 A Social Network Connection Game in Extensive Form

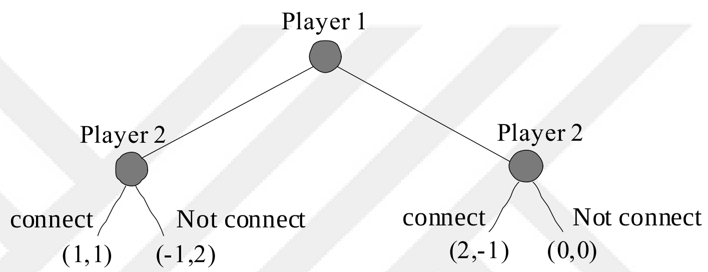

Figure 4.1. A Social Network Connection Game in Extensive Form

In a social network connection game in extensive form, branches of player 1 and player 2 illustrate strategies available for them, and vectors (1,1), (-1,2), (2,-1), (0,0) represent payoffs of players.

However, a social network connection game in extensive form cannot represent the situation in which players simultaneously act without any knowledge about opponents' future actions. Because, a social network connection game in extensive form game can only be applied for the cases where player 1 moves first, and player 2 moves after having observed the choice of player 1.

4.3.1 Evolutionary Stable Strategy of a Social Network Connection Game

In order to calculate evolutionary stable strategy of a social network connection game shown in Table 4.2., we use the calculation method of the body-size game (Easley and Kleinberg, 2010). It is assumed that in a random interaction, for some small positive number x, a $1 - x$ the fraction of the network population chooses connect and an x fraction of the network population chooses not connect.

With probability $1 - x$, if a player chooses connect and the opponent chooses connect as well, she obtains a payoff of 1, while with probability x, if the opponent chooses not connect; then, she obtains a payoff of -1. In this case, her expected payoff is $1(1 - x) + (-1) \cdot x = 1 - 2x$.

With probability $1 - x$, if a player chooses not connect and the opponent chooses connect; then, she obtains a payoff of 2, while with probability x, if the opponent chooses not connect as well, she receives a payoff of 0. In this case, her expected payoff is $2(1 - x) + 0 \cdot x = 2 - 2x$.

It is assumed that in a random interaction, for some very small positive number x, a $1 - x$ fraction of the network population chooses not connect and an x fraction of the network population chooses connect.

With probability $1 - x$, if a player chooses not connect and the opponent also chooses not connect; then, she obtains payoff of 0, while with probability x, if the opponent chooses connect, she obtains payoff of 2. In this case, her expected payoff is $0(1 - x) + 2 \cdot x = 2x$.

With probability $1 - x$, if a player chooses connect and the opponent chooses not connect; then, she obtains payoff of -1, while with probability x, if the opponent chooses connect as well, she obtains payoff of 1. In this case, her expected payoff is $(-1) \cdot (1-x) + 1 \cdot x = 2x - 1$

Since the expected payoff of not connect is higher than the expected payoff of connect in each situation ($2 - 2x > 1 - 2x$ and $2x > 2x - 1$), evolutionary stable strategy is not connect.

4.3.2 Accumulated Payoff Calculation of a Social Network Connection Game

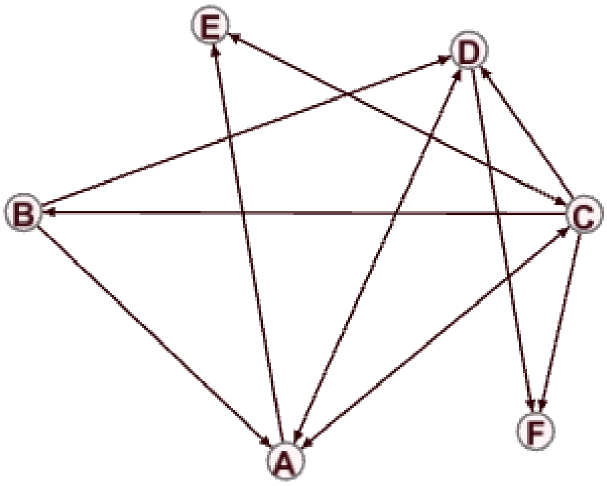

Figure 4.2. A sample of a graph to indicate the calculation of accumulated payoff of a social network connection game

Figure 4.2. illustrates a graph that is generated to demonstrate how to calculate accumulated payoff of a social network connection game. It is assumed that this graph might represent any state of a dynamic social network after it has started to grow.

To indicate calculation of accumulated payoff, nodes A, B, C, D, E and F are considered to be neighbors. Each node iteratively plays a social network connection game with their neighbors at each time step throughout the network lifetime; then, every node obtains an accumulated payoff (π):

- Node A plays with the nodes B, C, D, E and F, then obtains payoffs of $2, 1, 1, (-1), 0$, respectively, and accumulated payoff of node A is $(\pi A) = 2 + 1 + 1 + (-1) + 0 = 3$.
- Node B plays with the nodes A, C, D, E and F, then obtains payoffs of $(-1), 2, (-1), 0, 0$, respectively, and accumulated payoff of node B is $(\pi B) = (-1) + 2 + (-1) + 0 + 0 = 0$.
- Node C plays with the nodes A, B, D, E and F, then obtains payoffs of $1, (-1), (-1), 1, (-1)$, respectively, and the accumulated payoff of node C is $(\pi C) = 1 + (-1) + (-1) + 1 + (-1) = -1$.
- Node D plays with the nodes A, B, C, E and F, then obtains payoffs of $1, 2, 2, 0, (-1)$, respectively, and accumulated payoff of node D is $(\pi D) = 1 + 2 + 2 + 0 + (-1) = 4$.
- Node E plays with the nodes A, B, C, D and F, then obtains payoffs of $2, 0, 1, 0, 0$, respectively, and accumulated payoff of node E is $(\pi E) = 2 + 0 + 1 + 0 + 0 = 3$.
- Node F plays with the nodes A, B, C, D and E, then obtains payoffs of $0, 0, 2, 2, 0$, respectively, and accumulated payoff of node F is $(\pi F) = 0 + 0 + 2 + 2 + 0 = 4$.

4.3.3 Applicability of a Social Network Connection Game to Dynamic Spatial Social Networks

Two Nash equilibria occur (2,-1) and (-1, 2) in a social network connection game due to its symmetric structure. If not connect strategy is played against connect strategy, not connect strategy dominates connect strategy (2>-1). However, mutual connect strategy yields payoff of 1, while mutual not connect yields payoff of 0. For this reason, a dilemma occurs in this game.

Since not connect strategy dominates connect strategy, and evolutionary stable strategy is not connect strategy, it can be said that not connect is defective strategy and connect is cooperative strategy in a social network connection game.

However, static solutions, such as evolutionary stable strategy and Nash equilibrium, seem meaningless if the purpose is both describing how individuals establish or reestablish their connections and how they constitute social networks.

A strategic decision-making situation between connect or not connect strategies can be represented with both a normal form game and an extensive form game. In social network connection game in normal, players cannot observe the other players' following choices and they can unconsciously choose cooperative strategy. If players of a social network connection game continuously play this game at each time step throughout network lifetime; then, they would see that cooperative strategy can yield either higher payoff value than defective strategy or the worst payoff value of this game after any round. Thus, rational strategic decision-making abilities of individuals regarding risky situations can be examined with a social network connection game in normal form (strategic form).

On the other hand, a social network connection game in extensive form represents a situation in which player 1 has chance to observe the choice of player 2. Therefore, a social network connection game in extensive form is not proper to investigate evolution of cooperation due to the advantageous position of player 1.

As a result, if the purpose is both examining evolution of a cooperative strategy in dynamic spatial social networks and analyzing decision makers' choices, application of a social network connection game in normal form seems appropriate.

4.4 Discussion of Results

It is indicated that the evaluation criterion of evolutionary game theory is obviously unfair to less-connected individuals in social networks, because a randomly chosen individual can be removed by her high-fitness neighbor after any round of continuous 2x2 symmetric games (Zhang *et al*., 2015).

Furthermore, it is stated that application of evolutionary game-theoretic models to social networks seems to depend on Darwinian natural selection mechanism which would eventually eliminate cooperation from the network populations (Zhang *et al*., 2015; Nowak, 2006). However, cooperation is an essence of human society, even it is a costly behavior (Fu et al., 2008). On this basis, one can deduce that application of evolutionary game theory to dynamic social networks can lead to controversial results in terms of the examination of human interactions.

Moreover, it is inferred that cooperation levels hinge upon network topologies (Poncela *et al*., 2009; Ifti, Killingback and Doebeli, 2004). Additionally, Poncela *et al*. (2009) had drawn attention to the fact that the evolutionary dynamics alone cannot lead to high levels of cooperation in scale-free networks. It is also demonstrated that cooperation levels decrease after the growing phase of evolving networks (Gómez-Gardeñes *et al*., 2008; Poncela *et al*., 2009).

Besides, Iyer and Killingback (2016) reported that certain structural properties, which are common in real-world social networks, considerably affect the increase of cooperation levels that is present in the prisoner’s dilemma, hawk-dove and coordination classes of games.

On the other hand, the results of a human experiment indicated that cooperation levels are same in both scale-free network and square lattice in heterogeneous networks (Gracia-Lázaro *et al*., 2012). Given this evidence, it might be said that choices of decision makers in real world differ from artificial decision-making algorithms that are synthetically generated to examine effects of network topologies on evolution of cooperative strategy. Additionally, contrary to other works (e.g., Zhang *et al*., 2015) it might be inferred that evolution of cooperation can be observed with the method of accumulated payoff calculation in dynamic social networks.

In addition to analyzing the effects of social network topologies on emergence and evolution of cooperation, cooperation levels of humans from different social groups, genders, etc. might be compared with each other using the concept of continuous 2x2 symmetric games. That is to say individuals from same social groups or classes can play a game among each other; then, cooperation levels of each different groups or classes can be contrasted with each other. This may allow researchers to compare decision-making skills of individuals from different social groups, social classes, etc. regarding risky positions.

Consequently, from evolutionary game theory perspective, a 2x2 symmetric social connection game seems proper for application to examine rational choices of decision makers if this game is continuously played in dynamic spatial social networks, where individuals are represented with nodes and their interactions are represented with edges. However, in sociological context, results of the researches (e.g., Berkowitz, 1972; Kaplan, Grünwald and Hirte, 2016) showed that social interactions of individuals can be formed by various social variables in real world. In this sense, it can be said that a 2x2 symmetric game might be restrictive in terms of representing the whole certain underlying factors that form socioemotional interactions of people in real world. But, to examine decision-making abilities of humans, continuous 2x2 symmetric games might be applicable to dynamic socio-spatial networks where players have to choose one of the two strategies: connect or not connect.

Conclusion

Network topological measures have been used to determine the importance of nodes (e.g. degree distribution, centrality), as well as to understand structure of networks (e.g. clustering coefficient, average path length and diameter) in complex networks. Moreover, it is known that properties, such as nonlinearity, hierarchical organization, feedback, robustness, or emergence affect evolutionary dynamics of complex systems. Additionally, complex social networks exhibit different characteristics (e.g. six degrees of separation) that represent how people interact with each other in dynamic social networks (Easley and Kleinberg, 2010).

However, if the purpose is examining either altruistic behaviors of humans (e.g. emergent behaviors) or rational decision-making skills of individuals in social networks, combination of evolutionary game theory and network science seems proper for application.

Besides the examination of network topological effects on interactions of humans, evolutionary game-theoretic models might be applied in order to compare cooperation levels of humans from different cultures, professions, ages, etc.

In this sense, a social network connection game is articulated and we content that defective strategy (not connect) dominates cooperative strategy (connect), and defective strategy (not connect) is evolutionary stable. In a social network connection game, if not connect strategy is chosen against connect strategy; then, not connect strategy results in higher payoff value than connect strategy. On the other hand, mutual choice of connect strategy yields higher payoff value than mutual choice of not connect strategy. The indication is therefore that a dilemma occurs if players do not have knowledge about opponents' future choices.

It is obvious that both connect and not connect strategies have to be sustained to analyze interactions of individuals in dynamic social networks, and a social network connection game theoretically enables maintenance of cooperation due to the occurrence of a dilemma.

In a nutshell, although intricate socioemotional interactions of humans can be formed by countless factors in real world, such as homophily, by means of the concept of continuous 2x2 symmetric games (e.g. continuous prisoner's dilemma games) "rationality" of decision makers might be examined with 2x2 symmetric games in dynamic social networks as indicated with a social network connection game.

REFERENCES

Albert, R., et al. (2000) Error and attack tolerance of complex networks. Nature 406, 378-382

Aumann, R.J., Hart, S., (1992). Handbook of game theory with economic applications. Elsevier.

Aumann, R. (1995). Backward induction and common knowledge of rationality. *Games and Economic Behavior*, 8(1), pp.6-19.

Axelrod, R. (1984). *The evolution of cooperation*. New York: Basic Books.

Bachmaier, C., Brandes, U. and Schreiber, F. (2010). *Biological networks*. Konstanz: Bibliothek der Universität Konstanz.

Barabasi, A., Albert, R. and Jeong, H. (1999). Mean-field theory for scale-free random networks. *Physica A: Statistical Mechanics and its Applications*, 272(1-2), pp.173-187.

Barabasi, A. (2012). *Network science*. [online] Available at: http://barabasilab.neu.edu/networksciencebook/downlPDF.html

Barabasi, A., Jeong, H., Néda, Z., Ravasz, E., Schubert, A. and Vicsek, T. (2002). Evolution of the social network of scientific collaborations. *Physica A: Statistical Mechanics and its Applications*, 311(3-4), pp.590-614.

Barabasi, A. (2007). *The Architecture of Complexity*. [online] barabasi.com. Available at: http://barabasi.com/f/226.pdf

Baronchelli, A., Ferrer-i-Cancho, R., Pastor-Satorras, R., Chater, N. and Christiansen, M. (2013). Networks in Cognitive Science. *Trends in Cognitive Sciences*, 17(7), pp.348-360.

Benevenuto, F., Rodrigues, T., Cha, M. and Almeida, V. (2009). Characterizing user behavior in online social networks. *Proceedings of the 9th ACM SIGCOMM conference on Internet measurement conference - IMC '09*

Berkowitz, L. (1972). Social Norms, Feelings, and Other Factors Affecting Helping and Altruism. *Advances in Experimental Social Psychology Volume 6*, pp.63-108.

Borge-Holthoefer, J. and Arenas, A. (2010). Semantic Networks: Structure and Dynamics. *Entropy*, 12(5), pp.1264-1302.

BRANDES, U., ROBINS, G., McCRANIE, A. and WASSERMAN, S. (2013). What is network science?. *Network Science*, 1(01), pp.1-15.

Centola, D. (2010). The Spread of Behavior in an Online Social Network Experiment. *Science*, 329(5996), pp.1194-1197.

Christakis, N. and Fowler, J. (2010). *Connected*. Londres: HarperPress.

Cooney, D., Allen, B. and Veller, C. (2015). *Assortment and the evolution of cooperation in a Moran process with exponential fitness*. [online] Arxiv.org. Available at: http://arxiv.org/abs/1509.05757v2

Darity, W. (2008). *International encyclopedia of the social sciences*. Detroit: Macmillan Reference USA.

Das, A. (2010). Social Interaction Process Analysis of Bengalis' on Orkut®. *Handbook of Research on Discourse Behavior and Digital Communication*, pp.66-87.

Daskalakis, C. and Papadimitriou, C. (2015). Approximate Nash equilibria in anonymous games. *Journal of Economic Theory*, 156, pp.207-245.

Dutta, P. (1999). *Strategies and games*. Cambridge, Mass.: MIT Press.

Doebeli, M. and Hauert, C (2004). Spatial structure often inhibits the evolution of cooperation in the snowdrift game. *Nature Materials*, 428(6983), pp.643-646.

Doebeli, M. and Hauert, C. (2005). Models of cooperation based on the Prisoner's Dilemma and the Snowdrift game. *Ecology Letters*, 8(7), pp.748-766.

Easley, D. and Kleinberg, J. (2010). *Networks, crowds, and markets*. New York: Cambridge University Press.

Egghe, L. and Rousseau, R. (1990). *Introduction to informetrics*. Amsterdam: Elsevier Science Publishers.

Erdös, P. and Renyi, A. (1959). On random graphs, I. *Publicationes Mathematicae (Debrecen),*, [online] 6, pp.290-297. Available at: http://www.citeulike.org/group/3509/article/4012374 [Accessed 4 May 2016].

Fu, F., Hauert, C., Nowak, M.A. and Wang, L. (2008) ‘Reputation-based partner choice promotes cooperation in social networks’, *Physical Review E*, 78(2). doi: 10.1103/physreve.78.026117.

Gintis, H. (2005). Behavioral Game Theory and Contemporary Economic Theory. *Analyse & Kritik*, 27(1).

Gracia-Lazaro, C., Ferrer, A., Ruiz, G., Tarancon, A., Cuesta, J.A., Sanchez, A. and Moreno, Y. (2012) ‘Heterogeneous networks do not promote cooperation when humans play a prisoner’s dilemma’, *Proceedings of the National Academy of Sciences*, 109(32), pp. 12922–12926. doi: 10.1073/pnas.1206681109.ccccc

Granovetter, M. (1983). The Strength of Weak Ties: A Network Theory Revisited.*Sociological Theory*, 1, p.201.

Goldstein, J. (1999). Emergence as a Construct: History and Issues. *Emergence*, 1(1), pp.49-72.

Gómez-Gardeñes, J., Poncela, J., Mario Floría, L. and Moreno, Y. (2008). Natural selection of cooperation and degree hierarchy in heterogeneous populations. *Journal of Theoretical Biology*, 253(2), pp.296-301.

Hansen, D., Schneiderman, B. and Smith, M. (2011). *Analyzing social media networks with NodeXL*. Burlington, MA: Morgan Kaufmann.

Heylighen, F. (1989). *Self-organization, Emergence and the Architecture of Complexity*. [online] Transdisciplinary Research Group,. Available at: http://pespmc1.vub.ac.be/Papers/Self-OrgArchComplexity.pdf [Accessed 4 May 2016].

Ifti, M., Killingback, T. and Doebeli, M. (2004). Effects of neighbourhood size and connectivity on the spatial Continuous Prisoner's Dilemma. *Journal of Theoretical Biology*, 231(1), pp.97-106.

Iyer, S. and Killingback, T. (2016) ‘Evolution of cooperation in social dilemmas on complex networks’, *PLOS Computational Biology*, 12(2), p. e1004779. doi: 10.1371/journal.pcbi.1004779.

Junker, B. and Schreiber, F. (2008). *Analysis of biological networks*. Hoboken, N.J.: Wiley-Interscience.

Kaplan, S., Grünwald, L. and Hirte, G. (2016). The effect of social networks and norms on the inter-regional migration intentions of knowledge-workers: The case of Saxony, Germany. *Cities*, 55, pp.61-69.

Katz, N., Lazer, D., Arrow, H. and Contractor, N. (2004). Network Theory and Small Groups. *Small Group Research*, 35(3), pp.307-332.

Katzir, L. and Hardiman, S. (2015). Estimating Clustering Coefficients and Size of Social Networks via Random Walk. *ACM Trans. Web*, 9(4), pp.1-20.

Kirley, M. (2005). Complex networks and evolutionary game. *Complexity International*, [online] 12, pp.1-10. Available at: http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.324.565&rep=rep1&type=pdf [Accessed 4 May 2016].

Kleinberg, J. (2000). The small-world phenomenon. *Proceedings of the thirty-second annual ACM symposium on Theory of computing - STOC '00*.

Ladyman, J., Lambert, J. and Wiesner, K. (2012). What is a complex system?. *European Journal for Philosophy of Science*, 3(1), pp.33-67.

Lerman, K., Ghosh, R. and Kang, J. (2010). Centrality metric for dynamic networks. *Proceedings of the Eighth Workshop on Mining and Learning with Graphs - MLG '10*.

McNamara, J. M., & Weissing, F. J. (2010). Evolutionary game theory. In T. Székely, A. J. Moore, & J. Komdeur (Eds.), Social behaviour: Genes, ecology and evolution (pp. 109–133). Cambridge: Cambridge University Press.

Mesoudi, A. (2011). *Cultural evolution*. Chicago: University of Chicago Press.

Mitchell, M. and Newman, M. (2001). *Complex systems theory and evolution*. [online] Santa Fe Institute. Available at: http://s3.amazonaws.com/academia.edu.documents/30841539/10.1.1.63.2636.pdf?A

WSAccessKeyId=AKIAJ56TQJRTWSMTNPEA&Expires=1470316772&Signature=%2FC1Iswsg07mk6sIqjfXriOPj1YA%3D&response-content-disposition=inline%3B%20filename%3DComplex_systems_theory_and_evolution.pdf [Accessed 4 May 2016].

Myerson, R. (1991). *Game theory*. Cambridge, Mass.: Harvard University Press.

Nash, J. (1951). Non-Cooperative Games. *The Annals of Mathematics*, 54(2), p.286.

Newman, M. (2003). The Structure and Function of Complex Networks. *SIAM Rev.*, 45(2), pp.167-256.

Nowak, M. (2006). Five Rules for the Evolution of Cooperation. *Science*, 314(5805), pp.1560-1563.

Ōbayashi, S. (2007). *Evolutionary multi-criterion optimization*. Berlin: Springer.

Ore, Ø. and Wilson, R. (1990). *Graphs and their uses*. Washington: Mathematical Association of America.

Papacharissi, Z. (2011). *A networked self*. New York: Routledge.

Poncela, J., Gómez-Gardeñes, J., Traulsen, A. and Moreno, Y. (2009). Evolutionary game dynamics in a growing structured population. *New Journal of Physics*, 11(8), p.083031.

Putnam, R. (1995). Tuning In, Tuning Out: The Strange Disappearance of Social Capital in America. *PS: Political Science & Politics*, 28(04), pp.664-683.

Putnam, R. (2000). *Bowling alone*. New York: Simon & Schuster.

Rasmusen, E. (2001). *Games and information*. Malden, Mass.: Blackwell Publishers.

Rickles, D., Hawe, P. and Shiell, A. (2007). A simple guide to chaos and complexity. *Journal of Epidemiology & Community Health*, 61(11), pp.933-937.

Romp, G. (1997). *Game theory*. Oxford [England]: Oxford University Press.

Roos, P., Gelfand, M., Nau, D. and Lun, J. (2015). Societal threat and cultural variation in the strength of social norms: An evolutionary basis. *Organizational Behavior and Human Decision Processes*, 129, pp.14-23.

Sawyer, R. (2005). *Social emergence*. Cambridge: Cambridge University Press.

Scott, J. and Carrington, P. (2011). *The SAGE handbook of social network analysis*. London: SAGE.

Selten, R. (1975). Reexamination of the perfectness concept for equilibrium points in extensive games. *International Journal of Game Theory*, 4(1), pp.25-55.

Simon, H. (1991). The Architecture of Complexity. *Facets of Systems Science*, pp.457-476.

SMITH, J. and PRICE, G. (1973). The Logic of Animal Conflict. *Nature*, 246(5427), pp.15-18.

Strategy for an Army center for network science, technology, and experimentation. (2007). Washington, D.C.: National Academies Press.

Tucker, A. (1980). On Jargon: The Prisoner's Dilemma. *UMSP Journal*, 1, p.101.

Turocy, T. and Stengel, B. (2002). *Game Theory∗*. [online] Available at: http://s3.amazonaws.com/academia.edu.documents/30712690/TurocyStengelGameTheorySurvey.pdf?AWSAccessKeyId=AKIAJ56TQJRTWSMTNPEA&Expires=1470321001&Signature=uFEXY6NZOcmF8A91DapkfhG%2BzFk%3D&response-content-disposition=inline%3B%20filename%3DTexas_A_and_M_University.pdf [Accessed 4 May 2016].

Travers, J. and Milgram, S. (1969). An Experimental Study of the Small World Problem.*Sociometry*, 32(4), p.425.

Van Steen, M. (2010). *An Introduction to Graph Theory and Complex Networks*. [online] Available at: http://pages.di.unipi.it/ricci/book-watermarked.pdf [Accessed 4 May 2016].

Varian, H. (1992). *Microeconomic analysis*. New York: Norton.

Vincent, T. and Brown, J. (2005). *Evolutionary game theory, natural selection, and Darwinian dynamics*. Cambridge: Cambridge University Press.

Williams, P. (2013). *Security studies*. Abingdon, Oxon: Routledge.

WILSON, E. (2008). One Giant Leap: How Insects Achieved Altruism and Colonial Life. *BioScience*, 58(1), p.17.

Whitesides, G. (1999). Complexity in Chemistry. *Science*, 284(5411), pp.89-92.

Zhang, H. and Sundaram, S. (2012). Robustness of complex networks with implications for consensus and contagion. *2012 IEEE 51st IEEE Conference on Decision and Control (CDC).*

Zhang, Y., Chen, G., Guan, J., Zhang, Z. and Zhou, S. (2015). Unfavorable Individuals in Social Gaming Networks. *Sci. Rep.*, 5, p.17481.