

## Heuristic methods for postoutage voltage magnitude calculations

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**Abstract:** Power systems play a significant role in every aspect of our daily lives. Hence, their continuation without any interruption (or with the least duration of interruption due to faults or scheduled maintenances) is one of the key aims of electrical energy providers. As a result, electrical energy providers need to check in great detail the integrity of their power systems by performing regular contingency studies of the equipment involved. Among others, line and transformer outage simulations constitute an integral part of an electrical management system. Both accuracy and calculation speed depend on the branch outage model and/or the solution algorithms applied. In this paper, the local constrained optimization problem of the single-branch outage problem is solved by intelligent methods: particle swarm optimization, differential evolution, and harmony search. Simulations of IEEE 14-, 30-, 118-, and 300-bus systems are computed both by intelligent methods and by AC load flow. The results of the intelligent method-based simulations and AC load flow-based simulations are compared in terms of accuracy and computation speed.

**Key words:** Contingency studies, branch outage, intelligent methods, particle swarm optimization, differential evolution, harmony search

### 1. Introduction

Power systems operators should perform outage simulations regularly. Outages of transmission lines, transformers, or generation units may be harmful to the system. Limits of power system quantities, such as bus voltage magnitudes, active powers, or reactive power, can be exceeded. All possible outages need to be simulated individually; hence, the outcome of each on the investigated system can be determined. Postoutage voltage magnitudes and line flows should be determined as fast as possible so that remedial actions can be taken on time.

In order to determine the exact postoutage status, a full AC load flow is required. However, this is not practical for real applications due to long simulation times, even for a moderate size system. Hence, to achieve high solution speed, faster models are needed. DC load flow [1] is one of the fast methods, although it cannot handle reactive power flows. Consequently, postoutage voltage magnitudes cannot be determined. There are several other methods using preoutage initial state and linearized models [2–4]. However, none of these methods can provide the required accuracy for postoutage quantities such as reactive power flows, bus voltage magnitudes, etc.

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One approach to simulating branch outages is to increase the speed of the Newton–Raphson method. As is known, the Newton–Raphson method uses Taylor expansion of the second order, and higher derivatives are neglected. The second-order Newton–Raphson method, where the second-order derivatives were included in the computation process, was applied [5], and it was reported that this method gave tens of times faster results [6]. Another approach initially decreases the system size by focusing only on the voltage buses, and then applying the Newton–Raphson method [7]. The branch outage simulation results of this method were compared to the second-order Newton–Raphson method, and it was found that the former gave better results [5]. However, both methods require many iterations to converge, since they use the same base-case Jacobian matrix. A revised Jacobian matrix due to branch outages was also presented [8]. A similar study modeled the branch outage problem and solved it by using the direct solution technique [9]. However, it was reported that the methods given in [8] and [9] did not provide enough accuracy [10].

Postoutage bus voltage magnitudes and reactive power flows were computed by using sensitivities with linear and piecewise linear estimates. Simulation results showed that postoutage voltage magnitudes were computed accurately; however, computational speeds were not given [11]. Another sensitivity-based method provided sufficient computational accuracy but showed unacceptably high CPU times for large-scale systems. For example, CPU time for a single contingency calculation for the IEEE 300-test system was reported to be 28.2 times higher than that of the IEEE 14-bus test system [12].

Recently, the power systems branch outage problem has been modeled as a local constrained optimization problem by adding fictitious sources [13]. This model uses linearized reactive power equations for initial bus voltage magnitude estimations and revises them later with a local optimization cycle. The model is fast compared to the other models, because it only uses the variables of a bounded region, which includes the ones in the first-order neighborhood of outaged buses. In addition, calculated postoutage magnitudes are better in terms of accuracy than those obtained by employing the other models.

This paper solves the branch outage problem by using an existing branch outage model [13]. The local constrained optimization problem representing the branch outage phenomena is solved using nongradient-based method solutions. Our previous studies confirmed that 3 of the heuristic methods, namely particle swarm optimization (PSO), differential evolution (DE), and harmony search (HS), provided satisfactory results [14,15]. Therefore, we used those 3 methods for solving the branch outage problem and compared their computational accuracy and computational speed performances against conventional AC load flow.

The rest of the paper is organized as follows. In Section 2, the existing branch outage model is presented. In Section 3, the methods used for solving optimization problems, namely PSO, DE, and HS, are briefly introduced. Section 4 illustrates the results of the sample systems and, finally, Section 5 concludes the paper.

## 2. Branch outage model

In Figure 1, a transmission line's  $\pi$  equivalent circuit between 2 buses,  $i$  and  $j$ , and the corresponding reactive power flows are given. In the figure,  $y_{ij} = g_{ij} + jb_{ij}$  represents the line admittance between buses  $i$  and  $j$ , and  $b_{i0} = b_{j0}$  represents half of the line charging. On the other hand,  $Q_{ij}$ ,  $Q_{ij}^T$ , and  $Q_{Li}$  represent reactive power flowing through line  $i$ , transferred reactive power, and reactive power losses, respectively. These are mathematically expressed as follows:

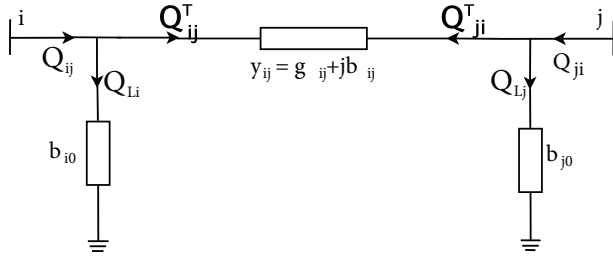
$$Q_{ij} = -[U_i^2 - U_i U_j \cos \delta_{ji}] b_{ij} + U_i U_j g_{ij} \sin \delta_{ji} - U_i^2 \frac{b_{i0}}{2} \quad (1)$$

$$Q_{ij}^T = -[U_i^2 - U_j^2] \frac{b_{ij}}{2} + U_i U_j g_{ij} \sin \delta_{ji} \quad (2)$$

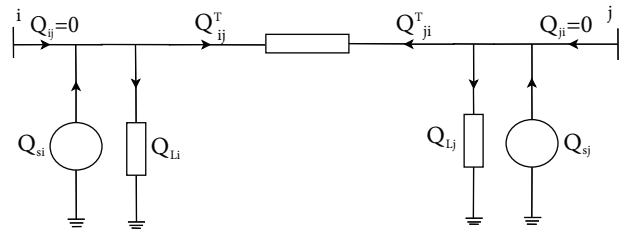
$$Q_{Li} = -[U_i^2 + U_j^2 - 2U_i U_j \cos \delta_{ji}] \frac{b_{ij}}{2} - [U_i^2 + U_j^2] \frac{b_{i0}}{4} \quad (3)$$

Here,  $U_i$  and  $U_j$  represent bus voltage magnitude,  $\delta_{ji}$  represents the difference between the bus voltage phase angles of bus  $j$  and bus  $i$ ,  $g_{ij}$  is the real part of line admittance between buses  $i$  and  $j$ , and  $b_{ij}$  represents the line susceptance.

A recent model [13] used 2 fictitious sources to simulate a branch outage:  $Q_{si}$ ,  $Q_{sj}$  as shown in Figure 2. Fictitious sources are selected in such a way that they create similar effects of an actual outage. That is, fictitious sources are determined in such a way that the injected power flows only through the outaged line.



**Figure 1.** The  $\pi$  equivalent of a transmission line and reactive power flows.



**Figure 2.** Branch outage modeling.

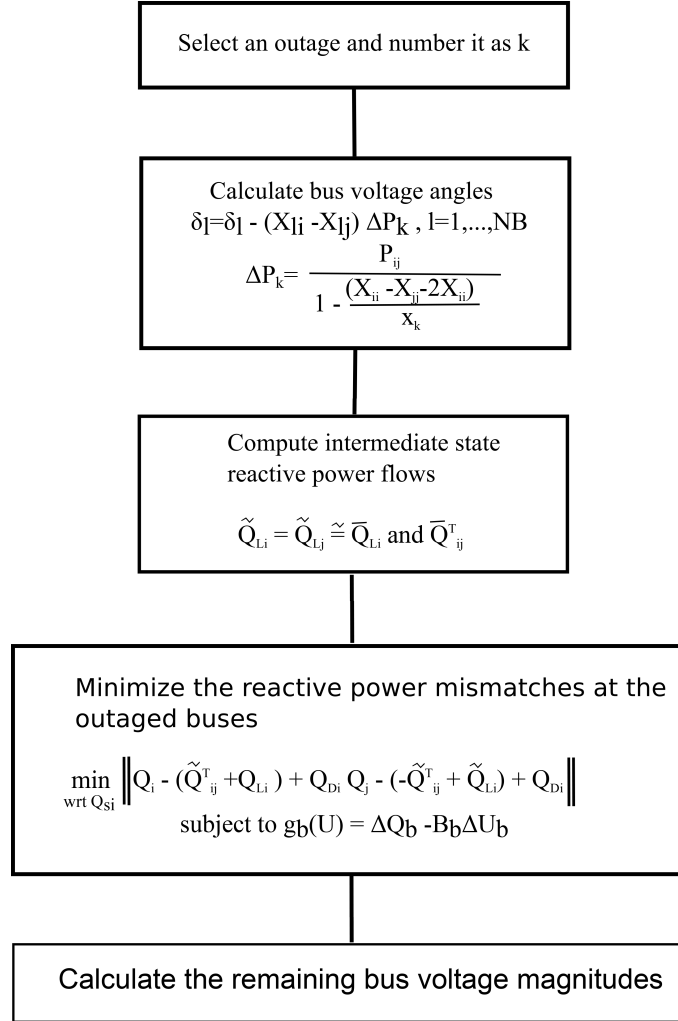
The approach to solving the branch outage problem involves the solution of 2 separate problems. The first is the solution of postoutage bus voltage angles and active power flows. Linear models provide satisfactory results for this problem. However, linear approaches cannot be applied to bus voltage magnitudes and reactive power flows because of high nonlinearity. Hence, branch outage is modeled as a local constrained optimization problem. With this approach, the network topology remains unchanged. In the local constrained optimization problem, the aim is to minimize the differences of real and simulated values of the outaged buses. The model only uses the variables within the first-order neighborhood of the outaged buses in the computations, and hence the speed of the model is expected to be appropriate for online computations.

The flowchart of the solution procedure for this model is given in Figure 3. Initially, preoutage system quantities are determined by running a base-case load flow. Then postoutage bus voltage angles are calculated by the equations given in the second step of the flowchart. The representation of the variables of the 2 equations given in the second step are defined as follows:  $\Delta P_k$  is the incremental active power injection due to a branch outage,  $X_{ij}$  is the entry of the sensitivity matrix describing the change of phase angle w.r.t. the change of bus real power,  $x_k$  is the reactance of the outaged line,  $P_{ij}$  is the preoutage active power flow through the line, and NB represents the number of buses. The next step is the computation of reactive power losses. The reactive powers at the outaged buses  $i$  and  $j$  will be different from the base-case load flow quantities due to the reactive power injections. This will increase load bus voltage magnitudes. These quantities can be determined by using the following equations:

$$\Delta U_b = B_b^{-1} \Delta Q_b \quad (4)$$

$$\Delta U_{new} = U_b + \Delta U_b \quad (5)$$

Here,  $\Delta U_b$  is the voltage magnitude increment vector due to reactive power changes in the outaged buses,  $B$  is the bus susceptance matrix, and  $\Delta Q$  is the reactive power change vector due to reactive power changes in the outaged buses. Note that all computations take place in the first-order neighbors of the outaged buses  $i$  and  $j$ , and this is represented by the subscript  $b$ .



**Figure 3.** Flowchart of the solution procedure of the branch outage model.

The computed new bus voltage magnitude vector is not equal to the postoutage bus voltage magnitude vector. Reactive power flow equations at outaged buses  $i$  and  $j$  also need to be satisfied. Therefore, the local constrained optimization problem to minimize reactive power mismatches, given in the fourth step of the flowchart, needs to be solved.

In the fourth step of the flowchart,  $\|\cdot\|$  represents the Euclidean norm. It is clear that there are only 2 nonzero values of the  $\Delta Q$  vector, because of the outaged buses  $i$  and  $j$ , and they are given as follows:

$$[\Delta Q] : [\Delta Q]_i = -[\Delta Q]_j = Q_{si} - Q_{ij}^T \quad (6)$$

If there is a transformer that has a tap ratio  $t$  between the outaged buses, then the values of Eq. (6) change,

as given below:

$$[\Delta Q]_k = \begin{cases} [\Delta Q]_i & = Q_{si} - Q_{ij}^T \\ [\Delta Q]_j & = \frac{tU_j}{-2U_i + tU_j} [\Delta Q]_i \\ [\Delta Q]_k & = 0 \text{ for } k \neq i, j \end{cases} \quad (7)$$

### 3. Heuristic methods

This study uses 3 heuristic methods as calculation tools to solve the optimization problem of the branch outage model. These are the PSO, DE, and HS methods. Details of the methods can be found in the following subsections.

#### 3.1. Particle swarm optimization

PSO mimics the behaviors of bird flocks and fish schools. It was developed by Kennedy and Eberhart in 1995 [16]. To date, this method has been applied to various power systems problems such as the economic dispatch problem [17], the state estimation problem [18], the optimal load flow problem [19], etc.

In PSO, solution candidates are called swarms, and each value of variables in a swarm is called a particle. A particle  $i$  in iteration  $k$  has 2 attributes: position and velocity.

Random initialization of the swarm positions can be performed as follows:

$$x_0^i = x_{\min} + rand(x_{\max} - x_{\min}) \quad (8)$$

Here,  $x_{\max}$ ,  $x_{\min}$  show the maximum and minimum positions that a variable can take,  $rand$  is a random number between 0 and 1, and  $x_0^i$  represents the position of a variable in the  $i$ th iteration. Swarm size is generally 15–30 times the number of variables. Random initialization of the velocity vector is shown below:

$$v_0^i = v_{\min} + rand(v_{\max} - v_{\min}) \quad (9)$$

Here,  $v_{\max}$ ,  $v_{\min}$  show the maximum and minimum velocities.

In each of the iterations, velocity and position vectors are updated according to Eqs. (10) and (11), respectively:

$$v_{k+1}^i = w v_k^i + c_1 rand(best_i - x_k^i) + c_2 rand(best_k^g - x_k^i) \quad (10)$$

$$x_{k+1}^i = x_k^i + v_{k+1}^i \quad (11)$$

Here,  $w$  is the inertial constant;  $best_i$  and  $best_k^g$  are the personal best and global best positions, respectively; and  $c_1$  and  $c_2$  are learning factors.

Finally, the algorithm stops if a specified stopping criterion is satisfied; otherwise, the algorithm restarts.

##### 3.1.1. Solution of branch outage problem by particle swarm optimization

The steps of the PSO algorithm to solve the branch outage problem can be given as shown below:

1. Perform a base-case load flow and use the obtained load bus voltage magnitudes of the buses that are in the bounded region as voltage magnitudes.

2. Form initial swarm  $Q_{si-initial}$ , elements of which are between  $Q_{ij}^T - limit$  and  $Q_{ij}^T + limit$ , where  $limit$  is a user-defined parameter.
3. Use either Eq. (6) or Eq. (7) to find the entries of the  $\Delta Q$  vectors. Solve the second equation of the minimization problem given in the fourth step of the flowchart in Figure 3 and revise load bus voltage magnitudes.
4. Evaluate the objective function given by the first part of the minimization problem given in the fourth step of the flowchart in Figure 3 for all particles in the swarm. Find the personal and global bests of the particles.
5. Calculate the new velocity and new position values for all particles using Eqs. (10) and (11).
6. If a specified stopping criterion is satisfied, then stop, or otherwise go to Step 2.

### 3.2. Differential evolution

DE is a genetic algorithm-based heuristic method. It was developed by Storn and Price [20]. Operators of DE are similar to those of genetic algorithms, such as crossover, mutation, and selection. It has only 3 control parameters, i.e. weight factor denoted as  $F$ , crossover rate denoted as  $CR$ , and population size denoted as  $N_p$ . DE has been applied to several power system problems, such as power system planning [21], transient stability-constrained optimal power flow [22], generation expansion planning [23], unit commitment [24], location of voltage collapse points [25], reactive power optimization [26], economic dispatch [27,28], etc.

In DE, the initial population consists of  $N_p$  randomly generated vectors between the lower and upper limits ( $x_{i(L)}$  and  $x_{i(H)}$ ), in accordance with Eq. (12):

$$x_i^{(G)} = x_{i(L)} + rand(x_{i(H)} - x_{i(L)}) \quad (12)$$

To find the best individual for the generation, all individuals' objective functions are calculated. Then mutation, crossover, and selection operations are repeatedly applied until a stopping criterion is satisfied.

The mutation phase of the algorithm is performed by adding a weighted difference of 2 randomly chosen vectors to another randomly chosen vector, where all 3 vectors should be distinct, for all individuals of the population. Hence,  $N_p$  new vectors are generated. This operation can be mathematically shown as in Eq. (13):

$$x_i'^{(G)} = x_{r_3}^{(G)} + F(x_{r_1}^{(G)} - x_{r_2}^{(G)}) \quad (13)$$

Here,  $i \neq r_1 \neq r_2 \neq r_3$ , and  $x_i'^{(G)}$  represents one of the generated mutant vectors.

Crossover is performed by generating a trial vector by using the elements of both the mutant and the target vectors. A sample crossover operation is mathematically expressed as:

$$x_i^{trial(G)} = \begin{cases} x_{ji}'^{(G)} & \text{if } rand \leq CR \text{ or } j = q \\ x_{ji}^{(G)} & \text{otherwise} \end{cases} \quad (14)$$

Here,  $x_{ji}'^{(G)}$  represents one element of a mutant vector,  $x_{ji}^{(G)}$  represents one element of a target vector, and  $x_i^{trial(G)}$  represents the generated trial vector.

In selection, the fitness values of the individuals of the trial vector are compared with the fitness values of the individuals of the corresponding target vectors one by one, as shown below. The new generation is selected according to the following criterion:

$$x_i^{(G+1)} = \begin{cases} x_i^{trial(G)} & \text{if } f(x_i^{trial(G)}) \leq f(x_i^G) \\ x_i^G & \text{otherwise} \end{cases} \quad (15)$$

Here,  $f(\cdot)$  is the fitness function.

### 3.2.1. Solution of branch outage problem by differential evolution

The steps of the DE algorithm to solve the branch outage problem can be given as shown below:

1. Perform a base-case load flow, and use the obtained load bus voltage magnitudes of the buses that are in the bounded region as voltage magnitudes.
2. Form an initial population between  $Q_{ij}^T - limit$  and  $Q_{ij}^T + limit$  representing  $N_p Q_{si}$  solutions.
3. Use either Eq. (6) or Eq. (7) to find the entries of the  $\Delta Q$  vectors. Solve the second equation of the minimization problem given in the fourth step of the flowchart in Figure 3 and revise the load bus voltage magnitudes.
4. Form  $N_p$  new mutant  $Q_{si}$  vectors by using Eq. (13).
5. Evaluate Eq. (14) and form  $Q_{si}^{trial}$  vectors.
6. By using the fitness function in the minimization problem given in the fourth step of the flowchart in Figure 3, decide whether the trial or the mutant vector will be included in the new generation.
7. If a specified stopping criterion is satisfied, stop, or otherwise go to Step 2.

### 3.3. Harmony search

HS is another recently developed heuristic method [29,30], which is inspired by the aim of finding the best harmony in jazz music. Since it is a recently developed algorithm, it has been applied to only few power systems problems: the economic dispatch problem [31], FACTS device optimal placement [32], and reactive power dispatch [33].

Following the determination of harmony memory size (HMS), harmony memory consideration rate (HMCR), and pitch adjusting rate (PAR) parameters, initial harmony memory (HM) selected from a feasible range is ordered according to the corresponding objective functions.

After initialization, an improvisation step comprising memory consideration, pitch adjustment, and randomization takes place. Details of the improvisation are given below.

In memory consideration, all decision variables are randomly selected from the existing elements and a new harmony vector is constructed. This restricts the solution values to the values of HM used in the past. In order to extend the solution space, HMCR is used as follows:

$$x_i' = \begin{cases} x_i' \in \{x_1', x_2', \dots, x_i^{HMS}\} & \text{with prob. } HMCR \\ x_i' \in X_i & \text{with prob. } HMCR \end{cases} \quad (16)$$

One can decide whether to pitch adjust the entries of the new harmony vector by using PAR values. This parameter can be used, as shown below:

$$\text{Pitch adjustment decision for } x'_i = \begin{cases} \text{YES with prob. } PAR \\ \text{NO with prob. } (1 - PAR) \end{cases} \quad (17)$$

Pitch adjustment for  $x'_i$  can be performed as:

$$x'_i \leftarrow x'_i + rand * bw \quad (18)$$

When the improvisation step is completed, the objective function's value at the newly obtained solution is computed. This value is compared to the worst value in HM and it replaces it if it is better.

Finally, the algorithm stops if a specified stopping criterion is satisfied; otherwise, it restarts from the improvisation step.

### 3.3.1. Solution of branch outage problem by harmony search algorithm

The HS algorithm for solving the branch outage problem is briefly given as shown below:

1. Perform a base-case load flow, and use the obtained load bus voltage magnitudes of the buses that are in the bounded region as voltage magnitudes.
2. Form HMS different  $Q_{si}$  solutions, elements of which are between  $Q_{ij}^T + limit$  and  $Q_{ij}^T - limit$ .
3. Use either Eq. (6) or Eq. (7) to find the entries of the  $\Delta Q$  vectors. Solve the second equation of the minimization problem given in the fourth step of the flowchart in Figure 3 and revise the load bus voltage magnitudes.
4. Form a new harmony vector  $Q'_{si}$  by using improvisation steps.
5. Calculate the values of the objective functions, and, depending on the comparison of these values against the new harmony members, decide whether these values exist in HM or whether the old values remain at their entries.
6. If a specified stopping criterion is satisfied, stop, or otherwise go to Step 2.

## 4. Tests and results

Branch outage heuristic solution algorithms are tested on IEEE 14-, 30-, 118-, and 300-bus test systems. IEEE 14-, 30-, and 118- bus test systems are used for computational accuracy tests, and all the test systems are used for testing the computational speed of the heuristic branch outage algorithms. MATLAB is used as a computational tool and a free power systems software package, and MATPOWER [34] is used for AC power flow tests. Heuristic algorithms are developed and power system branch outage simulation results of AC power flow are compared against the results in terms of computational accuracy and speed. Test simulations are performed on a laptop that has the following specifications: 2.20 GHz Core Duo CPU and 2.00 GB. Note that each branch outage simulation is run 100 times, and the mean values and standard deviations of the simulation results are reported.

As specified earlier, base-case load flow is run initially, and its results are then used as initial conditions for the branch outage conditions. The numerical values of the parameters of PSO, DE, and HS are given in Table 1.



These parameters are determined by investigating the effects of different parameters of heuristic methods to the results. From the experiences of [35] it is specified that setting an equal value of 2 for learning factors  $c_1$  and  $c_2$  in PSO will be sufficient, and hence these values are used here as learning parameters. For DE- and HS-based branch outage simulations, optimum parameter values are determined by simulating all possible branch outages using different parameter values. Each branch outage simulation for different parameters is run 100 times, and the largest mean value of a bus voltage magnitude percentage error and standard deviations are obtained. Because of limited space, only the results of DE-based branch outage simulations are illustrated in Tables 2 and 3. For DE-based branch outage simulations, Tables 2 and 3 show that 0.50, 0.90; 0.90, 0.90; and 0.90, 1.80 CR and F values are the most appropriate parameter values. For HS-based branch outage simulations, the most appropriate HMCR and PAR pair is found from different HMCR and PAR pairs as HMCR = 0.45, PAR = 0.85.

**Table 1.** Parameters of heuristic methods.

Method
PSO $c_1 = c_2 = 2$ , $N_p = 15$ , <i>maximum number of iterations</i> = 60
DE $F = 1.8$ , $CR = 0.9$ , $N_p = 15$ , <i>maximum number of iterations</i> = 60
HS $HMS = 15$ , $HMCR = 0.85$ , $PAR = 0.45$ , <i>brange</i> = 0.1, <i>maximum number of iterations</i> = 900
Note: Algorithm is stopped when there is no change in the computed solution for 5 consecutive iterations. For HS this is 75 iterations.

**Table 2.** The largest errors of all possible branch outage simulation results of IEEE 14-bus test system for different CR and F parameters of the DE method.

Outaged branch	CR, F 0.10, 0.10	CR, F 0.10, 0.90	CR, F 0.10, 1.80	CR, F 0.50, 0.10	CR, F 0.50, 0.90	CR, F 0.50, 1.80	CR, F 0.90, 0.10	CR, F 0.90, 0.90	CR, F 0.90, 1.80
1-5	0.18	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16
2-4	0.32	0.32	0.32	0.32	0.32	0.32	0.32	0.32	0.32
2-5	0.10	0.10	0.10	0.20	0.10	0.10	0.23	0.10	0.10
3-4	0.13	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10
4-5	0.73	0.73	0.73	0.73	0.73	0.73	0.73	0.73	0.73
4-7	0.79	0.79	0.79	0.79	0.79	0.79	0.79	0.79	0.79
4-9	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.12
5-6	0.87	0.87	0.87	0.87	0.87	0.87	0.87	0.87	0.87
6-11	0.82	0.41	0.39	0.47	0.41	0.41	0.41	0.41	0.41
6-12	0.88	0.88	0.86	0.88	0.88	0.88	0.88	0.88	0.88
6-13	1.07	1.07	1.07	1.08	1.08	1.07	1.07	1.07	1.07
7-9	0.60	0.60	0.60	0.74	0.60	0.60	0.60	0.60	0.60
9-10	0.16	0.14	0.14	0.14	0.14	0.14	0.14	0.13	0.14
9-14	0.38	0.07	0.09	0.29	0.07	0.07	0.07	0.07	0.07
10-11	0.25	0.14	0.14	0.14	0.14	0.14	0.25	0.14	0.14
12-13	0.24	0.06	0.07	0.17	0.06	0.06	0.07	0.06	0.06
13-14	0.37	0.36	0.35	0.36	0.36	0.36	0.36	0.36	0.36

Due to limited space, the results of 2 representative outages will be given for the IEEE 14-bus test system. They are chosen because they were heavily loaded during base-case load flows. Postoutage voltage magnitudes obtained by using AC load flow, PSO algorithm, DE algorithm, and HS algorithm, and the absolute percentage

errors of the heuristic algorithms are illustrated in Table 4 for the simulation of bus 7–9 outage. Table 5 shows similar values for the outage of transformer between buses 5 and 6. From Table 4, the largest absolute percentage error can be found as 0.60%, and from Table 5 the largest absolute percentage error is 0.90%. The largest maximum percentage error for all possible outages in the system is found as 1.07% for the simulation of bus 6–13 outage.

**Table 3.** Standard deviations of the largest percentage errors of all possible branch outage simulation results of IEEE 14-bus test system for different CR and F parameter values of the DE method.

Outaged branch	CR, F	CR, F	CR, F	CR, F	CR, F	CR, F	CR, F	CR, F	CR, F
	0.10, 0.10	0.10, 0.90	0.10, 1.80	0.50, 0.10	0.50, 0.90	0.50, 1.80	0.90, 0.10	0.90, 0.90	0.90, 1.80
1–5	0.081	0.001	0.001	0.000	0.000	0.001	0.016	0.000	0.000
2–4	0.009	0.000	0.004	0.000	0.000	0.000	0.000	0.000	0.000
2–5	0.002	0.001	0.004	0.309	0.000	0.000	0.403	0.000	0.000
3–4	0.109	0.000	0.003	0.006	0.000	0.000	0.000	0.000	0.000
4–5	0.006	0.000	0.001	0.000	0.000	0.000	0.024	0.000	0.000
4–7	0.000	0.000	0.006	0.000	0.000	0.000	0.006	0.000	0.000
4–9	0.000	0.001	0.002	0.000	0.000	0.000	0.000	0.000	0.000
5–6	0.000	0.000	0.002	0.013	0.000	0.000	0.000	0.000	0.000
6–11	0.284	0.001	0.043	0.263	0.000	0.000	0.000	0.000	0.000
6–12	0.000	0.003	0.024	0.000	0.000	0.000	0.001	0.000	0.000
6–13	0.003	0.001	0.015	0.035	0.000	0.000	0.005	0.000	0.000
7-9	0.000	0.000	0.005	0.458	0.000	0.000	0.000	0.000	0.000
9–10	0.043	0.001	0.010	0.000	0.000	0.000	0.037	0.000	0.000
9–14	0.636	0.000	0.074	0.714	0.000	0.000	0.006	0.000	0.000
10–11	0.330	0.001	0.013	0.018	0.000	0.000	0.388	0.000	0.000
12–13	0.536	0.001	0.033	0.312	0.000	0.000	0.022	0.000	0.000
13–14	0.000	0.001	0.022	0.000	0.000	0.000	0.000	0.000	0.000

**Table 4.** Voltage magnitudes and corresponding errors for outage of branches 7–9 in IEEE 14-bus test system.

Bus no.	Outage of lines 7–9						
	$U_{AC}$	$U_{PSO}$	%Err	$U_{DE}$	%Err	$U_{HS}$	%Err
1	1.0600	1.0600	0.00	1.0600	0.00	1.0600	0.00
2	1.0450	1.0450	0.00	1.0450	0.00	1.0450	0.00
3	1.0100	1.0100	0.00	1.0100	0.00	1.0100	0.00
4	1.0169	1.0178	0.09	1.0178	0.09	1.0178	0.09
5	1.0174	1.0196	0.22	1.0196	0.22	1.0196	0.22
6	1.0700	1.0700	0.00	1.0700	0.00	1.0700	0.00
7	1.0671	1.0699	0.26	1.0699	0.26	1.0699	0.26
8	1.0900	1.0900	0.00	1.0900	0.00	1.0900	0.00
9	1.0291	1.0353	0.60	1.0353	0.60	1.0353	0.60
10	1.0282	1.0338	0.54	1.0338	0.54	1.0338	0.54
11	1.0446	1.0480	0.33	1.0480	0.33	1.0479	0.33
12	1.0535	1.0539	0.04	1.0539	0.04	1.0539	0.04
13	1.0459	1.0472	0.12	1.0472	0.12	1.0472	0.12
14	1.0179	1.0225	0.46	1.0225	0.46	1.0225	0.46

Postoutage bus voltage magnitudes were also computed by other recently developed branch outage simulation methods [11,12]. Simulation results for the most heavily loaded branch 7–9 of the IEEE 14-bus test system was not provided in [11]. The largest absolute percentage error for this outage is found to be 0.44%, similar to that reported in [12]. The largest absolute percentage error for the outage of the transformer between bus 5 and bus 6 is found to be high: 2.44% for bus 12. Postoutage bus voltage magnitudes were not provided in [12] for this heavily loaded branch.

Heavily loaded branches during base case load flows are outage of the transmission line connected between buses 4 and 6 and outage of the transformer connected between buses 4 and 12 for the IEEE 30-bus test system. Due to limited space, only the largest absolute percentage errors of the simulations of the transmission line connected between buses 4 and 6, and outage of the transformer connected between buses 4 and 12, are given here. They are calculated as 0.52% and 0.64%, respectively. The simulation results of outage of the transmission line connected between buses 4 and 6 were also presented in [12]. Maximum error was found to be 0.19%, which was on the same order as this study. This outage was not simulated in [11]. Neither [11] nor [12] simulated the outage of the transformer connected between buses 4 and 12. Table 6 illustrates the maximum percentage errors for all possible outages in the system.

The simulation results for the 2 arbitrary outages in the IEEE 118-bus test system are shown in Tables 7 and 8, respectively. Table 7 shows the postoutage voltage magnitudes and their corresponding percentage errors for the outage of the transmission line connected between bus 52 and bus 53. Again, due to limited space, only the values of the buses with errors greater than 0.05% are illustrated in the table. Table 8 gives the postoutage voltage magnitudes and their corresponding percentage errors for the outage of the transformer between buses 30 and 17. Similarly, only the buses with errors greater than 0.1% are given. The largest absolute percentage errors are 0.16% and 0.53% for the line and transformer outage, respectively. Recently developed branch outage simulation methods in [11] and [12] did not provide the simulation results for any specific outages in the IEEE 118-bus test system.

**Table 5.** Voltage magnitudes and corresponding errors for outage of transformer 5–6 in IEEE 14-bus test system.

Bus no.	Outage of transformer 5–6						
	$U_{AC}$	$U_{PSO}$	%Err	$U_{DE}$	%Err	$U_{HS}$	%Err
1	1.0600	1.0600	0.00	1.0600	0.00	1.0600	0.00
2	1.0450	1.0450	0.00	1.0450	0.00	1.0450	0.00
3	1.0100	1.0100	0.00	1.0100	0.00	1.0100	0.00
4	1.0181	1.0268	0.85	1.0270	0.87	1.0273	0.90
5	1.0272	1.0347	0.73	1.0350	0.76	1.0356	0.82
6	1.0700	1.0700	0.00	1.0700	0.00	1.0700	0.00
7	1.0656	1.0657	0.01	1.0658	0.02	1.0660	0.04
8	1.0900	1.0900	0.00	1.0900	0.00	1.0900	0.00
9	1.0682	1.0600	0.77	1.0601	0.76	1.0603	0.74
10	1.0614	1.0544	0.66	1.0545	0.65	1.0546	0.64
11	1.0623	1.0587	0.34	1.0587	0.34	1.0588	0.33
12	1.0543	1.0554	0.10	1.0555	0.11	1.0555	0.11
13	1.0525	1.0510	0.14	1.0510	0.14	1.0510	0.14
14	1.0422	1.0381	0.39	1.0382	0.38	1.0383	0.37

**Table 6.** The largest bus voltage percentage errors for the branch outages in IEEE 30-bus test system.

Outaged branch	%Error PSO	% Error DE	%Error HS	Bus no.
1-3	0.06	0.06	0.07	3
2-4	0.07	0.07	0.07	3
3-4	0.06	0.06	0.06	6
2-6	0.24	0.24	0.24	4
4-6	0.52	0.52	0.52	30
5-7	0.21	0.21	0.21	7
6-7	0.14	0.14	0.13	7
6-8	0.11	0.11	0.10	30
6-9	0.37	0.37	0.37	10
6-10	0.15	0.15	0.14	30
9-10	0.56	0.56	0.56	10
4-12	0.64	0.64	0.64	16
12-14	0.44	0.44	0.43	14
12-15	0.24	0.24	0.24	15
12-16	0.05	0.05	0.05	17
14-15	0.03	0.03	0.02	14
16-17	0.08	0.08	0.08	17
15-18	0.28	0.28	0.28	18
18-19	0.05	0.05	0.05	19
19-20	0.47	0.47	0.47	19
10-20	0.26	0.26	0.26	20
10-17	0.26	0.26	0.26	17
10-21	0.33	0.33	0.33	21
10-22	0.09	0.09	0.09	22
21-22	0.05	0.05	0.05	22
15-23	0.33	0.33	0.32	23
22-24	0.35	0.35	0.35	24
23-24	0.05	0.05	0.05	29
24-25	0.19	0.19	0.20	25
25-27	0.24	0.24	0.24	27
28-27	1.28	1.28	1.18	30
27-29	0.31	0.31	0.30	29
27-30	0.13	0.13	0.13	30
29-30	0.13	0.13	0.13	30
8-28	0.07	0.07	0.07	28
6-28	0.53	0.53	0.53	30

**Table 7.** Voltage magnitudes and corresponding errors for outage of branch 52-53 in IEEE 118-bus test system.

Bus no.	Outage of lines 52-53						
	$U_{AC}$	$U_{PSO}$	%Err	$U_{DE}$	%Err	$U_{HS}$	%Err
51	0.9719	0.9726	0.07	0.9726	0.07	0.9726	0.07
52	0.9654	0.9669	0.15	0.9669	0.15	0.9669	0.15
53	0.9356	0.9371	0.16	0.9371	0.16	0.9371	0.16
58	0.9619	0.9623	0.05	0.9623	0.05	0.9623	0.05

**Table 8.** Voltage magnitudes and corresponding errors for outage of transformer 30–17 in IEEE 118-bus test system.

Bus no.	Outage of transformer 30–17						
	$U_{AC}$	$U_{PSO}$	$\%Err$	$U_{DE}$	$\%Err$	$U_{HS}$	$\%Err$
13	0.9668	0.9683	0.16	0.9683	0.16	0.9683	0.16
20	0.9538	0.9569	0.33	0.9569	0.33	0.9569	0.33
21	0.9531	0.9577	0.49	0.9577	0.49	0.9577	0.49
22	0.9639	0.9690	0.53	0.9690	0.53	0.9690	0.53
23	0.9970	0.9995	0.24	0.9995	0.24	0.9995	0.24
30	1.0131	1.0182	0.50	1.0182	0.50	1.0182	0.50

Absolute percentage error is computed as given below:

$$\%Err = 100 \frac{abs(U_{AC} - U_{method})}{U_{AC}} \quad (19)$$

Here,  $\%Err$  represents percentage error, and  $U_{AC}$  and  $U_{method}$  represent calculated bus voltage magnitude value by AC load flow and one of the proposed methods, respectively.

From Tables 4–8 it is seen that the percentage error values obtained by the 3 heuristic methods are of the same order. As stated earlier, each simulation is run 100 times, and the mean values of the simulations are reported. Additionally, the best parameters are determined before the simulations. The population numbers of all 3 heuristic methods are selected equally. Because of the above explanations, it is obvious to see small differences between the postoutage voltage magnitude percentage errors and the standard deviations of the methods.

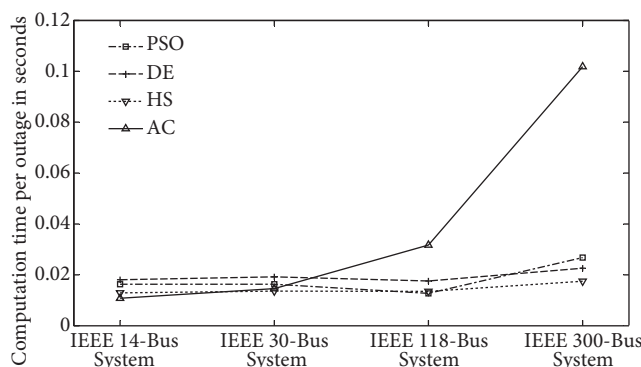
Table 9 shows average computation times per outage and their standard deviations for all test systems. Here,  $\sigma_{ac}$ ,  $\sigma_{pso}$ ,  $\sigma_{de}$ , and  $\sigma_{hs}$  represent standard deviation for average computation times per outage using AC load flow, PSO method, DE method, and HS method, respectively. Figure 4 illustrates the average computation time versus system size. Possible outage simulation lists for these test systems were 17, 36, 126, and 306, respectively. It is clear that the AC load flow is the fastest one for the IEEE 14-bus test system. However, as the system gets larger, the speed of the AC load flow simulation gets slower. In contrast, the speeds of all heuristic methods become faster as the system size becomes larger. HS is the fastest for the IEEE 30-, 118-, and 300-bus test systems. In addition, all heuristic methods can be sped up by parallel processing.

**Table 9.** Computation times for several test systems.

Test system	Number of outages simulations	Computation time per outage simulation						
		PSO	$\sigma_{pso}$	DE	$\sigma_{de}$	HS	$\sigma_{hs}$	AC
IEEE 14-bus	17	0.0163	0.0024	0.0181	0.0022	0.0129	0.0023	0.0108
IEEE 30-bus	36	0.0163	0.0015	0.0192	0.0017	0.0136	0.0015	0.0146
IEEE 118-bus	126	0.0127	0.0001	0.0176	0.0001	0.0135	0.0001	0.0317
IEEE 300-us	306	0.0268	0.0014	0.0226	0.0001	0.0175	0.0001	0.1018

Computational speed results were not provided in [11]. From the computational speed results of [12], it is clear that the proposed branch outage model in [12] is not independent from the system size. Simulation times increase as the system size increases. For instance, the CPU time of a single outage of the IEEE 300-bus test system is 28.2 times slower than the CPU time of a single outage of the IEEE 14-bus test system. It is seen from [12] that the single branch outage CPU time for the IEEE 300-bus test system is 0.0705 s, which is

approximately 4 times greater than the CPU time obtained for HS-based branch outage simulation. Hence, the HS-based branch outage solution algorithm can be used as a reliable branch outage simulation tool.



**Figure 4.** Computation times for several test systems.

## 5. Conclusion

This paper presented branch outage problem solutions with the nongradient-based method. The local constrained optimization-based power systems branch outage model brought a significant improvement in the calculation speed, since it reduced the solution space to a restricted region comprising the outaged bus and its first-order neighbors.

Further improvement was achieved by using 3 heuristic methods in this study. These methods, the PSO, DE, and HS algorithms, were selected according to the results of our previous studies. IEEE 14-bus, IEEE 30-bus, IEEE 118-bus, and IEEE 300-bus applications showed that the postoutage bus voltage magnitudes were computed with reasonable accuracy. The speed tests of the heuristic methods showed that the proposed formulation is not sensitive to system size. Therefore, heuristic methods get better than full AC load flow-based calculations as the system size increases. From the simulation tests it is observed that HS-based simulation results are slightly faster and more accurate than other heuristic methods. Therefore, the HS-based algorithm for the branch outage model can be used as a branch outage simulation tool. In addition, the proposed algorithms can easily be sped up by parallel processing implementation.

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