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ALGORITHM WITH LEAST SQUARE METHOD

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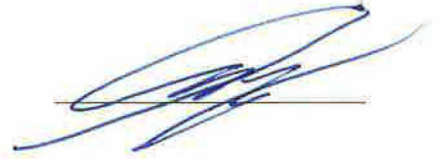
INDOOR POSITIONING SYSTEM DEVELOPMENT / TRIANGULATION ALGORITHM
WITH LEAST SQUARE METHOD

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INDOOR POSITIONING SYSTEM DEVELOPMENT / TRIANGULATION ALGORITHM WITH LEAST SQUARE METHOD

Abstract

Nowadays, smartphone market penetration continues to grow with developing technology. Accordingly, position detection in closed areas has become an important research area. For instance; finding a direct route to the gate based on location at an airport, determining a route to the destination that could be a shop or cafe at a shopping center or informing about sales discount to increase sales using location are several applicable areas of position estimation. In the thesis, I developed triangulation algorithm more efficient using least square method with the developments of Wi-Fi channel fixing, optimized A and n values used in log normal formula and more than 3 access points. I used synthetic data which is created from sample data and estimate location for comparison to analyzing success rate of algorithm. According to the measurement results, triangulation algorithm with least square method, channel fixing, optimized A and n values, more than 3 Access Points gives accurate location in closed areas more than simple triangulation algorithm does. The thesis will lead to detect position in closed areas and use it in daily lives using triangulation algorithm with least square method.

Key words: Indoor positioning development, triangulation algorithm, least square method

KAPALI ALANDA KONUM TESPİTİ / EN KÜÇÜK KARELER YÖNTEMİ İLE TRIANGULATION ALGORİTMASI

Özet

Gelişen teknoloji ile birlikte her geçen gün akıllı telefon kullanım oranı artmaktadır. Buna bağlı olarak kapalı alanda konum tespiti daha da önemli olmaktadır. Örneğin havaalanında konum bazlı olarak hangi kapıdan girileceğine göre yön tayini yapılması, büyük alışveriş merkezinde gidilmek istenilen mağazaya konum bazlı yön gösterilmesi ya da kişinin alışveriş merkezindeki lokasyonuna göre satışı arttırmak amacıyla, konuma göre indirim fırsatı sunulması kapalı alanda konum tespitinin kullanılacağı yerlerden birkaç tanesidir. Tezimde, üçgenleme algoritmasını en küçük kareler yöntemi ile birlikte kullanıp, Wi-Fi kanal sabitleme, log normal formülünde kullanılan A ve n değerlerinin optimizasyonu ve 3 ten fazla kullanımı geliştirmeleri ile daha verimli yapıyorum. Test ortamından elde ettiğim dataları kullanarak tahmin edilen konumları karşılaştırma amacıyla, sentetik olarak veri hazırlayıp konum tahmin ederek algoritmanın başarısını inceliyorum. Ölçüm sonuçları gösteriyor ki, üçgenleme algoritması en küçük kareler yöntemi, kanal sabitleme, optimize A ve n değerleri ve 3ten fazla modem kullanımı ile birlikte kapalı alanda konumu, yalın üçgenleme algoritmasına göre daha doğru buluyor. Bu tez, üçgenleme algoritması ile en küçük kareler yönteminin birlikte kullanılarak kapalı alanda konum tespit edilip, hayata geçirilmesi konusunda yol gösterici olacaktır.

Anahtar Kelimeler: Kapalı alanda konum tespiti, üçgenleme algoritması, en küçük kareler yöntemi

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1 INTRODUCTION

The research area concerning about techniques for indoor positioning system. With the increase in smartphone usage, many applications will depend on, or will be improved by indoor positioning system (IPS). Thus, position detection capability is added to the applications and people are more interested in determining indoor positioning system.

The importance of smartphones is increasing with the related technology development. Smartphones are used very frequently in social life. In addition to smartphone usage frequency, we spend a lot of time with our smartphones in different environments.

Smartphone innovations facilitate people's lives. Position detection is one of the innovation are available for using in life. With the location positioning system, people can find address easily.

Outdoor positioning system is an invaluable way to find the road, arrive at the destination quickly when lost or trying to find a location. Utilization rate of global positioning system (GPS) is increasing consistently and stay number one because of providing time and cost advantages. Like that outdoor positioning system, indoor positioning system gives facilities to people in different locations such as hospitals, shopping centers, hotels and other indoor locations.

Although IPS is a new technology, it is spreading rapidly. With IPS, all the features described below can be possible.

- *Sharing location in a closed area*
- *Sending location information*
- *Location based advertising*
- *Locating people in closed environments for example, finding closest staff more quickly instead of alarms system in hospital*

- *Finding addresses in closed environments such as the path to the gate in an airport, or the path to a desired shop in a shopping center*

In this thesis, I develop new IPS algorithms for triangulation algorithm which can provide more accurate position and resolve the disadvantages of simple triangulation algorithm. The proposed triangulation algorithm tries to minimize average error meter with least square method, Wi-Fi channel fixing, optimize A and n values and using more than 3 access points.

The rest of the paper is organized as follows: Chapter 2 introduces with Previous Works, Chapter 3 describes triangulation algorithm and log normal formula, Chapter 4 depict least square method and least square method formula. Result of the triangulation method with least square method and optimize A and n value is shown in Chapter 5. Also, the result of the more than 3 access point measurement shown in the same chapter. The result of the synthetic dataset which can comparison purposes is shown in Chapter 6 and finally Chapter 7 concludes this paper and suggests for future research on that thesis.

2 PREVIOUS WORK

Location positioning system is a major research area. Significant techniques have been proposed for finding or estimating location indoors and outdoors. The most known techniques are GPS, beacon and Wi-Fi. In this thesis, a system selected which is using efficiently in indoor positioning.

2.1 GPS

Smart phones become more important in our lives day by day. GPS which is brought by smart phone popularity and its biggest innovation can make a determination of a location. GPS gets its name from first letters of “Global Positioning System”. This system belongs to the United States Department of Defense and it consists satellites which are constantly revolving. Those satellites spread beam and GPS receiver catches these signals. The geolocation becomes possible in this way.

Further thought GPS comprise of at least 24 satellites (21 of them are actives and 3 of them are backups). Those satellites located 20,200 kilometers above the surface of the world. Satellites have quite wide field of view and they located properly for GPS receiver that always sees at least 4 of them [1].

This system basically involves the observation and calculations from an unknown location to known location. Known locations are GPS satellites. Unknowns are Cartesian coordination's of earth-fixed point of presence. (X, Y, Z) At this system data taken from 24 satellites but nearest 3 satellite's data is used at Triangulation Algorithm. GPS receiver scratches a sphere around those 3 satellites for the geolocation and 3 spheres interrupt each other at 2 points. One of those points is on the space and the other is on the earth. The point which is on the earth is the location that is desired to be detected [2].

The determination of the location with GPS is the widely used technology. But it is not possible to have good results with GPS at the places like underwater, tunnels or with the dense of

buildings. There is a need of open area to be able to see the satellites. Because of this reason GPS cannot be used for geolocation at the closed areas.

2.2 Beacon

Beacon is a location based small device which is generally works with a battery and uses Bluetooth low energy – BLE technology. These devices communicate with smart devices in their range through beam. Mobile devices catch signals coming from beacons and calculate location comparing the reference signal levels. To give an example for 1 meter of distance, reference signal level gets -60 dBm of signal strength. When signal level taken from beacon gets more than -60Dbm of signal strength, it can be determined that mobile device is less than 1 meter away from the beacon. Therefore location-based content and notification presented through mobile device [3].

Rather than Wi-Fi network, Low energy Bluetooth signals that are used by Beacon to detect location are not affected by physical barrier. However it can penetrate through walls. Moreover, BLE spends mobile device batteries less than the standard Bluetooth signal. iOS devices get BLE signals continually when it is in beacon' s range even though application is closed. Because Apple has own Beacon protocol so that when the application is closed, it can open related applications automatically. However android devices are not integrated operating system level beacons like iOS. Therefore, android devices must search particular BLE signals and the beacon application must be open in that device. Therefore, android devices battery will die fast, because this application will search continuous signal [4], [5].

Because of beacons give information and content according to the reference signal, geolocation is mostly done according to entering a specific area. For example it's possible that give information to people who visit to see pieces at the museum by placing beacon into pieces. This system does not step in according to the place where person is located but it does according to low signal reference taken by device from a beacon [6]. So the beacon system confirms that if there is an entry to the determined location or not. It doesn't use learned reference signals to determine a location.

2.3 Wi-Fi

Wi-Fi is the abbreviation of the word “Wireless Fidelity”. It means that wireless commitment or wireless connection. It shows that, products are capable of providing wireless connectivity with this technology. Wi-Fi is determined according to standard which are IEEE 802.11a, IEEE 802.11b, IEEE 802.11g and IEEE 802 [7].

Traditional location detection system based on Wi-Fi uses Wi-Fi fingerprint method which is one of the most common methods. This method requires a signal map which is created previously in different locations and displays signal strengths of Wi-Fi points. Therefore, Wi-Fi points which are around us and other Wi-Fi Points which have previously known location are compared. Thus, location is detected by fingerprint method. This method is used in closed areas, but for the first time, location detection in a new area is completed slower than the known area, because background information of Wi-Fi locations still exists in database. Additionally, fingerprint database must be regularly updated in closed areas, the reason behind this information is the following; for instance, when the furniture is moved another place, data of their locations in database must be also changed [8], [9].

Other most commonly used location detection method based on Wi-Fi is Wi-Fi triangulation. In this method, location detection is performed with the signal received from three access points [9]. The signals received from the access points create circles around themselves like GPS does. In this way, location of the mobile device is detected in where these circles conflict. The only difference between the GPS position and Wi-Fi triangulation method is that GPS receives signals from satellites but Wi-Fi based triangulation algorithm receives signals from access points. In this thesis, Wi-Fi based triangulation algorithm is selected and the algorithm is developed with least square method by adding new features, such as channel fixing to detect position.

2.4 Goal

The error rate in the indoor positioning system based on Wi-Fi is very high when the simple triangulation algorithm is used. The reason why the error rate is high is signal overlapping. In this thesis, simple triangulation algorithm has been developed with least square method to reduce the error rate. Thus, location detection is performed more accurately than simple

triangulation algorithm based on Wi-Fi. This thesis is a guide to detect location more accurate in closed areas using triangulation algorithm with least square method in indoor positioning system.



3 TRIANGULATION FOR INDOOR POSITIONING SYSTEM

3.1 Triangulation Algorithm

Triangulation is a powerful technique of finding location of unknown point, by measuring exact known location of two points. In this technique, reference objects are taken as basis for finding the unknown coordinates. After this step, the separation process performed on the triangle and detects the unknown coordinates approximately. Thus, position is calculated with measuring the intensity of the signal from several Wi-Fi hotspots.

Basically, indoor positioning system of triangulation works with the principle of the GPS system. Therefore, this algorithm also uses the same algorithm with the GPS triangulation. The only difference from the Wi-Fi technology, GPS technology uses satellites but Wi-Fi technology uses access points.

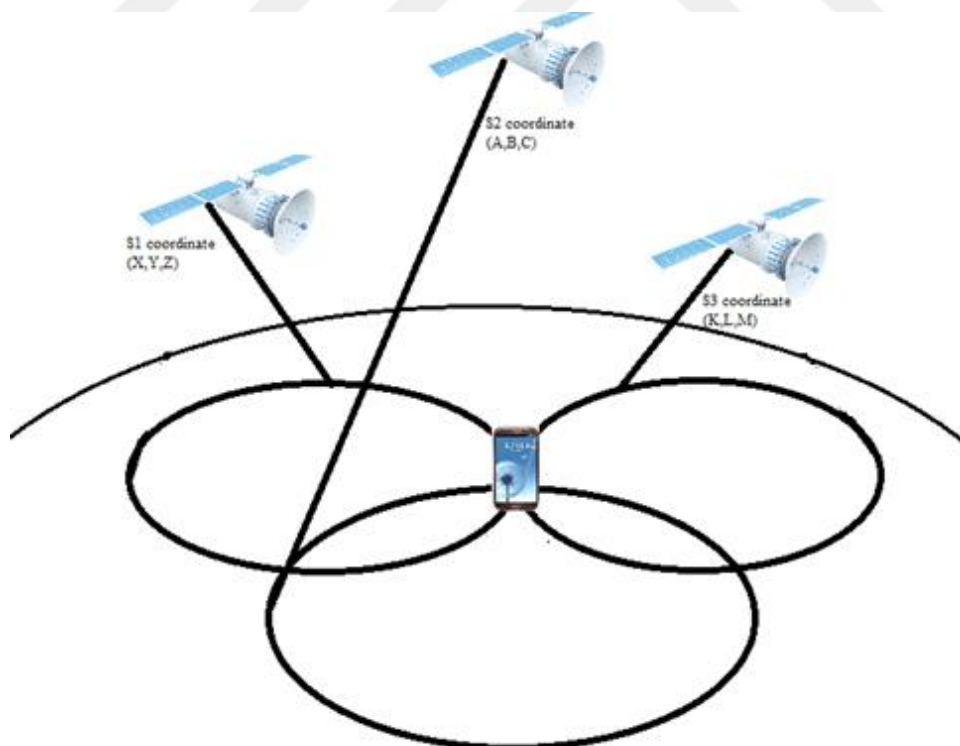


Figure 3.1 An example of GPS positioning using 3 satellites

As shown in figure 3.1, 3 satellites examined when the determining the position of the mobile phone. This phone is the located in the area of the intersection of the signal emitted from the satellite. When the location of the phone is determined, triangulation algorithm is used in outdoor positioning system.

Also GSM companies calculate the location information using triangulation algorithm with the base station is connected to mobile and surrounding base stations. On the related of the software must be installed on the mobile device in this system [10]. For the unknown position detection, the position of the base stations and the communication with the mobile device is so important.

At least three base stations in the GSM location detection technology are considered the reference point (tower point). The position of the unknown object is approximately determined through a mathematical algorithm using the coordinate values of these points. The only difference is the GSM algorithm (cellular communication network) is long distance wireless technology. However, Wi-Fi is middle distance wireless technology. Hence, these two technology works with geometric triangulation algorithm [11].

Today, internet usage rate has increased steadily in closed area day by day, with the development of the technology. Therefore access point has become common in indoor easily. So that Wi-Fi signal was started to be used more in closed area.

Wi-Fi signals should be used in closed area with the excess number of access point. If the signal received from an access point, it should work independently of the measured area regardless of whether the triangle, square, rectangular etc. The location of the object in a closed area is determined with the received signal being converted into the distance with the necessary technology.

In this thesis, geometric mathematical algorithm which is triangulation preferred because of too much access points in indoor location and the absence of reference points necessary to provide a specific geometric shape. When the conditions provided for triangulation algorithm, the position of objects in indoor areas were calculated approximately. Because of triangulation is often used in wireless communications, to identify the location of cell phones, walkie-talkies and many others.

Geometric triangulation algorithm requires at least three reference points like GPS. Three reference points, which known of x and y coordinates, are taken on a plane. Object's x and y coordinate is located approximately with mathematical formulas. Accordingly, approximate location of the object is determined. The location is known 3 reference points are the access points. This location of the object is determined with the Wi-Fi signals from the access points.

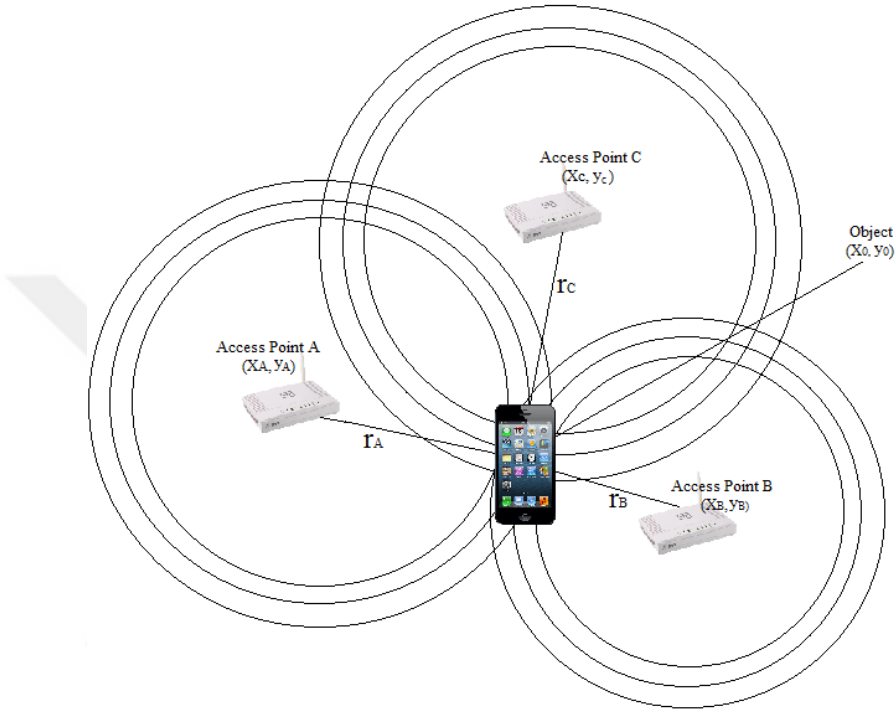


Figure 3.2 Three fixed access points and smartphone with unknown location

As shown in figure 3.2, access point A's coordinates are x_A, y_A ; access point B's coordinates are x_B, y_B and access point C's coordinates are x_C, y_C . r_A, r_B and r_C is the distance from the access point of the object. The location of the object is unknown and the coordinates of object in figure 3.2 are shown as x_0 and y_0 .

As shown in figure 3.2, each access point spread around their Wi-Fi signal their nearest position. These signals will create a circle around the access point. The coordinates of where the object is calculated by performing procedures below:

First off all, all distance is calculated from the access points:

$$r_A = \sqrt{(x_A - x_0)^2 + (y_A - y_0)^2}$$

$$r_B = \sqrt{(x_B - x_0)^2 + (y_B - y_0)^2}$$

$$r_C = \sqrt{(x_C - x_0)^2 + (y_C - y_0)^2}$$

then,

$$\begin{aligned} (r_A)^2 - (r_B)^2 &= -2x_A x_0 - 2y_A y_0 - x_B^2 - y_B^2 \\ &\quad + 2x_B x_0 + 2y_B y_0 + x_A^2 + y_A^2 \end{aligned}$$

$$\begin{aligned} (r_A)^2 - (r_C)^2 &= -2x_A x_0 - 2y_A y_0 - x_C^2 - y_C^2 \\ &\quad + 2x_C x_0 + 2y_C y_0 + x_A^2 + y_A^2 \end{aligned} \quad (3.1)$$

From the (3.1) to obtain

$$\begin{bmatrix} (r_A)^2 - (r_B)^2 + (x_B^2 + y_B^2 - x_A^2 - y_A^2) \\ (r_A)^2 - (r_C)^2 + (x_C^2 + y_C^2 - x_A^2 - y_A^2) \end{bmatrix} = \begin{bmatrix} 2(x_B - x_A) & 2(y_B - y_A) \\ 2(x_C - x_A) & 2(y_C - y_A) \end{bmatrix} \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} \quad (3.2)$$

Equations (3.2) can be used to extend for N pieces of sensor target as

$$\begin{bmatrix} (r_1)^2 - (r_2)^2 + (x_2^2 + y_2^2 - x_1^2 - y_1^2) \\ (r_1)^2 - (r_3)^2 + (x_3^2 + y_3^2 - x_1^2 - y_1^2) \\ \vdots \\ (r_1)^2 - (r_N)^2 + (x_N^2 + y_N^2 - x_1^2 - y_1^2) \end{bmatrix} = \begin{bmatrix} 2(x_2 - x_1) & 2(y_2 - y_1) \\ 2(x_3 - x_1) & 2(y_3 - y_1) \\ \vdots & \vdots \\ 2(x_N - x_1) & 2(y_N - y_1) \end{bmatrix} \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} \quad (3.3)$$

define

$$\bar{A} = \begin{bmatrix} 2(x_2 - x_1) & 2(y_2 - y_1) \\ 2(x_3 - x_1) & 2(y_3 - y_1) \\ \vdots & \vdots \\ 2(x_N - x_1) & 2(y_N - y_1) \end{bmatrix} \quad (3.4)$$

$$\bar{B} = \begin{bmatrix} (r_1)^2 - (r_2)^2 + (x_2^2 + y_2^2 - x_1^2 - y_1^2) \\ (r_1)^2 - (r_3)^2 + (x_3^2 + y_3^2 - x_1^2 - y_1^2) \\ \vdots \\ (r_1)^2 - (r_N)^2 + (x_N^2 + y_N^2 - x_1^2 - y_1^2) \end{bmatrix}$$

Hence the object's coordinates are concluded as

$$\begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = (\bar{A}^T \bar{A})^{-1} * (\bar{A}^T \bar{B}) \quad (3.5)$$

Finally, solve (3.5), the object's coordinate (x_0, y_0) can be obtained [12].

3.2 Triangulation Measurement

In this thesis, developed software passes through two separate processes in training and testing. In the training process, necessary parameters are calculated for the position prediction algorithm. In the test process, position is estimated by using these parameters.

For training process, at least 1 mobile phone that location is estimated must be in the test environment and this phone must be a wireless transceiver device. Also in closed environments with wireless access points, these access points must be set to position relative to a reference point.

In this thesis, Kadir Has University Cibali Center Campus was selected as the test environment. As a result of the measurements made in various areas, as the place where the metal is at least is the position of the Rezan Has Museum entrance area is obtained of the effective value. It is defined as the total of 36 m², the test environment that is 600*600 cm.

Firstly, battery powered access points are preferred for the test environment. However, the battery ends in a very limited time and also they close themselves. Therefore, wireless access points are used in this thesis. These wireless access points used are the same type of AirTies Air 5341 model. The strength of each wireless access point is set to 100%. All access points receive the same power electrical outlet is provided in the test environment.

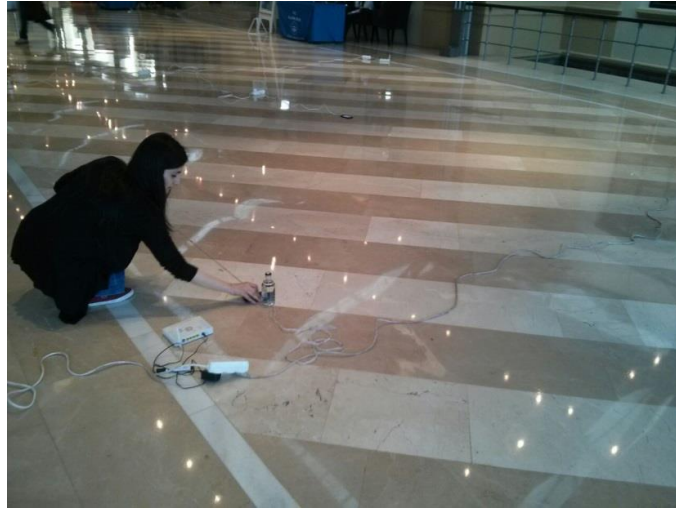


Figure 3.3 Measurement Area

As it is shown in figure 3.3, 3 access points are located position of the test environment corner. It is arranged at the (0, 0), (0,600) and (600, 0) cm. Thus, it is defined active area in 36 m². The active area is defined by a square of 40 cm distance from the wireless access points. So, this available test area is 27, 04 m² (520 cm * 520 cm) like shown in figure 3.4. In this area, 196 points have been identified with 40 cm intervals.

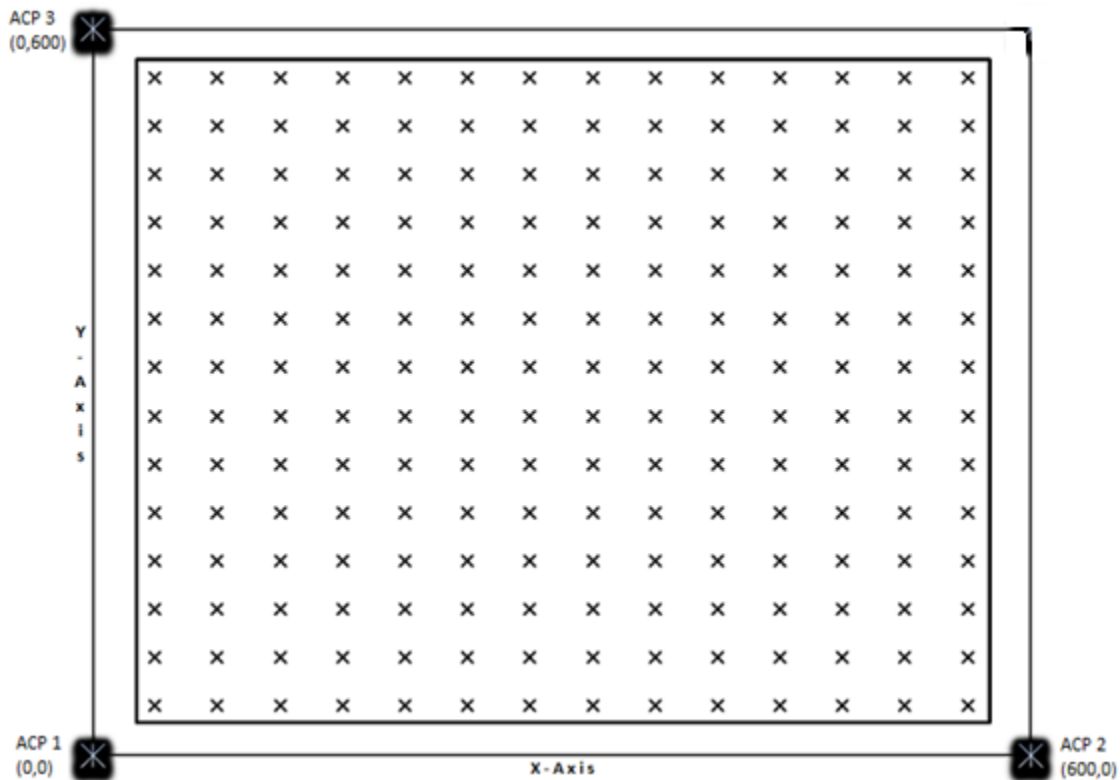


Figure 3.4 Test environment

In the test environment, signal measurement can be made with the smart phone. For the measure of access points', these access points must be defined in smartphone. Therefore, each access point's Media Access Control address must be saved in the smartphone for signal measurement.

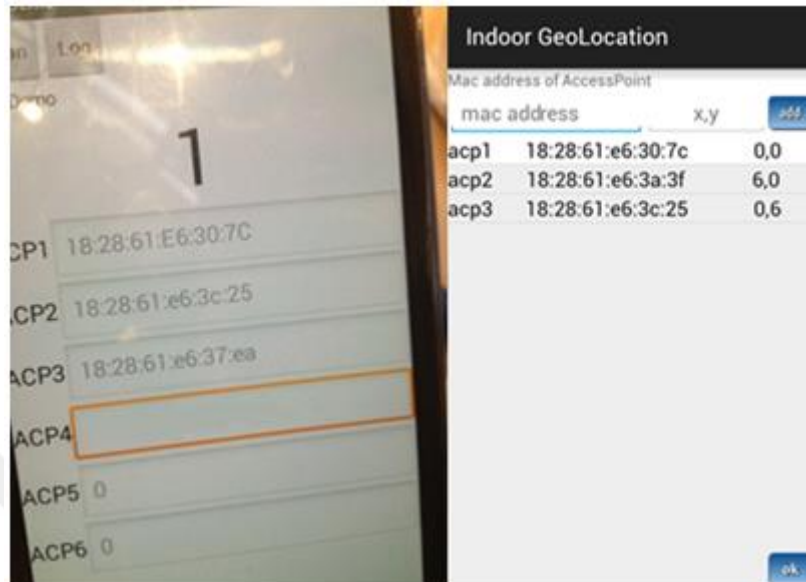


Figure 3.5 Access Point Mac Address Interface

As seen figure 3.4, each access point's MAC address has been registered in the application interface of the device for signal measurement. So, smartphone and access point connection has been established. Intensity was measured in the specified time interval for a particular point with Samsung S3 smartphone in this thesis. 12 seconds was measured in each point. In this period intensity value for each access point was recorded in the text file. Hence, for each point, 3 text files generated. Each text file contains each intensity values from the access point to that point.

3.3 RSSI and Log Normal

RSSI is the abbreviation of the word "Received Signal Strength Indicator". A brief definition of RSSI is the values measured at the receiving device from the signal emitted by the transmitter in wireless communication system and it measured by the dBm (decibel miliwatt).

RSSI is a value that can be changed quickly because of affecting by environment. If there is any object between two devices; RSSI value is affected too much negatively. Also this value depends on receiver gain, sensitivity and the signal power measured in the channel. Therefore

RSSI does not give accurate distance information because of the above item. However, it can provide information about the relative distance and is used. RSSI is how close to the zero value, the received signal strength is better. Therefore, for example, the value of RSSI -40 is better than the value of -50. The situation is opposite for noise. A low noise value is better than a high noise value in a test environment. For example, the value of -90 is better than the value of -80. If the noise increases, the signal quality decreases. The greater the difference between the RSSI and noise, the stronger the signal is [13].

In the measurement part, it was stated that the signal is measured with the access point. In this measurement, there are many negative numbers in a text file generated for each access point on the phone. These numbers demonstrate the intensity from each access point, and these values are RSSI values. Intensity measurement is made; individual RSSI values for each point of each access point were recorded. The obtained number of RSSI values is directly proportional to the waiting duration at that point. Location determination can be estimated using these RSSI values. Before the algorithm is run with these values, it must be calculated with the distance value and the propagation constant value.

The log normal formula is mostly used in indoor location based studies to calculate the RSSI value. In this thesis, the log normal distribution formula is used for the calculation of the distance value [14]. Thus, more efficient values were obtained for the triangulation algorithm. Although RSSI is minus, the lognormal variable is always a positive value. The 'location engine' uses the following model for the relation between distance and RSSI:

$$\text{RSSI (dBm)} = -10n \log(d) + A \quad (3.6)$$

The above formula is the lognormal formula. Unknown in this formula;

RSSI: is the RSSI value received (dBm)

d: distance in meters

A: received signal strength in dBm at 1 meter

n: propagation constant or path-loss exponent [15]

For calculating distance, the following formula is obtained:

$$d = 10^{\left(\frac{\text{RSSI}-(A)}{10 \cdot n}\right)} \quad (3.7)$$

In this thesis when distance values are measured, separate calculation is made for each access point. Although for indoor location the value is n fixed at 2.18, A values not constant for each access point [16]. Therefore, different values are calculated for 3 access points. A values are the average of the RSSI value measured at 1 meter away from the access points. This training phase is performed for 196 points and each distance values are obtained for algorithm.

In this thesis when distance values are calculated, separate calculation is made for each access point. Although for indoor location the value is n fixed at 2.18, values of A and RSSI are not constant for each access point. A values are the average of the RSSI value measured at 1 meter away from the access points for each access points. RSSI values that indicate the intensity values affected by the environment are average of RSSI values in exact point. Therefore, different values are calculated for 3 access points. This training phase is performed for 196 points and each distance values are obtained for algorithm.

3.4 Triangulation Algorithm Results

3.4.1 The result with real data

In this thesis, It was followed the steps for location estimation:

1. Access points were placed in corner of the test area. ((0,0), (0,600), (600,0)cm)
2. RSSI was measured for 196 points with smart phone
3. The average RSSI was calculated for each point and for each access point.
4. Each access point's RSSI value were measured at 1 meter for value of A shown in table 3.1
5. Distance value was obtained to RSSI value with log normal
6. This measurement is run triangulation algorithm and estimated locations are obtained.

Table 3.1 A and n Values

A values	N values
----------	----------

Access point 1	- 58.93 dBm of signal strength	2,18
Access Point 2	-56.32 dBm of signal strength	2,18
Access Point 3	-57.83 dBm of signal strength	2,18

In output, the minimum difference shows of error in the first line and the maximum difference of error on the last line (A. 1).

When this result is analyzed;

Average error : 8, 739 meter

Minimum error : 3, 295 meter

Maximum error : 13, 515 meter

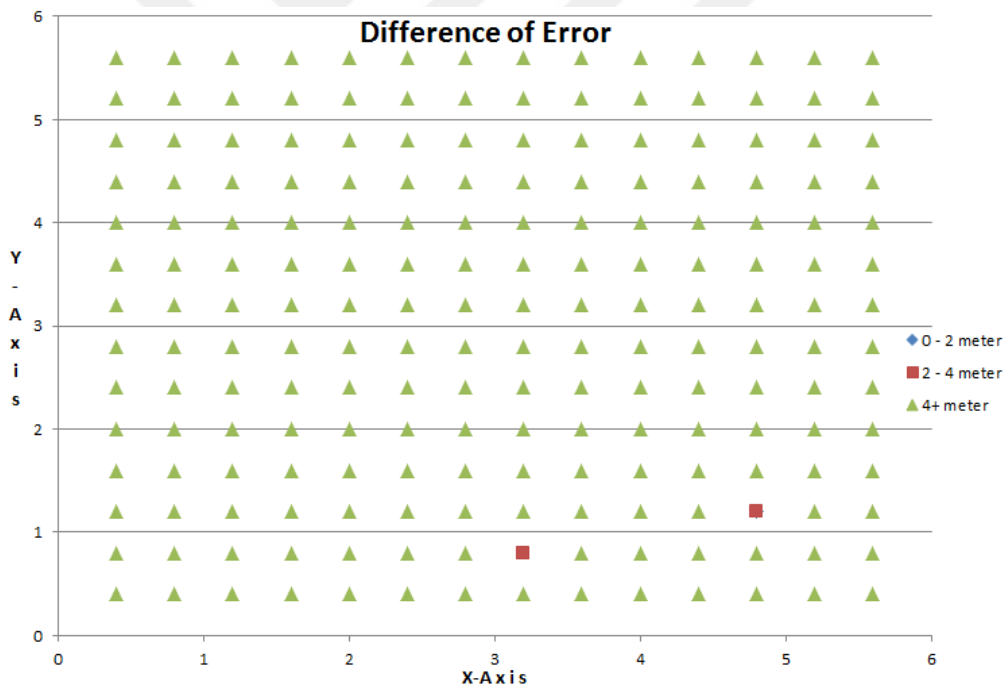


Figure 3.6 Difference of Error Distribution

Errors are classified at all points at the figure 3.5. These classifications are: 0 - 2 meter, 2 - 4 meter and 4+ meter. Each class is represented by a different shape and color. The class of blue which is 0 – 2 meter error is not in the error distribution figure because outputs have not included any 0 – 2 meter error. When all values are analyzed, the estimated point was determined to be too far from the real point.

The distance between the access point and receiver affects the performance directly. The transmission speed decreases when distance increases. However, this is not clearly observed in the measurements because the value of environment is a constant value.

Transmission distance of a signal depends on several factors. The following points should be considered to get a good performance from wireless networks:

- *Transmitter power,*
- *Two antenna locations to each other,*
- *Received antenna power,*
- *Receiver Sensitivity*

The building structure is one of the most important factors in indoor positioning system. Brick walls and metal partitions significantly intersect the signal. Glass and wood materials are more permeable. In addition, the effect of the other electronic devices is too much in the test environment [17]. For example, microwave ovens deflect too much signal because each machine broadcasts at different frequencies. This is a serious problem of confusion for indoor location.

3.4.2 The result with error free synthetic data

Signals are affected by many things such as wall structure, metal parts etc. in the measurement area like in this thesis as described in section 3.4.1. Nowadays, it is not possible to measure without any negative effects of signals in closed area. Therefore, synthetic data is generated to measure the success of triangulation algorithm.

As seen figure 3.11 and described in previous sections, RSSI value decreases with distance increase from the access point but it is not possible in measurement area because of noise and signal overlapping.

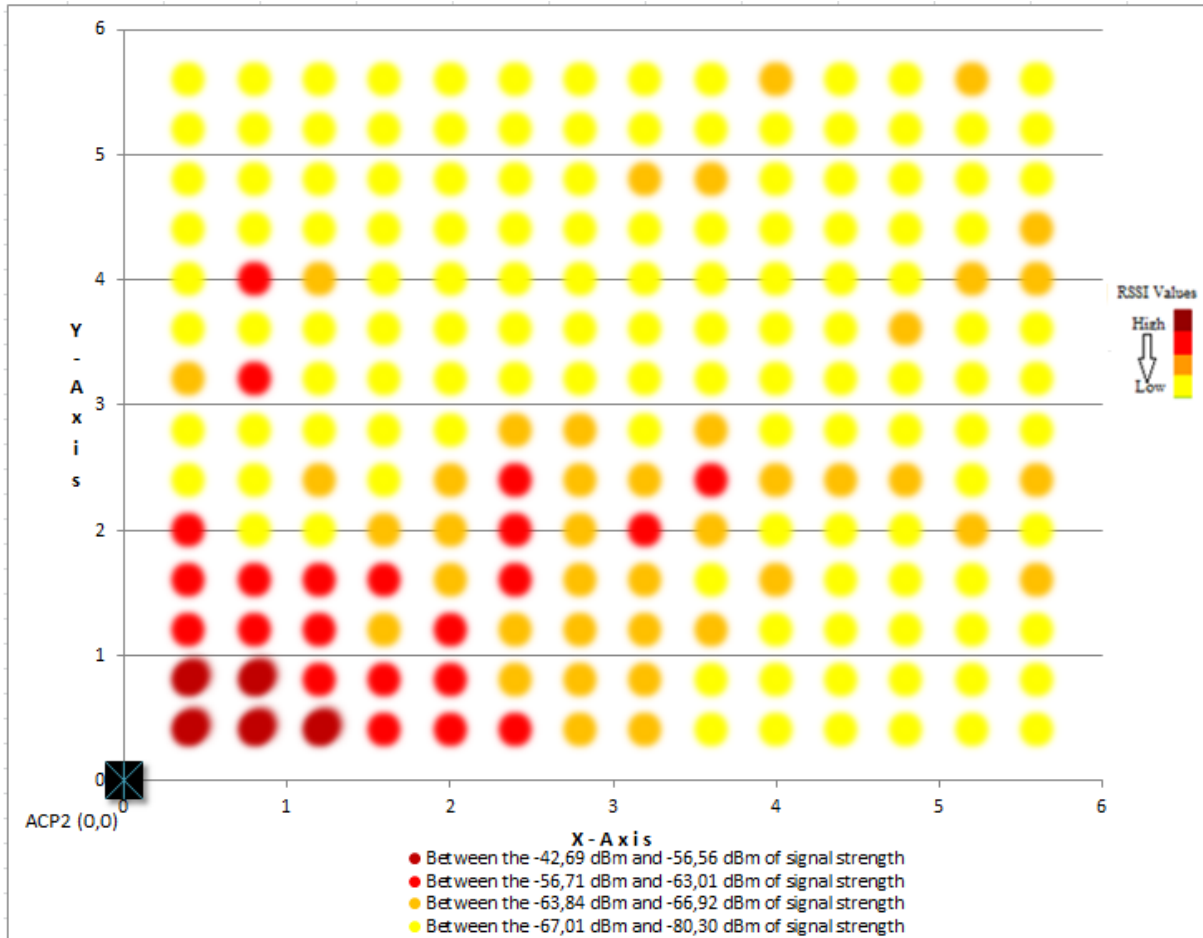


Figure 3.7 Signal decreasing move away from the access point 2 with real measurement

Therefore, synthetic data are generated depending on the distance and this operation is done with log normal formula.

$$\text{RSSI (dBm)} = -10n \log(d) + A \quad (3.8)$$

The value of n is taken as a constant 2.18 in the above log normal formula and the values of A are as follows for each access point:

Value of A for Acp 1: -52 dBm of signal strength

Value of A for Acp 2: -50 dBm of signal strength

Value of A for Acp 3: -54 dBm of signal strength

Value of A for Acp 4: -50 dBm of signal strength

Thus, synthetic RSSI value was obtained for each access point. Therefore, RSS values are calculated in the range of numbers shown in below for each access point:

Value of RSSI for Acp 1 between: -46.61 dBm and -71.59 dBm of signal strength

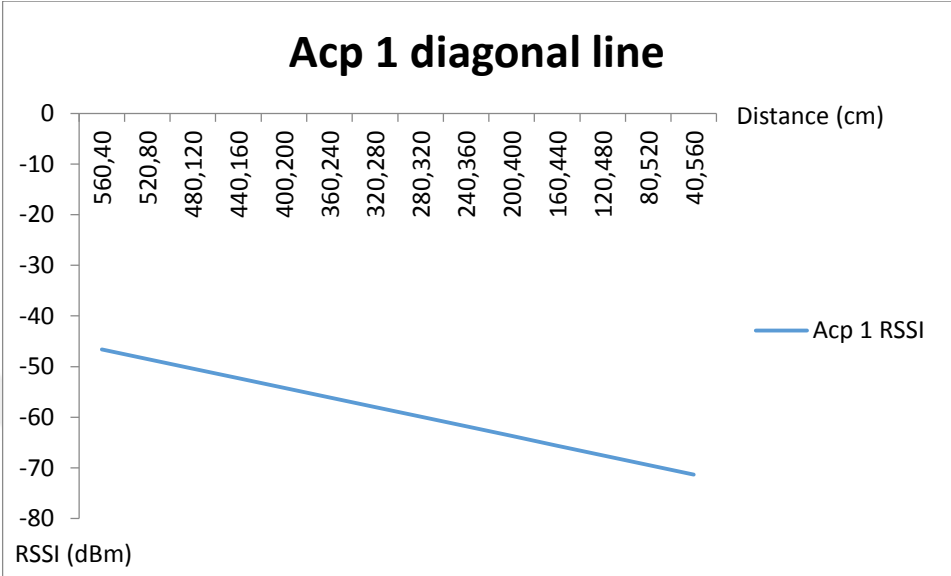


Figure 3.8 Value of RSSI for Acp 1 diagonal line

Value of RSSI for Acp 2 between: -44.61 dBm and -69.59 dBm of signal strength

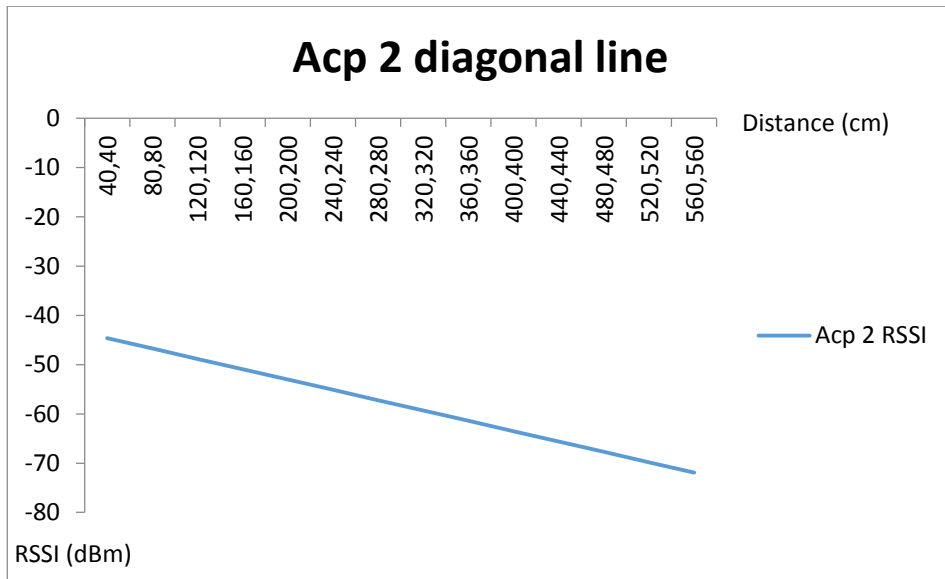


Figure 3.9 Value of RSSI for Acp 2 diagonal line

Value of RSSI for Acp 3 between: -48.61 dBm and -73.59 dBm of signal strength

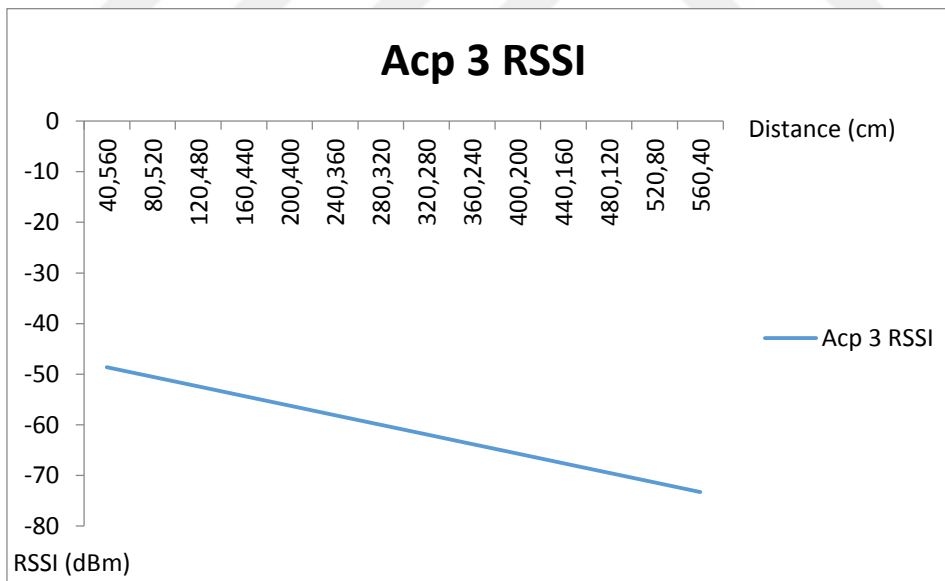


Figure 3.10 Value of RSSI for Acp 3 diagonal line

Value of RSSI for Acp 4 between: -44.61 dBm and -69.59 dBm of signal strength

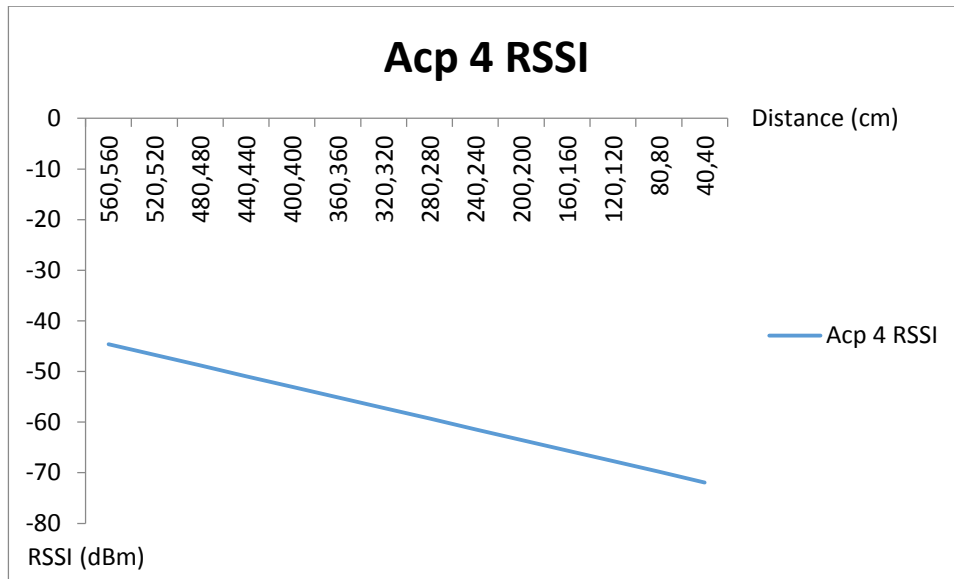


Figure 3.11 Value of RSSI for Acp 4 diagonal line

As seen from the charts 3.7, 3.8, 3.9 and 3.10 the move away on the diagonal, RSSI values decreased at the same rate for each access point. For examine in detail, figure 3.11 shows that signal decreasing form the access point 2.

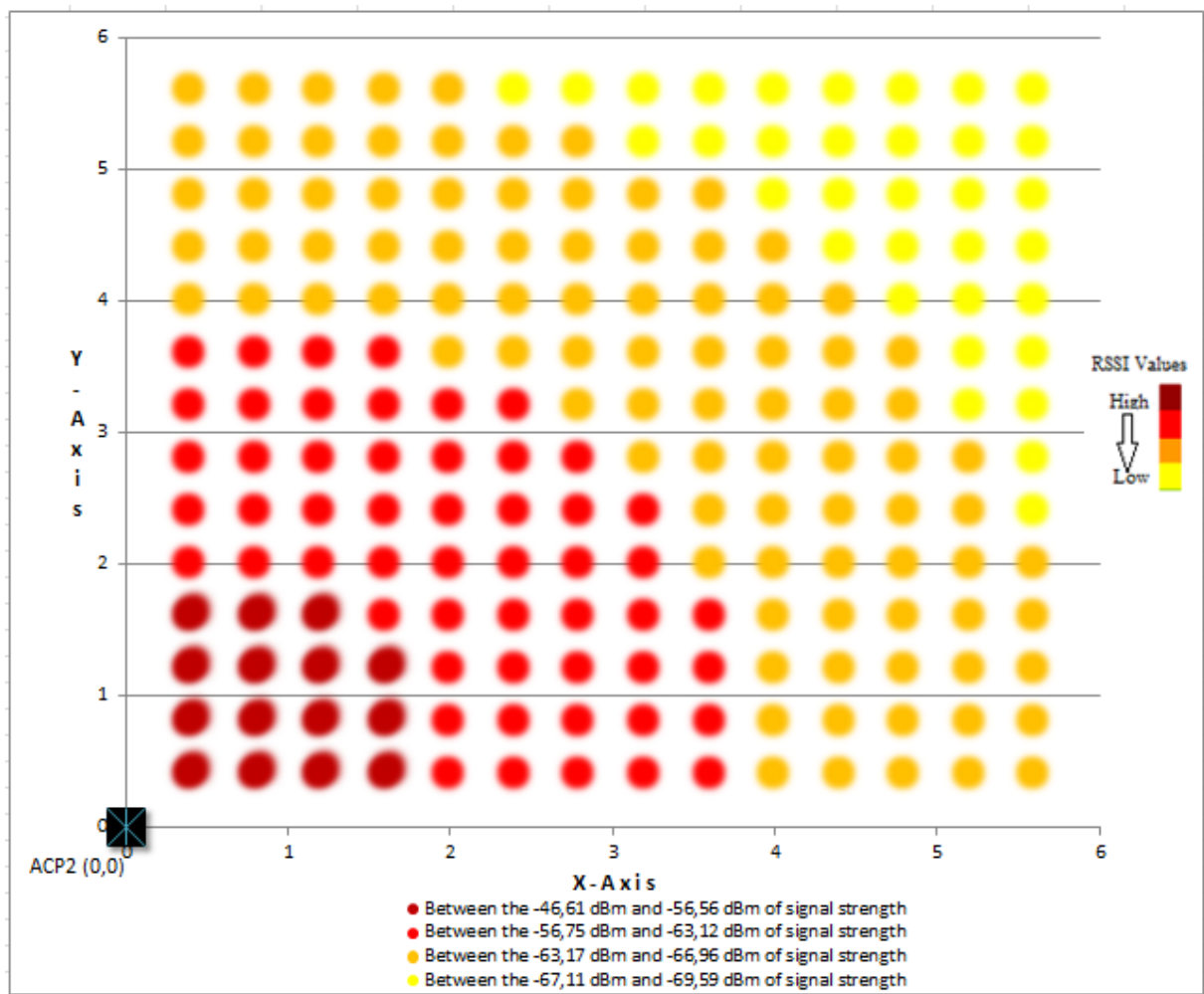


Figure 3.12 Signal decreasing move away from the access point 2 with synthetic data

To examine the results of synthetic data, distance values were obtained of the RSSI values with log normal formula and the obtained data were run through triangulation algorithm.

When the algorithm run with synthetic data with access point 1,2 and 3, output shows that proportional decrease in the signal is affected location position accordingly.

When get the output and this result is analyzed;

Average error : 0 m

Minimum error : 0 m

Maximum error : 0 m

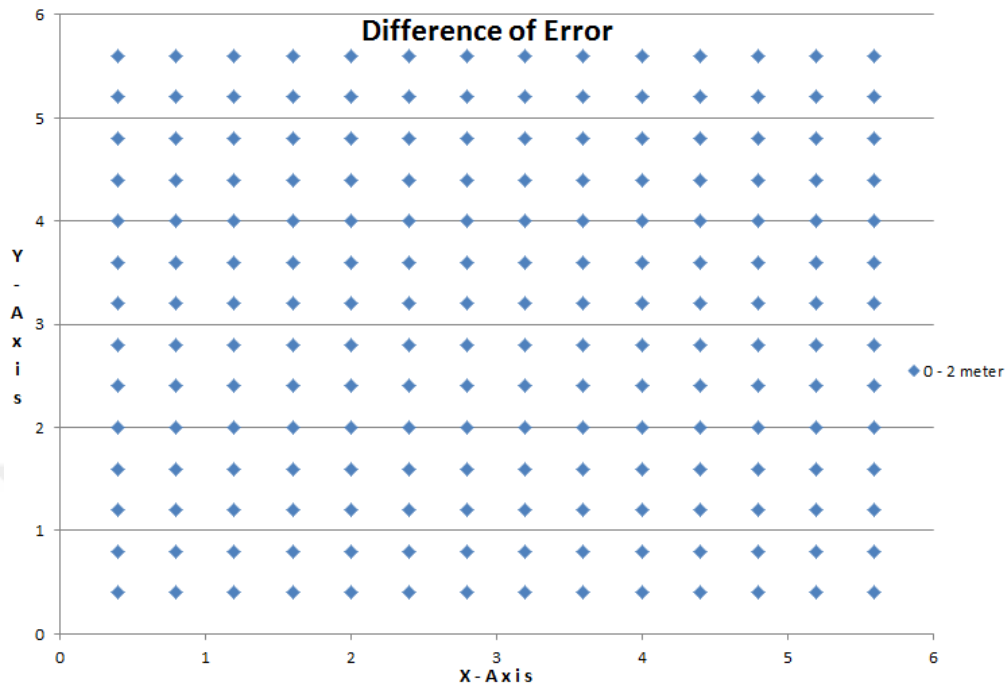


Figure 3.13 Difference of Error Distribution

As seen of figure 3.10, results of the test environment data indicate more accurate position. Errors are classified at all points at this figure. These classifications are: 0 - 2 meter and 2 - 4 meter and 4+ meter. Each class is represented by a different shape and color. In this result, it is not seen any error more than 0 meters. Therefore it shows that, algorithm is working successfully.

Due to, the distance between the access point and receiver affects the performance directly. And then the transmission speed decreases when distance increases. However, this is not clearly observed in the real measurement because the value of environment is not a constant value in real measurement.

3.4.3 The result with synthetic data with 5 dBm of noise

In probability theory, the normal distribution is a very common continuous probability distribution. It is synonyms for Gaussian distribution. This distribution is important in statistics.

It is often used in the natural and social sciences to represent real-valued random variables whose distributions are not known [18].

A normal distribution has a bell-shaped density curve described by its mean μ and standard deviation σ . The density curve is symmetrical, centered about its mean, with its spread determined by its standard deviation [19].

Normal distribution calculates by following formula:

$$\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \quad (3.9)$$

However NORMINV standard deviations excel formula is used for easily generating dataset in measurement area.

Formula is used as follows:

NORMINV (probability, mean, standard deviation)

Probability: A probability corresponding to the normal distribution.

Mean: The arithmetic mean of the distribution.

Standard deviation: The standard deviation of the distribution [20]

To examine the signal overlap in 5 dBm of noise, the synthetic data calculated by the following formula in excel document:

$$=NORMINV (RAND (); 0; 5) \quad (3.10)$$

In this measurement, it was prepared as a new dataset the following formula:

$$\text{RSSI (dBm)} = (-10n \log(d) + A) + \text{NORMINV}(\text{RAND}(); 0; 5) \tag{3.11}$$

Therefore, the only difference from the synthetic data set is NORMINV (RAND (); 0; 5) calculation.

When the triangulation algorithm runs with obtained data, output shows that signal overlap in 5dBm of noise is more inaccurate than error free synthetic data.

In output, the minimum difference shows the error in the first line and the maximum difference of error on the last line (A. 3).

When this result is analyzed;

- Average error : 9,797 m
- Minimum error : 3,488 m
- Maximum error : 15,180 m

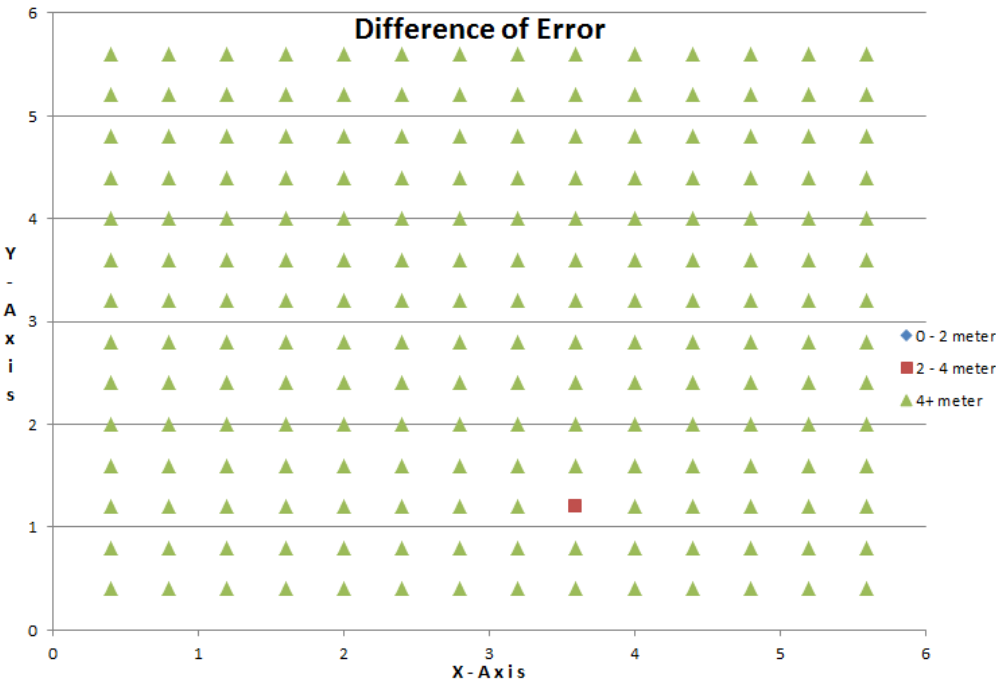


Figure 3.14 Difference of Error Distribution

As seen of figure 3.11 errors are classified at all points. These classifications are: 0 - 2 meter, 2 - 4 meter and 4+ meter. Each class is represented by a different shape and color. Like triangulation with real data output, the class of blue which is 0 – 2 meter error is not in the error distribution figure because outputs have not included any 0 – 2 meter error. Although A and n are the same in error-free synthetic dataset in this measurement, the result of the 5 dBm of noise test environment data results was too much affected negatively.



4 TRIANGULATION WITH LEAST SQUARE METHOD

4.1 Least Square Method and History

In our daily life, the generalizations can be requested from single points without using any mathematical data. The function, which will be modeled with this generalization, may then be used in studies. Thus, there is no need to collect the data again. The obtained function may not be a function of a matching table. The aim is to find the best matching function closer to our table.

The dataset obtained from experimental studies are point. Without a continuous function of these points, the only point does not anything. It is not possible to have the definition of a continuous function which could direct the desired value dataset. However, it is necessary to uncover the functions finds the closest value.

Best fitting function is called regression analysis to find the process to a data table. While regression analysis using, the most widely used method is the least squares method. In this method, depending on the actual equation with each other's closest connection between the two values changing.

One of the most commonly used method is the least squares method while regression analysis. This method was developed great mathematician Gauss at 1795 when he was 18 years old. The first time was used to determine the orbit of the Cres asteroid at 1801. Also for the first time, it was published in the second volume of the publication of Gauss's total works at 1809. French mathematician A. Legendre (1805) and American mathematician R. Adrian (1808) discovered same method unannounced and independently from Gauss [21].

Actual regression model and data generation process should be ideal for this method to be more accurate because most of the data set does not provide the ideal conditions. However, it can create a benchmark point. Therefore knowing ideal conditions is very important for using this method. So status can be determined whether requirements are met and the formation of the deviation can be minimized [22].

4.2 Least Square Method Formula

Least Square method is to produce a continuous function of the existing dataset. With this method, erroneous datasets obtained for unknown points minimized.

Consider first the following model to describe the least square method:

$$\varepsilon = Y - (\beta_0 + \beta_1 X) \quad (4.1)$$

Here;

Y: Dependent variable

X: Independent variable

ε : Random term

β_0 : Invariable (the point where the regression line cut the y-axis)

β_1 : Regression coefficient. (The slope of the regression line)

ε , the obtained values of Y,

$Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i$ represents the vertical distance on the truth. If β_0 and β_1 is known in this equation, it would have to be done to find the $\sum \varepsilon^2$.

4.1 equality in terms of convenience

$$L = \sum_{i=1}^n \varepsilon_i^2 = \sum_{i=1}^n (Y_i - \beta_0 - \beta_1 X_i)^2 \quad (4.2)$$

can define form. n indicates the number of the data obtained. L is representing the sum of the squared in this equation. Conditions that need to minimize L, L's partial derivatives are zero for each parameter (β_0, β_1).

$$\frac{dL}{d\beta_0} = 2 \sum_{i=1}^n (Y_i - \beta_0 - \beta_1 X_i)(-1) = 0 \quad (4.3)$$

and

$$\frac{dL}{d\beta_0} = 2\sum_{i=1}^n (Y_i - \beta_0 - \beta_1 X_i)(X_i) = 0 \quad (4.4)$$

obtained. It would be enough to solve the 4.5 equation for obtain β_0 and β_1

$$\sum_{i=1}^n (Y_i - \beta_0 - \beta_1 X_i) = 0 \quad (4.5)$$

If we expand on both sides of the 4.5 equation with X_i ,

$$\sum_{i=1}^n (X_i Y_i - X_i \beta_0 - \beta_1 X_i^2) = 0 \quad (4.6)$$

From here

$$n\beta_0 = \beta_1 \sum_{i=1}^n X_i = \sum_{i=1}^n Y_i \quad (4.7)$$

and

$$\beta_0 \sum_{i=1}^n X_i + \beta_1 \sum_{i=1}^n X_i^2 = \sum_{i=1}^n X_i Y_i \quad (4.8)$$

obtained.

The solution of 4.7 and 4.8 equations, least square estimator β_0 and β_1 ,

$$\widehat{\beta}_1 = \frac{\sum X_i Y_i - (\sum X_i)(\sum Y_i)/n}{\sum X_i^2 - (\sum X_i)^2/n} \quad (4.9)$$

and

$$\widehat{\beta}_0 = \bar{Y} - \widehat{\beta}_1 \bar{X} \quad (4.10)$$

calculated like this.

From here,

$$S_{xy} = \sum_{i=1}^n (X_i - \bar{X})(Y_i - \bar{Y}) \quad (4.11)$$

and

$$S_{xx} = \sum_{i=1}^n (X_i - \bar{X})^2 \quad (4.12)$$

If written form

$$\widehat{\beta}_1 = \frac{S_{xy}}{S_{xx}} \quad (4.13)$$

as it has.

Linear model should be examined how fit the data with the obtained formula. That's why, epsilon needs to be minimized. For minimizing ε (The difference between observations and models that is in formula 4.14), the presence of the best regression model will provide appropriate data. Finding the best regression model in least square method, the sum of the square error occurs to make the smallest in formula 4.15

$$\varepsilon_i = Y_i - \widehat{Y}_i \quad (4.14)$$

$$\sum_{i=1}^n \varepsilon_i^2 = \sum_{i=1}^n (Y_i - \widehat{Y}_i)^2 = \min. [23] \quad (4.15)$$

4.3 Least Square Method in Triangulation Algorithm

As described in the previous section, least square method provides the most suitable function to the dataset. In this method, optimum function is used with the sample dataset. This function results in scattered spots that may form a line. Thus, it is appropriate to obtain the correct passing through spots.

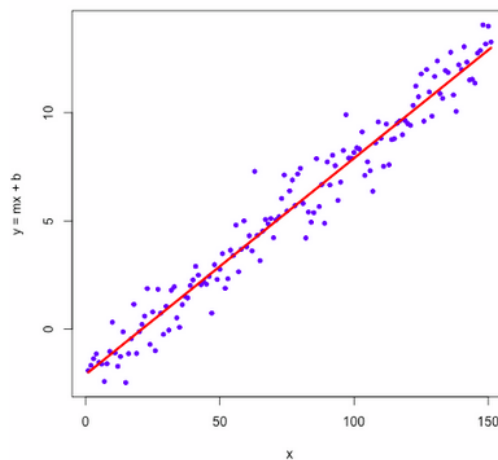


Figure 4.1 Points and fitting line

For example, suppose that the dataset events are blue dots. As it has seen, points were given at a rate of dissolution. Each point away from one another is not equal. It is not so easy to find the line that passing through all the dots. The red line in this figure shows in the least square method is obtained by fitting line.

When triangulation algorithm used to analyze values, it is observed that the data is scattered. The reason is that as the distance from the access point is not in proportion to the signal. Therefore, in this thesis least square method and triangulation algorithm are used together to reduce errors in the triangulation algorithm [5].

4.4 Wi-Fi Channels

Wireless networks have come a long way in the past 15 years. It has different wireless network protocols. These protocols are expressed as 802.11 b,g,c-f, h,j. The 802.11 standard is issued by the Institute of Electrical and Electronics Engineers (IEEE) in 1977. Products manufactured in accordance with the 802.11 standard operate at 2.4 GHz. This band is known as ISM (Industry, Science, and Medicine) in America and that can be used as unlicensed in Turkey. This band can access shared data transfer rate of up to 1 Mbps to 2 Mbps.

There are two types of technologies standardized by the IEEE: 802.11 and 802.11.b. 802.11 uses the Frequency Hopping Spread Spectrum (FHSS) modulation techniques in 2.4 GHz band and with this technique it can be reach up to 2 Mbps data transmission speed. Despite 802.11's limited bandwidth, it is providing robust and secure system infrastructure. It's affected quite low from the noise and interference because of used all bandwidth and frequency hopping technique. It is possible to use the 15 access points in the same coverage area. 802.11.b can reach up to 11 Mbps of shared bandwidth in 2.4 GHz bandwidth with Direct Sequence Spread Spectrum (DSSS) modulation technique. This modulation technique used simultaneous 3 separate access point in 3 separate channels in the same coverage area [24], . At first, wireless network phenomenon was not accepted sufficiently for several reasons: The first one is its total bandwidth speed limited such as 1-2 Mbps. The second one is adapters and access points produces by different firms was not compatible with each other. Third, this equipment was very expensive. However, these problems were solved in time and this technology has become quite popular in the world. First of all, unit prices went down with many companies produce the products according to these standards. Second, 802.11.b standard has become rivals to the

Ethernet. Therefore, incompatibility problems were fixed between different companies' products [25].

Although there are big differences between the protocols, 802.11b is the mostly preferred. 802.11.b has maximum 11 Mbit / sec raw speed and it is connected to each other as in the original equipment standards. Because the standards are accepted by more users and companies, more access points are generated using this protocol. Wireless local area networks gained its popularity and 802.11.b standard is used by the access points [26]. Below is the list of WLAN channels that are legally allowed wireless local area network channels using IEEE 802.11 protocols and Wi-Fi channels has 14 channels.

Table 4.1 Wi-Fi channels

Channel No.	Frequency (GHz)	Allowed Countries
1	2,412 GHz	Europe, ABD, Japan
2	2,417 GHz	Europe, ABD, Japan
3	2,422 GHz	Europe, ABD, Japan
4	2,427 GHz	Europe, ABD, Japan
5	2,432 GHz	Europe, ABD, Japan
6	2,437 GHz	Europe, ABD, Japan
7	2,442 GHz	Europe, ABD, Japan
8	2,447 GHz	Europe, ABD, Japan
9	2,452 GHz	Europe, ABD, Japan
10	2,457 GHz	Europe, ABD, Japan
11	2,462 GHz	Europe, ABD, Japan
12	2,467 GHz	Europe, Japan
13	2,472 GHz	Europe, Japan
14	2,484 GHz	Japan

As seen in table 4.1, using of 14 channels is not permitted in all countries. ABD uses 11 channels and Europe uses 13 channels. Like European countries, Turkey uses 13 channels. Only Japan uses 14 channels. Therefore, the devices manufactured for different countries are likely to cause incompatibility in some channels. The reason is that the international regulation for Wi-Fi is not the same in every country. However, these channels are set to be between 5 MHz in every country [7]. For example, between channel 1 and channel 2, there is 5 MHz gap. According to this calculation, there is 11 MHz difference between the channel 1 and channel 3 and this difference increase like this.

4.5 Channel Fixing and Measurement

Channel 11 is the most used channel in Turkey. Therefore, manufactured access points are set to use channel 11. However, Wi-Fi speeds are still a problem in a lot of different situations. Sometimes the signal interruptions or slowdowns experienced at especially in an apartment building or separate house. Fortunately, there is always a way to fix the signal interruptions or slow transfer speeds. This is because, radio waves actually not able to access the same distance in all directions. Walls, doors, people, elevator shafts and other obstacles cause irregular and deterioration of the radio frequency exposure pattern and irregular signals. Because it works in the unlicensed frequency range, Wi-Fi devices can interfere with other wireless devices or prevent each other's communication. Also the communication quality and speed of these devices reduces or completely blocked by Bluetooth, microwave ovens, cordless phones and similar devices working with radio signals. Thus, in the same environment, radio frequencies mixed or overlap with each other [27].

To avoid the signal conflict, wireless transmitting devices must be operated at least used channel. Herein, referred to above, the 5 MHz share between the two channels can be considered to avoid conflicts that may occur. However, these frequencies are affected by the environment. Therefore 1, 6 and 11 channels are separated theoretically by a wide range that will not interfere to each other. Thus, channel switching and channel retention operation is performed by the number of access points to be used. This channel fixing and channel changing operation are performed according to the number of access points to be used. This process has the basic settings of the wireless router. Therefore increasing the quality of the received signal is provided [27], [28].

Although channel settings can be seen as 13 pieces, it should be at least 3 channels between the 2 different channels. However 3 channel spacing is not very functional because homes are too close or there are too many devices in the shopping malls. This is why some countries have proposed as 5 channels spacing. Therefore, that is more logical use of 1, 5, 9 and 13 channels.

In this thesis, a new measurement was made with channel fixing for obtained more accurate signals. This measurement was performed in the same test environment with the previous measurement. As seen in figure 4.2, access points were placed in all corners of the test environment. 4 access points are used which positions are (0, 0), (0,600), (600, 0) and (600,600) cm. Thus, values obtained from the four-access point to the same point.

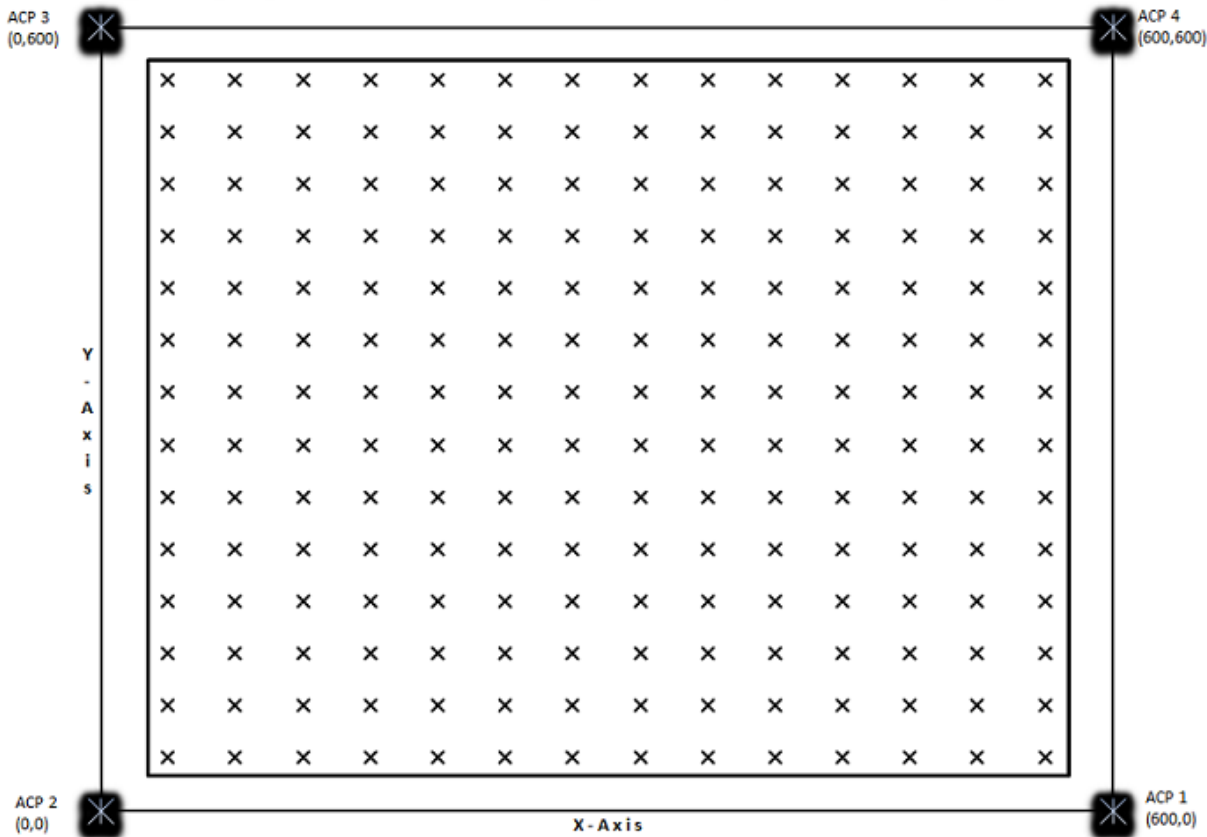


Figure 4.2 Measurement area with 4 access points

In this measurement, 1, 5, 9 and 13 channels which providing at least intersections with the frequency of each other are used to reduce the margin of error. Thus, 20 MHz portion is left between the two channels. This measure is determined with software developed by the 3rd party company.

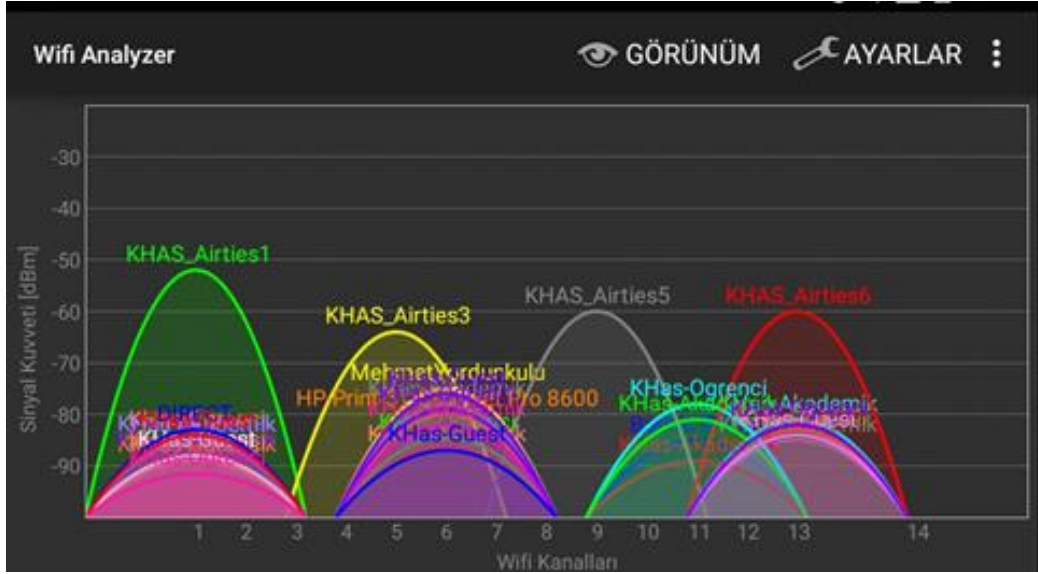


Figure 4.3 Access Points' Channels

As shown in figure 4.3, KHAS_Airties1 fixed channel 1, KHAS_Airties3 fixed channel 5, KHAS_Airties5 fixed channel 9, KHAS_Airties6 fixed channel 13. According to the previous measurements, which is fixing channel 6, signals are overlapped to a lesser extent. Therefore, it is planned to be reduced the error percentage.

4.6 Optimized A Values

The value needed for the lognormal formula, each access point provides an average of the RSSI value of 1 meter. However, signal distortions, noise and signal conflicts occur during the measurement in the test environment. Therefore, the error factor of the high rise, the value is measured for 12 seconds not during the measurement period.

The training phase is the same as the previous test environment. The only thing that is different to the value used in the measurement process, HTC One brand another smart phone was placed the center of the test environment. The coordinates of this phone are (300, 300) cm. During the measurement process, the software ran on this phone and received RSSI value of all the access points. Application is stopped in the phone at the end of the measurement process. The average of the RSSI values, which is different for each access point, was recorded for each access point and the average RSSI values were obtained.

Thus, using most appropriate for environment, optimized A values were calculated with this measurement.

4.7 Brute Force and Optimized n Values

Brute force is one of the principles of password cracking programs. This is a technique that can break even 128-bit encrypted passwords. This method is most commonly used to hack any password such as login passwords or RAR passwords. This attack tries to break the system password through trial and error method. Although Brute Force is a rapid process in locale, it is a tiring method according to the act on the web server's response.

Although, brute force technique is used preset programs, this technique carried out with malicious bots developed by attackers. To avoid this attack, first of all passwords must be include numbers, letters and signs. Secondly, if it is possible completely automated public turing test to tell computers and humans apart (captcha) security which aim of the project is to identify the behavior of people with computers is protect must be integrate. Thus the user is prompted to enter the security code. This will be prevented from brute force attacks. So, random security code is not estimated by program or malicious bots and it is prevented from brute force attacks.

- *This attack works fast in the local system.*
- *It maintains in multiple attack methods.*
- *It can be applied to many systems.*
- *It tries all combinations with the available data [29], [30].*

In this thesis, brute force technique was used for optimum n values. Because of the calculating the optimum value, the following steps must be apply for each point and these processes are very difficult to do without the use of an attack program.

For optimum n values with brute force algorithm:

- *Distance is calculated from the relevant access point for each point.*
- *Starting from $n=0$, the estimated distance is calculated equals to real distance for each RSSI values in the text files.*
- *Estimated distance and real distance value calculation is stopped when real and estimated distances equal each other.*

- The average of all values is calculated and n value is obtained for the corresponding point.
- The same process is done to the same access point for each point and the optimum n is obtained with calculating average all of n values.

Previous measurements were used at a fixed value of n . As a value, the value of n is calculated according to the environment, this development is expected to reduce the margin of error. Thus, the environment variable of n was calculated using brute force optimization.

4.8 Least Square Method with Triangulation Algorithm

Data that are obtained in the training process are points. Since these points are not connected with each other, they are not meaningful alone. It may not be a continuous function of these points will be used but a continuous function must be constituted.

Considering the signals received from the access point, it showed a decrease in proportion to the farther signal from the access point. The reason is that signal is affected by the environment. In this case, signal strength is less than expected, which makes it difficult to identify the location. Therefore, least square method was used in this thesis. To compare the single triangulation algorithm result, algorithm is run with access point 1, 2 and 3 dataset. Thus, fitting lines are generated all of 3 access points signals like shown in following figure 4.4, figure 4.5, and figure 4.6.

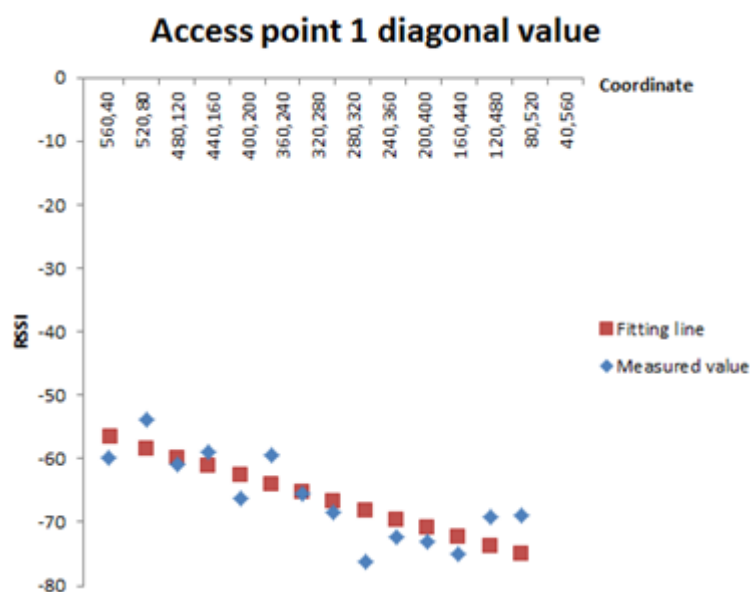


Figure 4.4 Access Points 1 Diagonal Value

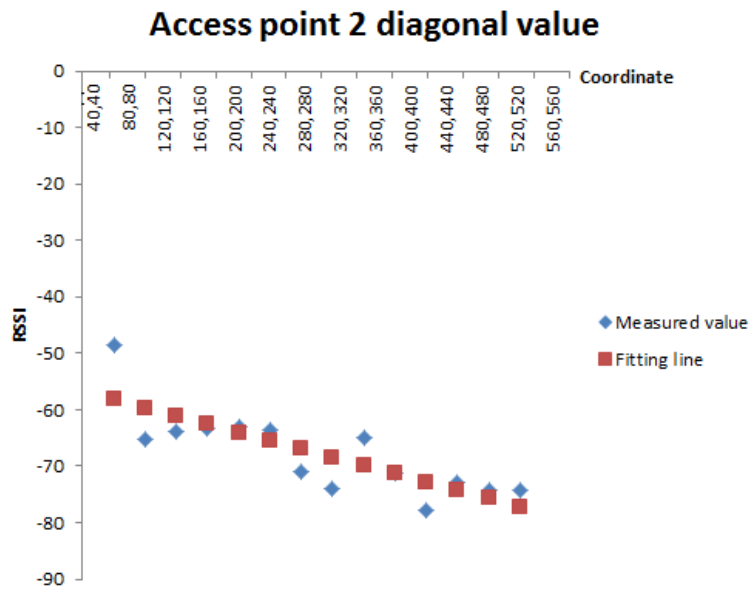


Figure 4.5 Access Points 2 Diagonal Value

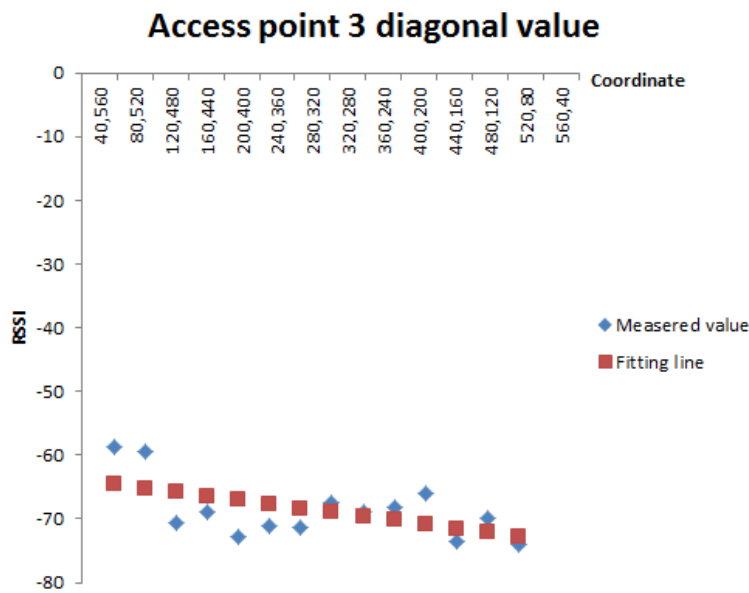


Figure 4.6 Access Points 3 Diagonal Value

Access point 1 shown in figure 4.4, access point 2 shown in figure 4.5, access point 3 shown in figure 4.6 with diagonal intensity values are shown in blue. Some points that are shown in red are the segment fit. These red spots that created by the signals on the diagonal for each access

points are continuous functions. With least square method, a reduction calculated in proportion to the distance from the access point. Thus, it aimed to decrease the error percentage.

4.9 Triangulation Algorithm with Least Square Method Results- 3 Access Points

4.9.1 The result with real data

For compare the single triangulation algorithm result, this algorithm is run for access point (0, 0), (0,600), (600, 0) cm and using A and n values in table 4.2 .It seems that output was varying when comparing simple triangulation output.

Table 4.2 A and n Values

	A values	N values
Access point 1	-57.69 dBm of signal strength	1.974
Access Point 2	-56.93 dBm of signal strength	1.983
Access Point 3	-52.75 dBm of signal strength	2.073

In output, the minimum difference shows the error in the first line and the maximum difference of error on the last line (A. 3).

When this result is analyzed;

- Average error : 2,826 m
- Minimum error : 0,004 m
- Maximum error : 7,513 m

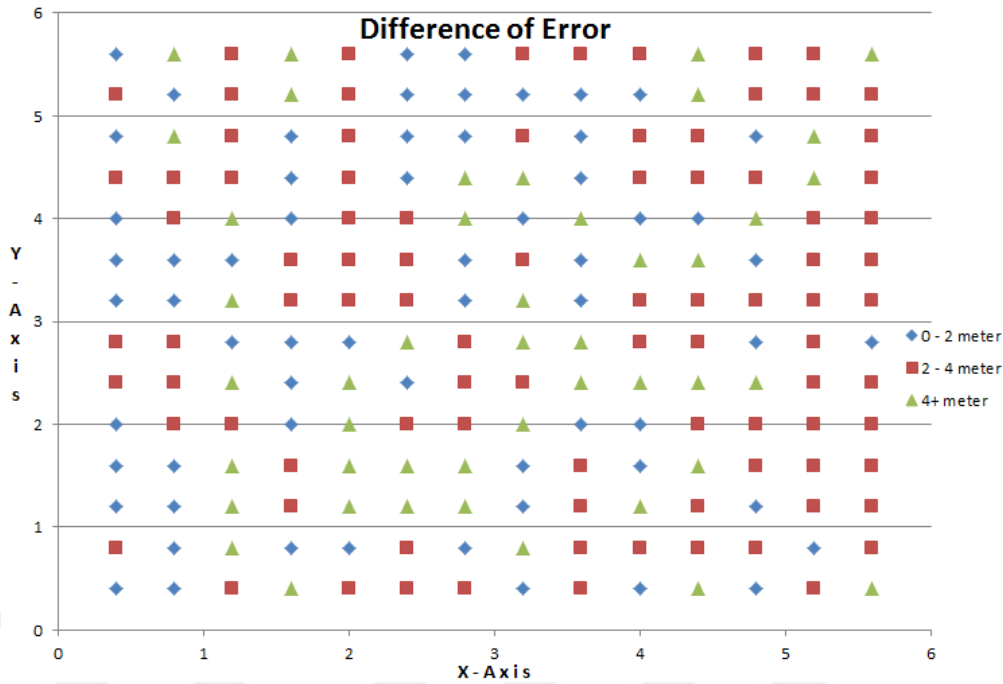


Figure 4.7 Distance of Error Distribution

Errors are classified at all points at the figure 4.7. These classifications are: 0-2 meters, 2-4 meters and 4+ meters. Each class is represented by a different shape and color. When all values are analyzed, all the improvements described in the previous chapter are completed, the output from the previous measurement is observed that has changed too much.

The difference from the previous measurement:

- Channel fixing
- Optimized A and n values
- Least square method: In the first measurement results were calculated in meters but in the second measurement, least square method was used and results were calculated units. In this method 1 unit were calculated as $40\sqrt{2}$ cm. Taken from the program output, it was multiplied by $40\sqrt{2}$. 56,56 were used as an approximate value of $40\sqrt{2}$ and so the location was found to be more accurate. In addition to all the improvements made when it is described, location was detected within average 2,826 m error. Thus, in $600\text{ cm} * 600\text{ cm} = 360.000\text{ cm}^2 = 36\text{ m}^2$ measurement area, position has been detected minimum 0,004 m and maximum 7,513 m error.

4.9.2 The result with error free synthetic data

When the triangulation algorithm with least square method runs with synthetic data which is used in chapter 3.4.2, the success of the algorithm was seen in output.

When this result is analyzed;

Average error : 0 m

Minimum error : 0 m

Maximum error : 0 m

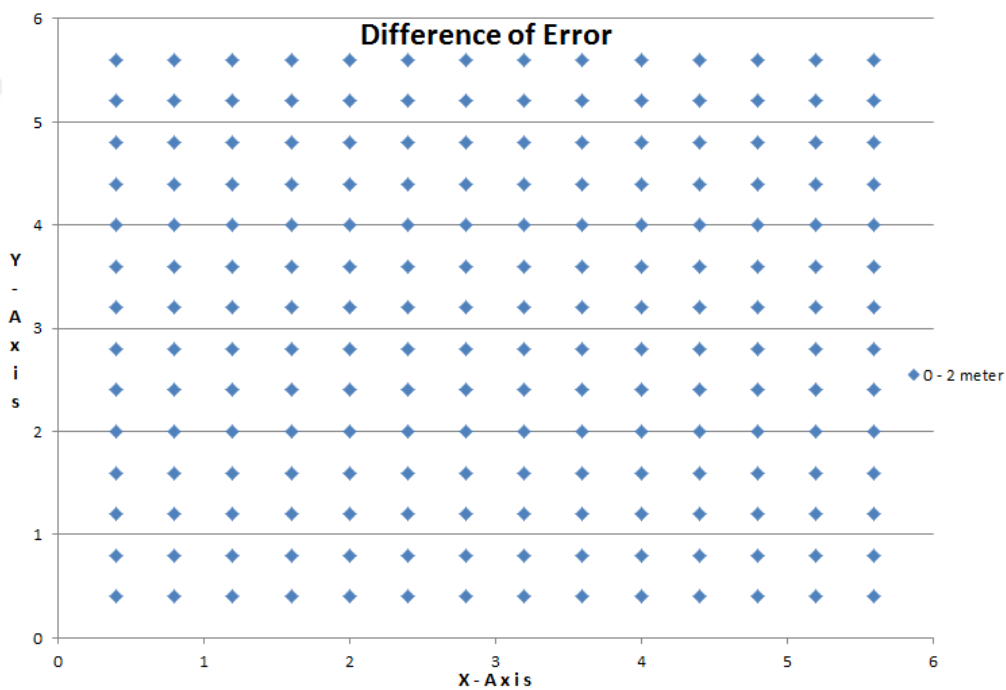


Figure 4.8 Distance of Error Distribution

Errors are classified at all points at the figure 4.8. These classifications are: 0 - 2 meter, 2 - 4 meter and 4+ meter. Each class is represented by a different shape and color. When all values are analyzed, it is not seen any error more than 0 meters. Therefore, output shows that, algorithm is working very successfully.

4.9.3 The result with synthetic data with 5 dBm of noise

When the algorithm runs with synthetic data with 5dBm of noise, output shows that additional improvements in triangulation algorithm are very successful to compare the single triangulation algorithm with synthetic data with 5dBm of noise.

In output, the minimum difference shows the error in the first line and the maximum difference of error on the last line (A. 4).

When this result is analyzed;

Average error : 4,881 m

Minimum error : 0,414 m

Maximum error : 11,325 m

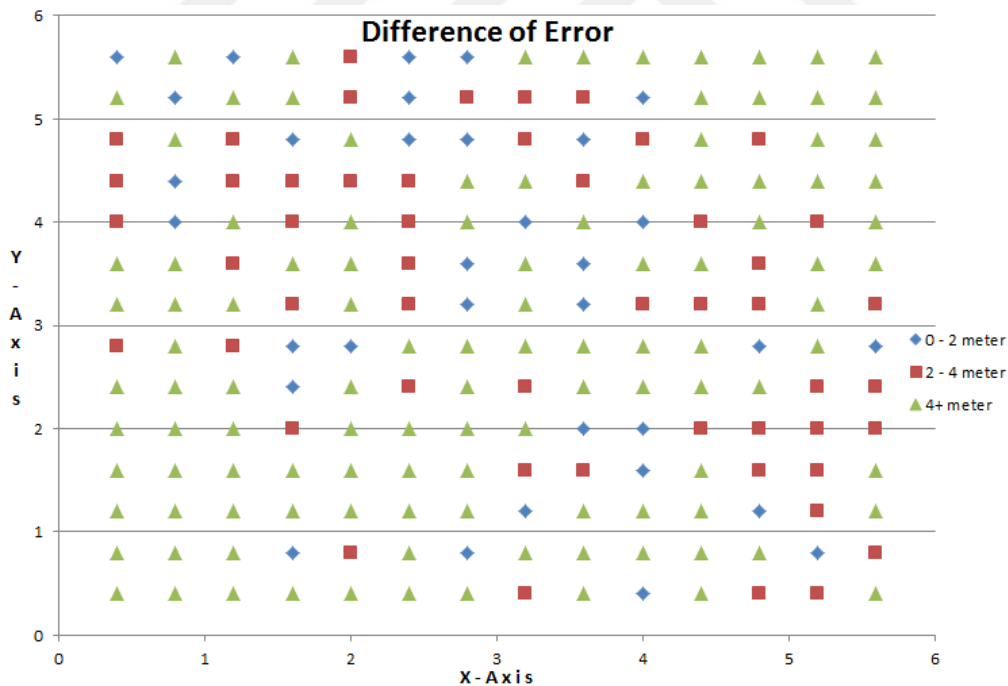


Figure 4.9 Distance of Error Distribution

Errors are classified at all points at the figure 4.9. These classifications are: 0 - 2 meter, 2 - 4 meter and 4+ meter. Each class is represented by a different shape and color. When all values

are analyzed, the estimated point was determined to be close compare the single triangulation algorithm with synthetic data with 5dBm of noise.



5 TRIANGULATION WITH LEAST SQUARE METHOD AND MORE THAN 3 ACCESS POINTS

5.1 More Than 3 Access Points

Triangulation algorithm, as the name suggests, takes the value of 3 access points and then it determines the position with these values. To determine the desired position to be detected with more accurate signals, in addition to channel fixing, optimize a and n values, and least square method, data were used obtained from 4 access points in the previous measurement.

For determine the impact more than 3 access points, algorithm is developed using 4 access points' values. In the testing phase, 4 values given to the algorithm but the algorithm choose 3 values for each access point. So, algorithm is used combination 4 of 3. The value of unused access point is ignored. This development aimed to reduce the margin of error.

5.2 Triangulation Algorithm with Least Square Method and More Than 3 Access Points Result

5.2.1 The result with real data

When the algorithm run with measurement data with A and n values which are showing table 5.1, it is observed that finds the closest value algorithm for real dataset in this thesis. In this

Table 5.1 A and n Values

	A values	N values
Access point 1	-57.69 dBm of signal strength	1.974
Access Point 2	-56.93 dBm of signal strength	1.983
Access Point 3	-52.75 dBm of signal strength	2.073
Access Point 4	-58.96 dBm of signal strength	2.352

In output, the minimum difference shows of error in the first line and the maximum difference of error on the last line (A. 5).

When this result is analyzed;

Average error : 1,352 m
 Minimum error : 0,186 m
 Maximum error : 3,001 m

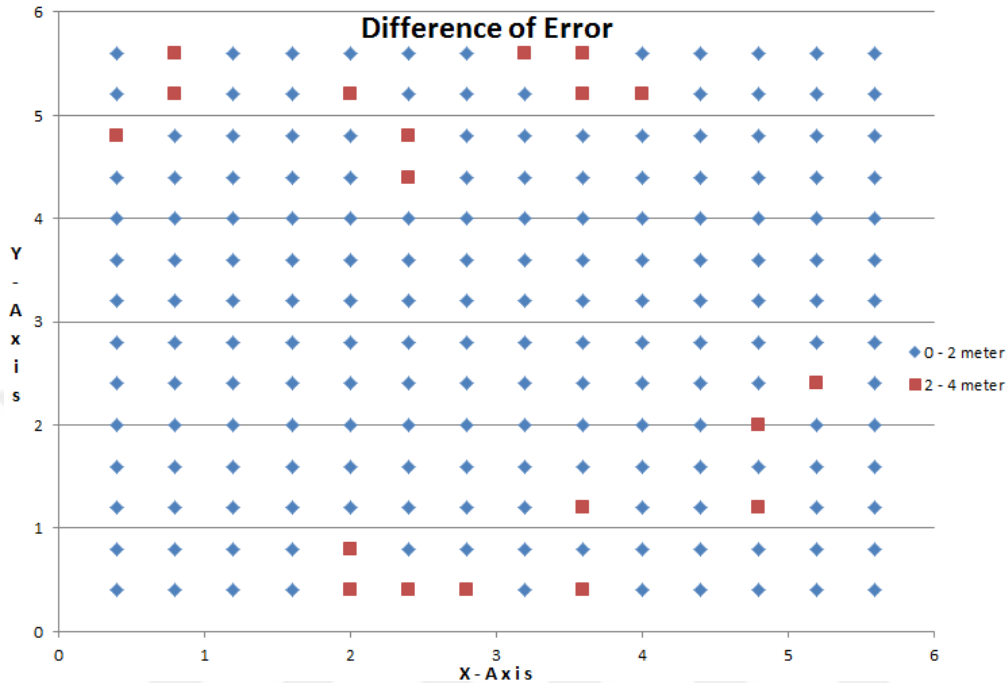


Figure 5.1 Difference of Error Distribution

Errors are classified at all points at the figure 5.1. These classifications are: 0-2 meter, 2-4 meter and 4+ meters. Each class is represented by a different shape and color. The only difference is the whole previous outputs; it has not more than 4 meter error.

When all values are analyzed, more than access points is very successful improvement. In this way, the output compare from the result of the triangulation algorithm with least square method is observed that 52% improvement. Therefore, this output shows that, it is the best indoor positioning system output with real data in this thesis.

5.2.2 The result with error free synthetic data

When the triangulation with least square method and more than 3 access point algorithm runs with synthetic data which is used in chapter 3.4.2, the success of the algorithm was seen in output.

When this result is analyzed;

Average error : 0 m

Minimum error : 0 m

Maximum error : 0 m

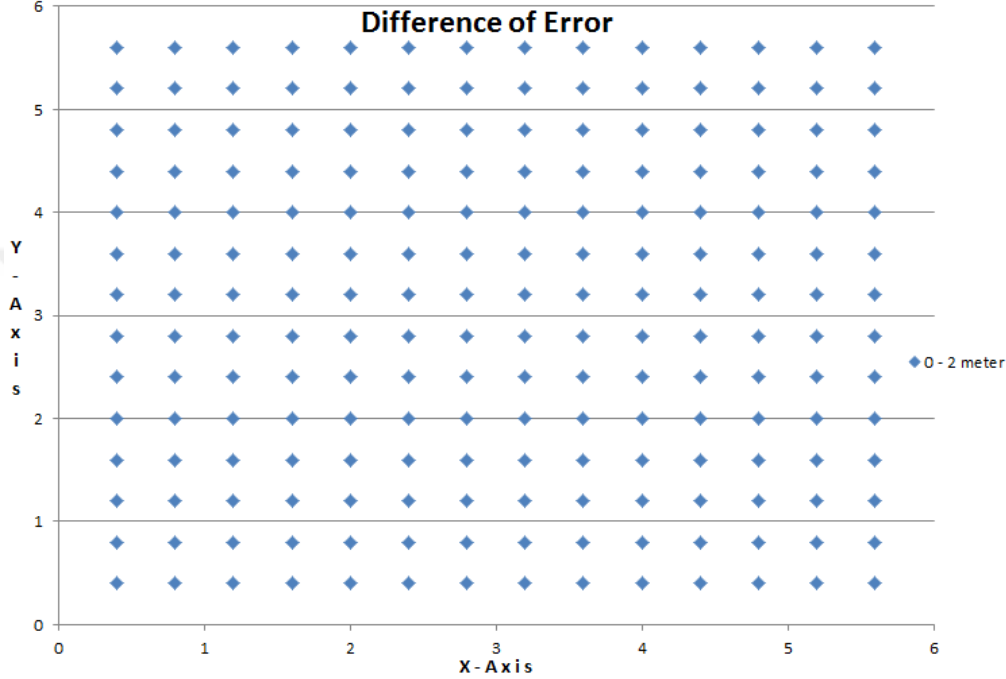


Figure 5.2 Difference of Error Distribution

As seen of figure 5.2 the result of the test environment data indicate more accurate position. Errors are classified at all points at this figure. These classifications are: 0-2 meter. Therefore, output shows that this algorithm estimates the position very successfully.

5.2.3 The result with synthetic data with 5 dBm of noise

When the algorithm run with synthetic data with 5 dBm of noise, output shows that triangulation with least square method with more than 3 access points are very successful to compare the triangulation algorithm with synthetic data with 5dBm.

In triangulation with least square method with more than 3 access point synthetic data with 5 dBm of noise output, the minimum difference shows the error in the first line and the maximum difference of error on the last line (A. 6).

When this result is analyzed;

- Average error : 2,319 m
- Minimum error : 0,196 m
- Maximum error : 5,166 m

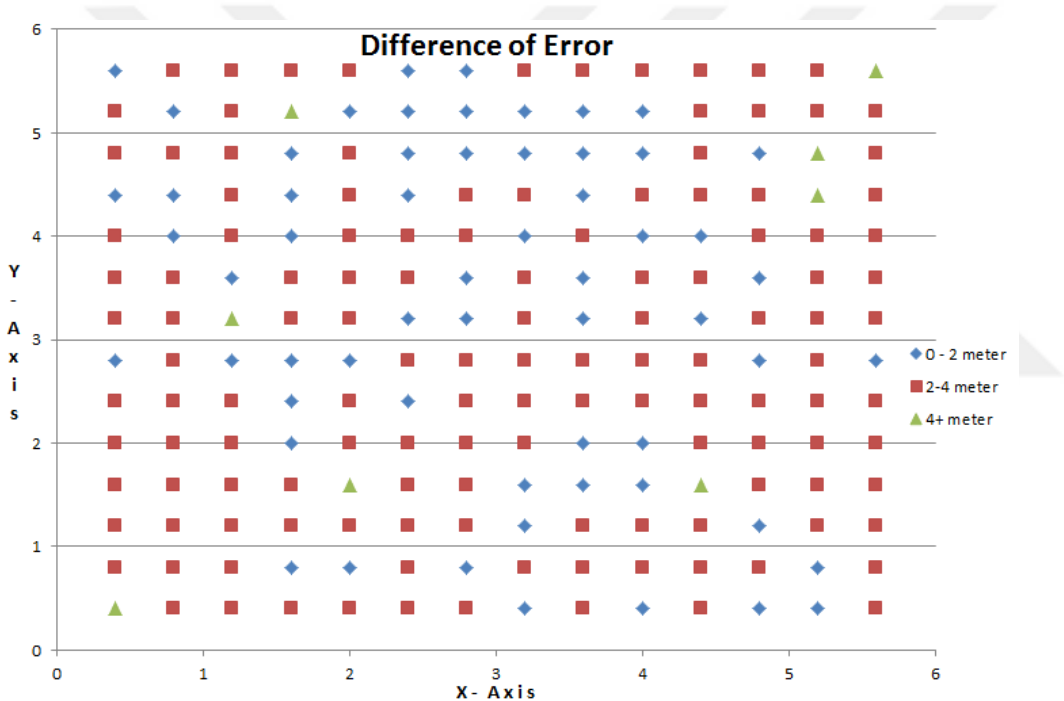


Figure 5.3 Difference of Error Distribution

As seen of figure 5.3, errors are classified at all points. These classifications are: 0 - 2 meter, 2 - 4 meter and 4+ meter. Each class is represented by a different shape and color. This output shows that, it is the best indoor positioning system output with 5dBm of noise in this thesis.

6 CONCLUSION

According to the results of all measurements in this thesis;

In measurements made with simple triangulation algorithm output (without any development in triangulation algorithm and measurement feature), it is determined that there are too many errors.

The result of the measurements with least square method development in triangulation algorithm and measurement feature, it was seen that error was too low.

The result of (4, 3) combination shows that the percentage of error decreased comparing to the previous measurements.

Errors free dataset outputs showed the accurate position, because there has been no signal overlapping in test environment. With least square developed algorithm also with combination of (4, 3) access points, it was seen that all algorithm is working successfully.

Synthetic dataset with 5 dBm of noise output shows that, signal overlapping in noisy area (5 dBm of noise). In these measurements, they have been showed that, signal strength is not decreased in proportion to distance from the signal access point for that signal overlapping.

To see all the results together, all algorithm outputs show in table 6.1.

Table 6.1 Output of all running algorithm

	6 meter * 6 meter								
	Triangulation			Triangulation + LSM			Triangulation + LSM + Access Points		
	Synthetic 0 dBm	Synthetic with 5 dBm of noise	Real	Synthetic 0 dBm	Synthetic with 5 dBm of noise	Real	Synthetic 0 dBm	Synthetic with 5 dBm of noise	Real
Average (meter)	0	9,797	8,739	0	4,882	2,826	0	2,319	1,352
Max (meter)	0	15,180	13,515	0	11,326	7,514	0	5,166	3,001
Min (meter)	0	3,488	3,295	0	0,414	0,004	0	0,196	0,186
0- 2 meter error (piece)	196	0	0	196	33	62	196	59	177
2 - 4 meter error (piece)	0	1	2	0	51	89	0	129	19
4+ meter error (piece)	0	195	194	0	112	45	0	8	0

Results are compared with fuzzy algorithm; fuzzy algorithm indoor location is seen that the location is less defective [12]. However, 4 access points must be in measurement area for fuzzy algorithm. These access points should be placed on a square measurement area. There is no such necessity for triangulation like fuzzy. Triangulation algorithm is done regardless of the geometry of the measuring area with a minimum of 3 access points.

When simplify the above table to make it more understandable, table 6.2 is obtained. This table shows that triangulation with least square method with more than 3 access points is the best indoor positioning system in this thesis. Also it displayed that, after all the accuracy is further improved as the error 84% reduced with the real dataset compared to the simple triangulation algorithm.

Table 6.2 Average error meters of all outputs

	Average error meter		
	Synthetic 0 dBm	Synthetic with 5 dBm of noise	Real
Triangulation	0 m	9,797 m	8,739 m
Triangulation + LSM	0 m	4,882 m	2,826 m
Triangulation + LSM + Access Point	0 m	2,319 m	1,352 m

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8 Curriculum Vitae

Ebru Alp was born on 15 January 1989, in Ankara. She received her BS degree in Computer Engineering in 2013 from Kadir Has University and also she was accepted M.S. degree in Computer Engineering from Kadir University in same year. She has been working in the Vodafone Telecommunication Company since July, 2013.

Appendix A - Outputs of Algorithms

A.1 Triangulation algorithm output with real data

Coordinate of Real x	Coordinate of Real y	Difference of error (meter) = $\sqrt{(X_{\text{Real}}-X_{\text{Estimation}})^2 + (Y_{\text{Real}}-Y_{\text{Estimation}})^2}$
4,8	1,2	3,294891197
3,2	0,8	3,412814235
1,2	5,2	4,037757051
4,8	0,4	4,072932236
5,2	2,4	4,180637511
5,6	0,8	4,864186057
5,2	0,8	5,053153174
5,6	1,2	5,062897293
2,4	1,2	5,445458016
4	0,8	5,452721339
3,2	1,2	5,491564895
5,2	1,2	5,49461773
5,2	2	5,664387875
4,8	2,4	5,797110746
2,8	2,4	5,907496001
1,6	5,6	5,993680005
3,6	0,8	6,018898903
4,4	3,2	6,031372149
2,8	4,8	6,12213729
4,8	2	6,128353286
0,8	5,2	6,185835513
5,2	1,6	6,186561646
5,6	1,6	6,28997186
4	2,8	6,328803441
3,6	5,2	6,474242504
4,4	2	6,514206168
1,6	0,4	6,536850694
2,8	4,4	6,552323099
5,6	2	6,705495806
2	5,2	6,711031217
2,8	1,2	6,782737574
5,6	2,4	6,800448882

4,4	5,2	6,853986942
2,8	0,8	6,926797601
4,4	0,4	6,933527601
3,6	4	6,966290548
5,6	0,4	6,994126178
3,2	4	7,058222156
4,8	0,8	7,06933639
4,8	3,2	7,101784564
4	4	7,193388979
5,2	0,4	7,254097394
4,4	4	7,327797213
4,8	5,6	7,338553877
2,4	3,6	7,362931617
1,2	5,6	7,43597539
4	2,4	7,449819662
0,4	5,6	7,5637299
1,6	5,2	7,600001053
1,6	2,4	7,606207268
2,8	0,4	7,69305661
3,2	5,2	7,69375669
3,2	4,4	7,771456749
4,4	4,4	7,7730705
1,6	3,6	7,810471945
2,4	0,8	7,816330661
0,8	3,6	7,825451105
1,2	1,6	7,839623524
5,6	2,8	7,851373192
5,2	4,4	7,856810676
3,6	4,8	7,912583017
3,6	3,2	7,916406066
0,8	5,6	7,921824285
4,8	4,4	8,014526936
2,4	0,4	8,026294413
0,4	2	8,031309358
2,8	5,6	8,036906992
0,8	2	8,048729713
2	4	8,066775626
5,2	2,8	8,162798111
0,4	1,2	8,184399306
3,2	0,4	8,240689898
2	0,8	8,251693766
2,4	3,2	8,287006034
2,8	2	8,288754128
3,2	2,4	8,400243449
4,8	2,8	8,401956498
0,4	1,6	8,436660121
4,8	4,8	8,462698033
2,8	3,2	8,470589354
1,6	3,2	8,502548912
2	5,6	8,502761081
4,8	4	8,505851163
2	2,4	8,508143922

0,4	4,8	8,513158051
0,8	4,8	8,555925432
4,4	0,8	8,592534667
4	2	8,663998673
2,4	4	8,710069632
3,2	4,8	8,722024134
4,8	3,6	8,736396568
2	2	8,777361164
0,4	5,2	8,810614337
5,6	5,6	8,815949297
3,6	2,4	8,819960091
4,8	1,6	8,824532736
2	4,4	8,833586191
4	4,4	8,857711499
1,2	4	8,901688042
2,4	4,8	8,948310455
1,6	4,4	8,95348569
4	1,6	8,96387472
4,4	5,6	8,988955501
1,2	2	8,989141895
2	1,2	9,00353936
3,6	1,2	9,040344297
3,2	2	9,042812892
0,4	3,6	9,055523673
3,6	4,4	9,05658037
3,6	1,6	9,072674799
1,2	2,8	9,087325734
0,8	3,2	9,093162541
5,2	5,2	9,128899441
2,4	5,6	9,146063197
2,8	3,6	9,149804042
4,4	1,2	9,173590355
4,4	2,8	9,214409422
4,4	4,8	9,216948953
5,6	5,2	9,230390728
1,2	1,2	9,317164054
5,6	3,2	9,337927447
0,8	2,4	9,366603333
0,4	4	9,403411349
3,2	5,6	9,458170859
4	3,2	9,477040941
1,6	2,8	9,510005889
0,8	4,4	9,513449112
3,2	1,6	9,540678854
1,2	2,4	9,543399028
5,6	4,4	9,555047933
2	2,8	9,573473769
4	1,2	9,588531952
1,2	4,4	9,592611532
1,6	4	9,653537227
0,4	0,8	9,656462706
2,8	1,6	9,739000359

4	5,2	9,802422405
3,6	2	9,887114139
1,2	3,6	9,891651126
5,2	4,8	9,894289515
3,6	3,6	9,8967818
4	0,4	9,899762472
0,8	1,2	9,901935013
2,4	4,4	9,909349323
2	3,6	9,915807834
0,4	4,4	9,955841552
4	5,6	9,962389372
1,6	4,8	9,990528815
0,4	0,4	10,01398148
2	1,6	10,04622242
2,4	2,8	10,06896842
1,6	2	10,07823819
5,6	4	10,09374187
5,2	3,6	10,13637139
5,2	4	10,14622437
1,6	1,6	10,1511324
3,6	0,4	10,17778217
0,8	2,8	10,2110259
1,2	0,4	10,25198303
3,2	3,2	10,26739655
4	4,8	10,3310771
4,8	5,2	10,33684173
5,6	3,6	10,439181
1,2	4,8	10,45101225
4,4	1,6	10,47225296
0,8	0,4	10,5466974
5,6	4,8	10,65537845
3,6	2,8	10,66060116
2	3,2	10,6878816
1,6	1,2	10,73432932
2,4	5,2	10,80220001
2,4	1,6	10,81610692
2,8	4	10,91441359
3,6	5,6	10,98665964
3,2	3,6	11,06422632
1,2	0,8	11,06799214
0,8	0,8	11,08028384
0,8	1,6	11,11633757
0,4	2,8	11,23333383
4,4	3,6	11,29773787
0,4	3,2	11,34031366
0,4	2,4	11,40482867
5,2	5,6	11,41964119
2	0,4	11,55919534
2,8	2,8	11,62176802
4,4	2,4	11,69280334
1,6	0,8	12,06643651
3,2	2,8	12,28105085

4	3,6	12,28869611
2,4	2	12,3559444
2,4	2,4	12,41267256
2,8	5,2	12,57194595
0,8	4	12,57325455
5,2	3,2	12,89053579
1,2	3,2	13,00073617
2	4,8	13,51499108

A. 2 Triangulation Algorithm output with synthetic data with 5dBm

Coordinate of Real x	Coordinate of Real y	Difference of error (meter) = $\sqrt{(X_{\text{Real}} - X_{\text{Estimation}})^2 + (Y_{\text{Real}} - Y_{\text{Estimation}})^2}$
3,6	1,2	3,488006795
4	5,6	4,144912138
1,2	5,6	4,227688667
0,8	1,2	4,383816068
3,6	3,2	4,596712781
1,2	0,4	5,621665979
0,4	1,6	5,636173458
0,8	2,4	5,636173458
2,8	5,6	5,636173458
3,6	2,8	5,706664741
4	3,2	5,722898854
2,4	2,8	5,739245846
2,8	2,4	5,767716273
4,8	0,8	5,888797749
4,8	4,8	6,030580202
1,2	2,4	6,058135141
5,6	0,4	6,134261456
2	2,8	6,244275704
2,4	1,2	6,267736805
2	1,2	6,267736805
3,2	3,2	6,407638866
5,2	1,6	6,553770887
2	4	6,559138109
1,2	0,8	6,598365523
2,4	2,4	6,599184501
1,6	1,6	6,70685511
5,6	2,4	6,8292988
1,2	1,6	6,849618957
3,6	3,6	6,925793431
1,6	0,8	6,973808211
2,4	4	7,008630849

2,8	4,4	7,026348533
1,6	4,8	7,030675933
1,6	5,6	7,031226266
5,2	5,6	7,054509093
0,8	3,6	7,061452426
3,2	0,4	7,078176658
0,4	1,2	7,083085007
4,8	4	7,083085007
2,4	5,6	7,116217307
3,2	2,4	7,146496243
4,8	3,2	7,146496243
3,6	4,4	7,194349773
2,8	2,8	7,199439681
4,4	2,8	7,205205302
4,4	0,4	7,216837533
1,2	2,8	7,221753437
1,2	4,8	7,233654955
2,8	3,2	7,242693207
4,4	0,8	7,294031881
2,4	0,8	7,294031881
2,8	3,6	7,355165202
0,4	4	7,372775754
3,6	0,8	7,392961724
0,8	4	7,397272263
1,2	2	7,412747671
4	5,2	7,503980236
4,4	2	7,642275198
3,2	4,8	7,643144367
3,2	1,6	7,700608557
2,8	4	7,703124116
1,6	1,2	7,735811459
0,8	1,6	7,810475693
4,4	4,8	7,824650456
5,6	2	7,868576261
2,8	0,4	7,881254209
4	4,4	7,95054269
4,4	5,6	7,955700091
4	3,6	7,993688314
2,4	4,4	8,059867867
4	0,8	8,079791591
1,2	3,6	8,097195436
1,6	4	8,140463057
2	0,4	8,1417741
5,2	3,6	8,1417741
2	1,6	8,17396506

2	2,4	8,178917152
5,2	0,8	8,218656008
0,8	4,4	8,219286977
1,2	4	8,307273553
4	2	8,457160204
2,4	1,6	8,554470378
3,2	4,4	8,568398947
1,6	2	8,781354605
2	5,6	8,78810269
4,4	1,6	8,812026685
3,6	0,4	8,8541894
2,4	0,4	8,941275896
2,4	4,8	9,015494308
4,8	5,2	9,046504765
1,2	1,2	9,056632666
1,2	4,4	9,08397795
0,8	2	9,096153398
4,8	2,4	9,105628507
0,4	4,4	9,118665471
5,6	4,8	9,126293318
5,2	3,2	9,154217858
5,6	2,8	9,154217858
3,6	5,6	9,170225412
2	2	9,208769386
2,8	2	9,215583799
3,6	2	9,255975481
4,8	4,4	9,263078249
4	2,4	9,26734291
5,6	1,2	9,275239067
4,4	4	9,355632035
2,8	1,2	9,396160439
0,4	2	9,41795872
5,2	2,8	9,514305229
5,6	5,6	9,617576513
3,2	4	9,686589274
0,4	3,6	9,704235027
5,2	4,8	9,729481523
3,2	3,6	9,813655783
3,2	1,2	9,852470743
2	4,4	9,852470743
0,4	3,2	9,863113648
5,2	1,2	9,915948209
2,8	0,8	10,13975755
0,8	2,8	10,15138444
0,8	0,8	10,17417887

4,4	2,4	10,24227009
2	3,6	10,25671287
2	5,2	10,31033991
1,2	3,2	10,35104577
3,2	5,6	10,4187746
4	1,2	10,43799227
3,6	2,4	10,49508908
5,2	0,4	10,49909263
0,8	5,6	10,76994484
2,4	2	10,81761602
4	2,8	10,93436802
0,4	0,8	11,0327478
3,2	0,8	11,03975297
1,6	2,4	11,04397296
4	1,6	11,04821245
4,4	4,4	11,04966988
5,6	4,4	11,18737493
2,4	3,6	11,39737035
0,8	0,4	11,49911245
3,2	5,2	11,55266831
4,8	1,2	11,55378299
1,6	4,4	11,65683186
3,6	4,8	11,73633669
4	0,4	11,73633669
4,4	3,6	11,80752872
5,2	2	11,83503405
2,8	4,8	12,07533703
3,6	5,2	12,28446979
4	4	12,31841603
5,2	4,4	12,33440815
1,6	0,4	12,74260949
4,8	3,6	12,76327801
5,6	0,8	13,21110063
2	0,8	13,41371022
3,6	4	13,52324146
5,6	5,2	13,55503075
3,2	2	14,04898589
2,8	1,6	14,07033709
2,4	5,2	14,09020625
2,4	3,2	14,1750073
4,4	3,2	14,1750073
5,6	4	14,19325713
0,4	2,4	14,24104038
5,2	5,2	14,26597661
4,8	0,4	14,30048708

0,4	2,8	14,35139936
1,6	2,8	14,35992646
5,6	1,6	14,39014284
0,4	0,4	14,43134004
0,8	4,8	14,43134004
3,2	2,8	14,44335772
1,6	3,2	14,55585021
1,6	3,6	14,57386936
2	3,2	14,58573902
4,8	2,8	14,5892518
0,4	5,6	14,59102897
5,6	3,2	14,59222816
4,8	1,6	14,59828675
0,8	5,2	14,6712196
0,8	3,2	14,71988514
4,4	5,2	14,76391036
2,8	5,2	14,78730627
5,6	3,6	14,78730627
4,4	1,2	14,79040623
5,2	4	14,83031998
4,8	5,6	14,85127862
0,4	4,8	14,88863131
3,6	1,6	14,94041026
5,2	2,4	14,96912316
2	4,8	15,01014228
4,8	2	15,06577683
0,4	5,2	15,11184452
1,6	5,2	15,16725144
1,2	5,2	15,18085804
4	4,8	15,18085804

A. 3 Triangulation Algorithm with least square method output with real data

Coordinate of Real x	Coordinate of Real y	Difference of error (meter) = $\sqrt{(x_{\text{Real}} - x_{\text{Estimation}})^2 + (y_{\text{Real}} - y_{\text{Estimation}})^2}$
5,6	2,8	0,004427
1,6	2,4	0,099187
4,8	2,8	0,174505
4,8	1,2	0,179406
2,4	5,2	0,253072
2,8	0,8	0,275483
3,6	3,6	0,303358
4	4	0,345874

3,6	3,2	0,394159
3,2	1,2	0,396732
2,8	3,6	0,421368
1,6	0,8	0,52824
1,6	4,8	0,577804
2,8	5,6	0,626465
2,8	4,8	0,666236
1,6	2,8	0,725345
4	5,2	0,727184
2,8	3,2	0,735175
0,4	5,6	0,770443
2,4	5,6	0,799302
3,2	4	0,799436
3,6	2	0,816812
2,8	5,2	0,835337
0,4	3,6	0,847614
4	1,6	0,922487
3,2	0,4	0,974868
3,6	4,8	0,975042
1,2	2,8	0,985023
3,6	5,2	1,002097
2	2,8	1,012248
2,4	4,8	1,013912
0,8	0,4	1,054366
0,8	5,2	1,08662
3,2	5,2	1,125816
0,4	1,2	1,186505
3,2	1,6	1,299386
1,6	4,4	1,331721
3,6	4,4	1,344532
5,2	0,8	1,354323
2,4	2,4	1,363817
0,8	3,6	1,368249
0,8	0,8	1,402087
0,4	4,8	1,407588
0,4	2	1,425239
0,8	1,2	1,431059
0,4	3,2	1,434446
4	2	1,436552
4,4	4	1,493297
2	0,8	1,527084
0,4	0,4	1,585735
4,8	0,4	1,593909
2,4	4,4	1,625416
1,6	2	1,627481

4,8	3,6	1,633564
0,4	4	1,712453
0,8	1,6	1,769384
1,6	4	1,771063
4,8	4,8	1,790196
4	0,4	1,902063
0,4	1,6	1,961996
0,8	3,2	1,972452
1,2	3,6	1,982991
0,8	4,4	2,022885
3,2	2,4	2,613031
0,8	2	2,047592
4,4	3,2	2,004459
5,2	0,4	2,079809
3,2	5,6	2,109399
5,6	2	2,111525
2,4	3,2	2,122344
0,8	2,8	2,127629
2,4	4	2,144049
4,8	3,2	2,147031
3,2	4,8	2,166724
4	4,8	2,19247
0,4	5,2	2,209448
2	5,2	2,264016
0,8	4	2,268352
2,4	3,6	2,291973
4	4,4	2,303145
4,8	1,6	2,30664
5,6	2,4	2,348931
4	3,2	2,353181
5,6	3,2	2,364011
1,6	1,6	2,36978
3,6	1,6	2,377924
5,2	1,2	2,389273
1,2	4,8	2,427586
1,2	5,2	2,445028
0,4	4,4	2,446116
0,4	2,4	2,488678
0,4	0,8	2,493556
1,6	1,2	2,501644
4,4	4,8	2,514739
2	5,6	2,566631
5,2	4	2,576834
5,2	2	2,622409
2,8	2,4	2,666134

4,4	2	2,679612
3,2	3,6	2,70565
2	4	2,710596
4,8	2	2,723641
2	3,6	2,760816
4,8	4,4	2,78331
4	5,6	2,788423
1,6	3,2	2,794961
2,4	0,8	2,826741
1,2	4,4	2,838902
2	4,4	2,884675
5,2	3,6	2,892976
5,2	2,4	2,908323
1,2	2	2,927767
0,4	2,8	2,936508
5,2	3,2	2,937152
4,8	5,2	2,960792
2,8	0,4	2,974219
1,2	5,6	2,986672
2,4	2	3,033158
2,8	2	3,043236
1,6	3,6	3,082818
5,2	1,6	3,119451
4,4	4,4	3,128433
5,6	5,2	3,144354
5,6	0,8	3,174327
0,8	2,4	3,201487
2,4	0,4	3,223388
3,6	1,2	3,284425
3,6	0,8	3,295915
1,2	0,4	3,301514
4,4	2,8	3,321935
5,6	4	3,329522
4	0,8	3,345095
5,6	1,2	3,353576
5,6	4,8	3,414987
4,4	0,8	3,420985
2	4,8	3,452923
2,8	2,8	3,464219
5,6	1,6	3,48203
4	2,8	3,501356
3,6	5,6	3,513164
4,4	1,2	3,535534
2	0,4	3,538558
2	3,2	3,581772

5,2	5,6	3,600947
4,8	0,8	3,657947
5,2	2,8	3,760701
3,6	0,4	3,769344
5,2	5,2	3,812927
5,6	3,6	3,825468
5,6	4,4	3,999577
4,8	5,6	3,999943
2,4	1,6	4,034738
2,8	4,4	4,05716
1,2	4	4,070082
1,2	1,2	4,09641
3,2	2,8	4,142278
3,2	4,4	4,145537
4,4	5,2	4,160657
2,4	1,2	4,186167
4	2,4	4,263697
0,8	5,6	4,281552
2	2	4,327542
1,2	2,4	4,353951
3,6	2,4	4,394323
4,4	0,4	4,434424
0,8	4,8	4,445901
3,6	4	4,452778
2,8	1,6	4,454221
3,2	3,2	4,695811
2,4	2,8	4,800376
2,8	1,2	4,842762
3,2	0,8	4,984227
1,6	5,6	5,039249
1,2	0,8	5,072198
4,8	4	5,130901
3,2	2	5,184244
2	2,4	5,42995
4,4	5,6	5,435406
4,4	3,6	5,443447
4,8	2,4	5,498831
1,2	1,6	5,512545
5,6	0,4	5,555458
4,4	2,4	5,611508
1,6	0,4	5,667771
3,6	2,8	5,753799
2,8	4	5,818712
4	3,6	6,016081
4	1,2	6,076707

4,4	1,6	6,122964
2	1,6	6,151722
2	1,2	6,329232
1,6	5,2	6,420329
5,2	4,4	6,710174
5,6	5,6	7,01685
1,2	3,2	7,43322
5,2	4,8	7,513542

A. 4 Triangulation algorithm with least square method output with synthetic data with 5dBm

Coordinate of Real x	Coordinate of Real y	Difference of error (meter) = $\sqrt{(X_{Real} - X_{Estimation})^2 + (Y_{Real} - Y_{Estimation})^2}$
0,4	5,6	0,414353429
2,8	0,8	0,442921376
3,6	3,2	0,481670141
5,2	0,8	0,556374261
2,4	5,6	0,700017583
4,8	2,8	0,767464716
0,8	4,4	1,016854711
3,6	2	1,090398186
3,2	4	1,094065908
0,8	4	1,103411369
1,6	4,8	1,155685091
3,2	1,2	1,162386043
3,6	3,6	1,191494604
5,6	2,8	1,199048879
2,8	3,6	1,206763078
1,6	0,8	1,277452666
2,4	5,2	1,278742632
0,8	5,2	1,33242873
2	2,8	1,334615436
4	4	1,336394121
1,6	2,4	1,357786808
4	1,6	1,42945698
4,8	1,2	1,472401253
2,8	4,8	1,531345192
4	2	1,573622912
3,6	4,8	1,575407768
4	5,2	1,616536329
4	0,4	1,690128393

2,8	3,2	1,734836938
1,2	5,6	1,757688271
2,4	4,8	1,78446426
1,6	2,8	1,901871233
2,8	5,6	1,940279641
3,2	5,2	2,009840238
3,6	4,4	2,146507675
5,2	0,4	2,216355729
1,6	4,4	2,295903623
2,4	3,2	2,296416724
0,4	4,4	2,337954525
4,8	3,6	2,365759871
4,8	0,4	2,439173721
1,6	2	2,45206428
4,4	4	2,473038084
2,8	5,2	2,476351348
3,6	5,2	2,507710814
3,2	0,4	2,531007844
0,4	4	2,574662637
0,4	2,8	2,857482906
2	5,6	2,86026649
5,6	2	2,930172162
5,6	2,4	2,971607708
1,2	2,8	2,994056186
3,2	1,6	3,038896668
4,4	3,2	3,058538332
4,8	4,8	3,233266242
2	5,2	3,322252207
5,6	0,8	3,357393328
1,2	4,8	3,358884665
1,2	3,6	3,42246522
4,8	3,2	3,427664593
4,8	2	3,44651916
0,4	4,8	3,500355809
3,6	1,6	3,52759404
5,6	3,2	3,58203445
5,2	1,2	3,585728036
2	0,8	3,597405677
5,2	2	3,621474827
5,2	2,4	3,628664553
1,6	3,2	3,640148066
2	4,4	3,642648237
4	3,2	3,664349001
2,4	4,4	3,669643085
2,4	3,6	3,670897438

5,2	4	3,694360506
5,2	1,6	3,718929189
4	4,8	3,728733457
2,4	2,4	3,756331399
4,8	1,6	3,792441083
3,2	4,8	3,801031391
4,4	2	3,8274842
3,2	2,4	3,846823339
1,2	4,4	3,870488467
1,6	4	3,899533341
2,4	4	3,944977525
4,4	4,8	4,001894187
0,4	3,6	4,078758724
4	4,4	4,098114797
3,2	5,6	4,107909329
5,2	3,2	4,234547899
4,8	4,4	4,306223454
5,2	3,6	4,355588658
4	0,8	4,356010064
1,2	5,2	4,475425999
2	4	4,538647336
4,4	0,8	4,633730254
3,2	3,6	4,678173906
4,4	4,4	4,681981436
0,8	2,4	4,715245679
4,8	5,2	4,746502699
5,6	4	4,751053101
4	5,6	4,779546495
1,6	1,6	4,90547679
1,6	1,2	4,906195668
5,6	5,2	4,916097615
2,8	2	4,93751951
2,8	2,4	4,952058804
5,6	1,6	5,062220968
4,4	1,2	5,066020315
0,4	1,2	5,070491973
2	3,6	5,105889592
2,8	0,4	5,200792917
4	2,8	5,229025221
2,4	2	5,271550342
0,4	1,6	5,277444146
2,4	0,8	5,406608648
0,4	5,2	5,533317468
5,6	1,2	5,549087741
2,4	0,4	5,582556056

1,6	3,6	5,622007774
2,8	2,8	5,732529192
3,6	5,6	5,734187523
0,8	5,6	5,744136921
3,6	1,2	5,768236932
5,6	3,6	5,775685525
2,4	1,6	5,776165615
0,8	1,6	5,776421704
4,4	2,8	5,804514192
0,8	2,8	5,841674189
5,6	4,4	5,844950989
5,2	2,8	5,884816285
5,6	4,8	5,91091501
5,2	5,6	6,031720354
4	2,4	6,050024788
3,6	0,8	6,069508544
5,2	5,2	6,085498671
2	3,2	6,089242755
0,4	3,2	6,104888021
1,2	0,4	6,146127023
1,2	2	6,220818672
0,4	2,4	6,244619789
4,8	0,8	6,373775588
4,8	5,6	6,55390283
2,8	4,4	6,625641326
2	4,8	6,648271391
3,2	2,8	6,705118697
1,2	4	6,736587023
4,4	5,2	6,739866811
0,8	4,8	6,74864913
3,6	4	6,948717535
3,2	4,4	7,021940106
1,2	1,2	7,024420416
2,4	1,2	7,041498821
2	2	7,079361155
3,6	0,4	7,15567041
4,4	0,4	7,240063851
2	0,4	7,387899667
0,8	0,8	7,50716661
1,2	2,4	7,514742721
0,8	2	7,647086339
1,6	5,6	7,668052504
0,8	3,2	7,698360653
2,8	1,6	7,761332308
0,8	0,4	7,83109822

3,6	2,4	7,835615982
3,2	3,2	7,851505098
2,4	2,8	7,963620501
4,8	2,4	8,136076148
0,8	1,2	8,165977765
4,4	5,6	8,219938355
0,8	3,6	8,225825448
1,2	3,2	8,281233804
5,6	0,4	8,34147081
3,2	0,8	8,470775054
2,8	1,2	8,520292015
3,2	2	8,529057317
3,6	2,8	8,588346667
4,4	3,6	8,631775994
4,4	2,4	8,683780025
0,4	2	8,725819151
2,8	4	8,803330876
0,4	0,8	8,855976986
1,6	5,2	8,872592885
4	3,6	8,914458594
4,8	4	8,920192278
1,6	0,4	9,005708425
2	1,2	9,050856714
2	2,4	9,277490809
0,4	0,4	9,440753819
1,2	0,8	9,485449108
4	1,2	9,712132468
5,2	4,4	9,714131394
2	1,6	9,769569011
1,2	1,6	9,84326547
4,4	1,6	10,22521128
5,6	5,6	10,92799847
5,2	4,8	11,32594132

**A. 5 Triangulation algorithm with least square method and more than 3 access points
output with real data**

Coordinate of Real x	Coordinate of Real y	Difference of error (meter) = $\sqrt{(X_{\text{Real}} - X_{\text{Estimation}})^2 + (Y_{\text{Real}} - Y_{\text{Estimation}})^2}$
3,2	2,8	0,186383394
2,8	4	0,21804022
2,4	3,6	0,242727431

5,2	1,6	0,378896159
3,2	4,4	0,389723205
4	1,2	0,432823348
4	4	0,433529582
4	2,8	0,443785374
3,6	3,6	0,506649795
4	0,8	0,508799672
3,6	2,8	0,51482311
5,6	2	0,525192032
2,8	3,6	0,533236876
4,4	4,8	0,545737375
5,2	3,2	0,546814965
4,4	3,2	0,547854654
4,4	2,4	0,618815754
0,4	0,8	0,624833518
5,6	2,8	0,635099533
4	3,6	0,635815531
2	2	0,638739518
2,4	3,2	0,678340243
2,4	2,8	0,706188555
2	4,8	0,717401325
2,8	2,4	0,717831435
4	2,4	0,732969586
2,8	4,4	0,735658899
4,8	5,2	0,753875922
0,8	0,8	0,75762913
5,2	4	0,767796179
4	3,2	0,791365005
4,4	2,8	0,807208212
4,4	4	0,810588028
3,2	3,2	0,826012833
4,8	2,4	0,827739399
2	3,2	0,828156342
0,8	2	0,838912798
4,8	2,8	0,858303403
5,6	4,4	0,858643867
1,6	4	0,866567681
2	2,8	0,870140235
1,6	2,4	0,871908833
3,6	3,2	0,896397207
2	2,4	0,897018633
1,6	3,6	0,915424911
4,4	2	0,919016934
5,6	1,2	0,924684275
4,4	4,4	0,957757194

5,6	2,4	0,967188433
2,8	3,2	0,987706651
3,2	3,6	0,993497444
4,8	0,4	1,001139515
1,6	2,8	1,010323859
4,8	4	1,01555671
1,2	1,6	1,024670816
0,4	0,4	1,02690166
1,2	3,6	1,026950545
5,2	5,2	1,040859441
5,2	0,4	1,040996309
4,4	5,2	1,042484847
1,6	4,8	1,047600406
2,8	2,8	1,061406466
2,4	2	1,0626895
1,2	1,2	1,066664354
0,8	0,4	1,070889905
5,6	0,8	1,074491022
4,8	1,6	1,100892797
2	3,6	1,102855999
2	4	1,123101202
0,4	1,2	1,133147696
4,8	5,6	1,140828988
2,8	1,6	1,141093557
0,4	2,8	1,148663297
5,6	0,4	1,18409538
5,6	1,6	1,186630868
3,6	1,6	1,190627954
0,8	1,2	1,20131671
3,6	2,4	1,21019949
4	2	1,218017149
2,8	4,8	1,225016772
3,2	0,8	1,230445424
3,2	4,8	1,2468373
5,6	4	1,255484154
5,2	0,8	1,25585721
0,8	2,8	1,256841979
2	1,2	1,259382975
4	4,4	1,261361348
4,8	0,8	1,267679471
2	4,4	1,274820207
4,8	3,2	1,274938739
4,4	1,2	1,275647213
5,6	4,8	1,293293866
3,6	4,4	1,304934736

2,8	1,2	1,306593694
5,2	2	1,309312016
3,2	1,2	1,320081858
1,2	4,8	1,322481669
5,2	1,2	1,325639619
3,2	0,4	1,351585974
2,8	2	1,355195983
4	4,8	1,357820624
4,4	0,8	1,371679373
0,4	2,4	1,3771895
3,2	2,4	1,403821707
1,6	1,2	1,408726128
2,4	2,4	1,413293568
5,2	2,8	1,415283882
3,2	5,2	1,42363029
0,4	1,6	1,430156607
0,4	3,2	1,431434925
3,2	1,6	1,441682556
1,2	4,4	1,449608353
4,4	3,6	1,461469389
1,2	4	1,462131805
3,6	4,8	1,472460939
1,2	3,2	1,478954297
3,6	0,8	1,481518547
2,4	5,2	1,484803222
5,6	5,6	1,485837264
2,4	4	1,497870067
5,2	4,8	1,513988185
4,8	4,8	1,518804655
1,6	2	1,523440427
4,8	3,6	1,53199723
4,4	1,6	1,537960153
0,4	3,6	1,540600045
0,4	4	1,549241914
0,8	4	1,558170109
2,4	5,6	1,577368783
2	5,6	1,582342906
5,6	5,2	1,601744022
2,4	1,2	1,603710295
5,2	4,4	1,606710179
2	1,6	1,611201551
1,2	2,8	1,612320164
0,4	4,4	1,619605185
0,8	3,2	1,6229159
1,2	2,4	1,628134552

3,2	4	1,629616861
1,2	5,6	1,639907067
1,2	2	1,640664219
2,8	5,6	1,648228792
5,2	3,6	1,654565326
4	0,4	1,670483354
3,2	2	1,672646976
5,2	5,6	1,697539613
0,4	2	1,709631314
0,8	3,6	1,713292559
1,6	1,6	1,724851876
1,2	5,2	1,725439805
1,6	3,2	1,73991379
3,6	2	1,744670422
0,8	4,4	1,762416807
3,6	4	1,777725266
1,6	0,4	1,806102935
4,8	4,4	1,815877422
4	1,6	1,817100664
2,8	0,8	1,818253348
1,6	4,4	1,81842009
0,4	5,2	1,827842973
0,8	2,4	1,830424518
2,4	0,8	1,838917365
2,8	5,2	1,843995141
1,6	5,6	1,864520826
1,2	0,8	1,865208117
5,6	3,2	1,869315058
1,6	0,8	1,882032674
4,4	0,4	1,890642611
0,8	4,8	1,901518559
5,6	3,6	1,913950561
4	5,6	1,940820251
1,2	0,4	1,949940696
0,4	5,6	1,959670398
4,4	5,6	1,972264844
2,4	1,6	1,977458631
1,6	5,2	1,978014718
0,8	1,6	1,99235754
0,4	4,8	2,022776021
3,6	1,2	2,050649088
5,2	2,4	2,073945955
4,8	2	2,139670018
4	5,2	2,157275865
3,2	5,6	2,221714188

2,4	4,4	2,266295469
2,4	0,4	2,274901707
0,8	5,2	2,294249594
2	0,8	2,305404843
3,6	0,4	2,30831336
3,6	5,2	2,332954558
2	0,4	2,401657914
2,4	4,8	2,482119543
0,8	5,6	2,483961032
2,8	0,4	2,533388461
2	5,2	2,580650795
4,8	1,2	2,593381699
3,6	5,6	3,001000534

**A. 6 Triangulation algorithm with least square method and more than 3 access points
output with synthetic data with 5dBm**

Coordinate of Real x	Coordinate of Real y	Difference of error (meter) = $\sqrt{(x_{\text{Real}} - x_{\text{Estimation}})^2 + (y_{\text{Real}} - y_{\text{Estimation}})^2}$
2,4	5,2	0,1960662
4,8	1,2	0,2298217
2,8	0,8	0,3807621
5,6	2,8	0,426091
1,6	2,4	0,4699958
3,6	3,2	0,5432771
4,8	2,8	0,543979
3,6	3,6	0,6391136
1,6	0,8	0,6630747
2,8	3,6	0,7166379
4	4	0,7314261
3,2	1,2	0,7682362
1,6	4,8	0,7714701
2,8	5,6	0,8074525
3,2	4	0,8711136
3,6	2	0,8940906
2,4	5,6	0,9075353
2	2,8	0,9124673
2,8	4,8	0,9159358
2,4	4,8	0,9357603
1,6	2,8	0,9603425
0,8	4	0,9627366
3,6	4,8	0,964354
2,8	3,2	0,9705292

4	5,2	0,9733293
0,4	5,6	1,067608
3,2	5,2	1,090165
2,8	5,2	1,094394
4	1,6	1,142998
0,8	5,2	1,167325
5,2	0,8	1,167574
1,2	2,8	1,177616
0,4	2,8	1,206038
3,6	4,4	1,209068
4	2	1,209252
3,2	0,4	1,211176
3,6	5,2	1,216411
0,8	4,4	1,292201
4,4	4	1,321922
3,2	1,6	1,434152
2	0,8	1,436649
4,8	3,6	1,451828
4,8	0,4	1,454856
2,4	2,4	1,518045
1,6	4,4	1,543296
1,2	3,6	1,706908
4	0,4	1,744751
1,6	2	1,748022
4,4	3,2	1,748245
1,6	4	1,803502
2,4	4,4	1,83658
2,4	3,2	1,84134
0,4	4,4	1,841461
3,2	4,8	1,869025
4,8	4,8	1,89655
2	5,2	1,967291
5,2	0,4	1,972462
4	4,8	1,975337
3,6	1,6	1,999916
3,2	5,6	2,062886
2,4	4	2,066401
1,2	4,8	2,070704
4,8	3,2	2,130319
3,2	2,4	2,194487
5,6	2	2,196537
5,2	1,2	2,218063
4,4	4,8	2,221155
1,6	1,6	2,22621
2	5,6	2,229705

4	4,4	2,230304
5,2	4	2,231278
4	3,2	2,233106
5,6	2,4	2,255941
0,4	4	2,256773
5,6	3,2	2,257569
4,4	2,8	2,259072
5,6	1,6	2,301803
4,8	1,6	2,337638
3,2	3,6	2,341853
4,4	0,8	2,364631
5,6	1,2	2,365594
2,4	3,6	2,369468
4,8	2	2,397037
2	3,6	2,399445
1,6	3,2	2,404656
2	4,8	2,413583
0,8	1,6	2,423732
0,4	1,2	2,425144
1,6	1,2	2,430115
4,8	4,4	2,432029
1,2	5,2	2,432304
5,6	4,8	2,434986
5,2	2,4	2,436113
2,8	2,4	2,437045
5,2	2	2,441509
2	4	2,443419
4	5,6	2,44396
0,4	4,8	2,445898
4,4	2	2,46288
4,8	5,2	2,473283
2,8	0,4	2,47866
0,8	0,8	2,491708
5,2	5,2	2,494489
0,8	2,8	2,496911
2	3,2	2,500416
5,2	2,8	2,504324
5,2	5,6	2,506151
4,8	5,6	2,506509
2	0,4	2,512989
5,6	4,4	2,540597
1,2	5,6	2,544578
0,4	3,2	2,549257
4,8	0,8	2,549317
2,8	4,4	2,549318

4,4	4,4	2,552041
5,2	1,6	2,556857
2,8	2	2,557535
2	4,4	2,559897
0,8	2,4	2,56438
5,6	0,8	2,578513
0,4	2,4	2,600432
5,6	3,6	2,601262
3,6	0,4	2,619483
1,6	3,6	2,622336
0,4	1,6	2,626804
0,8	0,4	2,629985
4	2,4	2,63162
2,4	1,6	2,632823
4,4	5,2	2,63581
1,2	4,4	2,640852
2,4	1,2	2,641957
0,8	2	2,650321
1,2	4	2,659426
2,4	2	2,685537
5,2	3,6	2,692913
0,8	4,8	2,694776
5,6	5,2	2,698068
0,4	3,6	2,7029
2,4	0,4	2,70663
5,2	3,2	2,714564
2,4	0,8	2,714775
3,2	4,4	2,735126
3,2	2,8	2,747197
1,2	1,2	2,757197
1,2	2	2,764222
4	0,8	2,764643
3,6	5,6	2,824769
4	2,8	2,839586
5,6	4	2,839946
1,2	2,4	2,847355
4,4	0,4	2,850564
0,8	5,6	2,854806
2,8	1,6	2,856018
3,6	2,4	2,858296
2,4	2,8	2,865209
1,2	0,4	2,868925
3,6	4	2,872067
3,2	3,2	2,878178
2	2	2,881992

0,4	5,2	2,882795
3,6	1,2	2,883545
4,4	1,2	2,893729
0,8	1,2	2,894649
3,6	0,8	2,912015
2,8	2,8	2,915034
0,8	3,2	3,063094
3,2	0,8	3,068568
2,8	1,2	3,088078
1,6	5,6	3,163542
0,4	2	3,238332
0,8	3,6	3,253217
1,2	0,8	3,323577
3,2	2	3,331535
2	2,4	3,332407
4,8	2,4	3,347069
4,4	5,6	3,352888
5,6	0,4	3,387736
0,4	0,8	3,391484
4,8	4	3,398088
4,4	3,6	3,461214
4,4	2,4	3,475235
1,2	1,6	3,594924
2,8	4	3,603863
3,6	2,8	3,651054
4	3,6	3,692429
1,6	0,4	3,778641
4	1,2	3,850814
2	1,2	3,970847
0,4	0,4	4,174771
4,4	1,6	4,176265
2	1,6	4,193788
1,6	5,2	4,197289
5,2	4,4	4,616683
5,6	5,6	4,832418
5,2	4,8	5,011327
1,2	3,2	5,165994