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Explicit solutions of two-variable scattering equations describing lossless low-pass two-ports with mixed lumped and distributed elements

Metin Şengül[1] | **Göker Eker**[2]

[1] Department of Electrical and Electronics Engineering,, Kadir Has University, Istanbul, Turkey

[2] Graduate School of Engineering, Department of Electronics Engineering, Kadir Has University, Istanbul, Turkey

Correspondence
Metin Şengül, Department of Electrical and Electronics Engineering, Kadir Has University, Cibali-Fatih, Istanbul 34083, Turkey.
Email: msengul@khas.edu.tr

Summary
One of the methods to describe mixed lumped and distributed element two-port networks is to use two-variable scattering equations. In literature, the solutions of these explicit descriptive equations for some classes of low-order ladder networks are derived under some restrictions. In this paper, the complete and explicit solutions of the equations are derived to describe lossless low-pass two-port networks with mixed lumped and distributed elements, up to four elements, without any restrictions.

KEYWORDS
broadband networks, lossless networks, mixed-element networks, scattering parameters, two-port networks

1 | INTRODUCTION

Mixed lumped and distributed element network design has been an important issue for microwave engineers.[1] The interconnections of lumped elements can be assumed to be transmission lines and used as circuit components. Also, the parasitic effects and discontinuities can be embedded in the design process by utilizing these kinds of structures.

Since these networks have two different kinds of elements, their network functions can be defined by using two variables: $p = \sigma + j\omega$ (the usual complex frequency variable) for lumped elements and $\lambda = \tanh(p\tau)$ (the Richard variable) for distributed elements, where τ is the equal delay length of distributed elements. In the earlier studies, since there is a hyperbolic dependence between p and λ, transcendental functions were used to express these kinds of network functions. But then p and λ were assumed as independent variables, and the network functions with two variables were used to describe two-port networks with mixed elements.[2-5] Although there are lots of studies in the literature about mixed element networks, a general analytic procedure to solve transcendental or multivariable approximation problems to design mixed element networks does not exist. But to describe lossless two-ports with mixed elements, there is a semianalytic technique.[6-15] In this approach, two-variable scattering functions are used. But it is applicable for the restricted circuit topologies; LC (inductor-capacitor) ladders cascaded with commensurate transmission lines (Unit Elements, UEs).

In this paper, the complete and explicit solutions are derived for lossless low-pass mixed-element topologies, up to four elements, without any restrictions.

2 | TWO-VARIABLE SCATTERING DESCRIPTION OF LOSSLESS TWO-PORTS FORMED WITH MIXED ELEMENTS

By means of two-variable polynomials g, h, f, the scattering parameters for a two-port with mixed lumped and distributed elements can be expressed as follows[6-17]:

$$S(p,\lambda)=\begin{bmatrix} S_{11}(p,\lambda) & S_{12}(p,\lambda) \\ S_{21}(p,\lambda) & S_{22}(p,\lambda) \end{bmatrix}=\frac{1}{g(p,\lambda)}\begin{bmatrix} h(p,\lambda) & \mu f(-p,-\lambda) \\ f(p,\lambda) & -\mu h(-p,-\lambda) \end{bmatrix}, \tag{1}$$

where $|\mu| = 1$ is a constant.

In Equation (1), $\lambda = \Sigma + j\Omega$ and $p = \sigma + j\omega$ are the Richards variable related with commensurate transmission lines and the usual complex frequency variable related with lumped elements, respectively.

The scattering Hurwitz polynomial $g(p,\lambda)$ with real coefficients[16] can be written as $g(p,\lambda) = \mathbf{P}^T \Lambda_g \boldsymbol{\lambda} = \boldsymbol{\lambda}^T \Lambda_g^T \mathbf{P}$. where

$$\Lambda_g = \begin{bmatrix} g_{00} & g_{01} & \cdots & g_{0n_\lambda} \\ g_{10} & g_{11} & \cdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ g_{n_p0} & \cdots & \cdots & g_{n_pn_\lambda} \end{bmatrix}, \quad \begin{matrix} \mathbf{P}^T = \left[\, 1 \;\; p \;\; p^2 \;\; \ldots \;\; p^{n_p} \right] \\ \boldsymbol{\lambda}^T = \left[\, 1 \;\; \lambda \;\; \lambda^2 \;\; \ldots \;\; \lambda^{n_\lambda} \right]. \end{matrix} \tag{2}$$

Similarly, the polynomial $h(p,\lambda)$ with real coefficients[16] can be written as $h(p,\lambda) = \mathbf{P}^T \Lambda_h \boldsymbol{\lambda} = \boldsymbol{\lambda}^T \Lambda_h^T \mathbf{P}$,where

$$\Lambda_h = \begin{bmatrix} h_{00} & h_{01} & \cdots & h_{0n_\lambda} \\ h_{10} & h_{11} & \cdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ h_{n_p0} & \cdots & \cdots & h_{n_pn_\lambda} \end{bmatrix}. \tag{3}$$

In Equations (2) and (3), n_p and n_λ are the number of lumped and distributed elements, respectively.

$f(p,\lambda)$ is a real polynomial and can be formed via the transmission zeros of the two-port.[16] Then it can be written as

$$f(p,\lambda) = f_L(p) f_D(\lambda). \tag{4}$$

Here, the polynomials $f_L(p)$ and $f_D(\lambda)$ are constructed by means of the transmission zeros of the lumped- and distributed-element sections, respectively.

If UEs are cascaded, then the polynomial $f_D(\lambda)$ can be expressed as[16]

$$f_D(\lambda) = \left(1-\lambda^2\right)^{n_\lambda/2}. \tag{5}$$

If only the zeros at DC are used, then the polynomial $f_L(p)$ can be written as

$$f_L(p) = p^k, \tag{6}$$

where k denotes the number of transmission zeros at DC.[16]

Finally, a practical form of the polynomial $f(p,\lambda)$ can be described as follows:

$$f(p,\lambda) = p^k \left(1-\lambda^2\right)^{n_\lambda/2}. \tag{7}$$

If $\lambda = 0$ is substituted in the polynomials $h(p,\lambda)$, $g(p,\lambda)$, and $f(p,\lambda)$, then the resulting polynomials describing the lumped element section have single variable p. The first column coefficients of the matrices Λ_h and Λ_g are the coefficients of the resulting polynomials $h(p,0)$ and $g(p,0)$, respectively. In a similar manner, if $p = 0$ is substituted in the polynomials $h(p,\lambda)$, $g(p,\lambda)$, and $f(p,\lambda)$, in this case, the resulting polynomials describing the distributed element section have single variable λ. The first row coefficients of the matrices Λ_h and Λ_g are the coefficients of the resulting polynomials $h(0,\lambda)$ and $g(0,\lambda)$, respectively. These single variable polynomials completely describe the related section.

Since the two-port network is lossless, then the following relation is valid:

$$S(p,\lambda) S^T(-p,-\lambda) = I, \tag{8}$$

where I is the identity matrix.[16] If Equation (1) is substituted in Equation (8), the following equation is obtained:

$$G(p,\lambda) = g(p,\lambda)g(-p,-\lambda) = h(p,\lambda)h(-p,-\lambda) + f(p,\lambda)f(-p,-\lambda). \tag{9}$$

The two-variable polynomial $G(p,\lambda) = g(p,\lambda)g(-p,-\lambda)$ given in Equation (9) must be factorized explicitly in designing lossless two-ports with mixed elements.

Alternatively, if the coefficients of the same powers of the complex frequency variables in Equation (9) are equated, the following equation set, which is called as fundamental equation set (FES), is reached[6,9]:

$$g_{0,k}^2 + 2\sum_{l=0}^{k-1}(-1)^{k-l}g_{0,l}g_{0,2k-l} = h_{0,k}^2 + f_{0,k}^2 + 2\sum_{l=0}^{k-1}(-1)^{k-l}\left(h_{0,l}h_{0,2k-l} + f_{0,l}f_{0,2k-l}\right) \text{ for } k = 0, 1, \ldots, n_\lambda, \tag{10a}$$

$$\sum_{j=0}^{i}\sum_{l=0}^{k}(-1)^{i-j-l}g_{j,l}g_{i-j,2k-1-l} = \sum_{j=0}^{i}\sum_{l=0}^{k}(-1)^{i-j-l}\left(h_{j,l}h_{i-j,2k-1-l} + f_{j,l}f_{i-j,2k-1-l}\right) \text{ for } i = 1, 3, \ldots, 2n_p - 1,\ k = 0, 1, \ldots, n_\lambda, \tag{10b}$$

$$\sum_{j=0}^{i}(-1)^{i-j}\left(g_{j,k}g_{i-j,k} + 2\sum_{l=0}^{k-1}(-1)^{k-l}g_{j,l}g_{i-j,2k-l}\right) = \sum_{j=0}^{i}(-1)^{i-j}\left(h_{j,k}h_{i-j,k} + f_{j,k}f_{i-j,k} + 2\sum_{l=0}^{k-1}(-1)^{k-l}\left(h_{j,l}h_{i-j,2k-l} + f_{j,l}f_{i-j,2k-l}\right)\right) \tag{10c}$$

$$\text{for } i = 2, 4, \ldots, 2n_p - 2, \quad k = 0, 1, \ldots, n_\lambda,$$

$$g_{n_p,k}^2 + 2\sum_{l=0}^{k-1}(-1)^{k-l}g_{n_p,l}g_{n_p,2k-l} = h_{n_p,k}^2 + f_{n_p,k}^2 + 2\sum_{l=0}^{k-1}(-1)^{k-l}\left(h_{n_p,l}h_{n_p,2k-l} + f_{n_p,l}f_{n_p,2k-l}\right) \text{ for } k = 0, 1, \ldots, n_\lambda. \tag{10d}$$

The solution of Equation (10) for the coefficients g_{ij} of the polynomial $g(p,\lambda)$ is equivalent to the factorization of the two-variable polynomial $G(p,\lambda) = g(p,\lambda)g(-p,-\lambda)$.

A practical circuit topology with mixed elements is the low-pass ladder sections connected with unit elements (LPLU) depicted in Figure 1.

If $\lambda = 0$ and $p = 0$ are substituted simultaneously in the polynomials $h(p,\lambda)$, $g(p,\lambda)$, and $f(p,\lambda)$, then the resulting polynomials describing LPLU network at DC are $h(0,0) = h_{00}$, $g(0,0) = g_{00}$, and $f(0,0) = 1$. If these polynomials are used in Equation (1) to obtain $S_{11}(p,\lambda)$, the following equation is reached, $S_{11}(0,0) = h_{00}/g_{00}$. Also, the input impedance can be written as $Z_{in}(p,\lambda) = \dfrac{1+S_{11}(p,\lambda)}{1-S_{11}(p,\lambda)}$. Then $Z_{in}(0,0) = \dfrac{g_{00}+h_{00}}{g_{00}-h_{00}}$. If h_{00} is selected as $h_{00} = 0$, then $g_{00} = 1$ from Equation (9). So the input impedance at DC is $Z_{in}(0,0) = 1$, which is equal to the normalized load resistance if the normalization resistance is selected as the given load resistance value at DC. As a result, there is no need to use a transformer.

In the literature, for the selected LPLU topologies given in Figure 2, FES is formed by using Equations (10) and (7), and then it is solved algebraically for the unknown coefficients. For the coefficient h_{00} is restricted as $h_{00} = 0$ for a transformerless design, the explicit relations for the entries of Λ_h and Λ_g matrices up to total degree $n = n_p + n_\lambda = 5$ are found in Aksen and Yarman.[9] But in the solutions for $n = 5$ seen in Aksen and Yarman,[9] the given g_{10} equation depends on g_{20} and the given g_{20} equation depends on g_{10}.

But in our paper, without any restrictions, the explicit coefficient relations are obtained algebraically and given in Table 1 for $n = 2$ to $n = 4$. For $n = 5$, the following procedure has been followed: If $h(p,0)$ is initialized and $f(p,0)$ is

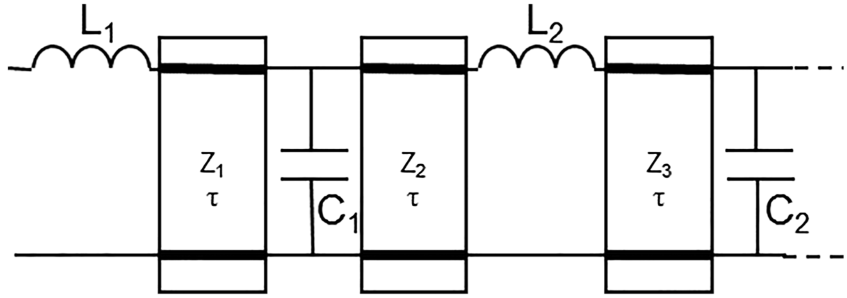

FIGURE 1 Low-pass ladder with unit elements (LPLU)[9]

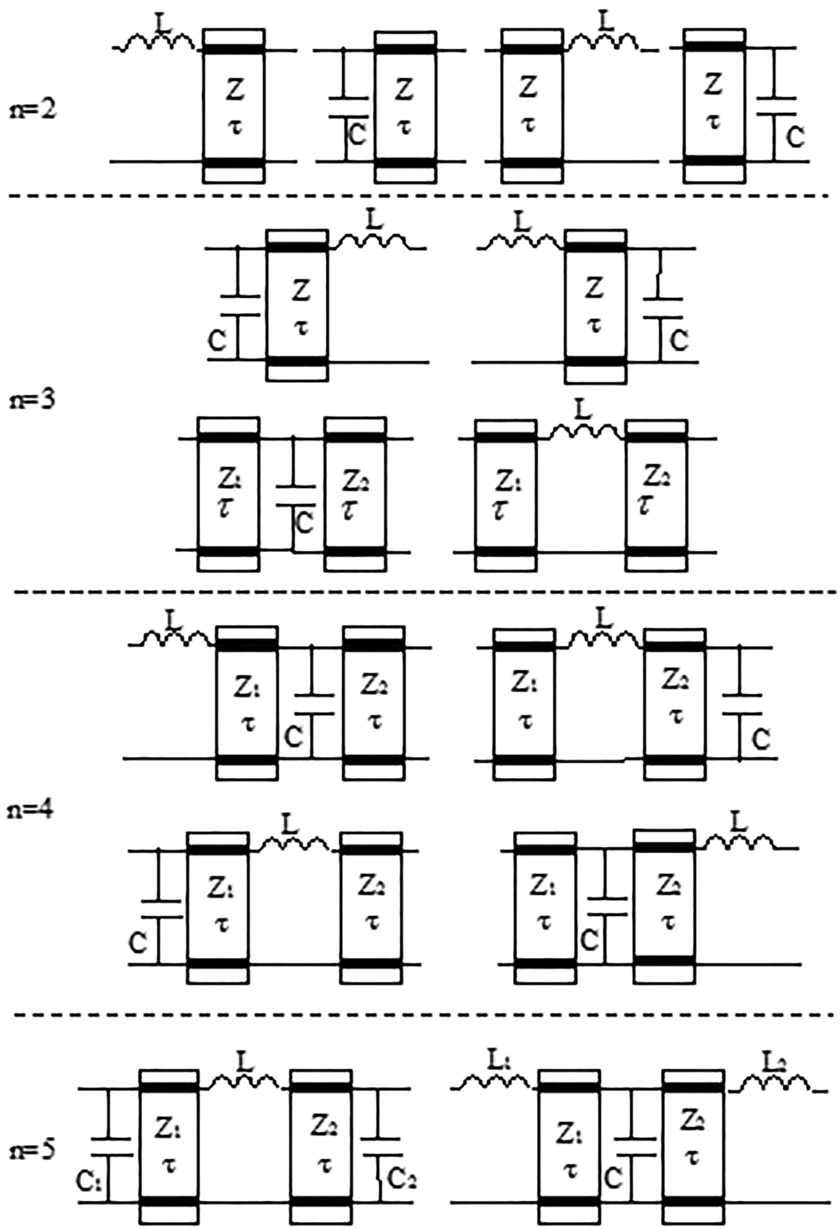

FIGURE 2 Low-order LPLU structures[9]

formed via Equation (7), then the strictly Hurwitz polynomial $g(p,0)$ can be calculated via Equation (9). Similarly if $h(0,\lambda)$ is initialized and $f(0,\lambda)$ is formed via Equation (7), again the strictly Hurwitz polynomial $g(0,\lambda)$ can be obtained via Equation (9). Then after solving FES algebraically for the remaining unknown coefficients of Λ_h and Λ_g matrices without any restrictions, the explicit equations for $n = n_p+n_\lambda \leq 5$ seen in Table 1 have been found.

As an example, suppose $n_p = 1$ and $n_\lambda = 1$, which corresponds to one lumped element and one UE in the LPLU. So the polynomials $h(p,\lambda)$, $g(p,\lambda)$, and $f(p,\lambda)$ can be written as

$$g(p,\lambda) = g_{00} + g_{01}\lambda + g_{10}p + g_{11}p\lambda, \tag{11a}$$

$$h(p,\lambda) = h_{00} + h_{01}\lambda + h_{10}p + h_{11}p\lambda, \tag{11b}$$

$$f(p,\lambda) = \left(1-\lambda^2\right)^{1/2}. \tag{11c}$$

TABLE 1 Complete and explicit solutions for low-order LPLU topologies

n	Coefficient relations
2	h_{00}, h_{01}, h_{10} independent coefficients $g_{00} = \sqrt{1+h_{00}^2}$, $g_{01} = \sqrt{1+h_{01}^2}$, $g_{10} = \lvert h_{10}\rvert$, $g_{11} = (g_{01}g_{10} - h_{01}h_{10})/(g_{00} - \mu_2 h_{00})$, $h_{11} = \mu_2 g_{11}$
3 (1 unit-element)	$h_{00}, h_{01}, h_{10}, h_{20}$ independent coefficients $h_{21} = g_{21} = 0$ $g_{00} = \sqrt{1+h_{00}^2}$, $g_{01} = \sqrt{1+h_{01}^2}$, $g_{10} = \sqrt{h_{10}^2 + 2(g_{00}g_{20} - h_{00}h_{20})}$, $g_{20} = \lvert h_{20}\rvert$, $g_{11} = (g_{01}g_{10} - h_{01}h_{10})/(g_{00} - \mu_2 h_{00})$, $h_{11} = \mu_2 g_{11}$
3 (1 lumped-element)	$h_{00}, h_{01}, h_{10}, h_{02}$ independent coefficients $g_{00} = \sqrt{1+h_{00}^2}$, $g_{02} = \sqrt{1+h_{02}^2}$, $g_{01} = \sqrt{2 + h_{01}^2 + 2(g_{00}g_{02} - h_{00}h_{02})}$, $g_{10} = \lvert h_{10}\rvert$, $\alpha = g_{01} - \mu_2 h_{01}$, $\beta = g_{10} - \mu_2 h_{10}$, $g_{11} = 2g_{02}\beta/\alpha$, $h_{11} = 2h_{02}\beta/\alpha$, $g_{12} = (g_{11}g_{02} - h_{11}h_{02})/\alpha$, $h_{12} = \mu_2 g_{12}$
4	$h_{00}, h_{01}, h_{02}, h_{10}, h_{20}$ independent coefficients $h_{22} = g_{22} = 0$ $g_{00} = \sqrt{1+h_{00}^2}$, $g_{02} = \sqrt{1+h_{02}^2}$, $g_{01} = \sqrt{2 + h_{01}^2 + 2(g_{00}g_{02} - h_{00}h_{02})}$, $g_{20} = \lvert h_{20}\rvert$, $g_{10} = \sqrt{h_{10}^2 + 2(g_{00}g_{20} - h_{00}h_{20})}$, $\alpha = g_{01} - \mu_2 h_{01}$, $\beta = g_{10} - \mu_2 h_{10}$, $\gamma = g_{01}g_{10} - h_{01}h_{10}$, $\dfrac{h_{11} = h_{20}\frac{\alpha}{\beta} + h_{02}\frac{\beta}{\alpha} - \frac{h_{00}}{g_{00}}\left(g_{20}\frac{\alpha}{\beta} + g_{02}\frac{\beta}{\alpha}\right) + \frac{h_{00}}{g_{00}^2}\gamma}{1 - \frac{h_{00}^2}{g_{00}^2}}$, $g_{11} = \dfrac{\gamma + h_{00}h_{11}}{g_{00}}$, $g_{21} = (g_{11}g_{20} - h_{11}h_{20})/\beta$, $h_{21} = \mu_2 g_{21}$, $g_{12} = (g_{11}g_{02} - h_{11}h_{02})/\alpha$, $h_{12} = \mu_2 g_{12}$
5 (2 unit-element)	$h_{00}, h_{01}, h_{02}, h_{10}, h_{20}, h_{30}$ independent coefficients $g_{00}, g_{01}, g_{02}, g_{10}, g_{20}, g_{30}$ calculated via Equation (9) $h_{22} = g_{22} = h_{31} = g_{31} = h_{32} = g_{32} = 0$ $\alpha = g_{01} - \mu_2 h_{01}$, $\beta = g_{10} - \mu_2 h_{10}$, $\gamma = g_{01}g_{10} - h_{01}h_{10}$, $\dfrac{h_{11} = h_{20}\frac{\alpha}{\beta} + h_{02}\frac{\beta}{\alpha} - \frac{h_{00}}{g_{00}}\left(g_{20}\frac{\alpha}{\beta} + g_{02}\frac{\beta}{\alpha}\right) + \frac{h_{00}}{g_{00}^2}\gamma}{1 - \frac{h_{00}^2}{g_{00}^2}}$, $g_{11} = \dfrac{\gamma + h_{00}h_{11}}{g_{00}}$, $g_{21} = (g_{11}g_{20} - g_{01}g_{30} - h_{11}h_{20} + h_{01}h_{30})/\beta$, $h_{21} = \mu_2 g_{21}$, $g_{12} = (g_{11}g_{02} - h_{11}h_{02})/\alpha$, $h_{12} = \mu_2 g_{12}$

Note. μ_2 will be assigned by the designer according to the desired connection order (see Table 2).

Then from Equation (9), the polynomial $g(p,\lambda)g(-p,-\lambda)$ is obtained as

$$(g_{00} + g_{01}\lambda + g_{10}p + g_{11}p\lambda)(g_{00} - g_{01}\lambda - g_{10}p + g_{11}p\lambda) = (h_{00} + h_{01}\lambda + h_{10}p + h_{11}p\lambda)(h_{00} - h_{01}\lambda - h_{10}p + h_{11}p\lambda) + \left(1-\lambda^2\right)^{1/2}\left(1-\lambda^2\right)^{1/2}. \tag{12}$$

If the coefficients of the corresponding degrees are equated, the following equation set is obtained.

$$g_{00}^2 - h_{00}^2 = 1, \tag{13a}$$

$$g_{01}^2 - h_{01}^2 = 1, \tag{13b}$$

$$g_{00}g_{11} - g_{01}g_{10} - h_{00}h_{11} + h_{01}h_{10} = 0, \tag{13c}$$

$$g_{10}^2 - h_{10}^2 = 0, \tag{13d}$$

$$g_{11}^2 - h_{11}^2 = 0. \tag{13e}$$

The first row and column coefficients of the matrix Λ_h are assumed to be known (they are going to be the initialized parameters); which are h_{00}, h_{01}, and h_{10}, then the other coefficients can be obtained from Equation (13) as given in Table 1 for $n = 2$. For higher degrees, the same procedure is followed.

TABLE 2 Connection order of the LPLU topologies

μ_1	μ_2	First element	Second element
+1	+1	Inductor	Unit element
+1	−1	Unit element	Inductor
−1	+1	Unit element	Capacitor
−1	−1	Capacitor	Unit element

If $h_{00} = 0$ is substituted in the solutions given in Table 1, the solutions given in Aksen and Yarman[9] is obtained. This clearly proves that the solutions given in Aksen and Yarman[9] corresponds the special case of the general solutions given in Table 1. For LPLU topologies with more elements, explicit solutions are not available. In this case, FES must be solved numerically.

The connection order for the LPLU topologies seen in Figure 2 is determined by the constants μ_1 and μ_2. All possibilities are given in Table 2, where $\mu_1 = h_{n_p,0}/g_{n_p,0}$ and μ_2 will be assigned by the designer according to the desired connection order.

The explicit solutions presented in this paper can be utilized to design practical microwave matching networks, amplifiers with mixed elements. In Şengül,[16] a new approach to design broadband matching networks with mixed lumped and distributed elements has been proposed. In this method, after supplying the initial coefficients of $h(p, \lambda)$ polynomial, the other unknown coefficients in Equations (2) and (3) are calculated by using the solutions given in the literature under the restriction of $h_{00} = 0$. But the solutions given in this work have been derived without any restrictions, and they can be used in the broadband matching approach presented in Şengül.[16]

Explicit solutions for different types of mixed element topologies can be derived. For instance, the solutions for high-pass ladder sections connected with unit elements (HPLU), band-pass ladder sections connected with unit elements (BPLU), and band-stop ladder sections connected with unit elements (BSLU) topologies are given in Sertbaş.[7] Also, the solutions for shunt capacitors separated by UEs can be found in Çakmak[18] and Şengül and Çakmak.[19]

3 | CONCLUSION

Since the mixed element networks have two different kinds of elements, their network functions can be defined by using two variables: p for lumped elements and λ for distributed elements, if p and λ were assumed as independent variables. In this paper, scattering parameters in terms of two-variable polynomials $g, \ h, \ f$ have been utilized. By using the losslessness condition, a fundamental equation set to describe the mixed element network is given. This equation set has been solved for low-pass ladders connected with unit elements without any restriction. Explicit design solutions are given in Table 1 up to four elements. For five-element case, after obtaining the first row and column coefficients of the two-variable polynomial g, explicit solutions are found for the unknown coefficients of Λ_h and Λ_g matrices without any restriction.

It is expected that the proposed solutions will be used to design two-variable networks such as broadband matching networks, microwave amplifiers.

ORCID

Metin Şengül https://orcid.org/0000-0003-1940-1456

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