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On (mis-)perception of probabilities in first-price sealed-bid auctions

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Abstract

We study a two-stage probability weighting model [see Tversky and Fox, 1995] in a first-price sealed-bid auction. We present the unique symmetric equilibrium and provide some experimental support for our model.

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1 Introduction

A considerable amount of real-life decision problems involve uncertain prospects as well as risky prospects (e.g., making investment, buying insurance, etc.). In such cases, individuals do not necessarily know the objective probabilities of occurrence, and it consequently becomes important how individuals perceive the probability of the event under consideration. For such scenarios involving both uncertain and risky prospects, Tversky and Fox (1995) proposed a *two-stage model* arguing that an individual first judges the probability of the event under consideration and then transforms this probability using a probability weighting function under risk (see also Fox and Tversky, 1998).

A first-price sealed-bid auction is an example of such scenarios, since lack of knowledge regarding the other bidders' valuations poses a risk, whereas lack of knowledge regarding the other bidders' bidding behavior, which in turn determines the auction outcome, poses an uncertainty. It is commonly observed in first-price auction experiments that subjects tend to bid higher than the risk neutral Nash equilibrium predictions (see Cox et al., 1988; Kagel, 1995, among others). Several models are presented to explain this well-documented *overbidding* phenomenon: ambiguity aversion (Salo and Weber, 1995), regret theory (Filiz-Ozbay and Ozbay, 2007), level-k thinking (Crawford and Iriberri, 2007), and loss aversion (Lange and Ratan, 2010), as well as subjective probability weighting (see Goeree et al., 2002; Armantier and Treich, 2009a,b; Keskin, 2016, among others).

With the current study, we aim to stimulate a discussion on subjective probability weighting in first-price auctions. In the models analyzed by Armantier and Treich (2009a) and by Keskin (2016), it is assumed that bidders subjectively weight their winning probabilities. On the other hand, in a recent experimental study, Armantier and Treich (2009b) reported that subjects perceive their winning probabilities differently, which implies that using probability weighting functions directly on the winning probabilities might be misleading. What if subjects have different winning probabilities in their minds based on their judgment of the uncertain situation they face, as described in the first part of the Tversky-Fox two-stage model? If so, this would imply that even if bidders do weight probabilities subjectively, they might have been considering weighted winning probabilities different from what the theory presumes they would. To our understanding, the experimental observations by Armantier and Treich (2009b) reveal an important shortcoming in the literature: either researchers misperceive the winning probabilities that subjects correctly perceive, or subjects misperceive the winning probabilities that researchers correctly calculate. Independent of which of these views is correct, it seems clear that these probability (mis-)perceptions create a disconnect between the theoretical analysis and the subject behavior observed in experiments. We believe that if one is able to identify the extent of this disconnect between the winning probabilities subjects perceive and the *objective* probabilities as calculated by researchers, then one would have a better understanding of the reasons behind overbidding.

In this paper, we conduct an experiment investigating subjects' beliefs about their winning probabilities in order to understand the extent of the above-described disconnect, and then we relate our observations to the Tversky-Fox two-stage model. Our findings indicate that the two-stage model is able to explain the observed behavior by a proper selection of the probability misperception and probability weighting functions. However, further experimental investigation is necessary for the elicitation of those functions.

2 The Model

Let N be the set of n bidders participating in a first-price sealed-bid auction. Each bidder $i \in N$ has a private valuation $v_i \geq 0$ and knows that the other bidders' valuations are independently drawn from $[0, \bar{v}]$ according to a cumulative distribution function F. All bidders simultaneously submit their bids, and the bidder with the highest bid wins the auction. Ties are broken randomly and with equal probabilities. The winner's payoff is equal to her own valuation minus her bid, whereas the other bidders receive no payoffs.

Consider a bidder $i \in N$ with valuation $v_i \geq 0$ and assume that each bidder $j \in N \setminus \{i\}$ submits her bid following some increasing bid function $\beta_j : [0, \bar{v}] \to [0, \infty)$. We can write that bidder i's bid $b \in [0, \infty)$ turns out to be greater than the bid of some other bidder $j \in N \setminus \{i\}$ with probability $F(\beta_j^{-1}(b))$. Accordingly, bidder i wins the auction with probability $\prod_{j \neq i} F(\beta_j^{-1}(b))$. This situation can be interpreted as if bidder i faces the following lottery[1]:

$$\Big(v_i - b\ ,\ \prod_{j \neq i} F(\beta_j^{-1}(b))\ ;\ 0\ ,\ 1 - \prod_{j \neq i} F(\beta_j^{-1}(b))\Big). \tag{1}$$

Surely, bidder i chooses the bid that induces the lottery with the highest expected utility.

We assume that each bidder's perception of her winning probabilities is in line with the two-stage model proposed by Tversky and Fox (1995): each bidder first judges the probability of the event under consideration (represented by $w_1 : [0,1] \to [0,1]$), after which this probability is transformed by the probability weighting function under risk (represented by $w_2 : [0,1] \to [0,1]$). To be more precise, assuming that a bidder's objective winning probability is $p \in [0,1]$, the bidder misjudges this probability and believes that she will win the auction with a probability of $w_1(p)$; but then she distorts this probability while making her bidding decision and *acts as if* her winning probability is $w_2(w_1(p))$. Utilizing a standard assumption in the literature on subjective probability weighting, which is that $w_1(0) = w_2(0) = 0$ and $w_1(1) = w_2(1) = 1$, the unique symmetric equilibrium can be written as in Proposition 1.

Proposition 1. *In a first-price sealed-bid auction with two-stage probability weighting, the unique symmetric Nash equilibrium is given by*

$$\beta(v_i) = v_i - \frac{\int_0^{v_i} w_2(w_1(F^{n-1}(y)))dy}{w_2(w_1(F^{n-1}(v_i)))}$$

if all bidders (mis-)perceive probabilities according to the same functions, w_1 and w_2.

Proof. The proof follows from Proposition 1 in Keskin (2016). To make the current paper self-contained, we provide a shorter version of the proof in the Appendix. □

In the following section, we provide some experimental support for the two-stage model in first-price sealed-bid auctions.

[1] A lottery is denoted by a tuple of pairs such that the former element of the pair is the outcome and the latter element is the respective probability of occurrence.

3 The Experiment

3.1 Experimental Design

The experiment was conducted at Bilkent University and involved three identical sessions with 24, 24, and 26 subjects. Each session took around 60 minutes. In the experiment we used a so-called *experimental currency (EC)*, with the exchange rate of 1 EC = 4 TRY, which was approximately 2 USD at the time of the experiment. Given this exchange rate, the subjects were paid an average of 14.05 USD.

The experiment consisted of three stages. In the first two stages, our subjects participated in a two-player first-price sealed-bid auction. These two stages differ only in the valuations subjects will possibly be assigned. In the third stage, our subjects faced a lottery framework which is —in essence— equivalent to the auction framework.[2]

In the first two stages, we collected the subjects' bids as well as their guesses about their winning probabilities. Anticipating that winning probabilities would be more transparent to subjects if the number of possible valuations is small, we employed a discrete setting, as in Goeree et al. (2002) and Armantier and Treich (2009b). Throughout the experiment we relied on the strategy method. In the first stage, each subject was given a list of valuations: 0 EC, 2 EC, 4 EC, 6 EC, 8 EC, 11 EC. For each valuation v from this list, each subject was asked to *submit* only one bid b and to *guess* her winning probability under the case in which the valuation v is realized and she bids b. We finalized the stage by randomly assigning a valuation to each subject, which in turn determined the auction outcome.[3] For an example, consider two subjects: i and j. Suppose that they are assigned the valuations v_i and v_j, respectively. If the bid subject i submitted under valuation v_i is greater than the bid subject j submitted under her own valuation v_j, then subject i wins. The winner is paid her own valuation minus her bid: $v_i - b_i$.

In the second stage, subjects were divided into *new* groups of two and were asked to complete the same set of tasks. The only difference from the first stage was the list of possible valuations: 0 EC, 3 EC, 5 EC, 7 EC, 9 EC, 12 EC.

In the third stage, we presented several lists of lotteries to our subjects. Each list is obtained from the auction framework in the first two stages of the experiment. For a concrete example, consider the first stage and a bidder i with valuation 6 EC. Assuming that the other bidder follows the risk neutral Nash equilibrium (RNNE) —which is bidding 0 EC, 1 EC, 2 EC, 3 EC, 4 EC, and 5 EC respectively—, this situation can be interpreted as if bidder i faces the lottery

$$(1 \text{ EC}\ ,\ 0.9167\ \ ;\ \ 0 \text{ EC}\ ,\ 0.0833)$$

if she bids 5 EC; and she faces

$$(2 \text{ EC}\ ,\ 0.7500\ \ ;\ \ 0 \text{ EC}\ ,\ 0.2500), \tag{2}$$

[2]The experimental design follows closely to those of Dorsey and Razzolini (2003) and Armantier and Treich (2009b).

[3]We carefully explained to our subjects that there will be separate randomizations for each subject. Furthermore, we emphasized that they should select their bids carefully, as one of them would directly influence their auction payoffs.

if she bids 4 EC; and so on. For instance, the lottery (2) utilizes the information that bidder i would win the auction if the other bidder submits a bid less than or equal to 4 EC, which occurs only if the other bidder (who bids according to the RNNE) has a valuation less than or equal to 8 EC. We asked our subjects to choose the lottery they prefer the most from each lottery list. The stage is finalized by randomly selecting a lottery list for each subject and by determining the outcome of the lottery the subject chose from that list. The subject is directly paid that outcome.

In theory, individual behavior in the auction framework should be consistent with that in the lottery framework. More precisely, if a subject with valuation 6 EC bids 4 EC in the auction stage, then she should choose the lottery (2) from the corresponding lottery list. If otherwise, this would mean that the subject —for some reason— does not perceive the lottery framework described above when participating the auction. As a matter of fact, we expect to find such differences, and we expect the reason behind those differences to be subjects' (mis-)perceptions of their winning probabilities in first-price auctions.

Table I: Observations in Stages 1 and 2

	Even				
Valuation	2	4	6	8	11
RNNE	1.00	2.00	3.00	4.00	5.00
Mean	1.05	2.50	3.95	5.50	7.53
Std. Dev.	0.28	0.53	0.77	1.10	1.51
Median	1.00	3.00	4.00	6.00	8.00
Mode	1.00	3.00	4.00	6.00	8.00
Obj. Prob. (%)	25.00	41.67	58.33	75.00	91.67
Mean†	29.78	38.69	46.08	52.36	61.57
Std. Dev.	22.08	20.48	18.82	17.95	22.37
Mean‡	30.40	31.71	42.82	44.00	58.84
Std. Dev.	22.11	16.70	19.94	15.95	20.89

	Odd				
Valuation	3	5	7	9	12
RNNE	1.00	2.00	3.00	4.00	5.00
Mean	1.74	3.24	4.74	6.20	8.36
Std. Dev.	0.60	0.64	1.05	1.39	1.83
Median	2.00	3.00	5.00	6.00	9.00
Mode	2.00	3.00	5.00	7.00	9.00
Obj. Prob. (%)	25.00	41.67	58.33	75.00	91.67
Mean†	29.30	36.62	44.04	49.70	59.22
Std. Dev.	22.69	20.22	18.99	20.28	23.57
Mean‡	16.10	33.87	35.02	40.40	53.84
Std. Dev.	10.79	24.11	17.53	19.25	21.74

† The respective objective probabilities are calculated under the assumption that each subject believes that the other bidder plays symmetrically

‡ The respective objective probabilities are calculated under the assumption that each subject believes that the other bidder follows the RNNE

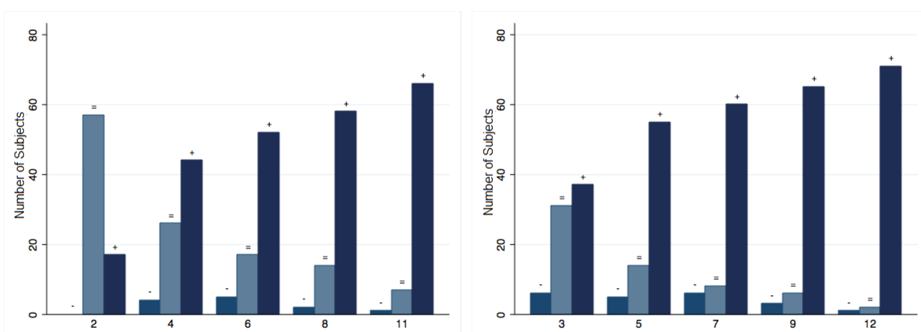
Finally, from another perspective, our experiment can be thought of as a game-theoretic version of the famous Ellsberg experiment in which the unknown urn is endogenously created (see Ellsberg, 1961).

3.2 Experimental Observations

In Table I, we report the bidding behavior and the probability predictions in stages 1 and 2. Our subjects tended to overbid when their valuations are high whereas their bids were in line with the RNNE predictions when their valuations were sufficiently small. In particular, there seems to be a clear overbidding when $v \geq 5$. Furthermore, subjects' guesses about their winning probabilities were significantly different than the objective probabilities.[4]

Armantier and Treich (2009b) argued that underprediction of winning probabilities leads to overbidding, but did not take a stand on the source of the disconnect between the objective and subjective probabilities. We take the analysis one step further. In our experiment, beside participating in the auction, each subject made decisions in a lottery framework obtained directly from the auction framework. In Figure 1, we report the observed lottery behavior as a comparison to the auction behavior. More precisely, in order to compare the lottery behavior with the bidding decisions, we first convert the lottery framework back into the auction framework by mapping every lottery to the corresponding bid amount. Then if a subject bids b when her valuation is v and if the same subject chose the lottery induced by an amount less/greater than b from the lottery list associated with valuation v, we report this switch using a "+"/"−" sign. In short, when subjects' behavior in the lottery framework is considered a baseline, a "+"/"−" sign represents overbidders/underbidders in the auction stages.

Figure 1: Comparison between Stages 1-2 and 3



(a) Even

(b) Odd

Figure 1 indicates that subjects' behavior in the lottery framework is mostly inconsistent with that in the auction framework. For instance, for each $v \geq 3$, most subjects chose a

[4] We employ two methods for the calculation of objective probabilities: (*i*) Each subject believes that the other bidder plays symmetrically. This means that each subject believes that she would win the auction if the other bidder ends up with a lower valuation, so that the subject's bids have no effect on the objective probabilities. (*ii*) Each subject believes that the other bidder follows the RNNE. This means that each subject believes that she could win the auction even when the other bidder ends up with a higher valuation. Accordingly, the subject's valuation has no effect on the objective probabilities.

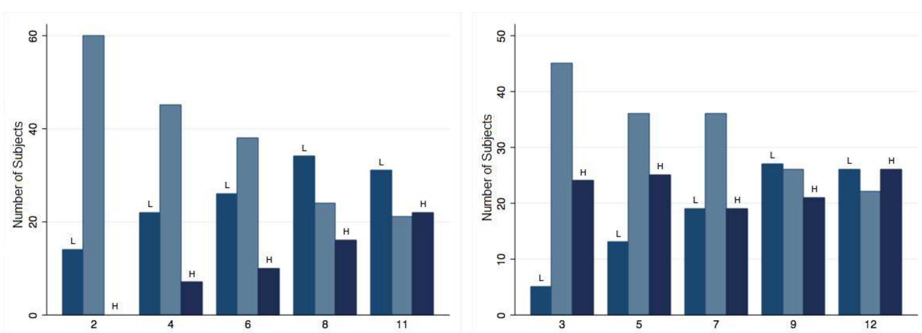
lottery induced by a bid amount lower than her actual bid in the auction framework. This highlights that when the objective probabilities were known to our subjects, their tendency to overbid was essentially suppressed.

Table II: Observations in Stage 3

	Even				
Valuation	2	4	6	8	11
Optimal Choice	1.00	2.00	3.00	4.00	5.00
Mean	0.81	1.70	2.64	3.54	4.51
Std. Dev.	0.39	0.77	1.12	1.38	1.43
Median	1.00	2.00	3.00	4.00	5.00
Mode	1.00	2.00	3.00	4.00	6.00
	Odd				
Valuation	3	5	7	9	12
Optimal Choice	1.00	2.00	3.00	4.00	5.00
Mean	1.25	2.17	2.97	3.71	4.52
Std. Dev.	0.57	0.93	1.32	1.51	1.64
Median	1.00	2.00	3.00	4.00	5.00
Mode	1.00	2.00	3.00	4.00	6.00

Using the observed probability predictions in stages 1 and 2, one can identify the lottery framework our subjects believed they were facing. Though, doing that one cannot *completely* explain the observations reported above. This indicates that there must be additional factors behind overbidding, and we relate those other factors to the second stage of the Tversky-Fox two-stage model.

Figure 2: Observations in Stage 3



(a) Even (b) Odd

Note: For a given valuation, if the theoretically optimal lottery is induced by the bid amount b, then L/H represents a subject who chose a lottery induced by b′ lower/higher than b.

As a matter of fact, since the objective probabilities were known in stage 3, subjects' behavior in the lottery framework can be regarded as another indicator for those other factors. Considering the mean and median values reported in Table II, subjects' behavior in this stage is quite different from that in the auction stages reported in Table I; and it is indeed very close to the theoretical predictions. However, we must also emphasize that Table II might be misleading. As further illustrated in Figure 2, especially when subjects faced a lottery list associated with a high valuation (e.g., $v \geq 8$), they struggled to choose the theoretically optimal lottery.

To sum up, our subjects overbid in the auction stages as they perceived their winning probabilities differently; yet even when the objective probabilities were explicitly presented, they still struggled to make the optimal choice, especially under high valuations. These observations are respectively related to the first (i.e., uncertainty) and second (i.e., risk) stages of the Tversky-Fox two-stage model. Accordingly, it can be claimed that such a two-stage model is able to explain the observed behavior by a proper selection of the probability misperception and probability weighting functions. Further investigation is surely necessary for the elicitation of those functions.

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Appendix

Proof of Proposition 1: We analyze symmetric Nash equilibrium. For bidder i with valuation $v_i \in [0, \bar{v}]$, the expected utility of bidding b when all the other bidders follow the *same* bidding function β is

$$w_2(w_1(F^{n-1}(\beta^{-1}(b))))(v_i - b) \ .$$

Bidder i aims to maximize this expected utility. The corresponding first order condition with respect to b is

$$\frac{\partial w_2(w_1(F^{n-1}(\beta^{-1}(b))))}{\partial \beta^{-1}(b)} \frac{\partial \beta^{-1}(b)}{\partial b}(v_i - b) - w_2(w_1(F^{n-1}(\beta^{-1}(b)))) = 0 \ .$$

As we search for symmetric equilibrium, $b = \beta(v_i)$ should be the maximizer of the objective function; that is, $b = \beta(v_i)$ should solve the equation above. Thus,

$$\frac{\partial w_2(w_1(F^{n-1}(v_i)))}{\partial v_i} \frac{1}{\beta'(v_i)}(v_i - \beta(v_i)) = w_2(w_1(F^{n-1}(v_i))) \ .$$

After arranging terms, one can obtain

$$\beta(v_i) = v_i - \frac{\int_0^{v_i} w_2(w_1(F^{n-1}(y)))dy}{w_2(w_1(F^{n-1}(v_i)))} \ .$$

To verify that β is indeed an equilibrium bidding function, one can show that it is increasing in v_i and that bidding above $\beta(\bar{v})$ is dominated for bidder i. Finally, suppose that bidder i bids as if her valuation is $z \in [0, \bar{v}]$ rather than v_i. By showing that $z = v_i$ is a best response in such a scenario, one can find that the bidding function β is the unique symmetric Nash equilibrium. $\square$

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