



## Double branch outage modeling and simulation: Bounded network approach



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### ABSTRACT

Energy management system operators perform regular outage simulations in order to ensure secure operation of power systems. AC power flow based outage simulations are not preferred because of insufficient computational speed. Hence several outage models and computational methods providing acceptable accuracy have been developed. On the other hand, double branch outages are critical rare events which can result in cascading outages and system collapse. This paper presents a double branch outage model and formulation of the phenomena as a constrained optimization problem. Optimization problem is then solved by using differential evolution method and particle swarm optimization algorithm. The proposed algorithm is applied to IEEE test systems. Computational accuracies of differential evolution based solutions and particle swarm optimization based solutions are discussed for IEEE 30 Bus Test System and IEEE 118 Bus Test System applications. IEEE 14 Bus Test System, IEEE 30 Bus Test System, IEEE 57 Bus Test System, IEEE 118 Bus Test System and IEEE 300 Bus Test System simulation results are compared to AC load flows in terms of computational speed. Finally the performance of the proposed method is analyzed for different outage configurations.

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### Introduction

Electric power systems have gone to restructuring process during the last two decades. One of the major challenges of the near future is the evolution to “smarter networks”. With this new evolution idea, measurement, control, protection and communication tasks have been improved and distributed generation facilities have been extended.

Since the outage of the components in this smart environment can cause significant problems, power system operators need to pre-simulate all possible contingencies. Those simulations provide the estimation of post outage voltage magnitudes and power flows by which they take the remedial actions on time. This can be achieved by resolving the AC load flow problem separately for each outage [1].

However it is a time-consuming process even for a moderate power system comprising hundreds of components. Therefore fast and accurate models have been developed for contingency analysis. DC load flow [2] was fast enough but it could not handle

reactive power flows. Other methods [3–5] suffered from insufficient accuracy due to using linearized models. On the other hand, Taylor series based methods required large number of iterations to converge [6].

Power injection and solution by sensitivity matrices were used in [7,8]. Simulation results of [7] showed that the proposed method did not provide accurate results for voltage magnitudes and reactive power flows. [8] was not fast enough for real time operation. One recent paper solved the line outage problem using piecewise linear estimates [9]. A faster and more accurate model was developed in [10] where the line outage phenomena was simulated by inserting two fictitious sources, and post outage state calculation was formulated as a local constrained optimization problem. Bus voltage magnitudes were initially determined solving linearized reactive power equations and they were later improved by a local optimization process. Since, the model used only limited number of network variables, it was fast enough for real time applications providing better accuracy than the traditional methods [10]. Single branch outage problem was later solved by genetic algorithms [11], by particle swarm optimization method [12], by differential evolution method and by harmony search method [13].

The number of outages in a contingency analysis is proportional with the number of branches for single branch outages whereas it

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is proportional with the square of the number of branches for double branch outages [14]. Computation speed, therefore, becomes more important for double branch outages. Multiple line outages were simulated in [8]. However, there were only a few "non-problematic" examples given for IEEE 118 Bus Test system.

Double branch outage modeling was initially proposed in [15], and solved by using differential evolution method. Later, it was solved by harmony search method [16]. This paper presents modeling of double branch outages by extending our previous work, formulating the process as a local constrained optimization problem and solution of the problem by differential evolution (DE) method and by particle swarm optimization (PSO) method. The validity of the proposed formulation is tested on IEEE 30 Bus Test System and on IEEE 118 Bus Test System. Post-outage voltage magnitudes calculated using the proposed formulation are compared with those of conventional full AC load flow results from the point of computational accuracy. Critical busses and outage types giving high computational bus voltage magnitude errors are identified and the reasons are criticized. Finally, comparisons are performed from the point of solution speed for several IEEE test systems.

The rest of the paper is organized as follows. In the second section, double line outage model is introduced. Adaptation of DE algorithm and PSO algorithm for double branch outage problem is briefly explained in the third section. Fourth section of the study illustrates the simulation results for several IEEE test systems. Fifth section concludes the paper.

### Double branch outage simulation

Single branch outage model introduced in [10] was selected as a starting point of modeling double branch outages. Assume that the branch between busses  $i$  and  $j$ , and the branch between busses  $k$  and  $l$  are simultaneously outaged. The first branch outage and the second branch outage are simulated using fictitious pairs  $Q_{si} - Q_{sj}$ , and  $Q_{sk} - Q_{sl}$  respectively. Double branch outage simulation is shown in Fig. 1.

The proposed double branch outage model is formulated for the union of the individual (marginal) bounded regions of the outaged branches. That is, it enforces all fictitious source reactive powers to circulate in the bounded region. Consequently, load bus voltage magnitudes of the busses in this union region are the parameters those will be optimized during the optimization cycle. Optimization cycle aims to minimize the additional reactive power flows between the bounded region and the remaining part of the network; that is, enforces all reactive power of the fictitious sources to circulate in the bounded region.

The steps of double branch outage solution can be given as follows [15],

- Select two lines to be outaged and assign them as:  $ij$  and  $kl$ .
- Compute bus voltage phase angles using the linearized active power equations as shown below.

$$\delta_m = \delta_m + (X_{mi} - X_{mj}) \Delta P_n + (X_{mk} - X_{ml}) \Delta P_r$$

$$m = 2, 3, \dots, NB$$

$$\Delta P_n = \frac{P_{ij}}{[1 - (X_{ii} + X_{jj} - 2X_{ij})/x_n]} \quad (1)$$

$$\Delta P_r = \frac{P_{kl}}{[1 - (X_{kk} + X_{ll} - 2X_{kl})/x_r]}$$

where,  $X_{ij}$  represents the  $i$ th row,  $j$ th column entry of the bus susceptance matrix,  $P_{ij}$  and  $P_{kl}$  are the pre-outage active powers flowing through the outaged branches, and  $x_n$  and  $x_r$  represent the reactances of the branches.

- Calculate the loss reactive power components,  $\tilde{Q}_{Li} \cong \tilde{Q}_{Lj}$ ,  $\tilde{Q}_{Lk} \cong \tilde{Q}_{Ll}$ , those will be used during the optimization.
- Minimize reactive power mismatches at busses  $i, j, k$  and  $l$ . This process is mathematically formulated by the following constrained optimization problem.

$$\begin{aligned} \min_{wrt Q_{si}, Q_{sk}} & \|Q_i - (\bar{Q}_{ij} + \bar{Q}_{Li}) + Q_{Di} \\ & Q_j - (-\bar{Q}_{ij} + \bar{Q}_{Li}) + Q_{Dj} \\ & Q_k - (\bar{Q}_{kl} + \bar{Q}_{Lk}) + Q_{Dk} \\ & Q_l - (-\bar{Q}_{kl} + \bar{Q}_{Li}) + Q_{Dl} \| \\ \text{subject to} & \quad g_q(\mathbf{V}_b) = \Delta \mathbf{Q}_b - \mathbf{B}_b \Delta \mathbf{V}_b = 0 \end{aligned} \quad (2)$$

where,  $\|\cdot\|$  is the Euclidean norm of the reactive power mismatch vector. Equality constraints of (2) are linearized reactive power equations for the load busses,  $\Delta \mathbf{Q}$  is the reactive power mismatch vector,  $\mathbf{V}$  is the load bus voltage magnitude vector and  $\mathbf{B}$  is the bus susceptance matrix. The subscript  $b$  signifies the variables included in the bounded region.

### DE and PSO algorithms for double branch outage problem solution

DE was first introduced by Storn and Price [17,18] and is a stochastic direct search optimization method. It is a population-based solution algorithm and uses the conventional operators of evolutionary algorithms. DE has been applied to several power system problems, such as, economic dispatch problem [19], power system planning [20], transient stability constrained optimal power flow [21], generation expansion planning [22], and unit commitment [23].

PSO was first introduced by Kennedy and Eberhart [24], mimicking the swarm behaviors of fish schools and birds. It was widely used in power system applications; such as, economic dispatch problem [25,26], transmission network expansion problem [27], and optimal load flow problem [28].

DE and PSO based solution procedure of double branch outage problem consists of two main stages. The first one includes the common steps for both methods; whereas the second one includes the individual steps of the methods.

#### Common steps of PSO and DE based double branch outage problem solution

1. Perform a power flow and determine pre-outage (base case) bus voltage magnitudes.
2. Create a random matrix,  $\mathbf{A}$  whose dimensions is  $N_p \times 2$ . The first column and the second column entries of  $\mathbf{A}$  are in the range of  $[Q_{ij} - \omega \quad Q_{ij} + \omega]$  and  $[Q_{kl} - \omega \quad Q_{kl} + \omega]$  respectively.  $\omega$  is a user defined parameter corresponding to half of the initial solution range.
3. Let the set of busses included in the bounded regions of the first outaged branch and the second outaged branch named as;  $BR_1$  and  $BR_2$ , respectively. The set of the busses included either in  $BR_1$  or in  $BR_2$  will constitute the bounded region of double branch outage:
 
$$BR = BR_1 \cup BR_2 \quad (3)$$
4. Perform the following computations for each entry of the first column of  $\mathbf{A}$ .

$$\begin{aligned} \Delta \mathbf{Q}_1 &= [0, \dots, A_{(1,i)}, \dots, A_{(1,j)}, \dots, 0]^T \\ \Delta \mathbf{Q}_1 &= [0, \dots, A_{(1,i)}, \dots, \bar{A}_{(1,i)}, \dots, 0]^T \\ \bar{A}_{(1,i)} &= -A_{(1,i)} + 2Q_{L1i} \end{aligned} \quad (4)$$

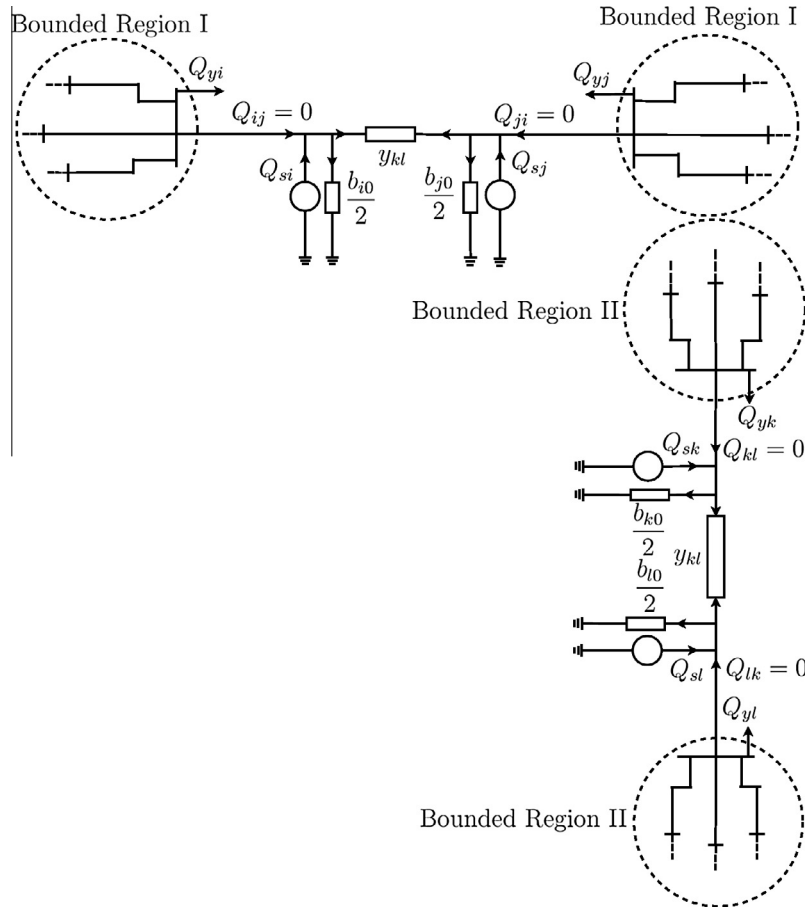


Fig. 1. Double branch outage simulation using fictitious reactive power sources.

$$\begin{aligned} \Delta \mathbf{V}_{BR} &= \mathbf{B}_{BR}^{-1} \Delta \mathbf{Q}_1 \\ \mathbf{V}_{BR_1} &= \mathbf{V}_{BR} + \Delta \mathbf{V}_{BR} \end{aligned} \quad (5)$$

Similarly, perform the following computations for each entry of the second column of  $\mathbf{A}$ ,

$$\begin{aligned} \Delta \mathbf{Q}_2 &= [0, \dots, A_{(2,k)}, \dots, A_{(2,l)}, \dots, 0]^T \\ \Delta \bar{\mathbf{Q}}_2 &= [0, \dots, A_{(2,k)}, \dots, \bar{A}_{(2,k)}, \dots, 0]^T \\ \bar{A}_{(2,k)} &= -A_{(2,k)} + 2Q_{L2k} \end{aligned} \quad (6)$$

$$\begin{aligned} \Delta \mathbf{V}_{BR} &= \mathbf{B}_{BR}^{-1} \Delta \mathbf{Q}_2 \\ \mathbf{V}_{BR_2} &= \mathbf{V}_{BR_1} + \Delta \mathbf{V}_{BR} \end{aligned} \quad (7)$$

5. Compute the objective function for the new bus voltage magnitudes obtained in step 4.

#### DE algorithm for double branch outage problem solution

6. Perform the following steps until the stopping criterion is met.

- Add the weighted sum of the two rows of  $\mathbf{A}$  to the third one. Do the similar linear combination for all elements in the population and create a new mutant matrix.

$$A_{i,:}^{(G)} = A_{(r3,:)}^{(G)} + F(A_{(r1,:)}^{(G)} - A_{(r2,:)}^{(G)}) \quad (8)$$

where,  $i \neq r_1 \neq r_2 \neq r_3$ , and  $r_1, r_2$  and  $r_3$  are random numbers between 1 and  $N_p$ , and  $F$  is a positive parameter that scales the difference vector.

- Create a trial matrix  $\mathbf{T}$  by the following perturbations.
  - Randomly select a number from the interval  $[1, N_p]$  for each individual in the population.
  - If this number is equal to the population index or smaller than another random number  $q$ , then the mutant element is inserted in the trial matrix otherwise corresponding element from matrix  $\mathbf{A}$  is inserted in the trial matrix

• Perform the following computations for each element of the first column of  $\mathbf{T}$ .

$$\begin{aligned} \Delta \mathbf{Q}_1 &= [0, \dots, T_{(1,i)}, \dots, T_{(1,j)}, \dots, 0]^T \\ \Delta \bar{\mathbf{Q}}_1 &= [0, \dots, T_{(1,i)}, \dots, \bar{T}_{(1,i)}, \dots, 0]^T \\ \bar{T}_{(1,i)} &= -T_{(1,i)} + 2Q_{L1i} \end{aligned} \quad (9)$$

$$\begin{aligned} \Delta \mathbf{V}_{BR} &= \mathbf{B}_{BR}^{-1} \Delta \mathbf{Q}_1 \\ \mathbf{V}_{BR_1} &= \mathbf{V}_{BR} + \Delta \mathbf{V}_{BR} \end{aligned} \quad (10)$$

Similarly, perform the following computations using the second column elements of  $\mathbf{T}$  matrix,

$$\begin{aligned} \Delta \mathbf{Q}_2 &= [0, \dots, T_{(2,k)}, \dots, T_{(2,l)}, \dots, 0]^T \\ \Delta \bar{\mathbf{Q}}_2 &= [0, \dots, T_{(2,k)}, \dots, \bar{T}_{(2,k)}, \dots, 0]^T \\ \bar{T}_{(2,k)} &= -T_{(2,k)} + 2Q_{L2k} \end{aligned} \quad (11)$$

$$\begin{aligned} \Delta \mathbf{V}_{BR} &= \mathbf{B}_{BR}^{-1} \Delta \mathbf{Q}_2 \\ \mathbf{V}_{BR_2} &= \mathbf{V}_{BR_1} + \Delta \mathbf{V}_{BR} \end{aligned} \quad (12)$$

- Determine whether the trial elements will be included in the new generation or not. This is done by computing the objective functions for the new bus voltage magnitudes and comparing them with the objective functions corresponding to the trial matrix elements. If the objective function for the trial matrix elements (row) is better, then corresponding row of  $\mathbf{A}$  is replaced by trial matrix elements.
7. Stop the algorithm if a predefined stopping criterion is met, otherwise go to step 2.

#### PSO algorithm for double branch outage problem solution

Steps of PSO algorithm to solve the double branch outage problem are as follows.

6. Perform the following steps until the stopping criterion is met.
- Compare the objective functions corresponding to each individual of the population with the local best values which were initially assigned as too big numbers. Replace the local best value with the cost function if the cost function is less than the local best. Assign the local best position, as the index of the new individual providing better objective function.
  - Compare the local best with the global best which was initially assigned as a too big value. If the minimum cost function is smaller than the global best value, replace the global best with this new cost function and assign the corresponding element of  $\mathbf{A}$  as the global best position.
  - Let,  $r_1$  and  $r_2$  be 2 random numbers between 0 and 1,  $C_1 = C_2 = 1$ ,  $w_{max} = 0.9$ ,  $w_{min} = 0.4$   $w = w_{max} - \frac{(w_{max} - w_{min})}{iter_{max}} \times iter_{no}$ . Update the velocity vector and  $\mathbf{A}$  as follows.

$$v(i, j) = w \times v(i, j) + C_1 \times r_1 (\text{local best position}(i, j) - A(i, j)) + C_2 \times r_2 (\text{global best position}(j) - A(i, j)) \quad (13)$$

$$A(i, j) = A(i, j) + v(i, j) \\ i = 1, \dots, N_p \quad j = 1, 2 \quad (14)$$

- Solve the following equations by using the first column entries of  $\mathbf{A}$ .

$$\Delta \mathbf{Q}_1 = [0, \dots, A_{(1,i)}, \dots, A_{(1,j)}, \dots, 0]^T \\ \overline{\Delta \mathbf{Q}_1} = [0, \dots, A_{(1,i)}, \dots, \overline{A_{(1,i)}}, \dots, 0]^T \\ \overline{A_{(1,i)}} = -A_{(1,i)} + 2Q_{L1i} \quad (15)$$

$$\Delta \mathbf{V}_{BR} = \mathbf{B}_{BR}^{-1} \Delta \mathbf{Q}_1 \\ \mathbf{V}_{BR_1} = \mathbf{V}_{BR} + \Delta \mathbf{V}_{BR} \quad (16)$$

Similarly, perform the following computations by using the second column entries of  $\mathbf{A}$ .

$$\Delta \mathbf{Q}_2 = [0, \dots, A_{(2,k)}, \dots, A_{(2,l)}, \dots, 0]^T \\ \overline{\Delta \mathbf{Q}_2} = [0, \dots, A_{(2,k)}, \dots, \overline{A_{(2,k)}}, \dots, 0]^T \\ \overline{A_{(2,k)}} = -A_{(2,k)} + 2Q_{L2k} \quad (17)$$

$$\Delta \mathbf{V}_{BR} = \mathbf{B}_{BR}^{-1} \Delta \mathbf{Q}_2 \\ \mathbf{V}_{BR_2} = \mathbf{V}_{BR_1} + \Delta \mathbf{V}_{BR} \quad (18)$$

- Compute the objective functions for the new bus voltage magnitudes.
7. Stop the algorithm if a predefined stopping criterion is met, otherwise algorithm go to step 2.

#### Tests and results

The proposed formulation and DE and PSO based post outage voltage magnitude calculations were tested on IEEE test systems. 30-Bus and 118-Bus test system results were illustrated for accuracy comparisons; whereas IEEE 14, 30, 57, 118 and 300 test system results were illustrated for computational speed comparisons.

Open source electrical power system package Matpower [29] and Matlab were used as computation tools. All simulations were run on a laptop that had a 2.20 GHz Core Duo CPU, and 2.00 GB Memory. DE parameters were selected as: population size = 15, scaling factor  $F = 1.8$ , and crossover rate  $CR = 0.9$ . PSO parameters were selected as: population size = 15, initial/final inertia weights 0.9/0.4, cognitive parameter  $c_1 = 2$  social parameter  $c_2 = 2$ . Maximum number of iterations was selected as 100 and optimization cycles were terminated when the absolute difference of the two successive solutions was less than 0.05 for both methods.

Branch outage simulations were performed for all possible double contingencies except those creating either any convergence problems or islanding conditions and except those resulting post-outage voltage magnitudes less than 0.8 p.u. 260, 1214, 4148, 15,312 and 87,614 double branch outages were simulated for IEEE 14 Bus, IEEE 30 Bus, IEEE 57 Bus, IEEE 118 Bus and IEEE 300 Bus test systems, respectively.

There were two different double branch outage configurations with respect to contents of marginal regions.

1. The set of busses included in the marginal bounded regions were mutually exclusive; i.e. there were no common busses in the two bounded regions.
2. There were some common busses in the bounded regions.

On the other hand, outaged branches could be classified into three groups with respect to outaged branch types:

- (a) Both of the branches were transmission lines/cables,
- (b) One of the outaged branch was a tap-changing transformer,
- (c) Both of the outaged branches were tap-changing transformers.

The number of simulations for test systems were given above. However, the results of some representative ones will only be shown in the following tables in order to limit the length of the paper. Note that, illustrated values are actually the arithmetic means of the results obtained from 500 consecutive simulations for each double branch outage.

Simultaneous outages of line 3–4 and line 21–22 in IEEE 30 Bus Test system was an example for 1(a)-type double line outage. Simulation results are shown in Table 1 together with full AC-load flow results and corresponding percentage bus voltage magnitude errors (PBVMEs). Note that critical busses showing

**Table 1**  
Post-outage voltage magnitudes for line 3–4/line 21–22 outage in IEEE-30 Bus Test System.

Bus No	$V_{(AC)}$	$V_{(DE)}$	Error DE %	$V_{(PSO)}$	Error PSO %
6	1.0343	1.0349	0.0580	1.0349	0.0580
7	1.0331	1.0336	0.0484	1.0336	0.0484
12	1.0525	1.0530	0.0475	1.0530	0.0475
21	1.0334	1.0328	0.0581	1.0329	0.0484
22	1.0394	1.0400	0.0577	1.0399	0.0481
24	1.0281	1.0286	0.0486	1.0286	0.0486
28	1.0336	1.0340	0.0387	1.0341	0.0484
Max. error	–	–	0.0581	–	0.0580

0.04% or higher PBVMEs are reported in the table. Bus No. represents the bus number,  $V_{(AC)}$ ,  $V_{(DE)}$  and  $V_{(PSO)}$  denote the post-outage bus voltage magnitudes calculated by full AC load flow, by DE based solution of the proposed formulation and by PSO based solution of the proposed formulation, respectively. Error DE % and Error PSO % represent the PBVME of DE based solution and PSO based solution, respectively.

Simultaneous outages of transformer 6–9 and line 14–15 in IEEE 30 Bus Test system was an example for 1(b)-type transformer-line outage. Table 2 illustrates the simulation results for this double branch outage. Note that the busses showing 0.3% or higher PBVMEs are reported in the table. There was not a 1(c) type double transformer outage for IEEE 30 Bus Test system.

Simultaneous outages of line 19–20 and line 16–17 was an example of 2(a)-type double line outage. Table 3, illustrates the post-outage bus voltage magnitudes for this double line outage. Note that the busses showing 0.1% or higher PBVMEs are reported in the table.

Outages of transformer 4–12 and line 10–22 was an example of 2(b)-type transformer-line outages for IEEE 30 Bus Test system. Table 4, illustrates the results for this transformer-line outage for the busses showing 0.6% or higher PBVMEs.

Outages of transformer 28–27 and transformer 6–10 was an example of 2(c)-type double transformer outage for IEEE 30 Bus Test system. Table 5, illustrates the simulation results for this double-transformer outage for the busses showing 0.6% or higher bus voltage magnitude errors.

One can easily realize from Tables 1–5 that both DE based solutions and PSO based solutions of the proposed constrained optimization formulation provide satisfactory results in terms of post-outage voltage magnitude accuracies. Among them, DE based solutions seem to be slightly better than the PSO based solutions. Illustrated results show that type-2 outages where the marginal bounded regions include common branches, are more critical from the point of accuracy.

It is clear that both the stopping criteria and the maximum number of iterations,  $N_{max}$ , affect the accuracy and the solution speed of the simulations. In order to quantify these affects, simulations were performed for IEEE 118 Bus test system where the stopping criteria was selected as not changing more than a specified value along  $N_k$  successive evolutions. Mean PBVME and its standard deviation for PSO based solutions with  $N_{max} = 100$  and  $N_k = 10$  were found to be 0.8724 and 1.64, respectively. Mean PBVME and its standard deviation for DE based solutions for the same  $N_{max}$  and  $N_k$  values were found to be 0.8518 and 1.51, respectively for the same. If  $N_k$  is decreased to 5, mean PBVME and its standard deviation for PSO based solutions increased to 1.16 and 2.37 respectively. Similarly, decreased  $N_k$  value, increased mean PBVME and its standard deviation for DE based solutions to 1.52 and 3.63, respectively.

Tables 6 and 7 illustrate the solution speeds of AC load flow solutions and evolutionary method based solutions of the proposed formulation for two different  $N_{max}$  and  $N_k$  pairs.

**Table 2**  
Post-outage voltage magnitudes for transformer 6–9/line 14–15 outage in IEEE-30 Bus Test System.

Bus No	$V_{(AC)}$	$V_{(DE)}$	Error DE %	$V_{(PSO)}$	Errors PSO %
10	1.0410	1.0448	0.3650	1.0448	0.3650
17	1.0358	1.0391	0.3186	1.0390	0.3089
20	1.0255	1.0286	0.3023	1.0286	0.3023
21	1.0294	1.0331	0.3594	1.0331	0.3594
22	1.0303	1.0338	0.3397	1.0338	0.3397
Max. error	–	–	0.3650	–	0.3650

**Table 3**  
Post-outage voltage magnitudes for line 19–20/line 16–17 outage in IEEE-30 Bus Test System.

Bus No	$V_{(AC)}$	$V_{(DE)}$	Error DE %	$V_{(PSO)}$	Error PSO %
15	1.0282	1.0269	0.1264	1.0268	0.1362
16	1.0448	1.0470	0.2106	1.0469	0.2010
17	1.0464	1.0448	0.1529	1.0450	0.1338
18	1.0046	1.0002	0.4380	0.9999	0.4678
19	0.9941	0.9875	0.6639	0.9871	0.7042
20	1.0505	1.0525	0.1904	1.0527	0.2094
23	1.0244	1.0234	0.0967	1.0233	0.1074
Max. error	–	–	0.6639	–	0.7042

**Table 4**  
Post-outage voltage magnitudes for transformer 4–12/line 10–22 outage in IEEE-30 Bus Test System.

Bus No	$V_{(AC)}$	$V_{(DE)}$	Error DE %	$V_{(PSO)}$	Error PSO %
16	1.0288	1.0223	0.6318	1.0224	0.6221
21	1.0286	1.0195	0.8847	1.0195	0.8847
22	1.0273	1.0172	0.9832	1.0172	0.9832
24	1.0156	1.0082	0.7286	1.0082	0.7286
25	1.0281	1.0209	0.7003	1.0209	0.7003
26	1.0106	1.0034	0.7124	1.0034	0.7124
27	1.0442	1.0372	0.6704	1.0372	0.6704
29	1.0248	1.0178	0.6831	1.0178	0.6831
30	1.0136	1.0066	0.6906	1.0066	0.6906
Max. error	–	–	0.9832	–	0.9832

**Table 5**  
Post-outage voltage magnitudes for transformer 28–27/transformer 6–10 outage in IEEE-30 Bus Test System.

Bus No	$V_{(AC)}$	$V_{(DE)}$	Error DE %	$V_{(PSO)}$	Error PSO %
21	1.0088	1.0152	0.6344	1.0152	0.6344
22	1.0078	1.0144	0.6549	1.0144	0.6549
23	0.9958	1.0021	0.6327	1.0021	0.6327
24	0.9737	0.9833	0.9859	0.9833	0.9859
25	0.9139	0.9303	1.7945	0.9303	1.7945
26	0.8942	0.9121	2.0018	0.9121	2.0018
27	0.8888	0.9069	2.0365	0.9068	2.0252
29	0.8656	0.8880	2.5878	0.8880	2.5878
30	0.8521	0.8774	2.9691	0.8774	2.9691
Max. error	–	–	2.9691	–	2.9691

**Table 6**  
Average double branch outage simulation time per simulation (cpu-second) for  $N_{max}=100, N_k = 10$ .

Test system	DE	PSO	AC
IEEE 14 Bus	0.0310	0.0373	0.0107
IEEE 30 Bus	0.0320	0.0353	0.0133
IEEE 57 Bus	0.0284	0.0336	0.0175
IEEE 118 Bus	0.0276	0.0264	0.0204
IEEE 300 Bus	0.0308	0.0309	0.0439

**Table 7**  
Average double branch outage simulation time per simulation (cpu-second) for  $N_{max} = 100, N_k = 5$ .

Test System	DE	PSO	AC
IEEE 14 Bus	0.0171	0.0210	0.0107
IEEE 30 Bus	0.0176	0.0205	0.0133
IEEE 57 Bus	0.0159	0.0189	0.0175
IEEE 118 Bus	0.0155	0.0157	0.0204
IEEE 300 Bus	0.0179	0.0186	0.0439



It is clear from the tables that the simulation time for the proposed formulation does not depend on the system size. The proposed method and optimization cycle is confined in a bounded region which comprises terminal busses of the outaged lines and their first order neighbors. Therefore, simulation time is mainly affected by the size of the bounded region rather than the size of the system. Note that average simulation time of a double branch outage for 118 Bus test system is less than those for the smaller size systems since it includes smaller bounded regions. One can also conclude that the proposed formulation is effective for large scale systems comprising 300 or more busses. On the other hand, decreased values of  $N_k$  improves the solution speed while increasing the PBVME.

Since DE based solutions provided slightly better results both from the point of accuracy and from the point of computational speed, the remaining simulation results will be given only for DE based solutions.

Figs. 2 and 3 show simulation results of all possible double branch outages by using the proposed method and full AC load flow. From the figures it is obvious that the simulation results of the proposed method are in concordance with the simulation results of full AC load flow. Performance of the concordance can be measured by the concentration of the points along  $y = x$  line. In other words, deviations from  $y = x$  line represent the errors of the proposed method. In fact, the aim of contingency analysis is to minimize the number of missing critical contingencies. The second important point is to minimize the number of missing alarms. There are 4 missed double branch outages from 1214, and 4 missed double branch outages from 15,312, for IEEE 30 Bus Test System and IEEE 118 Bus Test System, respectively. Total number of limit violations are found to be 67 from 36,420 ( $1214 \times 30$ ) magnitudes and 13 from 1,806,816 ( $15,312 \times 118$ ) voltage magnitudes for IEEE 30 Bus Test System and IEEE 118 Bus Test System, respectively. This shows that the proposed method provide better results for large scale systems not only from the computational speed but also from the point of computational accuracy.

Maximum PBVMEs are low enough providing satisfactory accuracy almost all of the simulated double line outages. For IEEE 30 Bus Test system, mean value and the standard deviation of the highest PBVMEs were computed as 0.873% and 1.792, respectively. For IEEE 118 Bus Test system they were found as 0.425% and 0.674% respectively.

The number of critical double branch outage simulations giving 5% or greater bus voltage magnitude errors were 10 and 21 for IEEE 30 Bus Test system and for IEEE 118 Bus Test system, respectively. Tables 8 and 9 illustrate those critical outages and corresponding

PBVMEs. They were generally 2-type outages where the marginal bounded regions of the outaged pairs include common bus(es).

The highest maximum PBVME for IEEE 30 Bus test system was found for the simultaneous outages of line 8–28 and transformer 28–27 (Table 8). Bus-28 was the common bus for this 2-b type outage pair and switching off two branches from the same bus increased the severity of the outage. On the other hand, bounded regions of the outaged branches were {6, 8, 27, 28} and {6, 8, 25, 27, 28, 29, 30}, respectively. That is, the first bounded region is a subset of the second one, and therefore their union bounded region is the second bounded region. Such a circumstances decrease the computational efficiency of the proposed method. The resulting post-outage voltage magnitude was calculated with an high error.

Similar case occurred for the simultaneous outages of the lines 10–21 and 10–22. As in the previous case, Bus-10 was a common bus for this 2-a type outage and lack of two branches from the same bus created a worse contingency. Consequently, post-outage voltage magnitude of bus-21 decreased to 0.87 p.u. Two bounded regions for this case were: {6, 9, 10, 17, 20, 21, 22} and {6, 9, 10, 17, 20, 21, 22, 24}, respectively. The second bounded region again covered the first one and decreased the computational efficiency of the proposed method. Due to the same reasons, post-outage voltage magnitude of bus-21 was calculated with a high error.

Another double line outage resulting high voltage magnitude errors was the simultaneous outage of the lines 6–28 and 8–28. It was again 2a-type contingency where bus-28 was a common terminal for both outaged lines. Due to the same reasons, actual post-outage voltage magnitude of bus-30 decreased to 0.86 p.u. Bounded regions for this double line outage were: {6, 8, 27, 28} and {2, 4, 6, 7, 8, 9, 10, 27, 28} respectively, and similarly the first bounded region was the subset of the second one. Therefore post-outage voltage magnitude of bus-30 included a high error of 17.23%.

Simultaneous outage of line 6–28 and transformer 28–27 was another 2b-type critical contingency, where Bus-28 was again a common bus of the two outaged branches. Post-outage voltage magnitude of bus-30 was therefore 0.86. Bounded regions for this case were: {2, 4, 6, 7, 8, 9, 10, 27, 28} and {6, 8, 25, 27, 28, 29, 30} respectively. None of the bounded regions was a subset of the other one. However, there were so many common branches which decreased the computational efficiency of the method. The resulting DE based PBVME was high.

Simultaneous outages of the branch pairs [(9, 10), (28, 27)], [(6, 28), (6 – 8)], [(12, 15), (12 – 14)], [(9, 10), (6 – 10)] and [(4, 6), (6 – 8)] were similar 2-type contingencies. The remaining outage

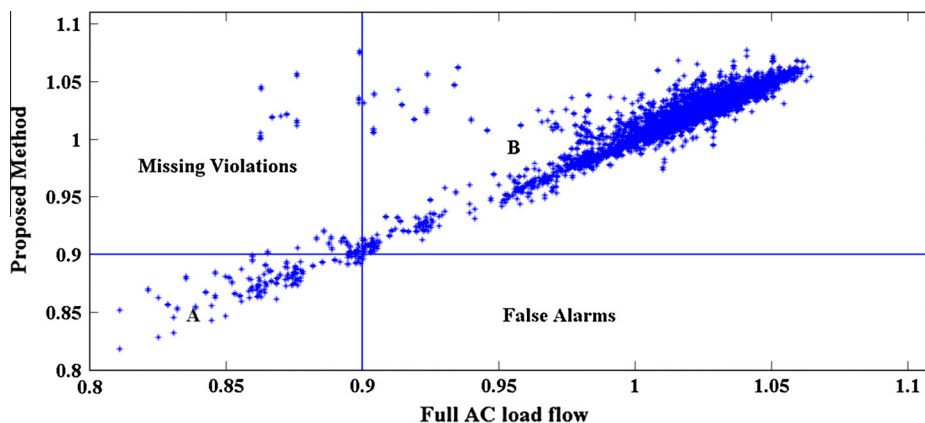


Fig. 2. IEEE 30 Bus Test System, double branch outage simulation results of Full AC Load Flow versus proposed method.

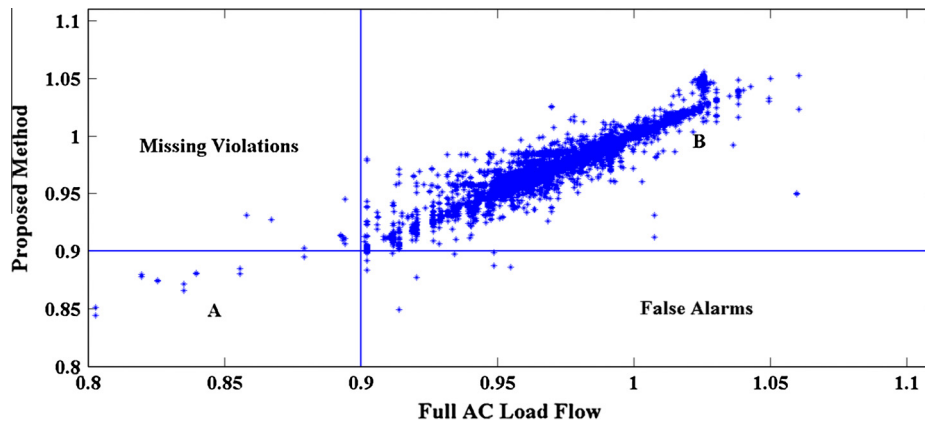


Fig. 3. IEEE 118 Bus Test System, double branch outage simulation results of Full AC load flow versus proposed method.

**Table 8**  
Critical double branch outage simulations giving 5% or more PBVMEs for IEEE 30 bus test system.

First outaged line	Second outaged line	Error %
4–6	6–8	5.223
9–10	6–10	5.280
15–23	28–27	5.496
9–10	28–27	6.049
12–15	12–14	8.329
6–28	6–8	14.62
6–28	28–27	16.56
6–28	8–28	17.23
10–21	10–22	17.60
8–28	28–27	21.35

**Table 9**  
Critical double branch outage simulations giving 5% or more PBVMEs for IEEE 118 bus test system.

First outaged line	Second outaged line	Error %
23–24	22–23	5.00
16–17	8–5	5.13
8–5	30–17	5.15
34–43	38–37	5.39
14–15	8–5	5.62
100–106	105–106	5.67
23–32	22–23	5.78
44–45	38–65	5.78
53–54	49–51	6.13
22–23	26–30	6.34
34–43	38–65	6.58
22–23	23–25	7.12
56–58	49–51	7.49
8–5	3–5	8.19
49–51	51–58	8.55
11–12	11–13	8.66
38–65	38–37	9.49
12–14	8–5	9.63
12–16	8–5	10.42
8–5	11–13	11.32
30–38	38–37	12.13

pair of [(15, 23), (28, 27)] was 1-type contingency and it was a severe outage pair resulting in a bus voltage magnitude of 0.82 for bus-30. Such a severe outage, can be hold by the proposed method with an acceptable error.

Bus voltage magnitude errors for IEEE 118-bus test system were generally less than those for IEEE 30-Bus test system. It was an expected circumstance since double branch outages for smaller-size systems would create more severe conditions. On

the other hand, percentage of 2-type outages will increase for smaller size test systems, which will decrease the computational accuracy of the proposed formulation. The highest voltage magnitude error in Table 9 was found for the simultaneous outages of the line 30–38 and the transformer 38–37. This 2b-type outage included a common bus (Bus no: 38) which increased the severity of the outage. Bounded regions were {8, 17, 26, 30, 37, 38, 65} and {30, 33, 34, 35, 37, 38, 39, 40, 65}. Maximum PBVME was found at Bus-38 as 12.13%.

The second critical case in Table 9 was the simultaneous outage of the line 11–13 and transformer 8–5. This was a 2b-type outage that included common buses 4, 5, 11 in the bounded regions: {4, 5, 11, 12, 13, 14, 15} and {3, 4, 5, 6, 8, 9, 11, 30}. The highest PBVME was found at bus 13, and the post-outage bus voltage magnitude for this bus was 0.90 p.u. while computed post-outage bus voltage magnitude was 1.0019 p.u.

Another critical 2a type contingency was the outages of the line 12–16 and the line 8–5. Bounded regions for this case were {2, 3, 7, 11, 12, 14, 16, 17, 117} and {3, 4, 5, 6, 8, 9, 11, 30} respectively. The highest PBVME was calculated for Bus-16 as 10.42% where AC load flow computed the post-outage magnitude as 0.93 p.u.

Maximum PBVME of the remaining 18 double branch outages were between 5% and 10 percent. 16 of them were 2-type and 2 of them were 1-type. 1-type double branch outages were the outages of line 14–15 and line 8–5, and outage of line 38–65 and 34–43. Since 7 of these double branch outages included branch 8–5, we can conclude that branch 8–5 was a critical branch for IEEE 118 Bus System. On the other hand, critical busses those can be inferred from critical outage pairs were 38, 37, 65, 51.

## Conclusion

This paper has presented a double branch outage model and a local constrained optimization problem representing the outage phenomena. Differential evolution algorithm and particle swarm optimization algorithm were used to estimate the post-outage voltage magnitudes solving the local constrained optimization problem. Double line outage simulations for IEEE 30 Bus Test System and for IEEE 118 Bus Test system were run, and the results of some sample outages were compared with AC load flow results from the point of computational accuracy. In addition, speed test results for IEEE 14, IEEE 30, IEEE 57, IEEE 118 and IEEE 300 Bus Test systems were illustrated and compared. Critical contingencies giving high PBVMEs were discussed for IEEE 30 Bus and IEEE 118 Bus Test systems.

Simulation results of the test systems showed that the proposed method computes the post-outage bus voltage magnitudes with acceptable accuracies. The number of missed critical double branch outages were less than 0.004% for IEEE 30 Bus Test System and less than 0.0003% for IEEE 118 Bus Test System. High PBVMs were generally found for the contingencies where the two branches connected to the same load bus were outaged. On the other hand, computational efficiency of the proposed method was decreased as the intersection of the marginal bounded regions increases. That is, if the number of buses included in both of the marginal bounded regions increases, accuracy of the proposed method decreases.

IEEE test system applications showed that the proposed bounded network formulation and its solution by DE and PSO methods found to be faster than the conventional AC load flow for large scale networks since they used only a bounded part of the network. Moreover, computation time is independent from the system size and the proposed formulation is more effective for large scale systems. Solution speed of the algorithm will later be increased by parallel computation.

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