Atmosferik Türbülans Altında Orbital Açısal Momentum Tabanlı Haberleşme Sistemleri
Orbital Angular Momentum Based Communication Systems Under Atmospheric Turbulence

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***Özetçe* —Orbital açısal momentum (OAM), yüksek veri hızı ve etkin frekans kullanımı avantajları nedeniyle gelecekteki iletişim sistemlerine aday olabilir. OAM uygun şekilde kullanıldığında, bir ışının kendi içinde çoklu ışınlara dönüştürülmesi ve aktarılacak birbirinden bağımsız veriler (diklik ilkesi sayesinde) haline gelmesi için bir çözüm sağlayabilir. Bununla birlikte, serbest uzay, OAM'nin ışın yayılımına girişim oluşturan atmosferik türbülans içerir ve bunun sonucu olarak ışık ışınlarındaki fotonlar ve OAM modlarının bağımsızlığı etkilenir. Bu yüzden, veri akışı diyafoniye maruz kalmaktadır. Bu çalışmada, zayıf ve güçlü atmosfer türbülansları ve bu türbülansların çeşitli OAM modlarını nasıl etkilediği incelenmiş ve OAM'nin önemli uygulama konuları tartışılmıştır.**

Anahtar Kelimeler—Orbital açısal momentum, atmosferik türbülans, diyafoni, girişim, diklik ilkesi.

***Abstract*—Orbital Angular Momentum (OAM) may be a candidate for future communication systems due to its advantages in terms of high data rates and effective frequency utilization. OAM can provide a solution where a beam is transformed into multiple beams within itself and becomes independent streams of data (pertaining to the principle of orthogonality) to be transferred when properly utilized. However, the free space contains atmospheric turbulence which interferes with the beam propagation of OAM and as a result the photons in the light beam are affected as well as the independence of each OAM mode. Therefore, the data stream suffers crosstalk. In this study, weak and strong atmospheric turbulences and how they affect a variety of OAM modes as they propagate through free space are investigated, and important OAM implementation issues are discussed.**

Keywords—Orbital angular momentum, atmospheric turbulence, crosstalk, interference, orthogonality principle.

I. INTRODUCTION

Beginning with the first radio signal transmitted and received in 1901 by Marconi [1] and the discovery of shaft like motion of a wave by Dr. Poynting in 1909 [2], there was an important discovery in 1992 that light constituting photons not only have linear momentum in them if linearly polarized as a physical quantity, but also have a shaft like motion that can attain spin and orbital momentum as well [3]. This has given rise to spin angular momentum (SAM) and orbital angular momentum (OAM), which induce a torque, that can be used to move infinitesimal objects or rotate them.

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With the discovery of OAM, the researchers were able to understand that light, when properly polarized or twisted in its phase, produces infinite number of replica of the very same light. This is actually the same light but split into different phases which are orthogonal to each other leading to these components not interfering with each other, allowing to carry data on them independently. This means an infinite number of data streams can be transmitted through one light beam as a carrier theoretically. However, since these beams propagate through free-space, the independent modes of each beam start to interfere with each other. On the other hand, this discovery has been an important candidate for the ever increasing demand for high data rates while using smaller bandwidths.

In the literature, different modulation techniques have been considered to support OAM, and the spectral efficiency has been increased as they are deemed multidimensional incorporation of increased data [4]–[7]. An experiment for transmitting a terabit of data has been conducted in [8], which further backs up the credibility of this technology for implementation. Also, there have been studies considering the atmospheric turbulence to provide better capacity to the channel [9]. In addition, there have been studies that consider the effects of turbulence on constellation points for higher mode detection [10]–[12]. Most of these studies are experiment-based and do not elaborate on the effects of atmospheric turbulence from the communications perspective.

Motivated by the above issues, the performance of OAM is studied for different atmospheric turbulence models in this paper. Specifically, weak and strong atmospheric turbulence models have been considered and the crosstalk probabilities between the modes are explored. In addition, the trade-off between data rate and crosstalk probabilities is discussed. The results and discussions provided are important for modeling the atmospheric turbulence and understanding its effects from the communication perspective.

II. ORBITAL ANGULAR MOMENTUM

The visible light spectrum is a collection of electromagnetic radiations visible to the human eye. What OAM proposes is that for a Gaussian beam, light is twisted and bent in a specific way to manipulate its phase and in turn an independent mode is generated which occupies the same space as the beam. This in turn allows to create infinite independent instances of the beam and makes it as a channel to transfer those modes that are generated.

The OAM beam is considered to be a Gaussian vortex beam with a hole in the center (like a doughnut shape) and the modes are obtained by increasing the rings/spirals around its circumference indicating a new mode being added [13], [14]. OAM is derived from the light and the spin on its photons. The generation of OAM based communication systems will be defined briefly in the following subsections. More information on OAM data transmission can be found in [15].

A. Holograms/Diffraction Patterns

Holograms are generated through computers which act as a mesh of a grid through where light can be shone on. Depending on the incidence angle and the distance, hologram dictates what pattern is developed as a result [3]. The pattern basically acts as a fork and twists the incoming beam with the correct parameters to achieve a desired result. The same procedure can be done with a hologram which nullifies a phase and lets through another observation. A simple Gaussian beam travels to the hologram/diffraction pattern and a resulting pattern can be obtained. They can be produced by computer programs and then developed onto photographic films. Gaussian or laser beams can be shone onto the material and the desired results can be obtained accordingly.

B. Spiral Phase Plates

Spiral phase plates act in the same way as the diffraction/hologram patterns, except that they are made from materials specially produced in labs to attain the desired effects. As the name suggests, they start off as being very thin as we move through the circumference and start to get thicker as we go on, as a result taking up a shape of somewhat a revolving shaft that increases in thickness as it revolves. When beams are incident on the material, the thinner part has less diffraction and the thicker parts have more. As a result, the phase changes are proportional to the thickness and we obtain higher order modes from thicker parts and lower order modes from the thinner parts. The optical thickness of the medium increases with azimuthal position. They are generally created from crystal, plastic, high density polyethene, etc. [16].

C. Laguerre Gaussian Beams

Helically phased light carries OAM irrespective of the radial distribution of the beam. However, it is useful to express most of beams in a complete basis set of orthogonal modes [11]. For OAM, we therefore use the Laguerre Gaussian (LG) mode set in this study. The best field distribution is described in cylindrical modes.

For a radial distance r from the propagation axis, azimuthal angle ϕ and propagation distance z, the field distribution is described as [7], [9], [11]

$$\begin{aligned} u(r,\phi,z) = {} & \sqrt{\frac{2p!}{\pi(p+|m|)!}}\frac{1}{w(z)}\left[\frac{r\sqrt{2}}{w(z)}\right]^{|m|} L_p^m\left[\frac{2r^2}{w^2(z)}\right] \\ & \times \exp\left[-\frac{r^2}{w^2(z)}\right]\exp\left[\frac{ikr^2z}{2(z^2+z_R^2)}\right] \\ & \times \exp\left[i(2p+|m|+1)\tan^{-1}\frac{z}{z_R}\right]\exp(-im\phi) \end{aligned} \tag{1}$$

where $w(z) = w(0) + \sqrt{1+(z/z_R^2)}$ is the beam with radius z, $w(0)$ is the radius of the zero-order Gaussian beam at the waist, $z_R = \pi w_0^2/\lambda$ is the Rayleigh range, λ is the optical wavelength and $k = 2\pi/\lambda$ is the propagation constant. The beam waist is at $z = 0$. The term L_p^m represents the generalized Laguerre polynomial, and p and m are the radial and angular mode/topological charge numbers, respectively [7]. With the help of this equation, we can produce an OAM mode to any extent that we want which will act as independent carriers of data to be transmitted. Given that as the modes increase in the beam, the beam starts to grow, and as the propagation distance is increased the beam grows as well.

III. ATMOSPHERIC TURBULENCE

The OAM modes are orthogonal to each other meaning that infinite number of streams can be transmitted. That is not the case when the propagation of OAM beams is in free space optical communication environments. The earth's atmosphere includes many gases and substances along with the Sun's rays, where there may be changes in temperatures and pressures all around the earth. Atmospheric turbulence is, small-scale, irregular air motions characterized by winds that vary in speed and direction. Turbulence is important since it mixes and churns the atmosphere and causes water vapor, smoke, and other substances, as well as energy, to become distributed both vertically and horizontally [17]. This results in the atmospheric turbulence interact with the photon and negatively affecting it. In the following section, how atmospheric turbulence is modeled and what affects these models have on the beams are explained [17].

A. Weak Atmospheric Turbulence

The modified Kolmogorov atmospheric turbulence model represents different levels of turbulence depending on the refractive-index structure parameter, C_n^2 measured in $m^{-2/3}$. The model can be given as [7]

$$\Phi(\kappa) = 0.033C_n^2(\kappa^2 + 1/L_0^2)f(\kappa,\kappa_l), \tag{2}$$

with

$$f(\kappa,\kappa_l) = \exp(-\kappa^2/\kappa_l^2)\left[1 + 1.802(\kappa/\kappa_l) - 0.254(\kappa/\kappa_l)^{7/6}\right] \tag{3}$$

where κ and C_n^2 denote spatial frequency and refractive-index structure parameter, respectively, $\kappa_l = 3.3/l_0$, and L_0 and l_0 represent the outer and inner scale of turbulence, respectively. The outer and inner scales are selected as $L_0 = 20$mm and $l_0 = 5$mm, respectively. Depending on the value of the refractive-index structure parameter, the strength of the atmospheric turbulence is determined. The typical range for C_n^2 is $C_n^2 \in [10^{-17}, 10^{-13}]$. When the atmospheric turbulence has a value of $C_n^2 \leq 10^{-15}$, the atmospheric turbulence is weak, whereas $C_n^2 \geq 10^{-14}$ implies that the atmospheric turbulence is strong [7]. In this study, we consider the weak atmospheric turbulence being modeled by the Johnson-SB statistic [18]. This model is a simpler one compared to the modified Kolmogorov model and we will discuss how well it can match it. The model is described in the form of a matrix where the desired channel is on the diagonal and the interfering signals, i.e., similar to atmospheric turbulence at the neighboring diagonals, are exponentially distributed. This model is based on the Johnson-SB distribution given as [18]

$$f_y(y|\gamma,\delta) = \frac{\delta}{\sqrt{2\pi}}\frac{1}{y(1-y)}\exp\left(\frac{1}{2}\left[\gamma + \delta\frac{y}{1-y}\right]\right) \tag{4}$$

where the parameters for the distribution of the diagonal elements are $\gamma = -2.7$ and $\delta = 1.004$. On the other hand, the entries of the neighboring diagonals are said to be interfering signals that exhibit exponential distribution [18]. Accordingly, first, second and third neighboring diagonals are exponentially distributed with parameters $1/\lambda = \{4.4 \times 10^{-2}, 5 \times 10^{-3}, 1.04 \times 10^{-3}\}$, respectively. Here, the effect of the turbulence can be shown by $\mathbf{H}$, which represents the transition matrix of the modes for non-diagonal elements.

B. Strong Atmospheric Turbulence

Strong turbulence is modeled by the modified Kolmogorov atmospheric turbulence model with the refractive index structure parameter $C_n^2 = 10^{-14} m^{-2/3}$. OAM beams pass through the modified Kolmogorov model [7], [9], [17], where the principle of orthogonality has been considered and the effect of turbulence has been assessed [7]. The received distorted OAM-carrying field is filtered according to the scalar product operation and the result is normalized by the transmit optical power [7]. Since the distorted field is in fact a linear superposition of a number of OAM fields scaled by random scalar coefficients, what is recorded is the square of the normalized scalar product. This value represents the fraction of optical power observed in received OAM state n. The resulting projection for each (m, n) corresponds to one instance of the channel, where m is the transmitted mode and n is the received mode.

Ideally, the channel matrix, $\mathbf{H}$, is an identity matrix, which resembles a matrix with no atmospheric turbulence. Also, it could be used as a comparison to a low refractive index structure parameter value such as $C_n^2 = 10^{-16}$ in which case almost no orthogonality is lost between the modes.

IV. SIMULATION RESULTS

In this section, the effect of atmospheric turbulence on different modes as they propagate, and the probability of them being detected are studied. The OAM modes start to lose orthogonality as the modes are affected by the atmospheric turbulence or in other words, crosstalk is induced in various channels. This crosstalk is harmful from the communication perspective. In the following, we will evaluate how the modes are affected by different levels of turbulence, and how the communication can be made more robust in the presence of strong turbulence. Note that the effect of noise is negligible compared to the effect of atmospheric turbulence.

A. Weak Atmospheric Turbulence Effect

For the weak atmospheric turbulence, the Johnson-SB model is considered for simulations. For modes $m \in \{0, 1, \ldots, 9\}$, a 10×10 matrix shows the crosstalk and the different probabilities of the transition matrix which depicts how well the OAM modes are received after experiencing atmospheric turbulence. This is presented in Fig 1.

Here, it can be seen that almost 91% correct reception is achieved for any mode. On the other hand, with the modified Kolmogorov model, for higher modes (e.g., $m \geq 10$) the probability of correct reception is lower. Nevertheless, the simple model of Johnson-SB distribution holds for lower modes during weak atmospheric turbulence. Note that Johnson-SB distribution does not depend on m and that similar probability of reception results are reported in [7] for $C_n^2 = 10^{-15} m^{-2/3}$ for low values of m. Next, the effect of strong atmospheric turbulence will be investigated.

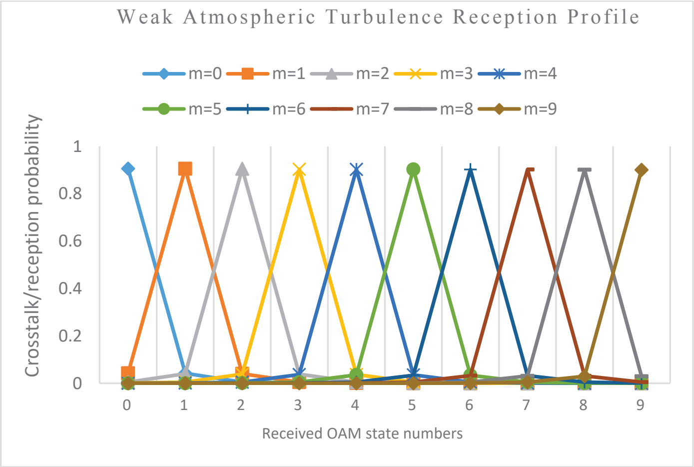

Fig. 1. Probability of reception of m through weak turbulence based on Johnson-SB distribution.

B. Strong Atmospheric Turbulence Effect

For the strong atmospheric turbulence, the modified Kolmogorov model is considered, where different mode reception profiles and the induced crosstalk in other receiving modes under $C_n^2 = 10^{-14}$ will be presented. We will refer to the case of $C_n^2 = 10^{-14}$ as the strong turbulence in this subsection. The crosstalk is obtained from the $\mathbf{H}$ matrix as given in [7]. Here, a contrast in the reception profile can be seen as compared to the weak atmospheric turbulence as depicted in Fig. 2.

It is shown in the figure that as higher order modes m are transmitted, there is a lower probability of detecting them. It should be further noted the probability of detection is low even for lower order modes, therefore, data rate may be traded-off by combining some transmission/reception modes.

Since the crosstalk is inevitable due to strong atmospheric turbulence, the data rate may be lowered by detecting neighboring modes. Assuming $m \in \{0, 1, \ldots, 7\}$, the probability of correct reception for a received state can be calculated in Table I for corresponding data rates. It can be observed that the probability of correct reception can be improved at the expense of reduced data rate. Furthermore, when the modes are combined for detection, it is better to transmit at lower modes and detect accordingly.

V. CONCLUSION

In this study, the effect of atmospheric turbulence on OAM based communication systems has been studied. The weak atmospheric turbulence has been modeled through the Johnson-SB statistical distribution, whereas the strong atmospheric turbulence has been modeled through the modified-Kolmogorov model. For the weak atmospheric turbulence $C_n^2 = 10^{-15}$, Johnson-SB model has been generated to validate the reported modified-Kolmogorov model results. The study has been conducted in terms of probability of correct reception. While the Johnson-SB model results are close to the reported results, it does not model the effect of modes. Nevertheless, probability of correct reception for weak atmospheric turbulence is high and the Johnson-SB model can be used for lower modes due to its simpler structure. For strong atmospheric turbulence $C_n^2 = 10^{-14}$, the reported modified-Kolmogorov model results have been processed. It is observed

TABLE I. PROBABILITY OF CORRECT RECEPTION WITH COMBINED MODES AT DIFFERENT RATES

Transmit and receive modes (m,n)	**OAM modes and probability of correct reception**															
Data rate (R)	0		1		2		3		4		5		6		7	
m,n	m=0	n=0	m=1	n=1	m=2	n=2	m=3	n=3	m=4	n=4	m=5	n=5	m=6	n=6	m=7	n=7
R	0.534		0.376		0.291		0.243		0.211		0.187		0.168		0.157	
m,n	m=0		n=0,1		m=2		n=2,3		m=4		n=4,5		m=6		n=6,7	
$R/2$	0.64				0.43				0.346				0.296			
m,n	m=1		n=0,1		m=3		n=2,3		m=5		n=4,5		m=7		n=6,7	
$R/2$	0.481				0.381				0.323				0.282			
m,n	m=0				n=0,1,2,3				m=4				n=4,5,6,7			
$R/3$	0.670								0.442							
m,n	m=3				n=0,1,2,3				m=7				n=4,5,6,7			
$R/3$	0.561								0.382							

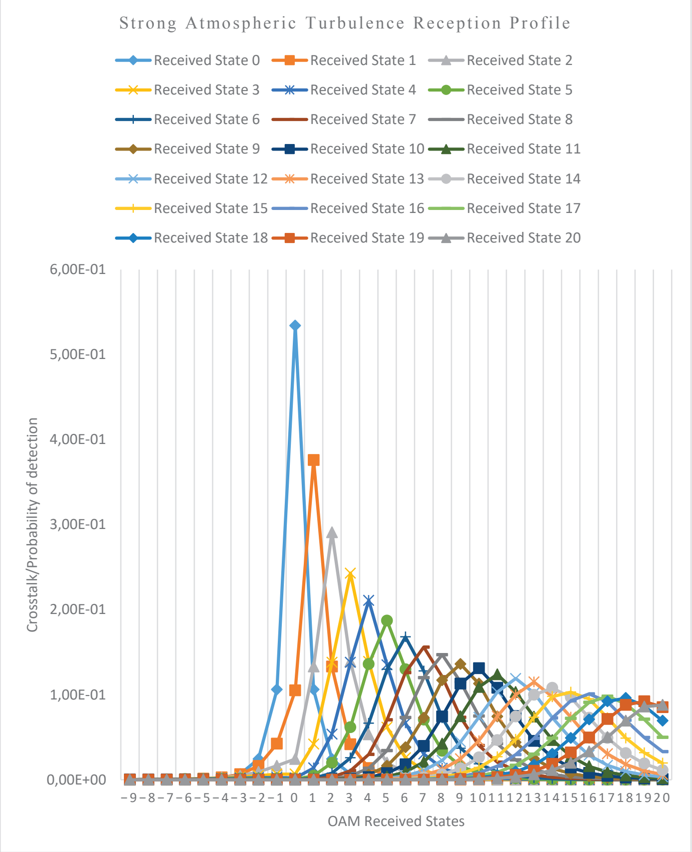

Fig. 2. Probability of reception of m through strong turbulence, $C_n^2 = 10^{-14}$.

that lower modes have higher probability of reception. In order to improve the probability of correct reception at the expense of data rate, some modes are combined and better detection probabilities have been obtained. This trade-off can be used effectively in the presence of strong turbulence.

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