Metin ŞENGÜL/ IU-JEEE Vol. 16(2), (2016), 3043-3047

Broadband Matching via Reflection Coefficient Modeling

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Abstract: *In this paper, a practical method is presented to design broadband matching networks via reflection coefficient modeling. In the proposed algorithm, reflection function values (ρ_2) at sample frequencies are optimized to get the desired gain level. At the same time, the corresponding reflection coefficient values (S_2) are calculated and modeled. Then matching network topology and element values are obtained via the formed reflection coefficient expression. An example is presented to explain the usage of the new method.*
Keywords: *Broadband Matching, Lossless Networks, Matching Networks, Modeling, Real Frequency Techniques.*

1. Introduction

Matching a generator to a complex load impedance is to design a lossless two-port network to maximize the power transfer from the generator to the load over an interested frequency band. The matching performance of the system is best measured by means of the transducer power gain (TPG) which is the ratio of power delivered to the load to the available power from the generator (Fig. 1).

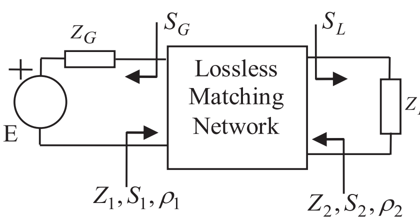

Figure 1. Double matching arrangement

In a matching problem, if the generator impedance is purely resistive and the load impedance is complex, then it is a single matching problem. But if both the generator and load impedances are complex, then the problem is a double matching problem.

In literature, there are lots of techniques to design broadband matching networks. But they can be grouped basically as the methods based on TPG optimization and the methods based on modeling. In the first group, the selected free parameters are optimized until reaching an acceptable gain level [1-7]. In the second group, firstly the values of any selected function are calculated usually via TPG optimization, and then a model is formed for the obtained data [8, 9].

But the problem of the methods in the second group is to obtain a realizable data set. Otherwise, the data set cannot be modeled. In the proposed method, both TPG optimization and modeling are realized at the same time, not sequentially. So it is guaranteed to obtain a realizable data set. In the next section, the rationale of the proposed method is described.

2. Rationala of the proposed method

Consider the double matching arrangement shown in Fig. 1. Input reflection function (ρ_1) can be defined as

$$\rho_1 = \frac{Z_1 - Z_G^*}{Z_1 - Z_G} \tag{1}$$

where Z_1 is the input impedance seen at port 1 when port 2 is terminated in the load (Z_L), Z_G is the generator impedance and the upper asterisk denotes complex conjugation.

In a similar manner, the reflection function (ρ_2) at port 2 can be defined as

Received on: 25.12.2015
Accepted on:10.05.2016

$$\rho_2 = \frac{Z_2 - Z_L^*}{Z_2 - Z_L} \quad (2)$$

where Z_2 is the output impedance seen at port 2 when port 1 is terminated in Z_G.

Since the two-port is lossless, on the imaginary axis of the complex frequency plane, it can be written that

$$|\rho_1|^2 = |\rho_2|^2 . \quad (3)$$

The transducer power gain at real frequencies can be defined as

$$TPG(\omega) = 1 - |\rho_1|^2 = 1 - |\rho_2|^2 . \quad (4)$$

Now let us define the reflection coefficients S_G and S_1 at port 1 as

$$S_G = \frac{Z_G - 1}{Z_G + 1}, S_1 = \frac{Z_1 - 1}{Z_1 + 1}. \quad (5)$$

Substituting the relationships from (5) in (1) yields the reflection coefficient at port 1 as a function of S_G and ρ_1 as follows

$$S_1 = \frac{\rho_1(S_G^* - 1) + S_G^*(S_G - 1)}{\rho_1 S_G(S_G^* - 1) + (S_G - 1)} . \quad (6)$$

In a similar manner, let us define the reflection coefficients S_L and S_2 at port 2 as

$$S_L = \frac{Z_L - 1}{Z_L + 1}, \ S_2 = \frac{Z_2 - 1}{Z_2 + 1} . \quad (7)$$

Substituting the relationships from (7) in (2) yields the reflection coefficient at port 2 as a function of S_L and ρ_2 as follows

$$S_2 = \frac{\rho_2(S_L^* - 1) + S_L^*(S_L - 1)}{\rho_2 S_L(S_L^* - 1) + (S_L - 1)} . \quad (8)$$

S_1 and S_2 can be expressed in terms of the scattering parameters (S_{ij} , $i, j = 1,2$) of the two-port, the reflection coefficient of the load and that of the generator as

$$S_1 = S_{11} + \frac{S_{12}S_{21}S_L}{1 - S_{22}S_L} , \quad (9)$$

$$S_2 = S_{22} + \frac{S_{12}S_{21}S_G}{1 - S_{11}S_G} . \quad (10)$$

Here the scattering parameters of the lossless matching network can be written in terms of three real polynomials by using the well known Belevitch representation as follows [7]

$$S(p) = \begin{bmatrix} S_{11}(p) & S_{12}(p) \\ S_{21}(p) & S_{22}(p) \end{bmatrix} = \frac{1}{g(p)} \begin{bmatrix} h(p) & \mu f(-p) \\ f(p) & -\mu h(-p) \end{bmatrix} \quad (11)$$

where $p = \sigma + j\omega$ is the classical complex frequency variable, h is a polynomial with real coefficients, g is a strictly Hurwitz polynomial, f is a real monic polynomial and μ is a unimodular constant ($\mu = \pm 1$). If the two-port is reciprocal, then the polynomial f is either even or odd and $\mu = f(-p)/f(p)$.

The polynomials f, g, h are related by the Feldtkeller equation [2]

$$g(p)g(-p) = h(p)h(-p) + f(p)f(-p) . \quad (12)$$

It is clear from (12) that the Hurwitz polynomial $g(p)$ is a function of $f(p)$ and $h(p)$. If the polynomials $f(p)$ and $h(p)$ are specified, then the scattering parameters of the two-port network, and then the network itself can be completely defined.

In almost all practical applications, the designer has an idea about the transmission zero locations of the matching network. Hence the polynomial $f(p)$ which is constructed on the transmission zeros is usually defined by the designer. For practical problems, the designer may use the following form of $f(p)$

$$f(p) = p^{m_1} \prod_{i=0}^{m_2} (p^2 + a_i^2) \quad (13)$$

where m_1 and m_2 are nonnegative integers and a_i 's are arbitrary real coefficients. This form corresponds to ladder type minimum phase structures, whose transmission zeros are on the imaginary axis of the complex p -plane.

So if the values of ρ_2 are initialized, they are optimized until reaching the desired TPG via (4). Also at each iteration, S_2 values are calculated via (8) and modeled simultaneously via (10) in terms of three real polynomials h , g and f . The crux of the method is to model the calculated S_2 data as ρ_2 values are optimized. A similar approach can be defined in terms of ρ_1 and S_1.

As the result, the following algorithm can be proposed to solve both single and double broadband matching problems with lumped elements. But the same algorithm can easily be adapted to design distributed or mixed element broadband matching networks.

3. Proposed Algorithm

Inputs:

- $Z_{L(measured)} = R_{L(measured)} + jX_{L(measured)}$, $Z_{G(measured)} = R_{G(measured)} + jX_{G(measured)}$: Measured load and generator impedance data, respectively.
- $\omega_{i(measured)}$: Measurement frequencies, $\omega_{i(measured)} = 2\pi f_{i(measured)}$.
- T_f : Desired flat TPG level.
- f_{norm}: Normalization frequency.
- R_{norm}: Impedance normalization number in ohms.

• $\rho_2 = \alpha_2 + j\beta_2$: Initial values of the reflection function at port 2 at all sample frequencies.
• $h_0, h_1, h_2, \ldots, h_n$: Initial real coefficients of the polynomial $h(p)$. Here n is the degree of the polynomial which is equal to the number of lossles lumped elements in the matching network. The coefficients can be initialized as ±1 in an ad hoc manner or the approach explained in [10] can be followed.
• $f(p)$: A polynomial constructed on the transmission zeros of the matching network. A practical form is given in (13).
• δ_c: The stopping criteria of the sum of the square errors.

Outputs:
• Analytic form of the input reflection coefficient of the lossless matching network, $S_{11}(p) = h(p)/g(p)$.
• Circuit topology of the lossless matching network with element values: The circuit topology and element values are obtained as the result of the synthesis of $S_{11}(p)$. Synthesis is carried out in Darlington sense. That is, $S_{11}(p)$ is synthesized as a lossless two-port which is the desired matching network [11]. Also the synthesis process can be carried out by using impedance based Foster or Cauer methods via $Z_{11}(p) = (1 + S_{11}(p))/(1 - S_{11}(p))$ as explained in [12].

Computational Steps:

Step 1: Normalize the measurement frequencies with respect to f_{norm} and set all the normalized angular frequencies $\omega_i = f_{i(measured)} / f_{norm}$.
Normalize the measured load and generator impedances with respect to impedance normalization number R_{norm};
$R_L = R_{L(measured)} / R_{norm}$, $X_L = X_{L(measured)} / R_{norm}$, $R_G = R_{G(measured)} / R_{norm}$, $X_G = X_{G(measured)} / R_{norm}$ over the entire frequency band.

Step 2: Obtain the strictly Hurwitz polynomial $g(p)$ from (12). Then calculate scattering parameters via (11).

Step 3: Calculate the values of the output reflection coefficient via (8) and (10) as $S_2^{(8)}$ and $S_2^{(10)}$, respectively..

Step 4: Calculate the errors via $\varepsilon_1(\omega) = (\mathrm{Re}(S_2^{(8)}) - \mathrm{Re}(S_2^{(10)}))^2$, $\varepsilon_2(\omega) = (\mathrm{Im}(S_2^{(8)}) - \mathrm{Im}(S_2^{(10)}))^2$ and $\varepsilon_3(\omega) = (1 - T_f - |\rho_2|^2)^2$, then $\delta = \varepsilon_1 + \varepsilon_2 + \varepsilon_3$, where $\mathrm{Re}(\cdot)$ and $\mathrm{Im}(\cdot)$ means the real and imaginary part of $(\cdot)$, respectively.

Step 5: If δ is acceptable ($\delta \le \delta_c$), stop the algorithm and synthesize $S_{11}(p)$. Otherwise, change the initialized values of ρ_2 and the coefficients of the polynomial $h(p)$ via any optimization routine and return to step 2.

4. Example

In this section, a double-matching example is presented for the design of a practical broadband matching network. The normalized load and generator impedance data are given in Table I. It should be noted that the given load data can easily be modeled as a capacitor $C_L = 4$ in parallel with a resistance $R_L = 1$ (i.e. $R_L // C_L$ type of impedance), and the generator data as an inductor $L_G = 1$ in series with a resistance $R_G = 1$ (i.e. $R + L$ type of impedance). Since the given impedance data are normalized, there is no need for a normalization step. The same example is solved here via SRFT and the method proposed in [7].

Table I
Given normalized load and generator impedance data

ω	R_L	X_L	R_G	X_G
0.0	1.0000	0.0000	1.0000	0.0000
0.1	0.8621	-0.3448	1.0000	0.1000
0.2	0.6098	-0.4878	1.0000	0.2000
0.3	0.4098	-0.4918	1.0000	0.3000
0.4	0.2809	-0.4494	1.0000	0.4000
0.5	0.2000	-0.4000	1.0000	0.5000
0.6	0.1479	-0.3550	1.0000	0.6000
0.7	0.1131	-0.3167	1.0000	0.7000
0.8	0.0890	-0.2847	1.0000	0.8000
0.9	0.0716	-0.2579	1.0000	0.9000
1.0	0.0588	-0.2353	1.0000	1.0000

The values of the reflection function at port 2 are initialized as $\rho_2 = \alpha_2 + j\beta_2 = 1 + j$ at all sample frequencies, and the polynomial $h(p)$ is initialized as $h(p) = p^4 + p^3 + p^2 + p + 1$ in an ad hoc manner. Also the polynomial $f(p)$ is selected as $f(p) = 1$, since a low-pass matching network is desired. In this example, using Fano's or Youla's relations [13, 14], the ideal flat gain level $T_{f,ideal}$ is computed as

$$T_{f,ideal} = 1 - e^{-2\pi / R_L C_L \omega_c} = 1 - e^{-2\pi / 1 \cdot 4 \cdot 1} = 0.7921 .$$

Then the desired flat TPG level is selected as $T_f = 0.8$. After running the proposed algorithm, the following scattering parameter of the matching network is obtained $S_{11}(p) = h(p)/g(p)$
where

$$h(p) = -2.8694p^4 - 2.6721p^3 + 0.0197p^2 - 1.7685p + 0.4937,$$

$$g(p) = 2.8694p^4 + 6.2213p^3 + 5.4806p^2 + 3.9157p + 1.1152,$$

If the obtained scattering parameter or the corresponding impedance function is synthesized, then the matching network seen in Fig. 2 is obtained.

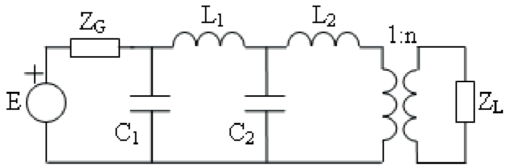

Figure 2. Designed lumped-element double matching network; Proposed: $L_1 = 1.7841$, $L_2 = 1.6678$, $C_1 = 1.6169$, $C_2 = 1.9165$, $n = 0.6220$, SRFT: $L_1 = 1.9422$, $L_2 = 1.8911$, $C_1 = 1.519$, $C_2 = 1.724$, $n = 0.5811$, Ref [7]: $L_1 = 1.9415$, $L_2 = 1.8861$, $C_1 = 1.5209$, $C_2 = 1.7261$, $n = 0.5817$, (normalized).

As seen in Fig. 3, initial performance of the matched system looks fairly good. However, it can be further improved via optimization utilizing the commercially available design package called Microwave Office of Applied Wave Research Inc. (AWR) [15]. For comparison purpose, the performance obtained via the proposed method here, via SRFT and via the proposed method in [7] are depicted in Fig. 3.

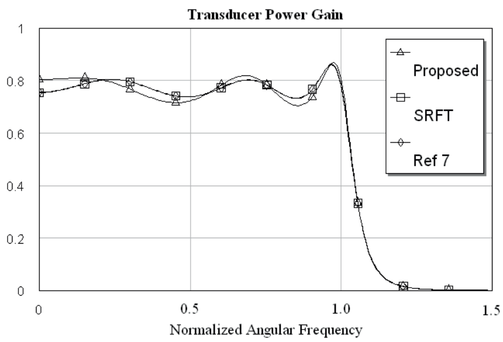

Figure 3. Performance of the matched system designed with lumped elements.

The algorithm is implemented via Matlab and the problem is solved ten times. The average elapsed time for this example is 67.9217 seconds. It is 20.0936 seconds via SRFT and 21.8058 sec via the method proposed in [7]. Since *TPG* optimization and modeling are implemented simultaneously at each iteration, it is a natural consequence to have the largest elapsed time for the proposed method here.

The ripple factor τ^2 for the curves in the passband can be calculated as

$$\tau^2_{proposed} = \frac{TPG_{\max} - TPG_{\min}}{TPG_{\min}} = \frac{0.8688 - 0.7050}{0.7050} = 0.2323,$$

$$\tau^2_{SRFT} = \frac{0.8606 - 0.7328}{0.7328} = 0.1744,$$

$$\tau^2_{\mathrm{Ref}[7]} = \frac{0.8626 - 0.7320}{0.7320} = 0.1784.$$

It can be said that the proposed method in [7] and SRFT have nearly the same performance. On the other hand, the ripple factor in the proposed method is a bit larger than the other ripple factors. But it can be stil concluded that the proposed method here generates pretty good initials for the commercially available design packages.

5. Conclusions

The proposed method consists of two major parts; *TPG* optimization and modeling. In the first part, for the selected flat transducer power gain level, reflection coefficient (S_2) values of the matching network is generated as a data set. In the second part, this data set is modeled as a bounded real reflection coefficient. The crux of the method is to realize these two parts at each iteration simultaneously to quarantee to get a realizable network.

Finally, the obtained reflection coefficient expression (S_{11}) or the corresponding impedance expression (Z_{11}) is synthesized as a lossless two-port yielding the desired matching network topology with initial element values. Eventually, the actual performance of the matched system is improved by means of a commercially available CAD tool.

The features of the proposed method can be explained as follows: The polynomial $f(p)$ is constructed by using the transmission zeros of the matching network, so they are under the control of the designer. Also the proposed method is applicable to solve both single and double matching problems with lumped, distributed or mixed elements.

An example has been presented here to construct a broadband matching network with lumped elements. It was shown that the proposed method generates very good initials to further improve the matched system performance by working on the element values.

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