KADİR HAS UNIVERSITY
GRADUATE SCHOOL OF SCIENCE AND ENGINEERING
PROGRAM OF ELECTRONICS ENGINEERING

EXPLICIT SOLUTIONS OF TWO-VARIABLE SCATTERING EQUATIONS AND BROADBAND MATCHING NETWORK DESIGN

GÖKER EKER

MASTER'S THESIS

İSTANBUL, JANUARY, 2019

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ACCEPTANCE AND APPROVAL

This work entitled EXPLICIT SOLUTIONS OF TWO-VARIABLE SCATTERING EQUATIONS AND BROADBAND MATCHING NETWORK DESIGN prepared by GÖKER EKER has been judged to be successful at the defense exam on and 04 January 2019 accepted by our jury as MASTER'S THESIS.

APPROVED BY:

Prof. Dr. Metin ŞENGÜL (Advisor)
Kadir Has University

Asst. Prof. Dr. Baran TANDER
Kadir Has University

Asst. Prof. Dr. Ertuğrul SAATÇI
İstanbul Kültür University

I certify that the above signatures belong to the faculty members named above.

Assoc. Prof. Dr. Ebru Demet AKDOĞAN

Dean of Graduate School of Science and Engineering

DATE OF APPROVAL:

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GÖKER EKER

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ABSTRACT

Mixed lumped and distributed element network design has been a significant issue for microwave engineers (Aksen, 1994). The interconnections of lumped elements can be assumed to be transmission lines and used as circuit components. Also the parasitic effects and discontinuities can be embedded in the design process by utilizing these kinds of structures.

Since these networks have two different kinds of elements, their network functions can be defined by using two variables; $p = \sigma + jw$ for lumped elements and $\lambda = tanh(p\tau)$ for distributed elements, where τ is the equal delay length of distributed elements. In the earlier studies, since there is a hyperbolic dependence between p and λ,transcendental functions were used to express these kinds of network functions. But then p and λ were assumed as independent variables, the network functions with two variables were used to describe two-port networks with mixed elements.

Although there are lots of studies in the literature about mixed element networks, a general analytic procedure to solve transcendental or multivariable approximation problems to design mixed element networks does not exist. But to describe lossless two-ports with mixed elements, there is a semi-analytic technique (Aksen, 1994). In this approach, two-variable scattering functions are used and practical solutions are obtained. But it is applicable for the restricted circuit topologies; LC ladders cascaded with commensurate transmission lines (Unit Elements).

In this thesis, the complete and explicit equations are derived for lossless low-pass mixed-element topologies, and by using the equations solved without any restriction,a broadband matching network design was made. The results were compared

with the results in the literature.

Keywords: Broadband networks, Lossless networks, Mixed-element networks, Two-port networks, Scattering parameters.

İKİ DEĞİŞKENLİ SAÇILMA DENKLEMLERİNİN ANALİZİ VE GENİŞBANT UYUMLAŞTIRICI TASARIMI

ÖZET

Karışık devre elemanı (toplu ve dağıtılmış eleman) içeren devreler mikrodalga mühendisliği için önemli bir konudur (Aksen, 1994). Toplu elemanlar arasındaki bağlantılar, iletim hattı olarak düşünülüp devre elemanı olarak tasarım sırasında denklemlere dahil edilirse, devrenin performansını bozmaları engellendiği gibi aynı zamanda devrenin istenen cevabı vermesi için kullanılmış olurlar.

Bu tür devrelerde, iki farklı tipte eleman bulunduğundan, devre fonksiyonları iki değişken kullanılarak tanımlanır. Devrede yer alan toplu elemanlar için $p = \sigma + jw$ klasik frekans değişkeni ve dağıtılmış elemanlar için $\lambda = tanh(p\tau)$ Richards değişkeni şeklinde tanımlanır(burada τ dağıtılmış elemanlar için gecikmedir). Dikkat edilirse bu iki değişken arasında hiperbolik bir bağımlılık vardır. Dolayısıyla bu tür devrelerin tanımlanmasında transandantal fonksiyonlar kullanılabilir. Fakat p ve λ bağımsız değişkenler olarak kabul edilirse karışık elemanlı devreler iki-değişkenli fonksiyonlar kullanarak tanımlanabilir.

Literatürde bu tür devreler üzerine birçok çalışma bulunmasına rağmen, bu denklemlerin çözümü için genel bir analitik method henüz bulunabilmiş değildir. Fakat yarı-analitik bir yaklaşım mevcuttur (Aksen, 1994). Bu yaklaşımda, iki-değişkenli saçılma denklemleri kullanılır ve sınırlı devre topolojileri için uygulanabilir durumdadır.

Literatürde, bahsedilen yarı-analitik yaklaşım ile düşük dereceli alçak-geçiren birim elemanlarla ayrılmış LC merdiven devreler için bazı kısıtlamalar altında saçılma denklemlerinin çözümleri verilmiştir. Fakat bu tezde, hiç bir kısıtlama olmadan çözülen denklemler kullanılarak, genişbant uyumlaştırma devresi tasarımı yapılmış, elde

edilen sonuçlar literatürde verilen denklemler kullanılarak tasarlanan uyumlaştırma devresi sonuçlarıyla karşılaştırılmıştır.

Anahtar Sözcükler: Saçılma denklemleri, İki-kapılı devreler, Genişbant devreleri, Uyumlaştırma devreleri.

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It has been a long way in my life. In this road, sometimes I had problems in my personel life and I can not get permission to join the lecturers from my work place. I was a soldier for a couple weeks. Therefore, in this long way I have a baby boy and he always makes me happy and my wife always supports me. She is a wonderful woman and mother.They make my feelings that all things are well and will be better than now. Also, I'm so lucky to have a supervisor Prof. Metin Şengül. He always supports and teaches me in way of academically and personally. He is a great teacher and father. All these things are so gratefully. I am so glad to have them and thank you each of them for participation in that though road.

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1. INTRODUCTION

There is a significant advantage of microwave circuits according to univariate structures of mixed-element, two-variable structures. The analytical solution of the filters having a mixed element structure and the broadband matching problem is not fully achieved. There is a need for defining mixed-element structures in using two-variable (rational form) functions. One of the methods to describe mixed lumped and distributed element two-port networks is to use two-variable scattering equations.

Since these networks have two different kinds of elements, their network functions can be defined by using two variables; $p = \sigma + jw$ (the usual complex frequency variable) for lumped elements and $\lambda = tanh(p\tau)$ (the Richard variable) for distributed elements, where τ is the equal delay length of distributed elements (Şengül,2018).

In the earlier studies, since there is a hyperbolic dependence between p and λ transcendental functions were used to express these kinds of network functions. But then p and λ were assumed as independent variables, the network functions with two variables were used to describe two-port networks with mixed elements.

Eventough there are lots of studies in the literature about mixed element networks, a general analytic procedure to solve transcendental or multivariable approximation problems to design mixed element networks does not exist. But to describe lossless two-ports with mixed elements, there is a semi-analytic technique.

In this approach, two-variable scattering functions are used. But it is applicable for the restricted circuit topologies Inductor-Capacitor ladders cascaded with commensurate transmission lines (Unit Elements, UEs).

In this thesis, the complete and explicit equations are derived for lossless low-pass mixed-element topologies, up to 4 elements, without any restrictions. The obtained results were compared with the literature.

2. PROPERTIES OF LOSSLESS TWO PORTS

This chapter contains about basic definitions of transmission lines and scattering parameter and matrix and canonical representation of them. Also fundamental properties of lossless lumped and distributed networks are summarized.

2.1 Defining the lossless two-port with scattering parameters

The behavior of lossless two-port circuits can be defined via matrices such as admittance, impedance and chain matrix. However, these matrices are defined for short or open circuit termination status. Using the concept of power in microwave circuit theory is more suitable than current or voltage concept. The scattering matrix is a very useful method to examine the power transfer characteristics of a circuit. Let us examine the scattering matrix properties and basic definitions of the two-port networks (Aksen,1994).

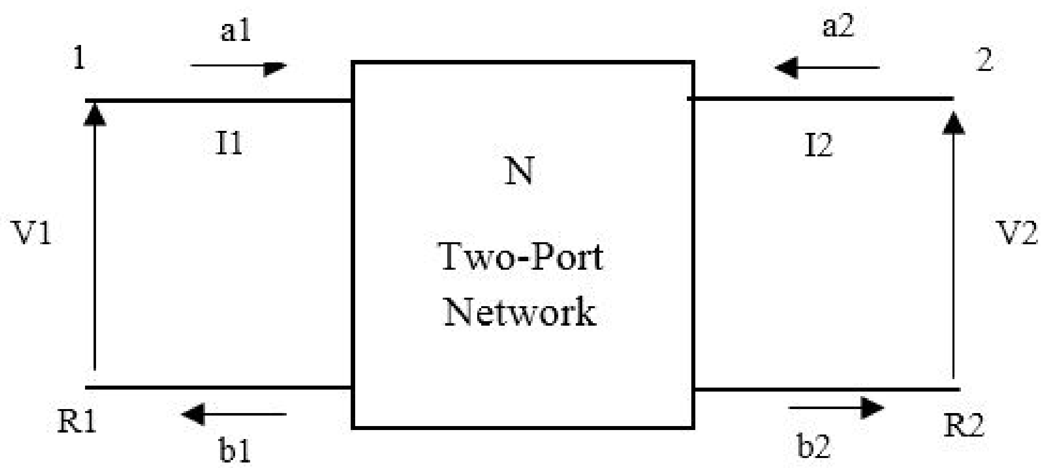

Figure 2.1 Two-Port Network

Scattering variables can be defined as follows ;

$$a_i = \frac{V_i + R_i I_i}{2\sqrt{R_i}} \tag{2.1}$$

$$b_i = \frac{V_i - R_i I_i}{2\sqrt{R_i}} \tag{2.2}$$

a_i and b_i variables are linear function of voltage and current variables defined to the same port (V_i,I_i). Normalized input wave is indicated by a_i, normalized reflected wave is indicated by b_i. (2.1) and (2.2) can be written as inverse relationship function as seen below equations :

$$V_i = (a_i + b_i).\sqrt{R_i} \tag{2.3}$$

$$I_i = \frac{(a_i - b_i)}{\sqrt{R_i}} \tag{2.4}$$

Scattering matrix of two ports (N) is as follows :

$$b = S.a \qquad b = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \qquad a = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \qquad S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \tag{2.5}$$

Elements of the S matrix are called scattering parameters.The following statements can be taken from the definitions in (2.5) for the physical interpretation of the scattering parameters;

$$S_{11} = \frac{b_1}{a_1}\bigg|_{(a_2)=0} \qquad S_{12} = \frac{b_1}{a_2}\bigg|_{(a_1)=0} \qquad S_{21} = \frac{b_2}{a_1}\bigg|_{(a_2)=0} \qquad S_{22} = \frac{b_2}{a_2}\bigg|_{(a_1)=0} \tag{2.6}$$

($a_i = 0$) condition shows that the termination resistance of the port "i" is equal to the reference normalization value R_i of the same port. S_{11} and S_{22} show input and output reflectance coefficients of the two-port. S_{21} and S_{12} show the forward and reverse transmission coefficients.

The meaning of the input reflection coefficient S_{11} can be found from (2.6).Z_1 is the input impedance of the two- port in figure 2.2 and current-voltage relationship for the first port is V_1=Z_1.I_1 When terminating condition a_2=0 is used ;

$$S_{11} = \frac{Z_1 - R_1}{Z_1 + R_1}, \tag{2.7}$$

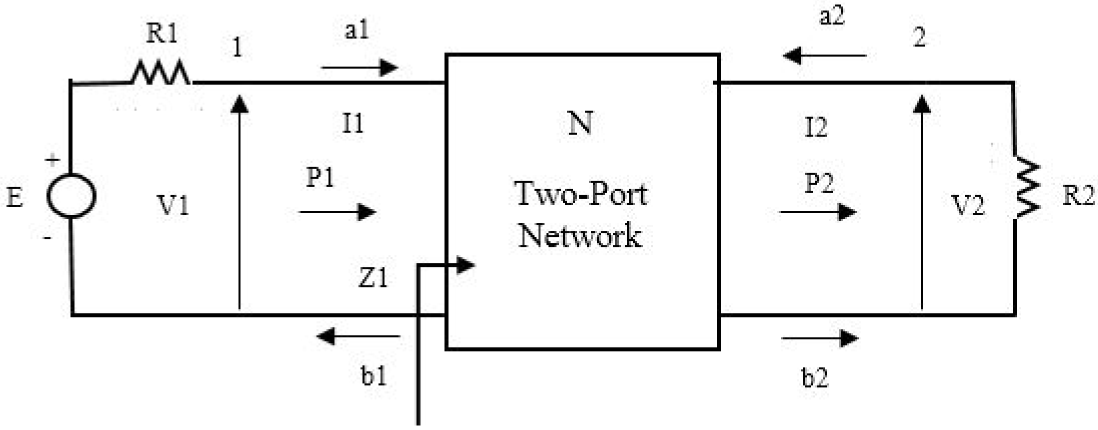

Figure 2.2 Doubly terminated two-port

(2.7) can be found.It shows the relationship between the input impedance and the input reflection of the two-port. A similar relationship can be found for the forward transmission coefficient (S_{21}).

$$S_{21} = 2\sqrt{\frac{R_1}{R_2}}.\frac{V_2}{E}. \tag{2.8}$$

2.2 Relationship between scattering parameters and power

Scattering parameters are useful to identify power transfer from source to load in losslessness two-port. Complex power in the first and second port will be as follows,

$$W_i = V_i(jw)I_i(jw) \quad (i = 1, 2) \tag{2.9}$$

If equations (2.3) and (2.4) are substituted in (2.8) and the real part is taken then the entering real power is found as ;

$$P_i = |a_i|^2 - |b_i|^2 \quad (i = 1, 2) \tag{2.10}$$

The net real power of the two- port is equal to the difference between the entering power and reflected power.

$$P_d = \sum_{i=1}^{2} a_i.a_i{}^* - \sum_{i=1}^{2} b_i.b_i{}^* \tag{2.11}$$

P_A is the power given to the circuit from excitation source in 1^{th} port and P_B is power distributed to 2^{nd} port.

$$P_A = \frac{|E|^2}{4R_1} \quad P_B = \frac{|V_2|^2}{R_2} \tag{2.12}$$

After squaring the forward transmission coefficient (S_{21}) and doing algebraic calculations, transfered power can be shown below :

$$|S_{21}|^2 = \frac{P_B}{P_A} \tag{2.13}$$

and the distributed power from source is,

$$P_A = |a_1| = \frac{|E|^2}{4R_1} \tag{2.14}$$

If the equations (2.5) and (2.10) combined and solved, expended power of two ports can be shown scattering parameters as below,

$$P_d = a^{*T}.(I - S^{*T}S).a \tag{2.15}$$

I represent unit matrix and ^{*T} is transpose of a matrix.

2.3 Scattering Transfer Matrix

Scattering parameter (Fettweis, 1982) for the explanations of power transfer is practical and useful tool for networks working at high frequencies. This method is used for finite values at output and input of the network. The tools use open circuits to find values of voltage and current of the network. Another difference between scattering parameter and others are structure of values in networks. In scattering parameters, the waves of voltage and current are utilized to calculate the efficiency of the network.

The waves are used to form for scattering parameters. The parameters a and b are used for the definition of the scattering parameters. Scattering parameters are the values of the scattering matrix of two port network as given below :

$$\begin{bmatrix} b_1 \\ a_1 \end{bmatrix} = T \begin{bmatrix} a_2 \\ b_2 \end{bmatrix}, \qquad T = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \tag{2.16}$$

T parameter relationship between scattering parameter can be defined as;

$$T_{11} = -\frac{det(S)}{S_{21}}, \qquad T_{21} = -\frac{S_{22}}{S_{21}}, \qquad T_{12} = -\frac{S_{11}}{S_{21}}, \qquad T_{22} = -\frac{1}{S_{21}}, \tag{2.17}$$

det(S) represent determinant of transfer scattering matrix and reciprocal scattering matrix means that if S_{12}=S_{21}, detarminant of T will be det(T)=1.

2.4 Canonic Representation of Scattering Transfer Matrix

Representation of scattering matrix in canonic polynomials (f,g and h) is published in the literature. The canonic forms of the scattering matrix and scattering transfer matrix are shown in below:

$$S = \frac{1}{g}\begin{bmatrix} h & \sigma f_* \\ f & -\sigma h_* \end{bmatrix}, \qquad T = \frac{1}{f}\begin{bmatrix} \sigma g_* & h \\ \sigma h_* & g \end{bmatrix} \tag{2.18}$$

S is representation of the scattering matrix and T is representation of scattering transfer matrix. In addition, canonic polynomials have some properties. Firstly,the polynomial $f = f(p)$, $g = g(p)$, and $h = h(p)$ are real and they are in the complex frequency p. g is the strictly Hurwitz polynomial means that if a single variable real polynomial has no zero in the right half plane, it is called the Hurwitz polynomial and in addition if there is no zero on the imaginary axis, it is the strictly Hurwitz polynomial. Then f,g and h polynomials have the following relation :

$$gg_* = hh_* + ff_*. \tag{2.19}$$

f is a polynomial whose highest coefficient is equal to 1.Also, it is a monic.σ is a constant form of unimodular. ($\sigma = +/-1$) If two-port has reciprocity property,the polynomial of f can be odd or even. Then if the $\sigma = -1$ polynomial of f is odd. If the $\sigma = +1$ polynomial of f is even. Therefore, $\sigma = f_*/f = +/-1$ can be written in the equation,if two-port has reciprocity property then,

$$gg_* = hh_* + \sigma f^2. \tag{2.20}$$

From (2.19), the following relation are also valid,

$$|g| \geq |h|\,, \qquad |g| \geq |f|\,, \tag{2.21}$$

As mentioned above, it can imply follow degree relations, and "deg" is representation of degree of a polynomial.

$$deg(g) \geq deg(h) \qquad deg(g) \geq deg(f) \tag{2.22}$$

The difference between deg(g) and deg(f) shows the number of transmission zeros at infinity and deg(g) refers to the degree of the lossless two-port.

3. TWO-VARIABLE CHARACTERIZATION OF MIXED ELEMENT STRUCTURES

Two-variable polynomials g, h, f, the scattering parameters for a two-port with mixed lumped and distributed elements can be shown as follows (Aksen,1994) where $|\mu| = 1$ is a constant :

$$S(p,\lambda) = \begin{bmatrix} S_{11}(p,\lambda) & S_{12}(p,\lambda) \\ S_{21}(p,\lambda) & S_{22}(p,\lambda) \end{bmatrix} = \frac{1}{g(p,\lambda)} \begin{bmatrix} h(p,\lambda) & \mu f(-p,-\lambda) \\ h(p,\lambda) & -\mu h(p,\lambda) \end{bmatrix} \tag{3.1}$$

In above equation, $p = \sigma + jw$ and $\lambda = \Sigma + j\Omega$ represent the Richards variable with transmission lines and the complex frequency related with lumped elements.

$(n_p + n_\lambda)^{th}$ shows the degree of the scattering Hurwitz polynomial $g(p,\lambda)$ with real coefficients. And it can be shown as $g(p,\lambda) = P^T \Lambda_g \lambda = \lambda^T \Lambda_g^T P$ can be shown as below :

$$\Lambda_g = \begin{bmatrix} g_{00} & g_{01} & g_{02} & \cdots\cdots & g_{0n_\lambda} \\ g_{10} & g_{11} & g_{12} & \cdots\cdots & g_{1n_\lambda} \\ g_{20} & g_{21} & g_{22} & \cdots\cdots & g_{2n_\lambda} \\ \cdots\cdots & \cdots\cdots & \cdots\cdots & \cdots\cdots & g_{3n_\lambda} \\ g_{n_p0} & \cdots\cdots & \cdots\cdots & \cdots\cdots & g_{n_pn_\lambda} \end{bmatrix} \tag{3.2}$$

$$P^T = \begin{bmatrix} 1 & p & p^2 & \ldots\ldots & p^{n_p} \end{bmatrix} \tag{3.3}$$

$$\lambda^T = \begin{bmatrix} 1 & \lambda & \lambda^2 & \ldots\ldots & \lambda^{n_\lambda} \end{bmatrix} \tag{3.4}$$

$(n_p + n_\lambda)^{th}$ shows the degree of the polynomial $h(p,\lambda)$ with real coefficients. And it can be shown as $h(p,\lambda) = P^T \Lambda_h \lambda = \lambda^T \Lambda_h^T P$ can be shown as below :

$$\Lambda_h = \begin{bmatrix} h_{00} & h_{01} & h_{02} & ...... & h_{0n_\lambda} \\ h_{10} & g_{11} & h_{12} & ...... & h_{1n_\lambda} \\ h_{20} & g_{21} & h_{22} & ...... & h_{2n_\lambda} \\ .... & .... & .... & ...... & h_{3n_\lambda} \\ h_{n_p0} & .... & .... & ...... & h_{n_p n_\lambda} \end{bmatrix} \tag{3.5}$$

$f(p,\lambda)$ is a real polynomial and it can be shown according to the tranmission zeros of two-port as can be written as $f(p,\lambda) = f_L(p) f_D(\lambda)$. and $f_L(p)$ and $f_D(\lambda)$ can be constructed by means of the transmission zeros of the transmission zeros of the lumped and distributed elements.If the two-port network is lossless, the relation can be written as $S(p,\lambda)S^T(-p,-\lambda) = I$ and I represents the identify matrix.

If (3.1) is substituted in $S(p,\lambda)S^T(-p,-\lambda) = I$, the following can be found $G(p,\lambda) = g(-p,-\lambda)g(p,\lambda) = h(-p,-\lambda)h(p,\lambda) + f(-p,-\lambda)f(p,\lambda)$. And this equation have to factorized explicity in designin lossless two-port with mixed elements. And if the coefficients of the similar powers of the complex frequency variable, the following equations can be set and it called as fundamental equation set (FES) is can be written :

$$g_{0,k} + 2\sum_{l=0}^{k-1} (-1)^{k-1} g_{0,l} g_{0,2k-l} = h_{0,k}^2 + f_{0,k}^2 + 2\sum_{l=0}^{k-1} (-1)^{k-1} (h_{0,l} h_{0,2k-1} + f_{0,l} f_{0,2k-l}) \tag{3.6}$$

for $k = 0, 1, ..., n_\lambda$

$$\sum_{j=0}^{i} \sum_{l=0}^{k} (-1)^{i-j-l} g_{j,l} g_{i-j,2k-1-l} = \sum_{j=0}^{i} \sum_{l=0}^{k} (-1)^{i-j-l} (h_{j,l} h_{i-j,2k-1-l} + f_{j,l} f_{i-j,2k-1-l}) \tag{3.7}$$

for $i = 1, 3, ..., 2n_p - 1$, $k = 0, 1, ..., n_\lambda$

$$\sum_{j=0}^{i} (-1)^{i-j} (g_{j,k} g_{i-j,k} + 2\sum_{l=0}^{k-1} (-1)^{k-l} g_{j,l} g_{i-j,2k-l}) \tag{3.8}$$

$$= \sum_{j=0}^{i} (-1)^{i-j} (h_{j,k} h_{i-j,k} + f_{j,k} f_{i-j,k} + 2\sum_{l=0}^{k-1} (-1)^{k-l} (h_{j,l} h_{i-j,2k-1} + f_{j,l} f_{i-j,2k-1})) \tag{3.9}$$

for $i = 2, 4, ..., 2n_p - 2$, $k = 0, 1, ..., n_\lambda$

$$g_{n_p,k}^2 + \sum_{l=0}^{k-1} (-1)^{k-l} g_{n_p,l} g_{n_p,2k-l} = h_{n_p,k}^2 + f_{n_p,k}^2 + \sum_{l=0}^{k-1} (-1)^{k-l} (h_{n_p,l} h_{n_p,2k-l} + f_{n_p,l} f_{n_p,2k-l}) \tag{3.10}$$

$k = 0, 1, ..., n_\lambda$

3.1 Explicit Formulas for Low-Order Mixed-Element Structures

In the literature, low-pass ladders connected (LPLU) structure which has fundamental equation set (FES) is formed by using by (Sertbaş A,2001) and (Sertbaş A,1997). For a transformerless design, it is solved algebraically for the unknown coefficients. In this design, coefficient of h_{00} is restrictred as equal 0 and the explicit relations for the entries of Λ_h and Λ_g matrices up to total degree $n = n_p + n_\lambda = 5$ are found in (Aksen A, Yarman 2001). Otherwise, g_{10} equation depends on g_{20} and g_{20} equation depends on g_{10} for $n = 5$ in (Aksen A, Yarman 2001). But in this thesis, the explicit coefficient relations are obtained algebraically and will be given from $n = 2$ to $n = 4$ without any restrictions.

The following procedure will be followed for $n = 5$; If $h(p, 0)$ is initialized and $f(p, 0)$ is formed via $f(p, \lambda) = p^k(1-\lambda^2)^{n_\lambda/2}$ then the strictly Hurwitz polynomial $g(p, 0)$ can be calculated via $G(p, \lambda) = g(-p, -\lambda)g(p, \lambda) = h(-p, -\lambda)h(p, \lambda) + f(-p, -\lambda)f(p, \lambda)$

In the same way, if $h(0, \lambda)$ is initialized then $f(0, \lambda)$ is formed via $f(p, \lambda) = p^k(1 - \lambda^2)^{n_\lambda/2}$ and the strictly Hurwitz Polynomial $g(0, \lambda)$ can be found via $G(p, \lambda) = g(-p, -\lambda)g(p, \lambda) = h(-p, -\lambda)h(p, \lambda) + f(-p, -\lambda)f(p, \lambda)$. After that, fundamental equation set is solved algebraically for the remaining unknown coefficients of Λ_h and Λ_g matrices without any restrictions, the explicit equations for $n = 5 \geq n_p + n_\lambda$ will be showed in the thesis.

3.1.1 Mixed element structure formed with one lumped element and one UE

The network that shown as below has one lumped element ($n_p = 1$) and one unit element ($n_\lambda = 1$).

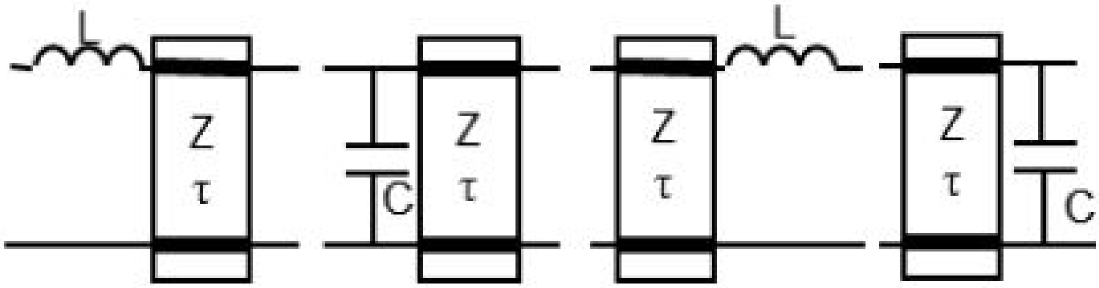

Figure 3.1 Mixed element structure formed with one lumped element and one UE

According to $gg_* = hh_* + ff_*$, two variable polynomials $h(p, \lambda)$ $f(p, \lambda)$ and $g(p, \lambda)$ can be shown as below :

$$g(p, \lambda) = g_{00} + g_{01}\lambda + g_{10}p + g_{11}p\lambda \tag{3.11}$$

$$h(p, \lambda) = h_{00} + h_{01}\lambda + h_{10}p + h_{11}p\lambda \tag{3.12}$$

$$f(p, \lambda) = (1 - \lambda^2)^{1/2} \tag{3.13}$$

Λ_h is a $2x2$ the matrix.

$$\Lambda_h = \begin{bmatrix} h_{00} & h_{01} \\ h_{10} & h_{11} \end{bmatrix} \tag{3.14}$$

First column coefficients describe the lumped element section and the first row coefficients describe the distributed element section.

Λ_g is a $2x2$ the matrix.

$$\Lambda_g = \begin{bmatrix} g_{00} & g_{01} \\ g_{10} & g_{11} \end{bmatrix} \tag{3.15}$$

First column coefficients describe the lumped element section and the first row coefficients describe the distributed element section.

The main goal is that G and H polynomials' coefficients can be calculated from algorithm formed by using Matlab. From the equation $G(p,\lambda) = g(-p,-\lambda)g(p,\lambda)$ $= h(-p,-\lambda)h(p,\lambda) + f(-p,-\lambda)f(p,\lambda)$, and $g(-p,-\lambda)g(p,\lambda)$ the following equation will be obtained as follows, $(g_{00} + g_{01}\lambda + g_{10}p + g_{11}p\lambda)(g_{00} - g_{01}\lambda - g_{10}p + g_{11}p\lambda) =$ $(h_{00} + h_{01}\lambda + h_{10}p + h_{11}p\lambda)(h_{00} - h_{01}\lambda - h_{10}p + h_{11}p\lambda) + (1-\lambda^2)^{1/2}(1-\lambda^2)^{1/2})$.

If the coefficients of the respective degrees are equal, the following equation set is obtained. With the help of these equations unknown coefficients can be calculated.

$$g_{00}{}^2 - h_{00}{}^2 = 1 \tag{3.16}$$

$$g_{01}{}^2 - h_{01}{}^2 = 1 \tag{3.17}$$

$$g_{00}g_{11} - g_{01}g_{10} - h_{00}h_{11} + h_{01}h_{10} = 0 \tag{3.18}$$

$$g_{10}{}^2 - h_{10}{}^2 = 0 \tag{3.19}$$

$$g_{11}{}^2 - h_{11}{}^2 = 0 \tag{3.20}$$

The unknown cofficients will be calculated with above equations. From the equation (3.6)

$$g_{00} = \sqrt{1 + h_{00}{}^2} \tag{3.21}$$

From the equation (3.7)

$$g_{01} = \sqrt{1 + h_{01}{}^2} \tag{3.22}$$

From the equation (3.8)

$$g_{11} = \frac{g_{01}g_{10} - h_{01}h_{10}}{g_{00} - \mu_2 h_{00}} \tag{3.23}$$

Table 3.1 Connection Order of the LPLU Topologies for one lumped element and one UE

μ_1	μ_2	**First Element**	**Second Element**
+1	+1	Inductor	Unit Element
+1	-1	Unit Element	Inductor
-1	+1	Unit Element	Capacitor
-1	-1	Capacitor	Unit Element

From the equation (3.9)

$$g_{10} = |h_{10}| \rightarrow \mu_1 = \frac{h_{10}}{g_{10}} \tag{3.24}$$

From the equation (3.10)

$$g_{11} = |h_{11}| \rightarrow \mu_2 = \frac{h_{11}}{g_{11}} \rightarrow h_{11} = \mu_2 g_{11} \tag{3.25}$$

3.1.2 Mixed element structure formed with two lumped elements and one UE

The network that shown as below has two lumped elements ($n_p = 2$) and one unit element ($n_\lambda = 1$).

Λ_h is a $3x2$ matrix and h_{00},h_{01}, h_{10}, h_{20} are independent coefficients and $h_{21} = 0$.

$$\Lambda_h = \begin{bmatrix} h_{00} & h_{01} \\ h_{10} & h_{11} \\ h_{20} & 0 \end{bmatrix} \tag{3.26}$$

First column coefficients describe the lumped element section and the first row coefficients describe the distributed element section.

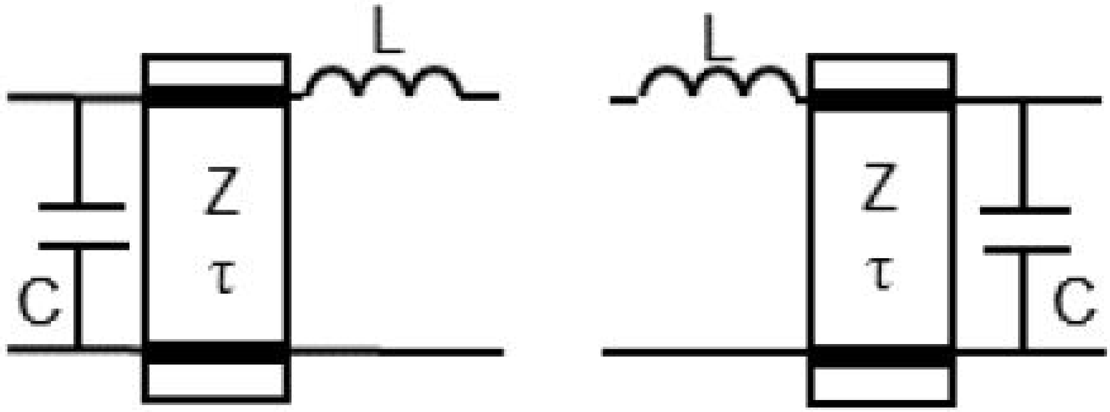

Figure 3.2 Mixed element structure formed with two lumped elements and one UE

Λ_g is a $3x2$ matrix and $g_{21} = 0$.

$$\Lambda_g = \begin{bmatrix} g_{00} & g_{01} \\ g_{10} & g_{11} \\ g_{20} & 0 \end{bmatrix} \tag{3.27}$$

First column coefficients describe the lumped element section and the first row coefficients describe the distributed element section.

The main goal is that G and H polynomials' coefficients can be calculated from algorithm formed by using Matlab. From the equation $G(p, \lambda) = g(-p, -\lambda)g(p, \lambda)$ $= h(-p, -\lambda)h(p, \lambda) + f(-p, -\lambda)f(p, \lambda)$, and $g(-p, -\lambda)g(p, \lambda)$ the following equation will be obtained as follows, $(g_{00} + g_{01}\lambda + g_{10}p + g_{11}p\lambda)(g_{00} - g_{01}\lambda - g_{10}p + g_{11}p\lambda) =$ $(h_{00} + h_{01}\lambda + h_{10}p + h_{11}p\lambda)(h_{00} - h_{01}\lambda - h_{10}p + h_{11}p\lambda) + (1 - \lambda^2)^{1/2}(1 - \lambda^2)^{1/2})$.

If the coefficients of the respective degrees are equal, the following equation set is obtained. With the help of these equations unknown coefficients can be calculated.

$$g_{00}{}^2 - h_{00}{}^2 = 1 \tag{3.28}$$

$$g_{01}{}^2 - h_{01}{}^2 = 1 \tag{3.29}$$

$$g_{00}g_{11} - g_{01}g_{10} - h_{00}h_{11} + h_{01}h_{10} = 0 \tag{3.30}$$

$$g_{10}{}^2 - h_{10}{}^2 - 2(g_{00}g_{20} - h_{00}h_{20}) = 0 \tag{3.31}$$

$$g_{11}{}^2 - h_{11}{}^2 + 2(g_{01}g_{21} - h_{01}h_{21}) = 0 \tag{3.32}$$

$$g_{11}g_{20} - g_{10}g_{21} - h_{11}h_{20} + h_{10}h_{21} = 0 \tag{3.33}$$

$$g_{20}{}^2 - h_{20}{}^2 = 0 \tag{3.34}$$

$$g_{21}{}^2 - h_{21}{}^2 = 0 \tag{3.35}$$

From the equation (3.18)

$$g_{00} = \sqrt{1 + h_{00}{}^2} \tag{3.36}$$

From the equation (3.19)

$$g_{01} = \sqrt{1 + h_{01}{}^2} \tag{3.37}$$

From the equation (3.20)

$$g_{00}g_{11} - g_{01}g_{10} - h_{00}h_{11} + h_{01}h_{10} = 0 \tag{3.38}$$

$$g_{00}g_{11} = g_{01}g_{10} + h_{00}h_{11} - h_{01}h_{10} \tag{3.39}$$

$$g_{11} = \frac{g_{01}g_{10} + h_{00}h_{11} - h_{01}h_{10}}{g_{00}} \tag{3.40}$$

From the equation (3.21)

$$g_{10}{}^2 - h_{10}{}^2 - 2(g_{00}g_{20} - h_{00}h_{20}) = 0 \tag{3.41}$$

$$g_{10} = \sqrt{h_{10}{}^2 + 2(g_{00}g_{20} - h_{00}h_{20}} \tag{3.42}$$

From the equation (3.22)

$$g_{11} = |h_{11}| \rightarrow \mu_2 = \frac{h_{11}}{g_{11}} \tag{3.43}$$

Table 3.2 Connection Order of the LPLU Topologies for two lumped elements and one UE

μ_1	μ_2	**First Element**	**Second Element**
+1	+1	Inductor	Unit Element
+1	-1	Unit Element	Inductor
-1	+1	Unit Element	Capacitor
-1	-1	Capacitor	Unit Element

From the equation (3.23)

$$g_{11}g_{20} - 0 - h_{11}h_{20} + 0 = 0 \tag{3.44}$$

$$g_{11}g_{20} - h_{11}h_{20} = 0 \tag{3.45}$$

$$g_{20}(g_{11} - h_{11}\frac{h_{20}}{g_{20}}) = 0 \tag{3.46}$$

$$g_{11} = \mu_1 h_{11} \tag{3.47}$$

From the equation (3.24)

$$g_{20} = |h_{20}| \rightarrow \mu_1 = \frac{h_{20}}{g_{20}}. \tag{3.48}$$

3.1.3 Mixed element structure formed with one lumped element and two UEs

The network that shown as below has one lumped element ($n_p = 1$) and two unit elements ($n_\lambda = 2$).

Λ_h is a $2x3$ matrix and h_{00},h_{01}, h_{10}, h_{02} are independent coefficients.

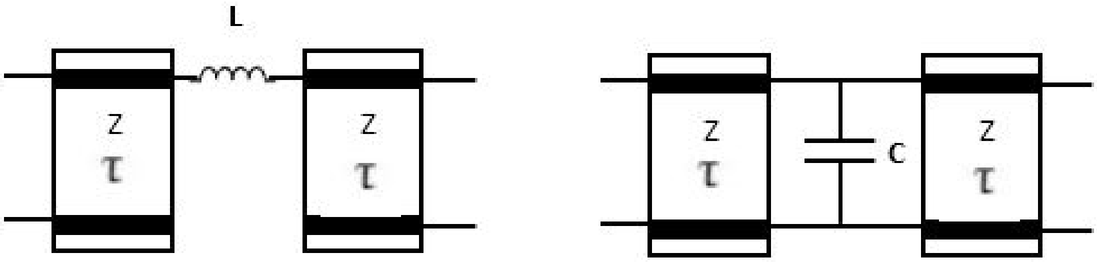

Figure 3.3 Mixed element structure formed with one lumped element and two UEs

$$\Lambda_h = \begin{bmatrix} h_{00} & h_{01} & h_{02} \\ h_{10} & h_{11} & h_{12} \end{bmatrix} \tag{3.49}$$

First column coefficients describe the lumped element section and the first row coefficients describe the distributed element section.

Λ_g is a $2x3$ matrix.

$$\Lambda_h = \begin{bmatrix} g_{00} & g_{01} & g_{02} \\ g_{10} & g_{11} & g_{12} \end{bmatrix} \tag{3.50}$$

First column coefficients describe the lumped element section and the first row coefficients describe the distributed element section.

The main goal is that G and H polynomials' coefficients can be calculated from algorithm formed by using Matlab. From the equation $G(p,\lambda) = g(-p,-\lambda)g(p,\lambda)$ $= h(-p,-\lambda)h(p,\lambda) + f(-p,-\lambda)f(p,\lambda)$, and $g(-p,-\lambda)g(p,\lambda)$ the following equation will be obtained as follows, $(g_{00} + g_{01}\lambda + g_{10}p + g_{11}p\lambda)(g_{00} - g_{01}\lambda - g_{10}p + g_{11}p\lambda) =$ $(h_{00} + h_{01}\lambda + h_{10}p + h_{11}p\lambda)(h_{00} - h_{01}\lambda - h_{10}p + h_{11}p\lambda) + (1-\lambda^2)^{1/2}(1-\lambda^2)^{1/2}$.

If the coefficients of the respective degrees are equal, the following equation set is obtained. With the help of these equations unknown coefficients can be calculated.

$${g_{00}}^2 - {h_{00}}^2 = 1 \tag{3.51}$$

$$g_{01}^2 - h_{01}^2 - 2(g_{00}g_{02} - h_{00}h_{02}) = 2 \tag{3.52}$$

$$g_{00}g_{11} - g_{01}g_{10} - h_{00}h_{11} + h_{01}h_{10} = 0 \tag{3.53}$$

$$g_{10}{}^2 - h_{10}{}^2 = 0 \tag{3.54}$$

$$g_{11}^2 - h_{11}^2 - 2(g_{10}g_{12} - h_{10}h_{12}) = 0 \tag{3.55}$$

$$g_{11}g_{02} - g_{01}g_{12} - h_{11}h_{02} + h_{01}h_{12} = 0 \tag{3.56}$$

$$g_{02}{}^2 - h_{02}{}^2 = 1 \tag{3.57}$$

$$g_{12}{}^2 - h_{12}{}^2 = 0 \tag{3.58}$$

From the equation (3.41)

$$g_{00} = \sqrt{1 + h_{00}{}^2} \tag{3.59}$$

From the equation (3.47)

$$g_{02} = \sqrt{1 + h_{02}{}^2} \tag{3.60}$$

From the equation (3.42)

$$g_{01}^2 - h_{01}^2 - 2(g_{00}g_{02} - h_{00}h_{02}) = 2 \tag{3.61}$$

$$g_{01} = \sqrt{2 + h_{01}^2 + 2(g_{00}g_{02} - h_{00}h_{02})} \tag{3.62}$$

From the equation (3.44)

$$g_{10} = |h_{10}| \to \mu_1 = \frac{h_{10}}{g_{10}}. \tag{3.63}$$

From the equation (3.48)

$$g_{12} = |h_{12}| \to \mu_2 = \frac{h_{12}}{g_{12}}. \tag{3.64}$$

From the equation (3.46)

$$g_{11}g_{02} - h_{11}h_{02} = g_{01}g_{12} - h_{01}h_{12} \tag{3.65}$$

$$g_{11}g_{02} - h_{11}h_{02} = g_{12}(g_{01} - \frac{h_{01}h_{12}}{g_{12}} \tag{3.66}$$

$$g_{11}g_{02} - h_{11}h_{02} = g_{12}(g_{01} - h_{01}\mu_2) \tag{3.67}$$

$$g_{12} = \frac{g_{11}g_{02} - h_{11}h_{02}}{\alpha} \quad where \quad \alpha = g_{01} - h_{01}\mu_2 \tag{3.68}$$

From the equation (3.45) if put the g_{12} and h_{12} in the equation

$$g_{11}^2 - h_{11}^2 = 2(g_{10}g_{12} - h_{10}h_{12}) \tag{3.69}$$

$$g_{11}^2 - h_{11}^2 = 2(\frac{g_{10}g_{11}g_{02}}{\alpha} - \frac{g_{10}h_{11}h_{02}}{\alpha} - \frac{h_{10}g_{11}\mu_2 g_{02}}{\alpha} + \frac{h_{10}h_{11}\mu_2 h_{02}}{\alpha}) \tag{3.70}$$

$$g_{11}^2 - h_{11}^2 = 2[g_{11}(\frac{g_{10}g_{02}}{\alpha} - \frac{h_{10}\mu_2 g_{02}}{\alpha}) - h_{11}(\frac{g_{10}h_{02}}{\alpha} + \frac{h_{10}\mu_2 h_{02}}{\alpha})] \tag{3.71}$$

$$g_{11}^2 - h_{11}^2 = 2[g_{11}(\frac{g_{02}}{\alpha}(g_{10} - h_{10}\mu_2)) - h_{11}(\frac{h_{02}}{\alpha}(g_{10} - h_{10}\mu_2))] \tag{3.72}$$

$$g_{11}^2 - h_{11}^2 = 2[g_{11}(g_{02}\frac{\beta}{\alpha}) - h_{11}(h_{02}\frac{\beta}{\alpha})] \tag{3.73}$$

$$g_{11}^2 - h_{11}^2 = g_{11}\frac{2g_{02}\beta}{\alpha} - h_{11}\frac{2h_{02}\beta}{\alpha}) \tag{3.74}$$

Where $\beta = g_{10} - h_{10}\mu_2$ and $\alpha = g_{01} - h_{01}\mu_2$,

$$g_{11} = \frac{2g_{02}\beta}{\alpha} \tag{3.75}$$

$$h_{11} = \frac{2h_{02}\beta}{\alpha} \tag{3.76}$$

Table 3.3 Connection Order of the LPLU Topologies for one lumped elements and two UEs

μ_1	μ_2	**First Element**	**Second Element**
+1	+1	Inductor	Unit Element
+1	-1	Unit Element	Inductor
-1	+1	Unit Element	Capacitor
-1	-1	Capacitor	Unit Element

3.1.4 Mixed element structure formed with two lumped elements and two UEs

The network that shown as below has two lumped element ($n_p = 2$) and two unit elements ($n_\lambda = 2$).

Λ_h is $3x3$ matrix and h_{00},h_{01},h_{02}, h_{10}, h_{20} are independent coefficients and $h_{22} = 0$.

$$\Lambda_h = \begin{bmatrix} h_{00} & h_{01} & h_{02} \\ h_{10} & h_{11} & h_{12} \\ h_{20} & h_{21} & 0 \end{bmatrix} \tag{3.77}$$

First column coefficients describe the lumped element section and the first row coefficients describe the distributed element section.

Λ_g is a $3x3$ matrix and $g_{22} = 0$.

$$\Lambda_g = \begin{bmatrix} g_{00} & g_{01} & g_{02} \\ g_{10} & g_{11} & g_{12} \\ g_{20} & g_{21} & 0 \end{bmatrix} \tag{3.78}$$

First column coefficients describe the lumped element section and the first row coefficients describe the distributed element section.

The main goal is that G and H polynomials' coefficients can be calculated from

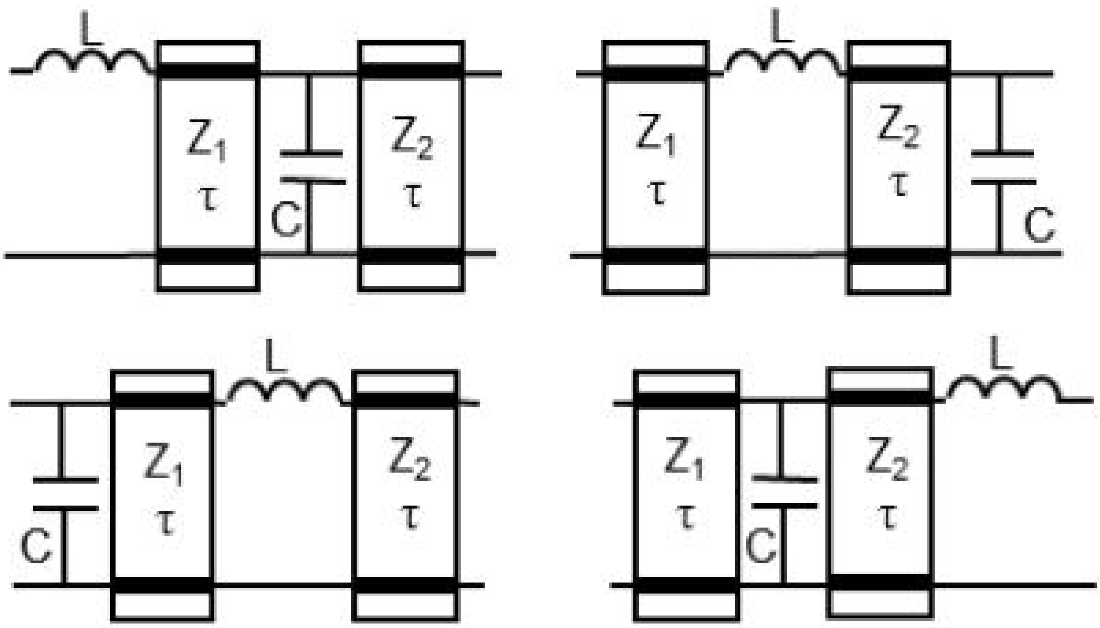

Figure 3.4 Mixed element structure formed with two lumped elements and two UEs

algorithm formed by using Matlab. From the equation $G(p,\lambda) = g(-p,-\lambda)g(p,\lambda)$ $= h(-p,-\lambda)h(p,\lambda) + f(-p,-\lambda)f(p,\lambda)$, and $g(-p,-\lambda)g(p,\lambda)$ the following equation will be obtained as follows, $(g_{00} + g_{01}\lambda + g_{10}p + g_{11}p\lambda)(g_{00} - g_{01}\lambda - g_{10}p + g_{11}p\lambda) =$ $(h_{00} + h_{01}\lambda + h_{10}p + h_{11}p\lambda)(h_{00} - h_{01}\lambda - h_{10}p + h_{11}p\lambda) + (1-\lambda^2)^{1/2}(1-\lambda^2)^{1/2})$.

If the coefficients of the respective degrees are equal, the following equation set is obtained. With the help of these equations unknown coefficients can be calculated.

$$g_{00}{}^2 - h_{00}{}^2 = 1 \tag{3.79}$$

$$g_{01}^2 - h_{01}^2 - 2(g_{00}g_{02} - h_{00}h_{02}) = 2 \tag{3.80}$$

$$g_{00}g_{11} - g_{01}g_{10} - h_{00}h_{11} + h_{01}h_{10} = 0 \tag{3.81}$$

$$g_{10}{}^2 - h_{10}{}^2 - 2(g_{00}g_{20} - h_{00}h_{20}) = 0 \tag{3.82}$$

$$g_{11}{}^2 - h_{11}{}^2 - 2(g_{01}g_{21} - g_{02}g_{20} + g_{10}g_{12} - g_{00}g_{22} - h_{01}h_{21} - h_{02}h_{20} - h_{10}h_{12} + h_{00}h_{22}) = 0 \tag{3.83}$$

$$g_{11}g_{02} - g_{01}g_{12} - h_{11}h_{02} + h_{01}h_{12} = 0 \tag{3.84}$$

$${g_{02}}^2 - {h_{02}}^2 = 1 \tag{3.85}$$

$${g_{12}}^2 - {h_{12}}^2 - 2(g_{02}g_{22} - h_{02}h_{22}) = 0 \tag{3.86}$$

$$g_{11}g_{20} - g_{10}g_{21} - h_{11}h_{20} + h_{10}h_{21} = 0 \tag{3.87}$$

$$g_{11}g_{22} - g_{12}g_{21} - h_{11}h_{22} + h_{12}h_{21} = 0 \tag{3.88}$$

$${g_{20}}^2 - {h_{20}}^2 = 0 \tag{3.89}$$

$${g_{21}}^2 - {h_{21}}^2 + 2(h_{20}h_{22} - g_{20}g_{22}) = 0 \tag{3.90}$$

$${g_{22}}^2 - {h_{22}}^2 = 0 \tag{3.91}$$

From the equation (3.69)

$$g_{00} = \sqrt{1 + {h_{00}}^2} \tag{3.92}$$

From the equation (3.75)

$$g_{02} = \sqrt{1 + {h_{02}}^2} \tag{3.93}$$

From the equation (3.70)

$$g_{01} = \sqrt{2 + 2(g_{00}g_{02} - h_{00}h_{02} + h_{01}^2)} \tag{3.94}$$

From the equation (3.79)

$$g_{20} = |h_{20}| \rightarrow \mu_1 = \frac{h_{20}}{g_{20}} \tag{3.95}$$

From the equation (3.80)

$$g_{21} = |h_{21}| \rightarrow \mu_2 = \frac{h_{21}}{g_{21}} \rightarrow h_{21} = \mu_2 g_{21} \tag{3.96}$$

From the equation (3.76)

$$g_{12} = |h_{12}| \rightarrow \mu_3 = \frac{h_{12}}{g_{12}} \rightarrow h_{12} = \mu_3 g_{12} \tag{3.97}$$

From the equation (3.78)

$$-g_{12}g_{21} + h_{12}h_{21} = 0 \tag{3.98}$$

$$\frac{h_{21}}{g_{21}} = \mu_2 = \frac{g_{12}}{h_{12}} = \mu_3 \rightarrow \mu_2 = \mu_3 \tag{3.99}$$

From the equation (3.74)

$$g_{11}g_{02} - h_{11}h_{02} = -h_{01}h_{12} + g_{01}g_{12} \tag{3.100}$$

$$g_{11}g_{02} - h_{11}h_{02} = g_{12}(g_{01} - h_{01}\frac{h_{12}}{g_{12}}) \tag{3.101}$$

and (3.87) equation shows that $\frac{h_{12}}{g_{12}}$ equal μ_2 so that,

$$g_{12} = \frac{g_{11}g_{02} - h_{11}h_{02}}{\alpha} \tag{3.102}$$

where $\alpha = g_{01} - h_{01}\mu_2$.

From the equation (3.77)

$$g_{11}g_{20} - h_{11}h_{20} = g_{10}g_{21} - h_{10}h_{21} \tag{3.103}$$

$$g_{11}g_{20} - h_{11}h_{20} = g_{21}(g_{10} - \mu_2 h_{10}) \tag{3.104}$$

$$g_{21} = \frac{g_{11}g_{20} - h_{11}h_{20}}{\beta} \tag{3.105}$$

where $\beta = g_{10} - \mu_2 h_{10}$.

From the equation (3.71)

$$g_{00}g_{11} - g_{01}g_{10} - h_{00}h_{11} + h_{01}h_{10} = 0 \tag{3.106}$$

$$g_{00}g_{11} = g_{01}g_{10} + h_{00}h_{11} - h_{01}h_{10} \tag{3.107}$$

$$g_{11} = \frac{g_{01}g_{10} + h_{00}h_{11} - h_{01}h_{10}}{g_{00}} \tag{3.108}$$

$$g_{11} = \frac{h_{00}h_{11} + \gamma}{g_{00}} \tag{3.109}$$

where $\gamma = g_{01}g_{10} - h_{01}h_{10}$.

From the equation (3.73)

$$g_{11}{}^2 - h_{11}{}^2 - 2(g_{01}g_{21} - g_{02}g_{20} + g_{10}g_{12} - g_{00}g_{22} - h_{01}h_{21} - h_{02}h_{20} - h_{10}h_{12} + h_{00}h_{22} = 0 \tag{3.110}$$

$$h_{11} = \frac{h_{20}\frac{\alpha}{\beta} + h_{02}\frac{\beta}{\alpha} - \frac{h_{00}}{g_{00}}(g_{20}\frac{\alpha}{\beta} + g_{02}\frac{\beta}{\alpha}) + \frac{h_{00}}{g_{00}^2}\gamma}{1 - \frac{h_{00}^2}{g_{00}^2}} \tag{3.111}$$

where $\gamma = g_{01}g_{10} - h_{01}h_{10}$.

From the equation (3.72)

$$g_{10}{}^2 - h_{10}{}^2 = 2(g_{00}g_{20} - h_{00}h_{20}) \tag{3.112}$$

$$g_{10}^2 = h_{10}^2 + 2(g_{00}g_{20} - h_{00}h_{20}) \tag{3.113}$$

$$g_{10} = \sqrt{h_{10}^2 + 2(g_{00}g_{20} - h_{00}h_{20})} \tag{3.114}$$

3.1.5 Mixed element structure formed with three lumped elements and two UEs

The network that shown as below has three lumped element ($N_p = 3$) and two unit elements ($N_\lambda = 2$).

Λ_h is a $4x3$ matrix and h_{00},h_{01},h_{02}, h_{10}, h_{20} and h_{30} are independent coefficients and $h_{22} = h_{31} = h_{32} = 0$

$$\Lambda_h = \begin{bmatrix} h_{00} & h_{01} & h_{02} \\ h_{10} & h_{11} & h_{12} \\ h_{20} & h_{21} & 0 \\ h_{30} & 0 & 0 \end{bmatrix} \tag{3.115}$$

Table 3.4 Connection Order of the LPLU Topologies for two lumped elements and two UEs

μ_1	μ_2	**First Element**	**Second Element**
+1	+1	Inductor	Unit Element
+1	-1	Unit Element	Inductor
-1	+1	Unit Element	Capacitor
-1	-1	Capacitor	Unit Element

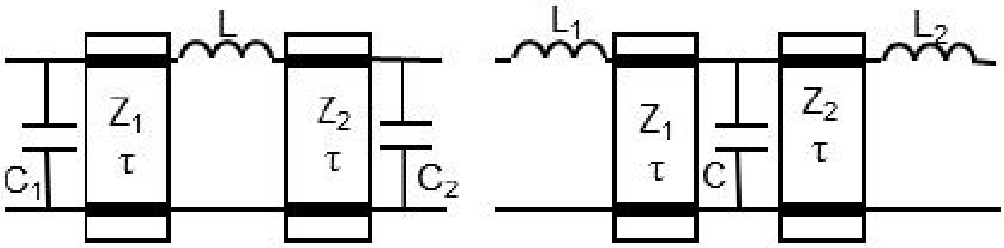

Figure 3.5 Mixed element structures formed with three lumped elements and two UEs

First column coefficients describe the lumped element section and the first row coefficients describe the distributed element section.

Λ_g is a $4x3$ matrix and $g_{22} = g_{31} = g_{32} = 0$.

$$\Lambda_g = \begin{bmatrix} g_{00} & g_{01} & g_{02} \\ g_{10} & g_{11} & g_{12} \\ g_{20} & g_{21} & 0 \\ g_{30} & 0 & 0 \end{bmatrix} \tag{3.116}$$

First column coefficients describe the lumped element section and the first row coefficients describe the distributed element section.

The main goal is that G and H polynomials' coefficients can be calculated from algorithm formed by using Matlab. From the equation $G(p, \lambda) = g(-p, -\lambda)g(p, \lambda)$ $= h(-p, -\lambda)h(p, \lambda) + f(-p, -\lambda)f(p, \lambda)$, and $g(-p, -\lambda)g(p, \lambda)$ the following equation will be obtained as follows, $(g_{00}+g_{01}\lambda+g_{10}p+g_{11}p\lambda)(g_{00}-g_{01}\lambda-g_{10}p+g_{11}p\lambda) = (h_{00}+h_{01}\lambda+h_{10}p+h_{11}p\lambda)(h_{00}-h_{01}\lambda-h_{10}p+h_{11}p\lambda)+(1-\lambda^2)^{1/2}(1-\lambda^2)^{1/2}$.

If the coefficients of the respective degrees are equal, the following equation set is obtained. With the help of these equations unknown coefficients can be calculated.

$$ {g_{00}}^2 - {h_{00}}^2 = 1 \tag{3.117} $$

$$ g_{01}^2 - h_{01}^2 - 2(g_{00}g_{02} - h_{00}h_{02}) = 2 \tag{3.118} $$

$$ g_{00}g_{11} - g_{01}g_{10} - h_{00}h_{11} + h_{01}h_{10} = 0 \tag{3.119} $$

$$ {g_{10}}^2 - {h_{10}}^2 - 2(g_{00}g_{20} - h_{00}h_{20}) = 0 \tag{3.120} $$

$$ {g_{11}}^2 - {h_{11}}^2 - 2(g_{01}g_{21} - g_{02}g_{20} + g_{10}g_{12} - g_{00}g_{22} - h_{01}h_{21} - h_{02}h_{20} - h_{10}h_{12} + h_{00}h_{22}) = 0 \tag{3.121} $$

$$ g_{11}g_{02} - g_{01}g_{12} - h_{11}h_{02} + h_{01}h_{12} = 0 \tag{3.122} $$

$$ {g_{02}}^2 - {h_{02}}^2 = 1 \tag{3.123} $$

$$ {g_{12}}^2 - {h_{12}}^2 - 2(g_{02}g_{22} - h_{02}h_{22}) = 0 \tag{3.124} $$

$$ g_{11}g_{20} - g_{10}g_{21} - g_{01}g_{30} + g_{00}g_{31} - h_{11}h_{20} + h_{10}h_{21} + h_{01}h_{30} - h_{00}h_{31} = 0 \tag{3.125} $$

$$ g_{11}g_{22} - g_{12}g_{21} - g_{01}g_{32} + g_{02}g_{31} - h_{11}h_{22} + h_{12}h_{21} + h_{01}h_{32} - h_{02}h_{31} = 0 \tag{3.126} $$

$$ {g_{20}}^2 - {h_{20}}^2 - 2(g_{10}g_{30} - h_{10}h_{30}) = 0 \tag{3.127} $$

$$ {g_{21}}^2 - {h_{21}}^2 - 2(g_{10}g_{32} + g_{12}g_{30} - g_{20}g_{22} + h_{10}h_{32} + h_{12}h_{30} - h_{20}h_{22}) = 0 \tag{3.128} $$

$$g_{22}{}^2 - h_{22}{}^2 - 2(g_{12}g_{32} - h_{12}h_{32}) = 0 \tag{3.129}$$

$$g_{20}g_{31} - g_{21}g_{30} - h_{20}h_{31} + h_{21}h_{30} = 0 \tag{3.130}$$

$$g_{22}g_{31} - g_{21}g_{32} - h_{22}h_{31} + h_{21}h_{32} = 0 \tag{3.131}$$

$$g_{30}{}^2 - h_{30}{}^2 = 0 \tag{3.132}$$

$$g_{31}{}^2 - h_{31}{}^2 - 2(g_{30}g_{32} - h_{30}h_{32}) = 0 \tag{3.133}$$

$$g_{32}{}^2 - h_{32}{}^2 = 0 \tag{3.134}$$

From the equation (3.107)

$$g_{00} = \sqrt{1 + h_{00}{}^2} \tag{3.135}$$

From the equation (3.108)

$$g_{01} = \sqrt{2 + h_{01}^2 + 2(g_{00}g_{02} - h_{00}h_{02})} \tag{3.136}$$

From the equation (3.109)

$$g_{00}g_{11} - g_{01}g_{10} - h_{00}h_{11} + h_{01}h_{10} = 0 \tag{3.137}$$

$$g_{00}g_{11} = g_{01}g_{10} + h_{00}h_{11} - h_{01}h_{10} \tag{3.138}$$

$$g_{11} = \frac{g_{01}g_{10} + h_{00}h_{11} - h_{01}h_{10}}{g_{00}} \tag{3.139}$$

$$g_{11} = \frac{h_{00}h_{11} + \gamma}{g_{00}} \tag{3.140}$$

where $\gamma = g_{01}g_{10} - h_{01}h_{10}$.

From the equation (3.110)

$$g_{10}{}^2 - h_{10}{}^2 = 2(g_{00}g_{20} - h_{00}h_{20}) \tag{3.141}$$

$$g_{10}^2 = h_{10}^2 + 2(g_{00}g_{20} - h_{00}h_{20}) \tag{3.142}$$

$$g_{10} = \sqrt{h_{10}^2 + 2(g_{00}g_{20} - h_{00}h_{20})} \tag{3.143}$$

From the equation (3.112)

$$g_{11}g_{02} - h_{11}h_{02} = -h_{01}h_{12} + g_{01}g_{12} \tag{3.144}$$

$$g_{11}g_{02} - h_{11}h_{02} = g_{12}(g_{01} - h_{01}\frac{h_{12}}{g_{12}}) \tag{3.145}$$

and (3.116) equation shows that $\frac{h_{12}}{g_{12}}$ equals μ_2 so that,

$$g_{12} = \frac{g_{11}g_{02} - h_{11}h_{02}}{\alpha} \tag{3.146}$$

where $\alpha = g_{01} - h_{01}\mu_2$.

From the equation (3.113)

$$g_{02} = \sqrt{1 + {h_{02}}^2} \tag{3.147}$$

From the equation (3.115)

$$g_{11}g_{20} - h_{11}h_{20} = g_{10}g_{21} - h_{10}h_{21} \tag{3.148}$$

and (3.116) equation shows that $\frac{g_{21}}{h_{21}}$ equals μ_2 so that,

$$g_{11}g_{20} - h_{11}h_{20} = g_{21}(g_{10} - \mu_2 h_{10}) \tag{3.149}$$

$$g_{21} = \frac{g_{11}g_{20} - h_{11}h_{20}}{\beta} \tag{3.150}$$

where $\beta = g_{10} - \mu_2 h_{10}$.

From the equation (3.116)

$$g_{12}g_{21} = h_{12}h_{21} \tag{3.151}$$

$$\frac{h_{12}}{g_{12}} = \frac{g_{12}}{h_{12}} = \frac{g_{21}}{h_{21}} = \frac{h_{21}}{g_{21}} = \mu_2 \tag{3.152}$$

Table 3.5 Connection Order of the LPLU Topologies for three lumped elements and two UEs

μ_1	μ_2	**First Element**	**Second Element**
+1	+1	Inductor	Unit Element
+1	-1	Unit Element	Inductor
-1	+1	Unit Element	Capacitor
-1	-1	Capacitor	Unit Element

From the equation (3.117)

$$g_{20} = \sqrt{h_{20}^2 + 2(g_{10}g_{30} - h_{10}h_{30})} \tag{3.153}$$

From the equation (3.120)

$$g_{21}g_{30} - h_{21}h_{30} = 0 \tag{3.154}$$

$$g_{30}(g_{21} - h_{21}\frac{h_{30}}{g_{30}}) = 0 \tag{3.155}$$

The connection order of the LPLU topologies for three lumped elements and two UEs table will be given below :

From the equation (3.111)

$$g_{11}{}^2 - h_{11}{}^2 - 2(g_{01}g_{21} - g_{02}g_{20} + g_{10}g_{12} - g_{00}g_{22} - h_{01}h_{21} - h_{02}h_{20} - h_{10}h_{12} + h_{00}h_{22}) = 0 \tag{3.156}$$

$$h_{11} = \frac{h_{20}\frac{\alpha}{\beta} + h_{02}\frac{\beta}{\alpha} - \frac{h_{00}}{g_{00}}(g_{20}\frac{\alpha}{\beta} + g_{02}\frac{\beta}{\alpha}) + \frac{h_{00}}{g_{00}^2}\gamma}{1 - \frac{h_{00}^2}{g_{00}^2}} \tag{3.157}$$

where $\gamma = g_{01}g_{10} - h_{01}h_{10}$.

From the equation (3.122)

$$g_{30} = |h_{30}| \rightarrow \mu_3 = \frac{h_{30}}{g_{30}} \tag{3.158}$$

4. BROADBAND MATCHING METHODS

4.1 Real Frequency Matching with Scattering Parameters

The matching problem is formulated by scattering parameters of the lossless equalizer network. That frequency scattering approach is Simplified Real Frequency Technique (SRFT) (Yarman, 1985). Matching network which is a lossless identified with the scattering parameters are formed with canonic polynomials f,g,h.

The canonic polynomials are represented with Belevitch representation.

$$S_{11} = \frac{h(p)}{g(p)} \qquad S_{12} = \sigma\frac{f(-p)}{g(p)} \qquad S_{21} = \frac{f(p)}{g(p)} \qquad S_{22} = -\sigma\frac{h(-p)}{g(p)} \qquad \sigma = \frac{f(p)}{f(-p)} \tag{4.1}$$

In the above equations, σ is a constant (+1 or -1), $f(p)$ is a real monic polynomial and $g(p)$ is a Hurtwitz polynomial. When the two port N is reciprocal, then f is either odd or even.

Relation in terms of degree between $f(p)$, $g(p)$ and $h(p)$ polynomials is that $g(p)$ polynomial can be bigger or equal than degree of $f(p)$ and $h(p)$ polynomials.

The relation $f(p)$, $g(p)$ and $h(p)$ can be seen below:

$$g(p)g(-p) = h(p)h(-p) + f(p)f(-p) \tag{4.2}$$

$f(p)$ and $h(p)$ polynomials are the parameters of Hurwitz polynomial $g(p)$.The network's definitions can be shown with $f(p)$ and $h(p)$ polynomials. $f(p)$ polynomial is the zeros of transmission of the matching two port network and it is depending on the distributed elements numbers and chosed lumped elements by the designer.

Numbers of the components and the equalizer network type of components can be determined. Also, $f(p)$ will be calculated with degree of n because of those selections. Then, the cofficients of $h(p)$ can be initialized and $g(p)$ can be calculated via (4.2). Polynomials' calculations can be used to calculate the value of scattering parameters in the equation (4.1).

Transducer power gain can be shown as below :

$$TPG(w) = \frac{(1-|S_{in}|^2)|S_{21}|^2(1-|S_L|^2)}{|1-S_{11}S_{in}|^2|1-S_{out}S_L|^2} \tag{4.3}$$

S_{in} represent of input reflection coefficient and it is terminated Z_L. S_L shows the load reflection coefficient. S_{out} represents the output reflection cofficient and also it is terminated Z_{in}.S_{22} shows the reflection coeffient of port 2 and S_{11} shows the reflection coeffient of port 1.

For the calculations of the transducer power gain, the real coefficients are needed to be initialized (Şengül and Çakmak,2018). Then, $f(p)$'s polynomial form should be selected and the degree have to equal or less than $g(p)$. After that, calculating $gg*$ via $ff*$+$hh*$ and finding roots of $G(p) = gg*$. With the known parameters of $f(p)$ and $h(p)$ polynomials, the scattering parameters can be calculated via (4.1) and reflection coefficients can be calculated to find TPG in the equation (4.3).

4.2 Parametric Representation of Brune Functions

Brune functions which the method is proposed for single matching problems are develop by Fettweis (Fettweis, 1979) and it is depended the parametric representation of the positive real impedance $Z_{out}(p)$ of a lossless network. Z_{out} is the positive real impedance. And it is identification of impedance while looking from the 2.port to the generator. It can be solved in a partial fraction expansion. Furthermore, the parameters can be used to identify the poles of Z_{out} to optimizate gain performance of the system for matching load network.

Z_{out} represents a positive real function and it has simple poles. It can be assumed that it shows minimum reactance function so it is identified from its even part. That means, Z_{out} equals $Z_{out}(p) = odd(p) + even(p)$ and e(p) equals $R_{out}(p)$. The equal equation shows that $R_{out}(p)$ is Hilbert transformation of the $Z_{out}(p)$ (Şengül and Çakmak,2018).

The positive real impedance function $Z_{out}(p)$ as shown below :

$$Z_{out} = C_0 + \sum_{i=1}^{k} \frac{C_i}{p - p_i} \tag{4.4}$$

C_0 represents real constant. Complex constant is p which is the distinct poles of Z_{out} with $Re(pi) < 0$.

$$Z_{out}(Even) = \frac{Z_{out}(p) + Z_{out}(p)}{2} = \frac{f(p)f(-p)}{n(p)n(-p)} \tag{4.5}$$

$f(p)$ and $Z_{out}(p)$ represent real polynomial and $n(p)$ is the Hurtwitz denominator of these polynomials. In lossless reciprocal two-ports, it can be odd or even polynomial. Whether $Z_{out}$is a minimum reactance function, the poles are located in the left half of the complex p plane.

$d(p)$ which is the Hurtwitz denominator polynomial can be shown as below and D_k represents non-zero conctant :

$$d(p) = D_k \prod_{i=1}^{k} (p - p_i) \tag{4.6}$$

If combine the equations (4.5) and (4.6), getting the below formula :

$$C_0 + \sum_{i=1}^{k} \frac{C_i}{p - p_i} = \frac{f(p)f(-p)}{n(p)n(-p)} \tag{4.7}$$

If combine the equations (4.6) and (4.7), getting the below formula,

$$C_i = -\frac{f(p_i)f(-p_i)}{p_1 D_k^2 \prod_{i=1}^{k} (p_i^2 - p_1^2)} \tag{4.8}$$

When $f(p)$ polynomial's degree is smaller than k, C_0 equals to 0. When $f(p)$ polynomial's degree equals to k, C_0 equals $\frac{1}{D_k^2}$. In the equation (4.28), Z_{out} can be shown depends on $f(p)$ and $d(p)$.

The monic polynomial can be represented as below :

$$f(p) = p^{k_1} \sum_{i=0}^{k_2} b_i p^{2i} \tag{4.9}$$

If the k_2 and k_1 show nonnegative integers, b_i will be equaled an arbitrary real coefficients. When $f(p)$ polynomial's zeros are located on the real frequency axis of the p plane, and $f(p)$ can be represented as:

$$f(p) = p^{k_1} \prod_{i=0}^{k_2} (p^2 - a_i^2) \tag{4.10}$$

When the number of poles equals odd, the poles should be chosen real. Otherwise, the number of poles are even, the poles can be thought as conjugate pairs.

As mentioned above, the output impedance parameters can be shown as :

$$R_{out} = -\sum_{i=1}^{k} C_0 + \frac{C_i p_i}{w^2 + p_i^2} \tag{4.11}$$

$$X_{out}(w) = -w \sum_{i=1}^{k} \frac{C_i}{w^2 + p_i^2} \tag{4.12}$$

4.3 Line Segment Technique for a Single Matching Problem

The network has a resistance at input port and a complex load at output port (Şengül and Çakmak,2018). To calculate a transducer power gain, the equalizer network have to be calculated.

Z_L represents the load impedance and Z_{out} represents output impedance as shown below :

$$Z_L(jw) = R_L(w) + jX_L \tag{4.13}$$

$$Z_{out}(jw) = R_{out} + jX_{out} \tag{4.14}$$

$$S_{out} = \frac{Z_{out}(jw) - Z_L(jw)}{Z_{out}(jw) - Z_L(jw)} \tag{4.15}$$

$$TPG(w) = 1 - |S_{out}|^2 \tag{4.16}$$

TPG can be obtained with the imaginary and real parts of load $Z_L(jw)$ and $Z_{out}(jw)$ is the output impedance. And transducer power gain can be written as ;

$$TPG(w) = \frac{4R_{int}R_{in}}{(R_{int} + R_{in})^2 + (X_{int} + X_{in})^2} = \frac{4R_{out}R_L}{(R_{out} + R_L)^2 + (X_{out} + X_L)^2} \tag{4.17}$$

R_{out} and X_{out} are parameters of output impedances and they can be calculated for maximum transducer power gain. The real frequency approach (Carlin and Yarman, 1983) can be showed to get these Z_{out} value.

Z_{out} has the unknown real parts and it represented a number of line segments R_{out} (Carlin, 1977).

$$R_{out} = k_0 + \sum_{j=1}^{n} b_j(w)k_J \tag{4.18}$$

$b_j(w)$ represents identification in R_{out} via to the sampling frequency $(w_j, j = 1, 2, 3, 4, ...n)$. Output impedance has the imaginary part and it can be calculated with Hilbert transformation (Carlin, 1977). X_{out} can be identified using the same line segments representation as shown below :

$$X_{out} = \sum_{j=1}^{n} c_j(w)k_J \tag{4.19}$$

And then $c_j(w)$ can be calculated via Hilberts transformation technique as show below :

$$c_j(w) = \frac{1}{\pi(w_j - w_{j-1})}I(w) \tag{4.20}$$

I(w) can be calculated as below :

$$I(w) = \int_{w_{j-1}}^{w_j} In|\frac{y+w}{y-w}|dy \tag{4.21}$$

The tranducer power gain equation (4.17) can be found after the calculation unknown output impedance. To minimize the difference between the actual power gain and the target, the least square method can be used as below :

$$E = \sum_{j=1}^{N_w} (T(w_j, k_j) - T_d)^2 \tag{4.22}$$

T_d represents the target value of TPG. E equals the difference of the actual and desired one. Number of sampling frequency representing by N_w.

4.4 Direct Computational Technique for Double Matching Problems

Direct computational technique is developed by Carlin and Yarman (Carlin and Yarman, 1983). It can be used for solving double matching problems. The method includes that the real part is simplified a real even rational function with the unknown coefficients to optimize the characteristic of the gain over a specified passband.

The technique is included the output impedance that is unknown parameter. Transducer power gain will be showed and S_{in} represents the complex normalized input reflection coefficient and it can be used for transducer power gain. As follows below, S_{int} can be seen and Z_{in} represents generator impedance at port one, Z_{int} shows the input impedance, Z_L represents the load impedance and Z_{out} represents the output impedance in the network.

$$S_{int} = \frac{Z_{int} - Z_{in}}{Z_{int} + Z_{in}} \tag{4.23}$$

S_{in} which represents the reflection coefficient of the generator can be shown as :

$$S_{in} = \frac{Z_{in} - 1}{Z_{in} + 1} \tag{4.24}$$

S_{int} which represents the reflection coefficient of the port one can be shown as :

$$S_{int} = \frac{Z_{int} - 1}{Z_{int} + 1} \tag{4.25}$$

TPG can be shown below in terms of S_{in} and S_{int} :

$$TPG(w) = \frac{(1 - |S_{in}|^2)(1 - |S_{int}|^2)}{|1 - S_{in}S_{int}|^2} \tag{4.26}$$

The aim is that to identify S_{in} in terms of a function of the impedance Z_2.Also, S_{int} can be identified in terms of the scattering parameters as below :

$$S_{int} = S_{11} + \frac{S_{12}^2 S_L}{1 - S_{22}S_L} = \frac{-S_L Q + S_{11}}{1 - S_L S_{22}} \tag{4.27}$$

where $Q = S_{11}S_{22} - S_{21}^2$. And the below equation can be found :

$$Q = \frac{S_{22}}{S_{11}} = \frac{S_{12}}{S_{21}} \tag{4.28}$$

Combine the equations, S_{int} can be represented as :

$$S_{int} = \frac{S_{12}(S_L S_{22}*)}{S_{21} * (1 - S_L S_{22})} \qquad S_{22} = \frac{Z_{out} - 1}{Z_{out} + 1} \qquad Z_{out} = \frac{n}{d} \tag{4.29}$$

Thanks to the equation (4.29), the even part of the Z_{out} can be seen as :

$$ev(Z_{out}) = \frac{1}{2}(Z_{out} + Z_{out}*) = \frac{n}{d}\frac{n*}{d*} = HH* \qquad H = \frac{f*}{d} \tag{4.30}$$

As mentioned above, S_{int} can be shown as :

$$S_{int} = \frac{H}{H*} = \frac{Z_L - Z_{out}*}{Z_L + Z_{out}} \tag{4.31}$$

And below equation means that Z_{out} impact to calculate the ratio of TPG with Z_L and Z_{in}.

5. BROADBAND MATCHING NETWORK DESIGN VIA EXPLICIT SOLUTIONS OF TWO VARIABLE SCATTERING EQUATIONS

In this chapter, we will present broadband double matching network design.

5.1 Broadband Double Matching Network Design

In this example, LPLU network which has four elements is employed to match the load and generator impedances. For this LPLU network, $n_p = 2, n_\lambda = 2$ and the frequency band is $1 \geq w \geq 0$. The broadband double matching network which improved (Aksen and Yarman,2001) will be compared with the low-pass mixed element broadband matching network (two lumped and two unit elements) has been designed by using the equations given the third part in the thesis.

τ represents the delay length which is chosen as the unknown coefficients and also the coefficicients (h_{00},h_{01},h_{02},h_{10},h_{20}) are chosen as the unknown coefficients. μ_2 is the constat for defining as -1. Another constant μ_1 will be obtained at the end of the optimization process by using the sign of h_{20}. If h_{20} is negative, μ_1 equals -1 and if h_{20} equals positive, μ_1 equals $+1$. The unknown coefficients of λ_h and λ_g matrices can be calculated by means of the explicit equations given the third part in the thesis.

The purpose is that trying an efficient network for transfer power via equalized network. And the network has a parallel inductor and a load capacitor. The normalized generator and load impedance datas are given below :

Table 5.1 Normalized Generator and Load Impedance Data

w	RG	XG	RL	XL
0.0	1.0000	0.0000	1.0000	0.0000
0.1	1.0000	0.1000	0.9901	0.1010
0.2	1.0000	0.2000	0.9615	0.2077
0.3	1.0000	0.3000	0.9174	0.3248
0.4	1.0000	0.4000	0.8621	0.4552
0.5	1.0000	0.5000	0.8000	0.6000
0.6	1.0000	0.6000	0.7353	0.7588
0.7	1.0000	0.7000	0.6711	0.9302
0.8	1.0000	0.8000	0.6098	1.1122
0.9	1.0000	0.9000	0.5525	1.3028
1.0	1.0000	1.0000	0.5000	1.5000

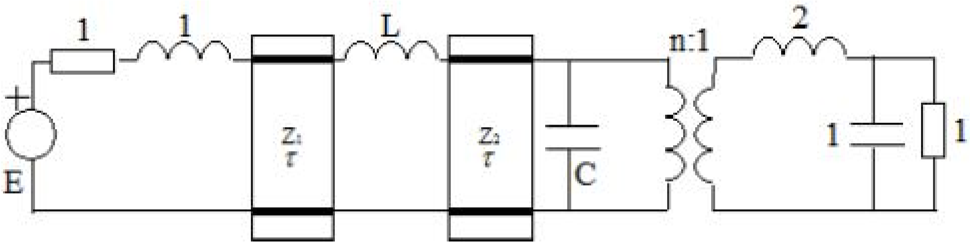

Figure 5.1 Designed mixed-element double matching network

Then completed the optimization process, the coefficient matrices which are showed below that completely describe the scattering parameters of the matching network under consideration are obtained :

$$\Lambda_k = \begin{bmatrix} -0.1076 & -3.4667 & -2.7720 \\ 0.3723 & -5.6308 & -11.6498 \\ 1.1199 & -8.2039 & 0 \end{bmatrix}, \qquad \Lambda_g = \begin{bmatrix} 1.0058 & 4.3988 & 2.9468 \\ 1.6225 & 8.9814 & 11.6498 \\ 1.1199 & 8.2039 & 0 \end{bmatrix} \tag{5.1}$$

The designed network with the normalized element values and the gain performance of the system are shown below :

Proposed Values: $L = 1.7916, C = 1.392, Z_1 = 0.137, Z_2 = 0.701, \tau = 0.2\ n = 0.8982$
Following reference values are taken from the (Aksen and Yarman,2001). Reference Values: $L = 2.126, C = 0.751, Z_1 = 0.161, Z_2 = 0.341, \tau = 0.21$

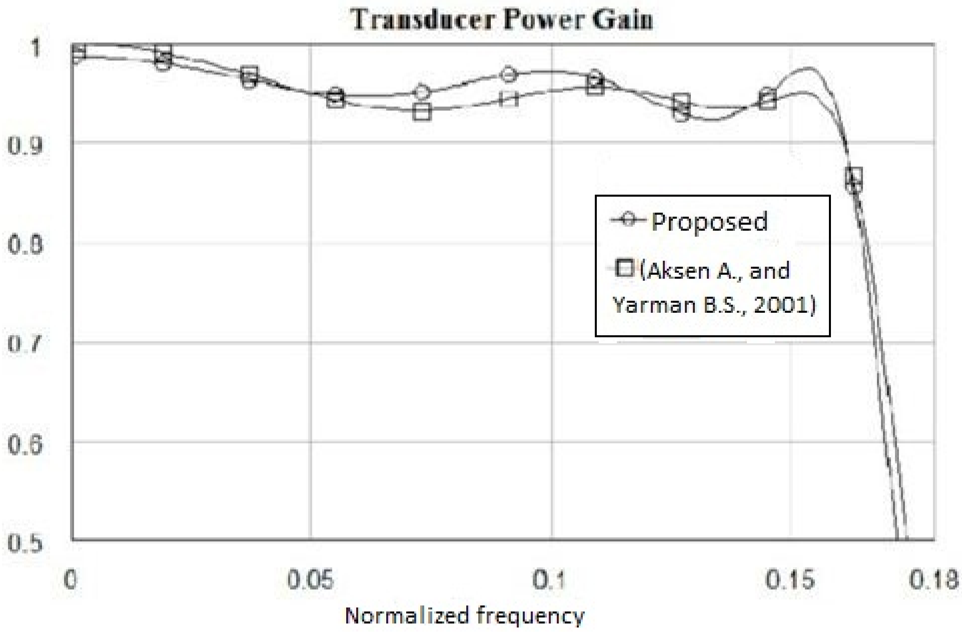

Figure 5.2 Performance of the matched system designed with mixed elements

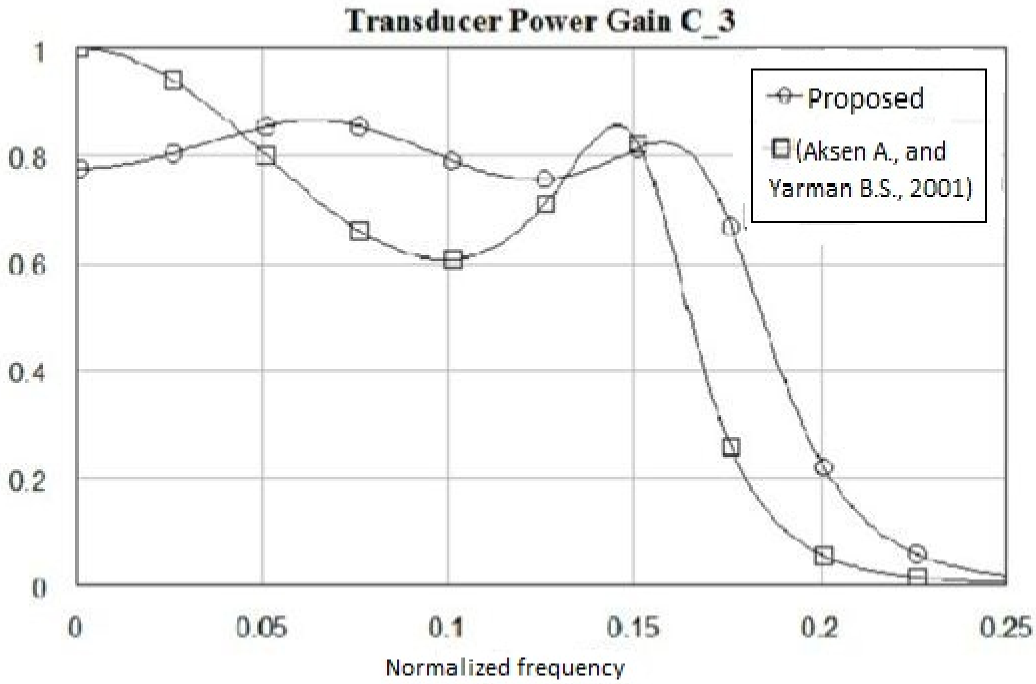

Figure 5.3 Performance of the matched system with $C = 3$

In the design, an ideal transformer is used which simply scales current and voltage. It does not have any inductance or frequency dependency. So DC passes through like all other frequencies. Then ideally there will be a power transfer at DC. But practically a transformer will not transfer any power at DC.

If the normalized capacitor value in the load is increased to 3, then the maximum available flat gain level equals about 0.8 (Fano,1950) (Youla,1964). But the transferred gain at DC will be unity. Then the gain will reduce dramatically to 0.8 levels in the passband as seen in the figure. On the other hand,a more flat transducer power gain curve fluctuating around 0.8 is obtained by means of the derived equations.

There is no transformer in (Aksen and Yarman,2001) (h_{00} is restricted and h_{00} equals 0),the low pass network is designed and the generator and load resistors are equal, the transferred gain is unity at DC. After that, the gain level reduces to approximately 0.95 (Fano,1950) (Youla,1964). For the maximum available gain for the selected load is ideally close to unity,this gain drop is not noticeable.

6. CONCLUSIONS

Mixed element networks are included different elements. The network which has mixed element structures can be defined by two variables which one of them is lumped element and the other is distributed element and were assumed as indepented variables. In this thesis, the complete and explicit equations are derived for lossless low-pass mixed-element topologies, and by using the equations solved without any restrictions, a broadband matching network design was made.

Explicit design equations have been solved up to four elements for LPLU without any restrictions. In case of five-element, it is obtained that the first row and column coefficients of the two-variable polynomial g, explicit equations are found for the unknown coefficients of Λ_g and Λ_h matrices without any restrictions. Then, the broadband matching network design was created. The results were compared with the results in the literature.

The utilization of the given explicit equations is demonstrated via a broadband double matching example. It is expected that the proposed equations will be used to design two-variable networks such as broadband matching networks, microwave amplifiers.

APPENDIX A: Matlab Codes

A.1 Matlab Codes for Main Program

```
clc
tic
clear
syms L fr
global m2 dist lump w SG SL T0 ZG ZL f_p mu

%*****source and load impedance, souce and
%load relection coefficient calculation*****
w=0:0.1:1;
z=i.*fr.*2+(1/(1+i*fr*1));
ZL=subs(z,fr,w);
r11=1;
r33=i.*fr.*1;
z11=r11+r33;
z11=simple(z11);
ZG=subs(z11,fr,w);
% ZG=ones(1,length(w));
SG=(ZG-1)./(ZG+1);
SL=(ZL-1)./(ZL+1);

%*****************************************************
```

```
%*****Initial values*******************************************
h0i=[1 -1 1]; %dist
hj0=[-1 1]; %lumped
T0=0.99; %gain
thau=0.2; %delay
%*******************************************************

%*****Optimisation vector construction***************************
dist=length(h0i)-1;
lump=length(hj0);
dimension=dist+lump;

if dist==lump
    m2=input('Enter m2 value (+1/-1):'); % ORNEK GIR
end

for a=1:dist+1;
   v(a)=h0i(a);
end
for a=1:lump;
   v(dist+1+a)=hj0(a);
end
v(dimension+2)=thau;

%*****************************************************
f_p=(1-L^2)^(dist/2);
mu=1;

%*****optimisation part************************************
const=length(v);
LB=[ones(1,const-1).*(-Inf) 0.2];
```

```
UB=ones(1,const).*Inf;

OPTIONS=optimset('MaxFunEvals',1000,'MaxIter',2500,'TolCon',1e-32,
'TolX',1e-32,'TolFun',1e-32);
v_new=fmincon(@error_srft,v,[],[],[],[],LB,UB,[],OPTIONS);
%*****************************************************

%*****Gettin h0i, hj0 and thau after optimisation****************
for a=1:dist+1;
    h0i(a)=v_new(a);
end;
h0i;

for a=1:lump;
    hj0(a)=v_new(dist+1+a);
end;
hj0(lump+1)=h0i(length(h0i));
hj0;

thau=v_new(dist+lump+2);

if hj0(1)>0
     m1=1;
else
     m1=-1;
end
%****************************************************

%*****Calculation of optimised h and g matrices*****************
if dist==1 & lump==1;
    [Ah,Ag]=hg11n(h0i,hj0,m1,m2,f_p);
```

```
elseif dist==1 & lump==2;
   m2=m1;
    [Ah,Ag]=hg12n(h0i,hj0,m1,m2,f_p);
elseif dist==2 & lump==1;
     if m1==1
          m2=-1;
     else
          m2=1;
     end
    [Ah,Ag]=hg21n(h0i,hj0,m1,m2,f_p);
elseif dist==2 & lump==2;
    [Ah,Ag]=hg22n(h0i,hj0,m1,m2,f_p);
elseif dist==2  & lump==3;
     m2=m1;
    [Ah,Ag]=hg23n(h0i,hj0,m1,m2,f_p);
end

Ah
Ag
thau
%***********************************************************

%****Calculation of load and source impedances,
% source and load reflection coefficients over new frequency range**
w=0:0.01:2;
ZL=subs(z,fr,w);
ZG=subs(z11,fr,w);
% ZG=ones(1,length(w));
SG=(ZG-1)./(ZG+1);
SL=(ZL-1)./(ZL+1);
```

```
%************************************************************

%*****According to optimised h and g matrices, getting the values
% of h, hpara, g, gpara and f*****
for a=1:length(w);
    d=i*tan(w(a)*thau);
    hv(a)=0;
    for b=1:lump+1;
        for c=1:dist+1;
            hv(a)=hv(a)+Ah(b,c)*((i*w(a))^(b-1))*(d)^(c-1);
        end
    end
end

for a=1:length(w);
    d=-i*tan(w(a)*thau);
    hpv(a)=0;
    for b=1:lump+1;
        for c=1:dist+1;
            hpv(a)=hpv(a)+Ah(b,c)*((-i*w(a))^(b-1))*(d)^(c-1);
        end
    end
end

for a=1:length(w);
    d=i*tan(w(a)*thau);
    gv(a)=0;
    for b=1:lump+1;
        for c=1:dist+1;
            gv(a)=gv(a)+Ag(b,c)*((i*w(a))^(b-1))*(d)^(c-1);
        end
```

```
    end
end

for  a=1:length(w);
    d=-i*tan(w(a)*thau);
    gpv(a)=0;
    for  b=1:lump+1;
        for  c=1:dist+1;
            gpv(a)=gpv(a)+Ag(b,c)*((-i*w(a))^(b-1))*(d)^(c-1);
        end
    end
end

%**********************************************************
fv=subs(f_p ,L,i.*tan(w.*thau));
fpv=conj(fv);

%*****Calculation  of  tpg  over  the  frequency  range****************
S22=-mu.*hpv./gv;
S12=mu.*fpv./gv;
S21=fv./gv;
S11=hv./gv;

SL=(ZL-1)./(ZL+1);
S1=S11+(S12.*S21.*SL)./(1-S22.*SL);
Z11=(1+S1)./(1-S1);
r1=(Z11-conj(ZG))./(Z11+ZG);
SG=(ZG-1)./(ZG+1);
S2=S22+(S12.*S21.*SG)./(1-S11.*SG);
Z22=(1+S2)./(1-S2);
```

```
tpg=(4.*real(ZL).*real(Z22))./((real(ZL)+real(Z22)).^2+(imag(ZL)
+imag(Z22)).^2);
%*****************************************************

%*****Plotting the result*****
hold on
renk=(round(rand(3,1)))';
div=(round(rand(3,1)))'+1;
color=[renk(1)/div(1) renk(2)/div(2) renk(3)/div(3)];
plot(w,T0,'r',w,tpg,'color',color)
axis([0 2 0 1])
%******************************************************
toc
```

A.2 Matlab Codes for Error Calculation

```
function eps=error_srft(v)

syms L fr

global m2 dist lump w SG SL T0 ZG ZL f_p mu

%*****error sub-program*****

%*****calculation of h0i, hj0 and thau from optimisation vector*****
dimension=dist+lump;

for a=1:dist+1;
    h0i(a)=v(a);
end

for a=1:lump;
    hj0(a)=v(dist+1+a);
end
hj0(lump+1)=h0i(length(h0i));

thau=v(dist+lump+2);
%*********************************************************
if hj0(1)>0
     m1=1;
else
     m1=-1;
end
%*****Calculation of h and g matrices*****************************
```

```
if dist==1 & lump==1;
    [Ah,Ag]=hg11n(h0i,hj0,m1,m2,f_p);
elseif dist==1 & lump==2;
    m2=m1;
    [Ah,Ag]=hg12n(h0i,hj0,m1,m2,f_p);
elseif dist==2 & lump==1;
     if m1==1
          m2=-1;
     else
          m2=1;
     end
    [Ah,Ag]=hg21n(h0i,hj0,m1,m2,f_p);
elseif dist==2 & lump==2;
    [Ah,Ag]=hg22n(h0i,hj0,m1,m2,f_p);
elseif dist==2  & lump==3;
     m2=m1;
    [Ah,Ag]=hg23n(h0i,hj0,m1,m2,f_p);
end
%****************************************************************

%*****calculation of h, hpara, g, gpara and f values*************
for a=1:length(w);
    d=i*tan(w(a)*thau);
    hv(a)=0;
    for b=1:lump+1;
        for c=1:dist+1;
            hv(a)=hv(a)+Ah(b,c)*((i*w(a))^(b-1))*(d)^(c-1);
        end
    end
end
```

```
for a=1:length(w);
    d=-i*tan(w(a)*thau);
    hpv(a)=0;
    for b=1:lump+1;
        for c=1:dist+1;
            hpv(a)=hpv(a)+Ah(b,c)*((-i*w(a))^(b-1))*(d)^(c-1);
        end
    end
end

for a=1:length(w);
    d=i*tan(w(a)*thau);
    gv(a)=0;
    for b=1:lump+1;
        for c=1:dist+1;
            gv(a)=gv(a)+Ag(b,c)*((i*w(a))^(b-1))*(d)^(c-1);
        end
    end
end

for a=1:length(w);
    d=-i*tan(w(a)*thau);
    gpv(a)=0;
    for b=1:lump+1;
        for c=1:dist+1;
            gpv(a)=gpv(a)+Ag(b,c)*((-i*w(a))^(b-1))*(d)^(c-1);
        end
    end
end

fv=subs(f_p,L,i.*tan(w.*thau));
```

```
fpv=conj(fv);
%*************************************************************

%*****calculation of tpg****************************************
S22=-mu.*hpv./gv;
S12=mu.*fpv./gv;
S21=fv./gv;
S11=hv./gv;

SL=(ZL-1)./(ZL+1);
S1=S11+(S12.*S21.*SL)./(1-S22.*SL);
Z11=(1+S1)./(1-S1);
r1=(Z11-conj(ZG))./(Z11+ZG);
SG=(ZG-1)./(ZG+1);
S2=S22+(S12.*S21.*SG)./(1-S11.*SG);
Z22=(1+S2)./(1-S2);

tpg=(4.*real(ZL).*real(Z22))./((real(ZL)+real(Z22)).^2 +
(imag(ZL)+imag(Z22)).^2);
%*************************************************************

eps=sum(((tpg-T0)./tpg).^2)
%*************************************************************
return
```

A.3 Matlab Codes for Mixed Element Structure Formed with One Lumped Element and One UE

```
function [Ah,Ag]=hg11n(h0i,hj0,m1,m2,f_p);
%*****hg11 sub-program*****

gj0=LLEL(hj0,[zeros(1,length(hj0)-1) 1]); %lumped
g0i=LLELd(h0i,f_p); %dist

h00=h0i(2);
h01=h0i(1);
g00=g0i(2);
g01=g0i(1);
h10=hj0(1);
g10=gj0(1);

g11=(g10*g01-h10*h01)/(g00-m2*h00);
h11=m2*g11;

Ah=[h00 h01;h10 h11];
Ag=[g00 g01;g10 g11];
%*****************************************************************

return
```

A.4 Matlab Codes for Mixed Element Structure Formed with Two Lumped Elements and One UE

```
function [Ah,Ag]=hg12n(h0i,hj0,m1,m2,f_p)
%*****hg12 sub-program*****

gj0=LLEL(hj0,[zeros(1,length(hj0)-1) 1]); %lumped
g0i=LLELd(h0i,f_p); %dist

h00=h0i(2);
h01=h0i(1);
h10=hj0(2);
h20=hj0(1);
g00=g0i(2);
g01=g0i(1);
g10=gj0(2);
g20=gj0(1);
g11=(g10*g01-h10*h01)/(g00-m2*h00);
h11=m2*g11;
h21=0;
g21=0;

Ah=[h00 h01;h10 h11;h20 h21];
Ag=[g00 g01;g10 g11;g20 g21];
%***************************************************************

return
```

A.5 Matlab Codes for Mixed Element Structure Formed with One Lumped Element and Two UEs

```
function [Ah,Ag]=hg21(h0i,hj0,m1,m2,f_p)
%*****hg21 sub-program*****

gj0=LLEL(hj0,[zeros(1,length(hj0)-1) 1]); %lumped
g0i=LLELd(h0i,f_p); %dist

h00=h0i(3);
h01=h0i(2);
h02=h0i(1);
h10=hj0(1);
g00=g0i(3);
g01=g0i(2);
g02=g0i(1);
g10=gj0(1);
alfa=g01-m2*h01;
beta=g10-m2*h10;
g11=2*g02*beta/alfa;
h11=2*h02*beta/alfa;
g12=(g11*g02-h11*h02)/alfa;
h12=m2*g12;

Ah=[h00 h01 h02;h10 h11 h12];
Ag=[g00 g01 g02;g10 g11 g12];
%****************************************************************

return
```

A.6 Matlab Codes for Mixed Element Structure Formed with Two Lumped Elements and Two UEs

```
function [Ah,Ag]=hg22n(h0i,hj0,m1,m2,f_p)
%*****hg22 sub-program*****

h00=h0i(3);
h01=h0i(2);
h02=h0i(1);
h10=hj0(2);
h20=hj0(1);

g00=sqrt(1+h00^2);
g02=sqrt(1+h02^2);
g01=sqrt(2+h01^2+2*(g00*g02-h00*h02));
g20=abs(h20);
g10=sqrt(h10^2+2*(g00*g20-h00*h20));
gama=g01*g10-h01*h10;
alfa=g01-m2*h01;
beta=g10-m2*h10;
h11=(h20*alfa/beta+h02*beta/alfa-(h00*g00)*(g20*alfa/beta+
g02*beta/alfa)+gama*h00/g00^2)/(1-(h00^2/g00^2));
g11=(gama+h00*h11)/g00;
g21=(g11*g20-h11*h20)/beta;
h21=m2*g21;
g12=(g11*g02-h11*h02)/alfa;
h12=m2*g12;
h22=0;
g22=0;
```

```
Ah=[h00 h01 h02;h10 h11 h12;h20 h21 h22];
Ag=[g00 g01 g02;g10 g11 g12;g20 g21 g22];

return
```

A.7 Matlab Codes for Mixed Element Structure Formed with Three Lumped Elements and Two UEs

```
function [Ah,Ag]=hg23n(h0i,hj0,m1,m2,f_p)
%*****hg23 sub-program*****

gj0=LLEL(hj0,[zeros(1,length(hj0)-1) 1]); %lumped
g0i=LLELd(h0i,f_p); %dist

h00=h0i(3);
h01=h0i(2);
h02=h0i(1);
h10=hj0(3);
h20=hj0(2);
h30=hj0(1);
g00=g0i(3);
g01=g0i(2);
g02=g0i(1);
g10=gj0(3);
g20=gj0(2);
g30=gj0(1);
gama=g10*g01-h10*h01;
alfa=g01-m2*h01;
beta=g10-m2*h10;
h11=(h20*alfa/beta+h02*beta/alfa-(h00/g00)*(g20*alfa/beta+
g02*beta/alfa)+h00*gama/g00^2)/(1-h00^2/g00^2);
g11=(gama+h00*h11)/g00;
g12=(1/alfa)*(g11*g02-h11*h02);
h12=m2*g12;
g21=(1/beta)*(g11*g20-h11*h20-g01*g30+h01*h30);
```

```
h21=m2*g21;
h22=0;
g22=0;
h31=0;
g31=0;
h32=0;
g32=0;

Ah=[h00 h01 h02;h10 h11 h12;h20 h21 h22;h30 h31 h32];
Ag=[g00 g01 g02;g10 g11 g12;g20 g21 g22;g30 g31 g32];
%*****************************************************************

return
```

CURRICULUM VITAE

Personal Information

Name Surname	: Göker Eker
Birthplace and Birthday	: Istanbul 20.06.1991

Education

Bachelor's Degree	: B.Sc. Electronics Engineering-2014 Kadir Has University
Master's Degree	: M.Sc. Electronics Engineering-2019 Kadir Has University
Languages	: English (Fluent)

Contact

Mobile phone	:+905368727645
E-mail	: eker.goker@gmail.com

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