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Component value calculations in a mixed element ladder network containing series capacitors separated by unit elements

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Summary
Lossless LC ladder networks have many applications. But at high frequencies, inductor-free designs are preferred since inductors are heavy and have limited values. An additional reason for not using the inductors (at high frequencies) consists in that the values of very small inductors could be quite imprecise. If LC ladder network is high-pass type, then parallel inductors must be removed. The resulting network will have only series capacitors, which can be combined as a single series capacitor. But if commensurate transmission lines separate the series capacitors, a high-pass mixed element network is obtained. In this paper, a procedure is proposed to compute the values of series capacitors and characteristic impedances of the transmission lines in the mixed element ladder network. The usage of the procedure is explained by means of the given example.

KEYWORDS
degenerate networks, lossless networks, passive networks, series capacitors, synthesis, unit elements

1 | INTRODUCTION

In the literature, there are many works about mixed lumped and distributed element networks. In Zhang et al,[1] a synthesis method by applying the Taylor series is proposed to design a variety of low-pass mixed lumped and distributed circuit networks, including low-pass filters and low-passing impedance transformers. In this work, the prototype of the distributed element filter contains open stubs between series transmission lines. Then, the open stub is replaced by shunt capacitors, and the transmission line between the two stubs is replaced by a scaled transmission line with two shunt capacitors on the two sides. When incorporating all the shunt capacitors at the same node point, the mixed circuit with shunt capacitors between transmission lines is obtained.

In Ghosh et al,[2] the design methodology of a four-branch line coupler using lumped and distributed elements is presented, and it is shown that this technique aims to improve the bandwidth.

In Zhuang et al,[3] a simple and effective method of designing wideband high-efficiency power amplifiers is presented. A distributed series and shunt low-pass ladder network is selected to implement the front-end and back-end matching networks which can satisfy the matching and filtering requirements simultaneously. The matching networks have lumped capacitors to provide electrical isolation between the radio-frequency signal and direct current. The matching networks can be treated as cascaded structures of several transmission lines interconnected with lumped DC block capacitors.

The objective in Juliet et al.[4] is to design and implement a hybrid microwave band-pass filter (BPF) by mixing two technologies such as microstrip line and lumped element and evaluate its performance. It is shown that the designed hybrid BPF has an improved performance and has reduced size compared with a filter designed using individual lumped and microstrip components.

In Zhang and Peroulis,[5] mixed lumped and distributed circuits (MCs) have been analyzed and applied to a wideband BPF synthesis procedure for spurious-response suppression. A new synthesis method for the MC-based wideband BPF has been proposed to precisely form the passband and effectively control the spurious response.

BPF are not preferred due to large circuit size and transmission loss for millimeter-wave (mm-wave) front ends. To solve this problem, in Shen et al,[6] a modified low-temperature cofired ceramic (LTCC)-based hybrid lumped and distributed resonator (HLDR) is introduced by effectively utilizing the advantages of lumped and distributed elements. In Shen et al,[7] the design of ultra-low-loss mm-wave LTCC BPF is proposed by merging the lumped and distributed circuits.

In Shen et al,[8] a design method of low-loss mm-wave monolithic-microwave-integrated-circuit (MMIC) BPF using lumped-distributed parameters is proposed for 5G mm-wave applications, and it is shown that the proposed MMIC BPF is a good candidate for 5G applications.

Consider the high-pass LC ladder network seen in Figure 1, which can be a filter or a broadband matching network. If the network is a filter, it can be designed via well-known methods given in many books about network synthesis.[9–11] On the other hand, if the network is a broadband matching network, then it can be designed via simplified real frequency technique or line segment technique.[12] In both cases, it is desired to obtain a driving point impedance $Z(p)$ at the end of the design procedure. Then, this impedance function is synthesized via continued-fraction expansion, and the network seen in Figure 1 is reached.[9,10] The first and last circuit element can be a parallel inductor or a series capacitor. If there are m series-connected capacitors and n parallel-connected inductors in Figure 2, there will be $m + n$ transmission zeros at DC.

For low-pass and band-pass cases, high-precision LC ladder synthesis algorithms are proposed in Kılınç and Yarman and Yarman and Kılınç,[13,14], respectively. Also, in Yarman et al,[15] a high-precision synthesis algorithm to include the extraction of finite frequency and right half-plane (RHP) transmission zeros of an impedance function as Brune/Darlington Type C sections are introduced.

In all the networks mentioned above, inductors are utilized. But especially at high frequencies, inductor-free networks are preferred, since they are heavy, bulky and difficult to implement at the desired values. An additional reason for not using the inductors (at high frequencies) consists in that the values of very small inductors could be quite imprecise.

So, at this point, the parallel inductors present in the high-pass LC ladder network can be removed, and then, the remaining network will have only series capacitors. Finally, all the series capacitors can be simplified as a single series capacitor.

If transmission lines separate the series capacitors (practically, they already exist between series capacitors), a mixed element network is obtained as seen Figure 2. If equal length transmission lines are used, which are called unit

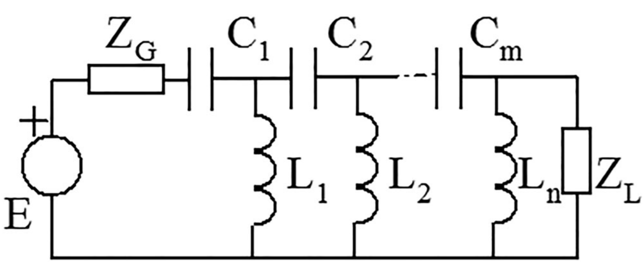

FIGURE 1 High-pass LC ladder network

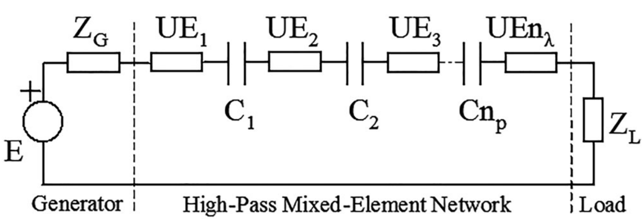

FIGURE 2 Mixed element ladder network

elements (UE) or commensurate lines, then the resulting network is very practical to fabricate. Additionally, open-ended series connected stubs can be used instead of series capacitors via Richards' transformation.[11]

In the literature, there are many works on mixed element networks. In Şengül and Eker,[16] low-pass, and in Sertbaş,[17] high-pass, band-pass and band-stop mixed element networks with inductors are analyzed. Inductor-free low-pass mixed element network is studied in Şengül and Çakmak.[18]

At the end of any design process, it is a necessity to compute the series capacitor values and characteristic impedances of the commensurate transmission lines by using the expression describing the mixed element network. In this work, a new method is proposed to compute the component values. In the next sections, after briefly summarizing the fundamental properties of the interested mixed element network, the proposed approach is explained.

2 | RATIONALE OF THE PROPOSED APPROACH

Let us consider the high-pass mixed element network seen in Figure 2. The first and last component can be a UE or a series capacitor.

This mixed element network can be defined via transfer scattering matrix, which may be expressed in terms of three polynomials g,h,f with two-variable as follows[12,19,20]:

$$T(p,\lambda)=\begin{bmatrix}T_{11}(p,\lambda) & T_{12}(p,\lambda)\\ T_{21}(p,\lambda) & T_{22}(p,\lambda)\end{bmatrix}=\frac{1}{f(p,\lambda)}\begin{bmatrix}\mu g(-p,-\lambda) & h(p,\lambda)\\ \mu h(-p,-\lambda) & g(p,\lambda)\end{bmatrix} \tag{1}$$

where μ is a constant such that $|\mu| = |f(-p,-\lambda)/f(p,\lambda)| = 1$. In the expression, two frequency variables are used: $p=j\omega$ for lumped element section and $\lambda = j\Omega$ for UE section.

The polynomial $g(p,\lambda)$ can be defined as $g(p,\lambda)=\mathbf{P}^T\Lambda_g\boldsymbol{\lambda}=\boldsymbol{\lambda}^T\Lambda_g^T\mathbf{P}$ where

$$\Lambda_g=\begin{bmatrix}g_{00} & g_{01} & \cdots & g_{0n_\lambda}\\ g_{10} & g_{11} & \cdots & .\\ . & . & . & .\\ g_{n_p0} & \cdots & \cdots & g_{n_pn_\lambda}\end{bmatrix},\quad \mathbf{P}^T=\left[1pp^2\ldots p^{n_p}\right],\ \boldsymbol{\lambda}^T=\left[1\lambda\lambda^2\ldots\lambda^{n_\lambda}\right]. \tag{2}$$

This polynomial is a real coefficient scattering Hurwitz polynomial, and its degree is n_p+n_λ.[20]

Similarly, $h(p,\lambda)$ can be defined as where

$$\Lambda_h=\begin{bmatrix}h_{00} & h_{01} & \ldots & h_{0n_\lambda}\\ h_{10} & h_{11} & \ldots & .\\ . & . & . & .\\ h_{n_p0} & \ldots & \ldots & h_{n_pn_\lambda}\end{bmatrix}. \tag{3}$$

This polynomial is also a real coefficient polynomial, and its degree is n_p+n_λ.

$f(p,\lambda)$ is formed by using all the transmission zeros of the high-pass mixed element network, and it is written as

$$f(p,\lambda)=f_L(p)f_D(\lambda) \tag{4}$$

where the polynomials $f_L(p)$ and $f_D(\lambda)$ are formed by using the transmission zeros of the lumped and UE sections, respectively. In Figure 2, n_p capacitors are connected in series; as a result, there are n_p transmission zeros at DC. Since UE are connected between series capacitors, it is not a degenerate network. On the other hand, if UE are removed, then there will be only one DC transmission zero due to its degenerate circuit topology.

From the detailed analysis of this high-pass mixed element ladder network,[21] it is seen that the polynomials $g(\lambda)$ and $h(\lambda)$ formed with the last row coefficients of Λ_g and Λ_h matrices describe the transmission line section of the mixed element network. On the other hand, $g(p)$ and $h(p)$ formed with the last column coefficients of Λ_g and Λ_h matrices represent the lumped element section. So, to calculate series capacitor values, some of the coefficients of $h(p)$ and $g(p)$ will be utilized. In a similar manner, characteristic impedances of the commensurate transmission lines will be calculated via the coefficients of the polynomials $g(\lambda)$ and $h(\lambda)$.

Driving point input reflectance $S_{11}(p,\lambda)$ of a two-variable lossless two-port can be described in terms of the polynomials $h(p,\lambda)$ and $g(p,\lambda)$ as follows:

$$S_{11}(p,\lambda)=\frac{h(p,\lambda)}{g(p,\lambda)}. \tag{5}$$

In the existing literature, unfortunately, realizability conditions which are known as the necessary and the sufficient conditions of the synthesis of $S_{11}(p,\lambda)$ are not yet determined for the general form of $S_{11}(p,\lambda)$. On the other hand, the necessary and the sufficient conditions for the synthesis of $S_{11}(p,\lambda)$ with low-degree polynomials up to degree $n_p + n_\lambda$ of five are explained in Yarman and Aksen.[22] The necessary conditions can be summarized as follows:

$$g(p,\lambda)\ \text{must be scattering Hurwitz and all the entries}\ g_{ij}\ \text{of}\ \Lambda_g\ \text{of 2 must be non}-\text{negative}, \tag{6a}$$

- $h(p,\lambda)$, $g(p,\lambda)$ and must satisfy the losslessness condition (or equivalently Feldtkeller condition) of

$$g(p,\lambda)g(-p,-\lambda)=h(p,\lambda)h(-p,-\lambda)+f(p,\lambda)f(-p,-\lambda). \tag{6b}$$

However, necessary conditions are not sufficient to complete the synthesis. On the other hand, in the next section, for the mixed element structure seen in Figure 2, they are obtained, and the proposed approach to calculate component values of the mixed element structure is explained.

3 | PROPOSED APPROACH

The transfer scattering matrix of the high-pass mixed element network is defined as[23–25]

$$T(p,\lambda)=T_1(p)T_{RN}(p,\lambda) \tag{7}$$

where $T_1(p)$ and $T_{RN}(p,\lambda)$ are the transfer scattering matrices of the extracted component and the remaining network, respectively.

Firstly, calculate a constant $\alpha=h_{0n_\lambda}/g_{0n_\lambda}=\pm 1$. Then, the first component is a commensurate transmission line or a series capacitor for $\alpha=-1$ or $\alpha=+1$, respectively.

If the first component is a series capacitor, its value is calculated via the following equation:

$$CV=\frac{g_{1n_\lambda}-\alpha h_{1n_\lambda}}{g_{0n_\lambda}+\alpha h_{0n_\lambda}}. \tag{8}$$

Here, CV must be positive. This restriction imposes relationships between independent variables h_{ij} of Λ_h of 3 and g_{ij} of Λ_g of 2 such that

$$\bullet\text{Either simultaneously}\ g_{1n_\lambda}-ah_{1n_\lambda}>0\ \text{and}\ g_{0n_\lambda}-ah_{0n_\lambda}>0, \tag{9a}$$

$$\bullet\text{or simultaneously}\ g_{1n_\lambda}-ah_{1n_\lambda}<0\ \text{and}\ g_{0n_\lambda}-ah_{0n_\lambda}<0. \tag{9b}$$

The above conditions are sufficient to make CV positive beyond the losslessness condition.

Then, the coefficients of the matrices Λ_h and Λ_g of the remaining network are calculated via the following expressions:

$$h_{x,y}^{RN} = h_{x+1,y} + \frac{1}{2 \cdot CV}\left(h_{x+2,y} - g_{x+2,y}\right), \tag{10a}$$

$$g_{x,y}^{RN} = g_{x+1,y} + \frac{1}{2 \cdot CV}\left(h_{x+2,y} - g_{x+2,y}\right) \tag{10b}$$

for $x = 0,1,\ldots,(n_p - 1)$, $y = 0,1,\ldots,n_\lambda$.

If the first component is a commensurate transmission line, its characteristic impedance is calculated via the following equation:

$$Z = \frac{\left(g_{n_p0} + g_{n_p1} + \ldots + g_{n_pn_\lambda}\right) + \left(h_{n_p0} + h_{n_p1} + \ldots + h_{n_pn_\lambda}\right)}{\left(g_{n_p0} + g_{n_p1} + \ldots + g_{n_pn_\lambda}\right) - \left(h_{n_p0} + h_{n_p1} + \ldots + h_{n_pn_\lambda}\right)}. \tag{11}$$

Here, Z must be positive, which in turn requires that

$$\text{Either simultaneously} \left(g_{n_p0} + g_{n_p1} + \ldots + g_{n_pn_\lambda}\right) + \left(h_{n_p0} + h_{n_p1} + \ldots + h_{n_pn_\lambda}\right) > 0 \text{ and } \left(g_{n_p0} + g_{n_p1} + \ldots + g_{n_pn_\lambda}\right) - \left(h_{n_p0} + h_{n_p1} + \ldots + h_{n_pn_\lambda}\right) > 0 \tag{12a}$$

$$\text{or simultaneously} \left(g_{n_p0} + g_{n_p1} + \ldots + g_{n_pn_\lambda}\right) + \left(h_{n_p0} + h_{n_p1} + \ldots + h_{n_pn_\lambda}\right) < 0 \text{ and } \left(g_{n_p0} + g_{n_p1} + \ldots + g_{n_pn_\lambda}\right) - \left(h_{n_p0} + h_{n_p1} + \ldots + h_{n_pn_\lambda}\right) < 0 \tag{12b}$$

The inequalities given in (9) and (12) are the sufficient conditions imposed on $S_{11}(p,\lambda)$ at the first step of the synthesis process beyond Feldtkeller condition.

Then, the coefficients of the matrices Λ_h and Λ_g of the remaining network are calculated via the following expressions:

$$h_{x,y}^{RN} = h_{x,y} + \frac{1}{2 \cdot Z}\left(h_{x,y-1} + g_{x,y-1}\right) + \frac{Z}{2}\left(h_{x,y-1} - g_{x,y-1}\right), \tag{13a}$$

$$g_{x,y}^{RN} = g_{x,y} - \frac{1}{2 \cdot Z}\left(h_{x,y-1} + g_{x,y-1}\right) + \frac{Z}{2}\left(h_{x,y-1} - g_{x,y-1}\right) \tag{13b}$$

for $x = 0,1,\ldots,n_p$, $y = 0,1,\ldots,(n_\lambda - 1)$.

Alternatively, all the characteristic impedances of the commensurate transmission lines can be calculated by using the approach proposed in literature.[26,27] Also, in Yarman et al,[28] a high-precision method is presented to synthesize a Richards immittance (which is a positive real function expressed in terms of the variable $\lambda = \Sigma + j\Omega$) as a lossless two-port constructed with cascade connections of commensurate transmission lines, as well as short and open stubs.

After extracting any component (a series capacitor or a commensurate transmission line), the remaining input reflectance $S_{11}^{RN}(p,\lambda)$ must satisfy the inequalities given in (9) and (12) until the last component is obtained as a positive quantity.

In Şengül and Çakmak,[29] a similar approach is proposed for the synthesis of ladder networks formed with cascaded shunt capacitors and commensurate transmission lines. In the approach, transfer scattering matrix of the remaining network is calculated via

$$T_{RN}(p,\lambda) = T_1^{-1}(p)T(p,\lambda) \tag{14}$$

where $T_1(p)$ is the transfer scattering matrix of the first component and is formed as follows:

Firstly, calculate a constant $\alpha = h_{n_p0}/g_{n_p0} = \pm 1$. Then, the first component is a shunt capacitor or a commensurate transmission line for $\alpha = -1$ or $\alpha = +1$, respectively.

If the first component is a shunt capacitor, its value is calculated via the following equation:

$$CV = \frac{g_{n_p0} + \alpha h_{n_p0}}{g_{(n_p-1)0} - \alpha h_{(n_p-1)0}} \tag{15}$$

Then, for the shunt capacitor, $T_1(p)$ is formed by using the following polynomials:

$$h_1(p) = -\frac{CV}{2}p,\ g_1(p) = \frac{CV}{2}p + 1,\ f_1(p) = 1. \tag{16}$$

If the first component is a commensurate transmission line, its characteristic impedance is calculated via 11, and $T_1(p)$ is formed by using the following polynomials:

$$h_1(\lambda) = \frac{Z^2 - 1}{2\cdot Z}\lambda,\ g_1(\lambda) = \frac{Z^2 + 1}{2\cdot Z}\lambda + 1,\ f_1(p) = \left(1 - \lambda^2\right)^{1/2}. \tag{17}$$

Then, after forming transfer scattering matrix of the extracted component ($T_1(p)$), the inverse of this matrix is obtained, and then a matrix multiplication is realized as seen in 14. So, all the polynomials forming transfer scattering matrix of the remaining network is obtained by means of this expression. If the coefficients of the polynomials forming the matrices ($T_1(p)$ and $T(p, \lambda)$) are not integers, the differences between the calculated coefficients and the coefficients they should be are very large after extracting a few components. This yields large errors in the calculated component values.

On the other hand, in the proposed approach here, there is no need to form the transfer scattering matrix of the extracted component ($T_1(p)$), and no need to obtain the inverse of the matrix, and no need to multiply the matrices. Namely, the transfer scattering matrix of the remaining network is not calculated via 14. All the coefficients of the matrices Λ_h and Λ_g of the remaining network are calculated separately via (10) and (13). As a result, component values calculated via these coefficients are very close to the component values they should be. So, the approach proposed here is more convenient for the calculation of component values. This approach can be modified to compute the component values of the structures formed with shunt capacitors (or series/shunt inductors) separated by commensurate transmission lines.

4 | EXAMPLE

A broadband impedance matching network is designed by using a high-pass mixed element network with cascaded series capacitors and UE in Yarman and Aksen.[22] In the design process, the generator impedance is formed as a parallel combination of an inductor $L_G = 1H$ and a resistor $R = 1\Omega$, and the load impedance is formed as an inductor $L_L = 2H$ in parallel with the series combinations of a capacitor $C_L = 2F$ and a resistor $R = 1\Omega$. All the component values are given as normalized.

In Yarman and Aksen,[22] the focus is on the calculation of the two-variable polynomials $h(p, \lambda)$ and $g(p, \lambda)$ to be able to obtain a broadband matching network, which means that the necessary and sufficient conditions of synthesis for the topology under consideration are already satisfied. In this work, the focus is on the calculation of component values by using the following Λ_h and Λ_g matrices obtained after completing the matching network design:

$$\Lambda_h = \begin{bmatrix} 0 & 0 & 0.0634 \\ 0 & 0.5610 & -0.1778 \\ 1.0185 & -0.8926 & 0.3284 \\ 0 & 0.4771 & -0.7524 \end{bmatrix}, \Lambda_g = \begin{bmatrix} 0 & 0 & 0.0634 \\ 0 & 0.5610 & 0.2935 \\ 1.0185 & 1.7293 & 0.7583 \\ 1 & 2.1750 & 1.2514 \end{bmatrix}.$$

It can be concluded from the matrices that the number of series capacitors is three and the number of commensurate transmission lines is two. This means that the polynomial $f(p,\lambda)$ is equal to $f(p,\lambda)=f_L(p)f_D(\lambda)=p^3\cdot(1-\lambda^2)$.

The polynomials $h(p,\lambda)$, $g(p,\lambda)$, and $f(p,\lambda)$ satisfy the necessary conditions for the synthesis given in (6).

Since $\alpha=h_{02}/g_{02}=0.0634/0.0634=+1$, the first component must be a series capacitor. Since the inequality given in (9a) is satisfied, a series capacitor with the following normalized value will be extracted: $C_1=\frac{g_{12}-\alpha h_{12}}{g_{02}+\alpha h_{02}}=\frac{0.2935-(+1)(-0.1778)}{0.0634+(+1)0.0634}=3.7141F$.

Then, the coefficients of the remaining Λ_h and Λ_g matrices after extracting C_1 can be calculated by using 10a and 10b. Two of the coefficients are calculated below to illustrate the calculations:

$$h_{0,0}^{RN}=h_{1,0}+\frac{1}{2\cdot C_1}\left(h_{2,0}-g_{2,0}\right)=0+\frac{1}{2\cdot 0.3741}(1.0185-1.0185)=0,$$

$$g_{1,0}^{RN}=g_{2,0}+\frac{1}{2\cdot C_1}\left(h_{3,0}-g_{3,0}\right)=1.0185+\frac{1}{2\cdot 3.7141}(0-1)=0.8839.$$

After extracting C_1, Λ_h and Λ_g matrices of the remaining network are given below:

$$\Lambda_h=\begin{bmatrix}0 & 0.2081 & -0.2356\\ 0.8839 & -1.1212 & 0.0586\\ 0 & 0.4771 & -0.7524\end{bmatrix},\Lambda_g=\begin{bmatrix}0 & 0.2081 & 0.2356\\ 0.8839 & 1.5008 & 0.4885\\ 1 & 2.1750 & 1.2514\end{bmatrix}.$$

At this point, it is necessary to check whether the remaining polynomials $h(p,\lambda)$, $g(p,\lambda)$, and $f(p,\lambda)$ satisfy the necessary conditions for the synthesis given in (6).

Since new $\alpha=h_{02}/g_{02}=-0.2356/0.2356=-1$, the second extracted component is a commensurate transmission line as expected. Since the inequality given in 12a is satisfied, the normalized characteristic impedance value of the commensurate transmission line is

$$Z_1=\frac{\left(g_{n_p0}+g_{n_p1}+...+g_{n_pn_\lambda}\right)+\left(h_{n_p0}+h_{n_p1}+...+h_{n_pn_\lambda}\right)}{\left(g_{n_p0}+g_{n_p1}+...+g_{n_pn_\lambda}\right)-\left(h_{n_p0}+h_{n_p1}+...+h_{n_pn_\lambda}\right)}=\frac{(2.1750+1.2514+1)+(0.4771-0.7524)}{(2.1750+1.2514+1)-(0.4771-0.7524)}=0.88289\Omega.$$

Then, the coefficients of the remaining Λ_h and Λ_g matrices after extracting Z_1 can be calculated by using 13a and 13b. Two of the coefficients are calculated below to illustrate the calculations:

$$h_{0,1}^{RN}=h_{0,1}+\frac{1}{2\cdot Z_1}\left(h_{0,0}+g_{0,0}\right)+\frac{Z_1}{2}\left(h_{0,0}-g_{0,0}\right)=0.2081+\frac{1}{2\cdot 0.88289}(0+0)+\frac{0.88289}{2}(0-0)=0.2081,$$

$$g_{1,1}^{RN}=g_{1,1}-\frac{1}{2\cdot Z_1}\left(h_{1,0}+g_{1,0}\right)+\frac{Z_1}{2}\left(h_{1,0}-g_{1,0}\right)=1.5008-\frac{1}{2\cdot 0.88289}(0.8839+0.8839)+\frac{0.88289}{2}(0.8839-0.8839)$$
$$=0.4996.$$

After extracting Z_1, Λ_h and Λ_g matrices of the remaining network are given below:

$$\Lambda_h=\begin{bmatrix}0 & 0.2081\\ 0.8839 & -0.1201\\ 0 & 0.6019\end{bmatrix},\Lambda_g=\begin{bmatrix}0 & 0.2081\\ 0.8839 & 0.4996\\ 1 & 1.1672\end{bmatrix}.$$

Then, after extracting all the components in a similar manner, mixed element network seen in Figure 3 is obtained.

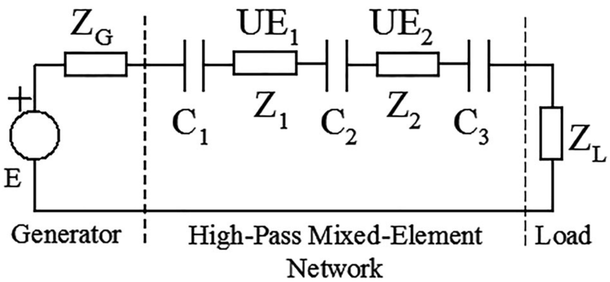

FIGURE 3 Obtained high-pass mixed element network, $C_1 = 3.7141F$, $C_2 = 1.4893F$, $C_3 = 0.9122F$, $Z_1 = 0.88289\Omega$, $Z_2 = 1.7691\Omega$

In Section 2, it is said that if the UE are removed, there will be only one DC transmission zero due to its degenerate circuit topology. This can be verified for the given example as follows: Firstly, let us write $S_{21}(p,\lambda)$ scattering parameter as

$$S_{21}(p,\lambda) = \frac{f(p,\lambda)}{g(p,\lambda)} = \frac{p^3(1-\lambda^2)}{g_{02}\lambda^2 + p(g_{11}\lambda + g_{12}\lambda^2) + p^2(g_{20} + g_{21}\lambda + g_{22}\lambda^2) + p^3(g_{30} + g_{31}\lambda + g_{32}\lambda^2)}.$$

Now, if the UE are removed, which means that $\lambda = 0$, then

$$S_{21}(p,0) = \frac{f(p,0)}{g(p,0)} = \frac{p^3}{p^2 g_{20} + p^3 g_{30}} = \frac{p^3}{p^2(g_{20} + p g_{30})} = \frac{p}{g_{20} + p g_{30}} = \frac{p}{1.0185 + p}.$$

After removing UE, $f(p,0) = p$ as expected. Using the coefficients of $g(p,0)$ and $h(p,0)$ polynomials can be calculated with the equivalent capacitor normalized value as follows:

$$\alpha = h_{20}/g_{20} = 1.0185/1.0185 = +1$$

$$C_{eq} = \frac{g_{30} - \alpha h_{30}}{g_{20} + \alpha h_{20}} = \frac{1-(+1)(0)}{1.0185 + (+1)1.0185} = 0.4910F.$$

Equivalent capacitor normalized value can be obtained also by using the calculated series capacitor values as follows:
$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{1}{3.7141} + \frac{1}{1.4893} + \frac{1}{0.9122}$, then $C_{eq} = 0.4910F$.

5 | CONCLUSION

In this work, component values of a high-pass mixed element network composed of series capacitors and UE are calculated. Since the lumped element section of the high-pass mixed element network consists of only series capacitors, this section is a kind of degenerate network. So, to be able to obtain the series capacitor values, a new approach is proposed.

Firstly, the mixed element structure is expressed by means of two-variable transfer scattering matrix. After deciding the first component type (a series capacitor or a UE), the value of the first extracted component is calculated by using some of the coefficients of $h(p,\lambda)$ and $g(p,\lambda)$. Then, by using the given expressions in the paper, the coefficients of the remaining Λ_h and Λ_g matrices are computed via the extracted component value and the coefficients of $h(p,\lambda)$ and $g(p,\lambda)$. This process is repeated until all component values are computed.

In the interested high-pass mixed element structure, the UE between the series capacitors are used as circuit elements. Additionally, series capacitors can be converted to open-ended series connected stubs by using Richards' transformation; as a result, the resulting circuit is very practical to fabricate.

DATA AVAILABILITY STATEMENT

Data sharing not applicable to this article as no datasets were generated or analyzed during the current study.

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