Research Article

Sabri Gokmen*

Stripped and layered fabrication of minimal surface tectonics using parametric algorithms

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Abstract: This article describes a parametric design and fabrication workflow influenced by Frei Otto's form-finding experiments on soap films. The research investigates minimal surface geometry by combining physical and digital experiments in a computational framework. Operating on mesh topology, various parametric design tools and plug-ins in Rhinoceros/Grasshopper are presented to discuss the translation of minimal surfaces to flat strips suitable for planar fabrication using flexible materials. These tools are tested on a case study to show the automated design and manufacture of double-curved surfaces as double-layered strips running in perpendicular directions that can be affixed at point connections for structural stability. The development of the parametric workflow, material constraints, and stripped fabrication of layers are discussed.

Keywords: form-finding, Frei Otto, minimal surface, digital fabrication, parametric modelling

1 Introduction

Since the 1990s, architectural practice has been transforming rapidly with the development of digital technologies that introduced computational workflows for advanced modelling and manufacture of parametric surfaces [1]. A key subject in this field is the use of parametric tools that can combine design, optimization, and fabrication of non-standard geometry through variables, principles, and constraints [2,3]. These systems are often driven by various performance-based criteria ranging between material properties, structural behaviour, geometric optimization, or fabrication constraints [4]. As designers are tasked with developing optimal solutions for various structural, formal, or geometric problems, the development of computational workflows that can automate design and manufacture is becoming an area of growing interest [5].

Current research for the design and manufacture of parametric surfaces focuses on the physical production and transformation of materials that establish a feedback mechanism between design and fabrication [6]. This notion of digital fabrication is tasked to develop techniques and tools for the translation of digital geometric information onto materials that can integrate numerous challenges for manufacturing. For instance, if a parametric surface is defined by a boundary condition or topology, then the subdivision and geometric description of its constituent parts can be interpreted according to various fabrication protocols or material constraints [7,8]. The description of this workflow requires parametric understanding of computational geometry for seamless digital fabrication that can automate decision-making during design development.

As mathematical descriptions of topology, minimal surfaces offer a novel connection between parametric modelling and the physical manufacture of forms [9]. Since minimal surfaces present zero mean curvature in all directions, they offer suitable structural candidate forms for stable architectural structures [10]. This key physical property leads one of the pioneering architects of the 20th century, Frei Otto, to research novel minimal surface forms using soap film studies that are documented and translated into tensile systems [11]. In his method, Otto used analogue computation using architectural models at different scales that can be used to translate experimental forms found in soap films onto other materials and structural systems. This translation became a point of interest for contemporary parametric applications on minimal surfaces, where the digital modelling and fabrication of curved and layered structures are investigated in diverse materials ranging from fabric and plastic to concrete formwork [6,8,12].

This article will present a parametric workflow and case study to discuss an integrated approach for the design, optimization, and manufacture of minimal surfaces as a double-layered structure. The development of parametric tools for the physics-based modelling of minimal surfaces and their rationalization for digital fabrication will be

* **Corresponding author: Sabri Gokmen,** Department of Architecture, Kadir Has University, Istanbul, Turkey, e-mail: sabri.gokmen@khas.edu.tr

presented using sample forms [13]. As a case study, an installation project titled "Occa" will be presented as a parametric application of minimal surfaces constructed from double-layered strips. The novelty of this study is the automated parametric translation of minimal surface mesh geometry as double-layered curved strips that can be manufactured out of planar flexible materials [6,8,12,14]. Key aspects of the parametric modelling of minimal surfaces and their transformation under digital fabrication and material constraints are discussed with an emphasis on the performance and construction of curved and layered structures [15,16].

2 Form-finding in computational architecture

Recent advancements in architecture and engineering show extensive interest in the use and development of computational tools for the design, optimization, and manufacture of complex structural systems [2,17]. An emerging strategy among computational designers is the integration of diverse parametric tools in a dynamic, process-oriented workflow for the automated production of a range of possibilities for given problems [7]. More recently, designers began addressing the need for the implementation of custom parametric design software to provide solutions for multiple design criteria, optimization protocols, material behaviour, and structural constraints [3,16]. An area of focus among these topics is "form-finding" strategies that can be explored through various computational workflows, emergent structural systems, and simulation-based constraint solvers [12,14,18]. To achieve this task, architects commonly prefer Rhinoceros and Grasshopper tools, while a physics-based particle solver – Kangaroo – is extensively used for parametric form-finding studies [19].

In architectural design, "form-finding" signifies the use of a material/mechanical or digital/computational system to develop the design under various physical, material, or performance-oriented constraints [3,20]. Historically, this method has been attributed to the innovative structural models developed by Gaudi, where the structural configuration of a building can be built as a network of hanging chains in tension that can be directly translated into a compression model built out of vaults following Hooke's law of inversion [21]. During the 20th century, this analogue technique has been expanded by Frei Otto's innovative physical models where the mechanical behaviour of a structural system can be tested using scaled models [22]. Today, these form-finding methods are replicated using digital tools for the design exploration and behaviour analysis of architectural structures [18].

A common field of interest for form-finding in architecture is explored on vaults and tensegrity structures. Computational exploration of compression-only vault forms acquired *via* the multi-body rope approach can be simultaneously tested through thrust network analysis (TNA) for structural validation [20]. Parametric exploration of tensegrity structures also shows alternative ways to combine structural members working in compression and tension. Palmieri *et al.* showed the development of a tensile-integrity structural system using arches, cables, and membranes that can be parametrically explored using T3 tensegrity modules [23]. These emerging computational processes require digitally integrated workflows combining multiple designs and analysis tools that are used to simulate structural behaviour and simultaneously give feedback on design configuration.

In architecture, form-finding experiments have been associated with the generative modelling of "minimal surfaces" due to their differential geometry, novel mathematical description, and generative possibilities [9,24]. One of the early experimental uses of minimal surfaces can be found in the work of Frei Otto, who developed soap film studies for form-finding experiments to develop strategies for the geometric design of lightweight, tensile, and cable-net structures [11,25,26]. These structures are often translated into grid systems that are formed of triangles or squares, to reduce material cost, waste, and labour. As new structural systems emerge, the building components in square and triangular grid systems become more industrialized, prefabricated, and modulated to accommodate ease of construction and design [27]. Today, non-standard production of minimal surfaces can be found in contemporary digital and bespoke productions with examples of fabric-based haptic pavilion structures [7], digital fabrication of cable-net and knitted form-work for concrete shells [6], computation of concrete shell bridge structures [28], and shell structures for architectural vaults [8].

Contemporary studies on form-finding design research present novel digital workflows where multiple parametric tools are integrated for the computational analysis and further design development of structures [4,5,15]. Fenu *et al.* revisited Musmeci and Nervi's historical structures, particularly concrete shell bridges to develop form-finding strategies combining particle-spring system and TNA implementing both vertical and horizontal forces to analyse structural behaviour under seismic loads [28]. Another interest in this area focuses on combining computer-aided design and computer-aided engineering methods inside an integrated parametric framework for design development. In this domain, Basic

Analysis of Taut Structures presents a custom plug-in development to automate the calculation of taut structures and funicular shells inside Grasshopper to enable a parametric definition of boundary conditions for form-finding studies [5]. Structural optimization of free-form and multi-layered architectural envelopes feature complex computational workflows and often use mesh geometry to develop parametric configurations of beams and tension-free nodes [15].

A physical component of form-finding research is its applicability to materiality and the determination of digital fabrication tools and processes for the manufacture of parametric forms [6,8]. These often require developable surface strategies that can be manufactured out of planar elements or three-dimensional modular production, the former offering a cheaper solution, while the latter requiring custom formwork [4]. A surface-based strategy combining parametric, mathematical, and physical investigation into the characteristics of origami structures shows the advantage of rule-based principles using scissor units for deployment strategies [29]. Menges explores the limits and capabilities of robotic manufacturing that transform digital models into flexible materials using woven carbon fibre and wood [3]. With ongoing research in this domain, form-finding strategies are being integrated with digital fabrication tools to automatically transform digitally acquired geometries onto physical materials where curvature, flexibility, and structural performance are considered.

3 Frei-Otto and form-finding using soap films

Frei Otto was one of the pioneering architects of the 20th century, often recognized for his innovative form-finding approach towards the design of tensile structures [26]. A Pritzker winner architect, he founded the Institute for Lightweight Structures (IL) in Stuttgart, Germany, in 1964, where he extended his innovative practice and research on natural forms and material systems. In his research, Otto used physical form-finding experiments to influence the construction of scaled physical models that can be built as cable-net, membrane and lightweight structures [25]. This analogue methodology offered a precursor to modern computation, where physical systems were translated onto large-scale material structures using experiments, photographs, and scaled models.

Otto's collaboration with tentmaker Peter Stromeyer during the 1950s and 1960s resulted in the development of various form-finding experiments to establish novel strategies for the design and construction of tensile structures [11]. One of these methods used soap films to mimic the physical behaviour of a pre-stressed and tensioned surface geometry to find conditions of equilibrium. As a dynamic medium, soap film uses a rationed mixture of soap, glycerin, sugar, and water to produce a highly tensile and durable surface in between various boundary conditions to observe the structural behaviour of the surface. The resulting "minimal" surfaces would exhibit equal tensile forces running in all directions, presenting optimal structural solutions for a given boundary configuration [9,14].

Otto used minimal surface strategies throughout his career on various structures such as German Pavilion in Expo (1967), Munich Stadium (1972), and Mannheim Multihalle (1974). In Mannheim, he developed a continuous double-layered thin wooden lattice system that can be adjusted to the double-curved surface geometry. This "grid shell" structure was formed out of 50 cm × 50 cm size squares that were interlocked using pin joints enabling the deformation of square grid cells into rhombi [11]. The initial design was first developed as an inverted hanging chain model on a 1:100 scale that was later photographed and processed to acquire the resting coordinates for grid nodes. This information enabled further development of architectural drawings and calculation of the structural geometry, albeit no mathematical accuracy. This aspect required the physical structure to be lifted and positioned *in situ* where the pin joints were affixed at the calculated positions to stabilize the gridshell.

Despite having a lack of computational rigour, Otto's revolutionary design approach enabled the use of model building and experimentation as a strategy to document and translate geometry onto material systems. In his practice, he was a follower of Hooke's law of inversion, where a scaled hanging chain model acting in tension can be translated into a rigid concrete arch working in compression [21]. This required the speculative use of material systems, like soap films, as a means of finding suitable forms of design, but a further inquiry into the resulting geometry leads to the deterministic calculation of lightweight structures in a similar equilibrium [26]. Thus, his contribution provides two key lessons for the form-finding strategy in construction: (i) a found form through experimentation can be directly translated as a geometry input for design (analogue computation) and (ii) a physical configuration in equilibrium, such as minimal surfaces in soap film, can be a guide for another material medium at a different scale, such as tensile structures made of steel cables and membranes [25]. Today, Otto's methodology can be replicated using various computational algorithms where a combination of abstract mathematical and geometrical models can be used for digital form-finding experiments.

4 Minimal surface tectonics

In mathematics, minimal surfaces are surfaces that present minimum surface area for a given boundary condition. These surfaces also present zero mean curvature among all sampled surface points [30]. A common physical example of the minimal surface is visualized using soap film where the surface boundary is restricted by a wire dipped into the solution produced by a mixture of soap, water, sugar, and glycerin [31]. In this medium, the buoyancy of the film rests in a temporary state where both compressive and tensile properties of the solution appear to be balanced.

Classical examples of minimal surfaces include the plane, the catenoid, and the helicoid. While the catenoid and the plane are the only minimal surfaces of revolution, the helicoid and catenoid are conjugated as their coordinate functions are harmonically related [10]. First discovered by Euler in 1741, the catenoid has a minimal surface with finite topology and two fixed circular ends [32]. This mathematical analysis was further simplified by Lagrange. While the minimization and analytic solution of surface area under a fixed boundary have historically been challenging for mathematicians, a few surface formulations have been developed such as the Enneper surface, Scherk's surface, and Costa's minimal surface. In 1970, Schoen discovered the "gyroid," a triply periodic minimal surface that also arises naturally in biology and material science due to its high surface area [33]. Minimal surfaces have since become a point of interest for the chemical composition of polymers and self-assembly of complex materials, due to the spatial infilling capacity while exhibiting symmetrical properties that are also observed in cellular structures and crystallography [34].

For continuous treatment of minimal surfaces, the soap film problem defines the minimal surface area that can be derived using differential equations. Schwarz provided the analytical solution for continuous minimum surfaces that have continuous derivatives for minimal surfaces bounded by skew quadrilaterals [35]. For minimal surfaces that are not analytically solved, Pinkall offered a method of triangulated discretization of surfaces that are relaxed using small perturbations on vertex positions [24]. This method offered a real-life approximation of minimal surface geometry where no closed mathematical expressions can be found. Using this generative toolbox, many tensile structures and free-form architectural surfaces can be mathematically modelled where the physical assembly and use of soap films appear limited [13].

The structural stability of minimal surfaces emerging in liquid films is affected by the interaction between surface stresses and the disjoining pressure, which act independently from each other [36]. This physical property is numerically analysed for the catenoid minimal surfaces emerging between two opposite rings separated along a horizontal axis. As the distance is increased, the catenoid dips towards the axis of separation, thus minimizing the area reaching an absolute minimum measurement when the distance is less than $1.056R$, where R is the radius of the rings. In 1831, Goldschmidt offered a coefficient where the surface discontinuously jumps to two disconnected planar discs at a separation ratio of $1.325R$, thus providing a discontinuous solution for the minimization problem of a surface of revolution [37]. This critical distance has also been shown through an *in situ* observation where the maximum distance between the disks to form a catenoid is defined in relation to ring radius: $h_c \simeq 1.33R$ [38].

5 Methodology

This section will describe a computational workflow for the design, development, and manufacture of minimal surfaces using parametric tools. This research is completed as part of an undergraduate parametric design seminar where students learned about Frei Otto's soap film experiments and carried out analogue and digital experiments to explore minimal surfaces for the design development and digital fabrication of an installation project. For the analogue experiments, the behaviour of soap films under various boundary conditions defined by metal rods is observed. These are then captured using a fixed camera and lighting setup to aid the development of design forms. For the digital experiments, students learned about Rhinoceros and Grasshopper tools and plug-ins oriented towards the production of form-finding strategies and parametric modelling of minimal surfaces. These plug-ins featured the physics simulation engine Kangaroo [19], mesh modeller Weaverbird [39] and mesh topology editor Stripper [40], and custom Python scripts that automated the digital fabrication process. These exercises lead to the production of an installation project titled "Occa," which is designed, manufactured, and built through the use of parametric tools. The computational process of minimal surface tectonics is described in the following section.

5.1 Analogue computation: replicating Frei-Otto's soap film experiments for form-finding

The first stage of the project focuses on replicating Frei Otto's from-finding experiments using soap films [25].

Student groups experimented with a liquid mixture composed of various percentages of sugar, soap, water, and glycerin to develop minimal surfaces in various boundary conditions. Experiments ranged from surfaces defined by single metal rods in arbitrary shapes to documenting surface formations occurring between double or triple boundary conditions. Furthermore, additional metal rods as "spikes" are used to develop holes bounded by wool thread rings. Each experiment is captured with a digital camera and diagrammed to understand how the bounding condition affects the physical formation of the surface structure (Figure 1).

A common preference among these tests is the use of rounded boundary conditions that allow for the stable construction of minimal surfaces. When boundaries with sharp edges or kinks are used the thin liquid films exhibit spontaneous failure, preventing continuous observation of surface construction and stability [37]. To avoid this issue, circular bounding profiles are preferred to control the consistent physical behaviour and formation of minimal surfaces. Another observation is the role of glycerin and sugar content in the soap film mixture, both of which improved the consistency, buoyancy, and durability of surface tension that enables continuous separation of metal rods and observation of minimal surfaces [31]. While most of these experiments are carried out intuitively, controlling the behaviour, distance, and alignment of boundary conditions is documented through pictures, which further influenced the general formal approach to the parametric development of surfaces.

5.2 Digital computation: parametric modelling of minimal surfaces

The second stage of the project focuses on translating analogue experiments on minimal surfaces as a continuous parametric model. This process is achieved using mesh modelling tools in Rhinoceros and parametric modelling scripts developed in Grasshopper with multiple plug-ins. A key component in this phase is the role of the physics engine plug-in Kangaroo, which is used for interactive simulation, form-finding, and constraint-solving problems [12,19]. While Kangaroo offers a benchmark tool for form-finding experiments, coupling this engine with other parametric tools for the automated manufacture of minimal surfaces is explored. A novelty in this approach is the integration of the Stripper plug-in to rationalize curved mesh surfaces into strips that can discretize minimal surface geometry as strips running in perpendicular directions (*U* and *V*) that allow digital fabrication using planar, flexible materials [40]. This workflow is prepared in Grasshopper to automate parametric modelling, rationalization, and fabrication of minimal surfaces.

The first phase of parametric modelling explores the geometric description of a mesh geometry that can be used for Kangaroo relaxation simulation. In Figure 2, this workflow is visualized using a four-legged mesh topology parametrically developed from a tetrahedron. Using Grasshopper tools, each face of the tetrahedron is scaled down

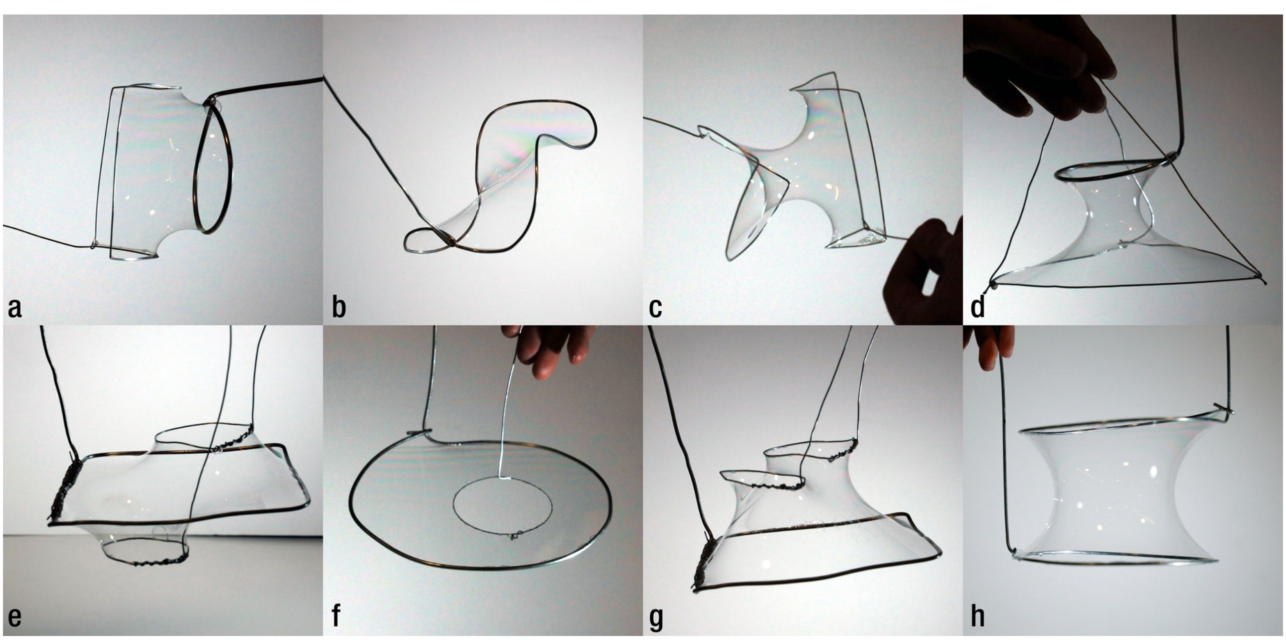

Figure 1: Various analogue form-finding experiments using soap films and metal rods: (a) vault and hole, (b) folded, (c) vault and bent hole, (d) triangular prism, (e) opposite holes, (f) flat hole, (g) pulled holes, and (h) catenoid.

and connected to a smaller tetra located at the centre using mesh quads defining three mesh quads per face (Figure 2a). These are then subdivided into smaller quads to increase mesh vertex and face count for physics-based form-finding in Kangaroo (Figure 2b). For subdivision, both Catmull-Clark (Weaverbird) and RefineStrips (Kangaroo) commands are used and compared [19,39,41]. While the Catmull-Clark subdivision divides a single quad face into four faces, the RefineStrips component divides a quad into two faces along the main curvature direction of the mesh strip. In this model, the outer strips are oriented towards the open faces of the mesh topology. After the quad mesh is recursively subdivided, it is transformed into a mass-spring model for Kangaroo simulation. During this operation, the outer edge profiles of the mesh topology are kept fixed to allow dynamic relaxation of inner quad vertices (Figure 2c). The coloured mapping of the resulting geometry shows higher displacement for vertices where mesh topology converges onto a point (red), compared to shared edges where transformation is mild (yellow), while green quads show the fixed boundary conditions of minimal surfaces (Figure 2d). This diagram provides general feedback on how the initial mesh topology and subdivision are effected by the physics-based relaxation (form-finding). The next step is to transform the resulting mesh quads into linear strips organized in two directions of curvature using Stripper tools [40]. These are visualized using black and white colours. The black coloured inner strips follow the *U* direction of quad mesh that are defined as "rings" running perpendicular to the mesh topology, while the white-coloured outer strips follow the *V* direction connecting minimal surface open faces (Figure 2e). Same colour strips in both directions are offset from each other to leave gaps, while opposite colour strips are offset along the mesh normals to separate and stack black and white layers. This protocol establishes a double-layered parametric construction of a minimal surface developed from a mesh topology.

The following steps of the parametric workflow also include a custom script written in Python that places a tag onto each unique strip running along the topology in *U* and *V* directions (Figure 2f) and unroll each mesh strip as a planar boundary representation for laser cutting (Figure 2g). The final geometry of the strips also requires the computation of rivet joint locations that specify connections for black (*U*) and white (*V*) strips. The location of these holes is computed using the base mesh geometry to populate circular holes that are perpendicular to the surface. These are subtracted from the strips using cylinders to calibrate hole positions before unrolling geometry. This operation both automates the part generation and accurately marks the hole positions on each strip using material thicknesses.

Figure 3 shows a detailed generation of strips using mesh geometry. During subdivision, each mesh quad face is recursively divided into four faces using the Catmull-Clark algorithm. The resulting mesh faces have consistent edge and vertex orders that can be traversed in

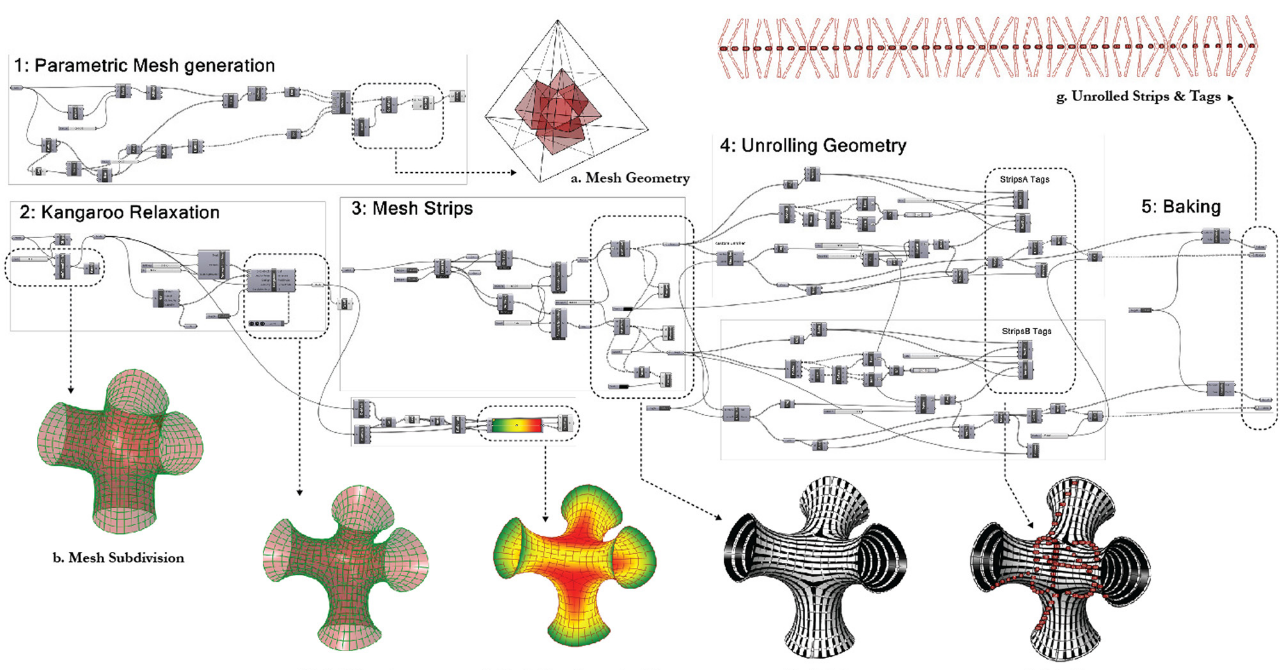

Figure 2: Parametric workflow of form-finding experiments in Grasshopper combining multiple plug-ins using a sample four-legged node.

perpendicular directions using the Stripper plug-in. With this tool, the *V* directions of the mesh topology can be extracted by connecting even edges of mesh faces (0–2), while the *U* directions connect odd edges (1–3). Both directions can be extracted as new mesh strips that be layered and offset according to material thicknesses for the digital fabrication of minimal surfaces.

Figure 4 shows the parametric modelling of various minimal surface samples based on analogue experiments from Figure 1 and other subsets of triply periodic minimal surfaces such as gyroid and six-legged node – a subset of Schwarz P surface. In every sample, the mesh topology is generated using quad mesh faces that are recursively subdivided using the Catmull-Clark subdivision by averaging neighbouring points to maintain surface topology while adding more faces in the form of algebraic expression [41]. This mesh is then used in the Kangaroo plug-in to develop minimal surfaces where the naked boundary edges of the meshes are kept fixed. In the final step, the minimal surfaces are transformed into double-layered strips running in perpendicular directions.

This parametric workflow poses some restrictions for the design exploration of minimal surfaces suitable for digital fabrication. The stripped discretization of meshes requires quad faces as triangular faces do not provide perpendicular *UV* strips and consistency on edge loop calculation. Among the sampled forms in Figure 3, meshes containing vertices with an odd number (3 or 5) of neighbouring faces show discontinuity in strips (folded, pulled holes, opposite holes), whereas vertices with an even number of adjacent faces (4 or 6) show consistency and continuous layering of strips (gyroid, 4-legged node, and catenoid). As the strips need to run continuously in perpendicular directions, this aspect restricts the mesh topology to an even number of adjacent face counts per vertex. Furthermore, the quad count in the mesh topology also becomes a focus of optimization for material and joints since each quad features a single joint connecting black and white strips. Additional remarks on key aspects of digital fabrication are discussed using the production of a case study below.

5.3 Physical computation: digital fabrication of doubly curved surfaces in strips and layers

This section discusses the digital fabrication of an installation project titled “Occa” using this parametric workflow on minimal surfaces. The design of the project stems from preliminary research on polyhedra geometries that are used for the parametric modelling of meshes and its form takes inspiration from glass sponges documented by Haeckel *et al.* (1998). For the installation, a branching topology defined by the packing and shared edges of four dodecahedrons is used (Figure 5a). This symmetrical topology is defined through four-legged connections that form a closed pentagonal ring in the middle. This line network is then transformed into a

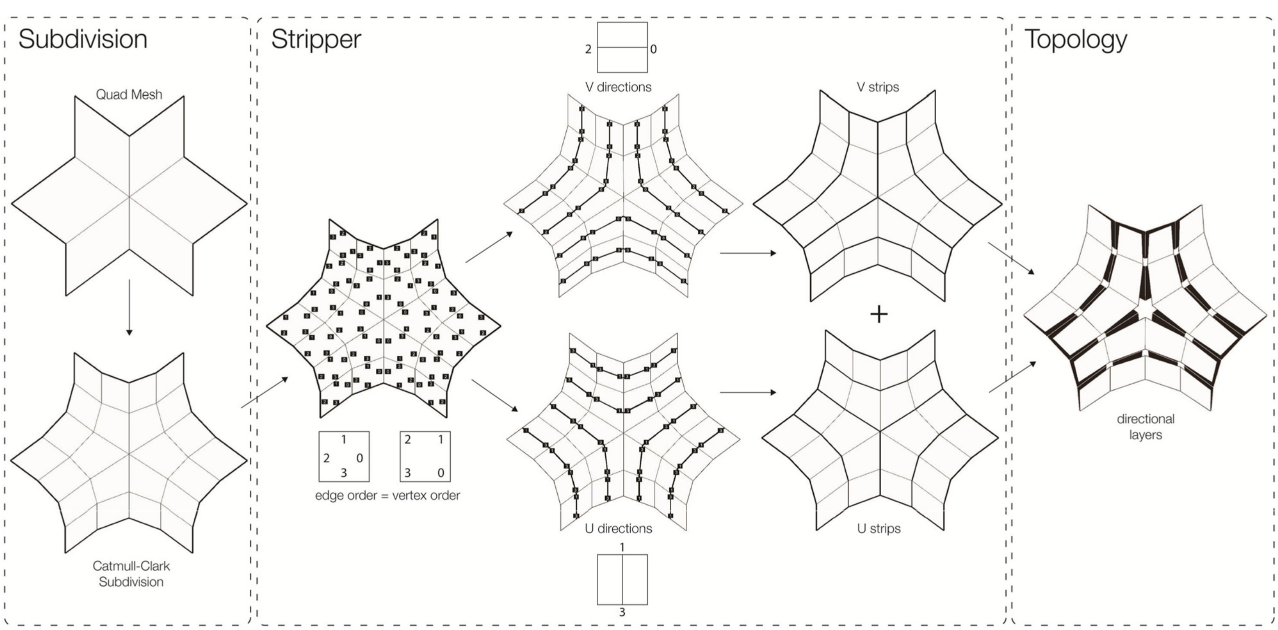

Figure 3: Subdivision and strip generation of mesh geometry.

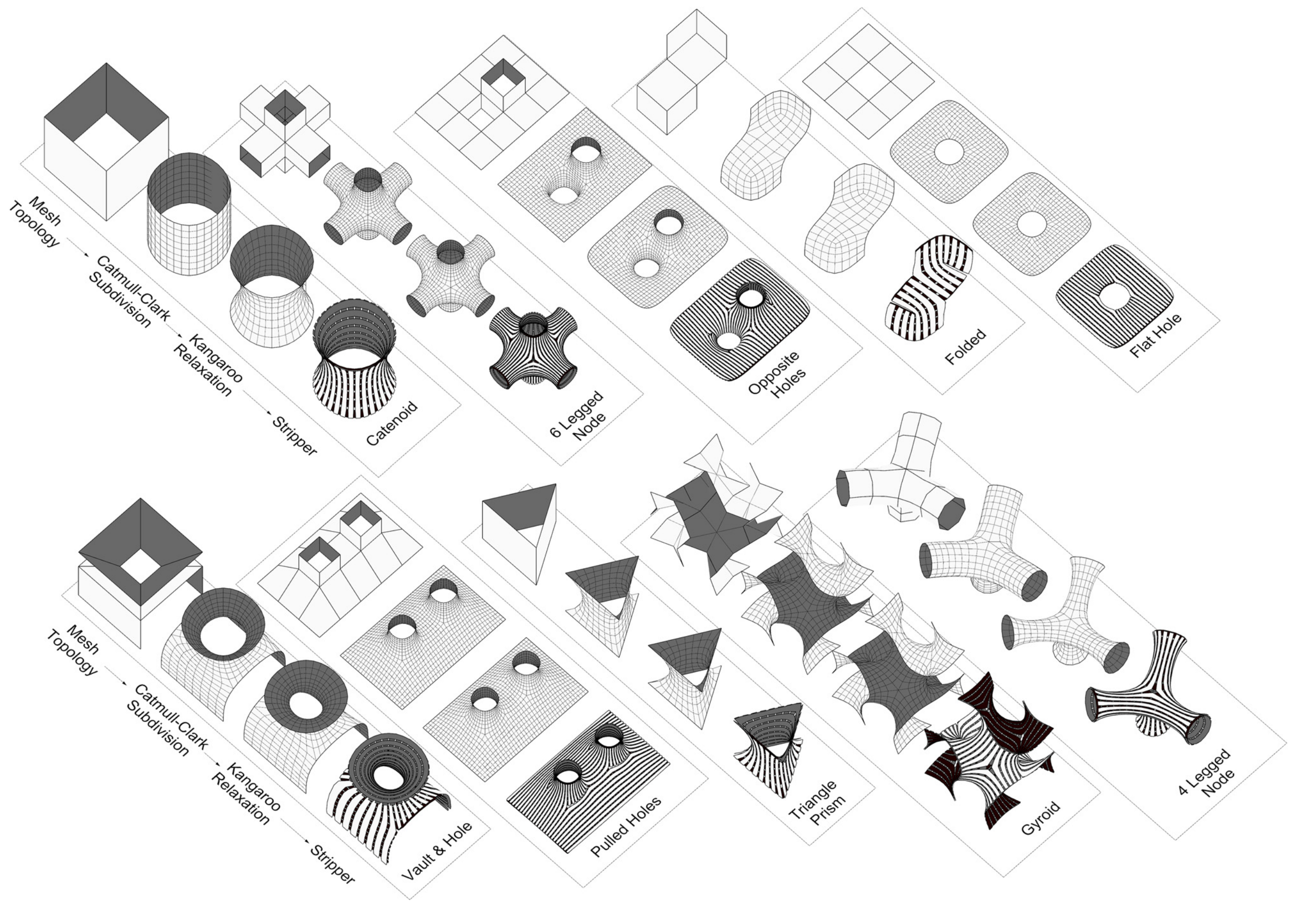

Figure 4: Parametric modelling of minimal surfaces using various samples. Back faces of meshes are shown in grey. Strips are shown in white and black.

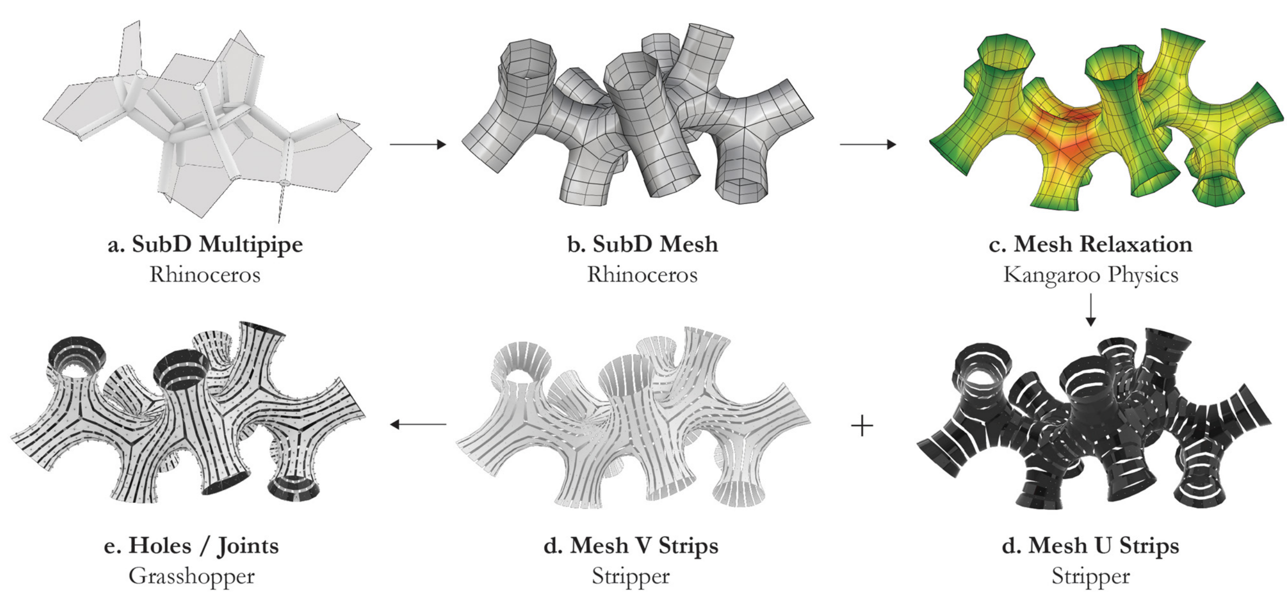

Figure 5: Parametric modelling workflow using Weaverbird, Kangaroo, and Stripper tools for the production of the case study.

subdivision surface using Rhinoceros native SubDMultiPipe command that transforms the four-legged node network into a mesh surface of quads (Figure 5b). The pipe function is controlled using a thickening radius that calculates a hollow tubular topology with open faces located at the end of open branches. The resulting structure has 8(*V*) × 4(*U*) quad faces per branch defining octagonal profiles along the topology. Then, each strip is first subdivided using the Catmull-Clark subdivision in Grasshopper to acquire 16(*V*) × 4(*U*) quads per branch to increase the vertex count required for the accurate computation of minimal surface using Kangaroo simulation (Figure 5c) [42]. This decision also allows for the optimization of total strips and rivet joints while developing a physical model that accurately captures the design topology.

The next phase of the project focuses on the digital fabrication of black and white strips running in perpendicular directions along the structure (Figure 5d). A geometric property of the design is the placement of four black strips in each branch of the topology, while 16 white strips connect multiple branches. This decision allows for the reduction of joints and total pieces per branch as black strips remain hidden inside the structure. In total, the structure is composed of 84 black strips running perpendicular to surface topology, all of which are connected end to end to form closed rings, and 112 open-ended white strips that are attached externally to black rings. A total of 1,344 rivet joints are shared between layered white and black strips, while no connection between the same coloured strips occurs. To place either coloured strip in the correct order along the structure, it needs to be connected to the opposing colour strip, black strips acting as a connector for white strips, and *vice versa*.

Figure 6 shows a diagram of rivet joints for double-layered strips calculated on resulting minimal surfaces. The mesh is defined through quad panels formed by groups of four vertices (Pi, Pj, Pk, and Pm) that are used for the calculation of *U* and *V* strips running in perpendicular directions using Stripper. The same quad mesh points also act as inputs for the mass-spring model in Kangaroo, where every edge of mesh faces is converted into a spring that is relaxed for form-finding. Every quad face holds a single connection between black (*U*) and white (*V*) strips. The centre (C) of this rivet hole is computed by taking the average geometric location of four vertices and projecting the point onto the mesh surface. The vector direction (*R*) is the perpendicular face normal of the mesh face. These centre points and vectors are used to model a cylinder with the radius of rivets protruding through strips to subtract a circular hole on each mesh face before unrolling and grouping operations. This operation enables the accurate calculation of rivet joints and alignment for layered digital fabrication of minimal surfaces (Figure 7).

The material selected for the construction of the project is 3 mm thick PVC sheets ordered in two colours (black and white) that are suitable for laser cutting. A key property of this plastic material is its flexibility and durability that when connected in the form of a ring provides rigidity to the structure. During the digital fabrication process,

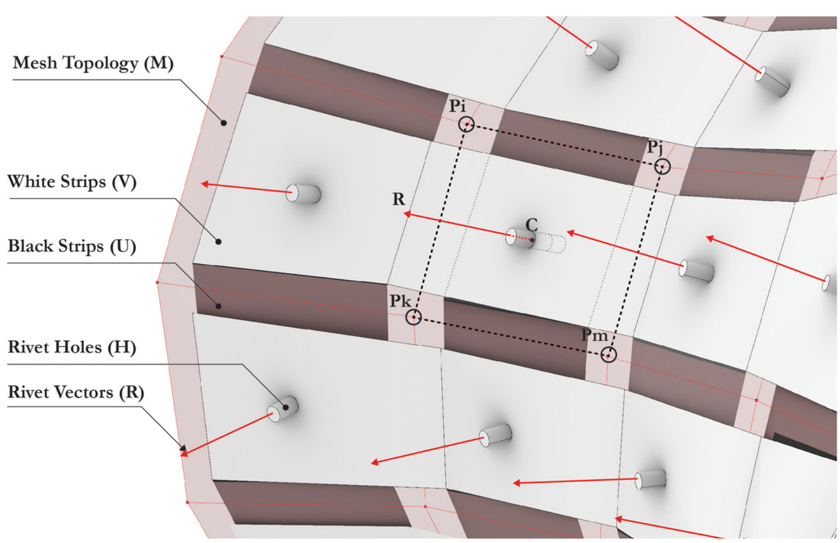

Figure 6: Details of calculation of strips and holes using mesh geometry.

Figure 7: Construction of a part of the installation showing alignment and connection of black and white strips using rivet guns.

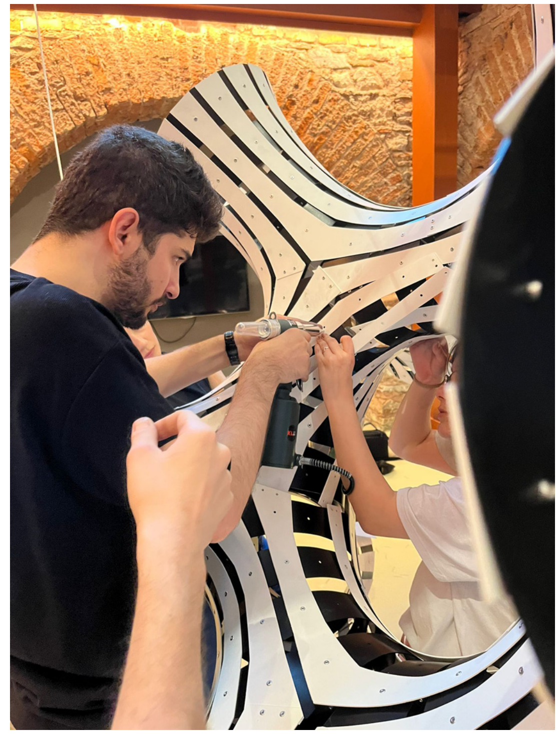

Figure 8: Connecting white and black strips on suspended installation using rivet gun.

white and black strips are laid out on sheets, laser-cut, and organized in groups. The structure presents white strips that do not fit into the production bay of the used CNC (100 cm × 160 cm), which are segmented and connected with additional end-to-end joints. Before the installation, all the pieces are joined into four pre-assembly groups in the fabrication laboratory to allow separate transportation and assembly at the final location of the installation.

Before the final assembly of the installation, the structure is temporarily kept in suspension to allow the connection of pre-assembled groups using rivets (Figure 8). During this process, the installation acquired durability and stability as more pieces and rivets are connected showing the structural performance of double-layered fabrication. The structure is then lifted in place and suspended from four connections to the steel beams located in space using steel cables running along two channels going through the topology (Figure 9). Due to the balancing weight and symmetry of the structure, four cable connections sufficed to hang and balance the installation. The structure is placed in an exhibition space with archaeological remains that can be seen from all sides and a glass ceiling (Figure 10).

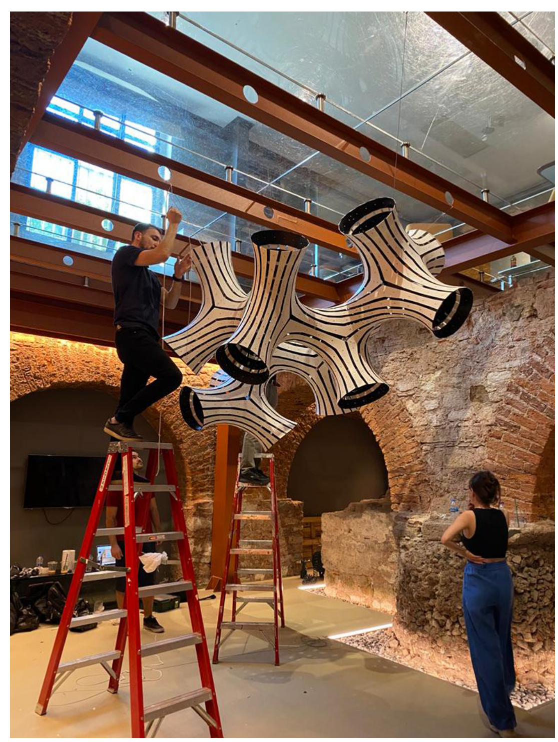

Figure 9: Suspension of installation.

Figure 10: View of the final installation "Occa" showing the completed assembly of layered and curved strips in suspension.

6 Conclusion

This study presented a computational workflow for the parametric modelling and digital fabrication of minimal surfaces by integrating Rhinoceros and Grasshopper tools. The overall aim and novelty of the research are to develop an automated parametric workflow for the geometric rationalization and transformation of minimal surface geometry into double-layered curved strips. This method can be used for the construction of minimal surfaces as mesh topology defined out of quads faces. While parametric modelling of minimal surfaces is a well-documented area of research [6,8,12,14] the transformation and rationalization of curved surface geometry for digital fabrication require topology optimization and consideration for material behaviour and performance [2,16,20]. These aspects are tested and shown using an installation project as a case study, where the surface topology is transformed into flexible linear strips built out of laser-cut PVC strips that are connected using aluminium rivets.

Revisiting Frei Otto's form-finding technique on soap films within the digital age brings up novel challenges to the parametric design and fabrication of curved structures. This research project extended an analogue technique towards a computational workflow that presents a novel application of minimal surface geometry as double-layered and curved structural construction. This protocol is addressed with the use of strips running in perpendicular directions along mesh topology with black (*U*) strips and outer white (*V*) strips that are connected at single rivet joints on mesh faces. While this aspect of fabrication poses challenges for the part identification, rivet alignment, and overall construction of strips, the minimal surface geometry offers a guiding topology for intuitive fabrication as single rivet joints automatically align when certain adjacent pieces are fixed in place. This shows that a historical perspective on developing a physical and intuitive approach based on Hooke's law can be replicated, simulated, and advanced with computational tools in contemporary digital practice.

One of the key learning outcomes of this research shows the potential of using consistent mesh topologies that can be rationally divided into rings or linear strips using edge loops. To achieve this, Rhinoceros subdivision tools are used to transform line networks into meshes made of quads. Transforming minimal surfaces into mesh topology offers both consistent subdivisions of surfaces and parametric extraction of strips that follow boundary edge loops. This topological property led the research to focus on structures that are formed out of four-legged joints, which are suitable for parametric translation into strips. These structures can be extracted from various polyhedra parametrically or built using line networks that can be transformed into subdivision surfaces in Rhinoceros/Grasshopper. While this approach shows the potential parametric exploration of minimal surface topologies, inquiries into alternative subdivision methods, the inclusion of triangular faces for mesh topologies, and the application of other materials for constructions are defined as avenues of future research.

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