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Detecting structural changes using wavelets ☆

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ABSTRACT

We propose a powerful wavelet method to identify structural breaks in the mean of a process. If there is a structural change in the mean, the sum of the squared scaling coefficients absorbs more variation, leading to unequal weights for the variances of the wavelet and scaling coefficients. We use this feature of wavelets to design a statistical test for changes in the mean of an independently distributed process. We establish the limiting null distribution of our test and demonstrate that our test has good empirical size and substantive power relative to the existing alternatives especially for multiple breaks.

1. Introduction

The primary goal of this paper is to present a test that can be used to identify structural breaks in the mean of an independently distributed process at unknown locations. We primarily focus on multiple structural breaks because most financial data exhibit multiple structural changes in a time window. A wavelet decomposition additively splits data into their local weighted averages (scaling coefficients) and local weighted differences (wavelet coefficients). If the process has a constant mean, then the variances of the wavelet and scaling coefficients have equal magnitude. If, however, there is change in the mean of the process, the variances of the wavelet and scaling coefficients diverge and there is a greater allocation to the variance of the scaling coefficients. We use this feature of the

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wavelet decomposition to design a statistical test to identify changes in the mean of an independently distributed process.[1] It is through these weighted local differences in moving windows that we construct our statistical test for no structural break under the null hypothesis. We derive the test's null distribution and demonstrate that it is asymptotically normally distributed.

The wavelet transformation operates in local time windows. The length of the moving window is determined by the length of the wavelet filter. If the filter length is two, as is the Haar filter, then localized differences amount to the differences between two consecutive observations. If the filter length is four, the length of the moving window is the weighted difference between the last two and first two observations in a window of four observations. A longer filter more accurately captures the local structural features of the data, but boundary issues may arise. In our test, we use a Haar filter that has a length equal to two and is a good compromise between localization in a local time window and cost in terms of boundary treatments.

Structural breaks can be permanent or temporary in nature. If a structural break is permanent, then there is a permanent change of indefinite duration in the mean or variance. In a temporary break, the mean or the variance shifts from its null value but eventually reverts to its null value. Whether such breaks are temporary or permanent in nature, they may occur abruptly or gradually. To capture such possibilities, we use Monte Carlo simulations to model break locations through sinusoidals and to allow for abrupt as well as gradual structural breaks. Most financial data exhibit multiple structural changes in a time window and these changes occur gradually rather than abruptly. Therefore we primarily focus on smooth, multiple structural breaks. Nevertheless our simulation framework is all-encompassing and allows for single and multiple breaks, including abrupt changes as well as smooth ones. Our Monte Carlo simulations indicate the presence of minimal empirical size distortions relative to their nominal sizes and significant power improvements compared to the existing structural break tests especially in the case of multiple structural breaks.

The literature on structural change tests is extensive. Several tests for structural breaks have been proposed in the literature. Chow (1960) derived F-tests for structural breaks with known break points. Brown et al. (1975) developed cumulative sum (CUSUM) and CUSUM-squared tests that are also applicable to cases where the time of the break is unknown. More recently, contributions by Ploberger et al. (1989), Hansen (2002), Andrews (1993), Inclan and Tiao (1994), Andrews et al. (1996), and Chu et al. (1996) have extended tests for the presence of breaks to account for heteroskedasticity. Methods for estimating the number and timing of multiple break points, as in Bai and Perron (1998), Bai and Perron (2003a), and Altissimo and Corradi (2003), have also been developed.[2] Chen and Hong (2012) and Öztürk and Stengos (2014) both proposed tests that may useful for detecting smooth structural breaks. These tests are formulated in the context of linear regression models, and they use the estimated residuals to detect the departure of parameters from constancy.[3]

The mechanism of our wavelet framework can be described in the following way. The wavelet decomposition yields localized weighted differences (wavelet coefficients) that are equal in number to the number of data points.[4] We square each wavelet coefficient to obtain its magnitude and calculate the sample average of the squared coefficients. We expect this sample average to be equal to one-half of the variance of the process under the null when there is no structural change.[5] However, the sample average of the squared wavelet coefficients should be significantly smaller than the null average when there are one or more structural breaks. We center and standardize the sample average of the squared wavelet coefficients to obtain the null distribution with no structural change.

Alternative tests, such as the CUSUM and moving sum (MOSUM), approach the problem through cumulative sums of either recursive residuals (i.e., one step ahead of prediction errors) or OLS

[1] Although we primarily focus on structural breaks in the mean for an independently distributed time series, our framework can be generalized to structural breaks in the variance and structural breaks in stationary and non-stationary time series.

[2] Bai and Perron (2003a) and Bai and Perron (2003b) provide versions of these tests that correct for heteroskedasticity and serial correlations.

[3] As will be mentioned below, our test can also be applied on the estimated residuals rather than the series itself without any change in the outcome.

[4] In this paper, the maximum overlap discrete wavelet transformation (MODWT) is used.

[5] The average of the squared wavelet coefficients and the average of the squared scaling coefficients are equal to the overall variance in a one-level wavelet decomposition.

residuals. They start from a given window to calculate the sequence of cumulated sums by increasing the window length one step at a time[6] and reject the null of no structural change when the properly standardized supremum of these cumulated sums crosses the critical lines or when the maximum cumulated sum is sufficiently large.[7] In a Sup-F type test,[8] splitting the data into two or more blocks and calculating the unrestricted and restricted residual sum of squares for all possible sub-samples for a given length are necessary to find the supremum in an F-test.

In the presence of multiple structural breaks, MOSUM- and CUSUM-type tests yield smaller maximum values of their cumulated sums of residuals (CSR). A similar argument applies to the Sup-F test, where the difference between the sums of squared residuals (SSRs) of the restricted and unrestricted models becomes small in the presence of multiple structural breaks. These tests rely on larger CSRs (or larger differences between restricted and unrestricted SSRs) to detect structural change, and smaller CSRs do not yield high power. Smaller CSRs occur because the OLS estimation goes through the average of multiple structural breaks (in particular, when such breaks are reverting), which yields smaller residuals that underestimate structural change locations. Our test, conversely, always operates in a local time window with excellent frequency localization features, where the imbalance between squared wavelet and scaling coefficients is preserved and the power of our test is not compromised in the presence of multiple structural breaks.

Several recent papers have successfully demonstrated the usefulness of wavelets in an econometric hypothesis-testing framework (see Gençay et al., 2001 for an overview). Gençay and Signori (2015), using wavelet transformation, recently introduced a new family of portmanteau tests for serial correlation with better power properties. Fan and Gençay (2010) proposed a unified wavelet spectral approach to unit root testing by providing a spectral interpretation of the existing Von Neumann unit root tests. Xue et al. (2014) proposed using wavelet-based jump tests to detect jump arrival times in high frequency financial time series data. These wavelet-based unit root, cointegration and jump tests have desirable empirical sizes and higher power relative to the existing tests. Gençay and Gradojevic (2011) utilized wavelets for errors-in-variables estimation.

The outline of this paper is as follows: Section 2 introduces the wavelet-based structural change test and its limiting null distribution. The Monte Carlo simulations are described in Section 3.

2. The wavelet test for structural change

Let a time series $\{y_t\}_{t=1}^{T}$ evolve according to the following data generation process (DGP):

$$y_t = \boldsymbol{\mu}(t) + \varepsilon_t, \tag{1}$$

where ε_t are normally, identically and independently distributed with mean zero and variance σ^2 for $t = 1, \ldots, T$. The structural changes in the mean of y_t occurring at (unknown) dates $t_1, t_2, \ldots, t_k$ can be formulated as follows:

$$\boldsymbol{\mu}(t) = \begin{cases} \mu_1 & \text{for } 1 \leqslant t \leqslant t_1, \\ \mu_2 & \text{for } t_1 < t \leqslant t_2, \\ \vdots & \vdots \\ \mu_z & \text{for } t_{z-1} < t \leqslant T. \end{cases} \tag{2}$$

[6] The MOSUM test uses moving windows but keeps their size constant.

[7] While the CUSUMs of the recursive residuals, properly standardized, have a distribution that tends to approach a standard Wiener process (Sen, 1982; Kramer et al., 1988), Ploberger et al. (1989) show that the OLS-based CUSUMs have a distribution that tends to approach a Brownian bridge. Because the OLS residuals sum to zero when there is an intercept in the regression, their cumulated sum cannot be expected to drift off after a structural change, as often occurs with recursive residuals, which provides the rationale for the standard CUSUM test. No matter how large a structural shift has occurred, the cumulated OLS residuals will return eventually to the origin; Ploberger et al. (1989) provide critical lines that are parallel to the horizontal axis of the plot of cumulated residuals, unlike the positively and negatively sloped critical lines of the conventional CUSUM test.

[8] The Sup-F test for structural breaks was proposed by Andrews (1993) and generalized by Bai and Perron (1998).

Under the null hypothesis, it is assumed that there is no structural change in the mean, $H_0 : \mu_1 = \mu_2 = \cdots = \mu_z = \mu$. Under the alternative hypothesis, (H_1), there may be one or more breaks in the mean at unknown locations, such that there exists at least one $i \in \{1, \ldots, z\}$ such that $\mu_i \neq \mu$.

2.1. Wavelet and scaling coefficients

Consider the unit scale Haar maximum overlapping discrete wavelet transformation (MODWT) of $\{y_t\}_{t=1}^{T}$, where T is the number of observations. The wavelet and scaling coefficients for this transformation are given by

$$W_t = \frac{1}{2}(y_t - y_{t-1}); \quad V_t = \frac{1}{2}(y_t + y_{t-1}), \quad (t = 1, 2, \ldots, T; \quad \text{mod } T), \tag{3}$$

where we assume that the boundary condition is circular. As we approach the end of the sample, if enough data are not available for a given filter length, we use data from the beginning of the sample to complete the analysis. This type of boundary treatment is innocuous, as we use a Haar filter that has two coefficients and the data are assumed to be stationary.[9]

The wavelet coefficients, $\{W_t\}_{t=1}^{T}$, capture the behavior of $\{y_t\}$in the high frequency band $\left[\frac{1}{2}, 1\right]$, while the scaling coefficients, $\{V_t\}_{t=1}^{T}$, capture the behavior of $\{y_t\}$ in the low frequency band $\left[0, \frac{1}{2}\right]$. Accordingly, the variance (energy) of $\{y_t\}$ is given by the sum of the energies of $\{W_t\}_{t=1}^{T}$ and $\{V_t\}_{t=1}^{T}$, where

$$\sum_{t=1}^{T} y_t^2 = \sum_{t=1}^{T} W_t^2 + \sum_{t=1}^{T} V_t^2.$$

2.2. Statistical properties of wavelet tests

We propose a test statistic based on the squared average of the wavelet coefficients, namely,

$$\delta_m^2 = \frac{1}{m}\sum_{t=j}^{m+j} W_t^2 \tag{4}$$

for $j \in 1, 2, 3, \ldots, T - m$, where m is an arbitrary length of the window that is used to test for structural changes. In Eq. (4), δ_m^2 is defined as the average of the squared wavelet coefficients over an interval, m. When $m = T$, then δ_T^2 is the average of the squared wavelet coefficients for the whole sample. In the remainder of this paper, we explore the statistical properties of δ_T^2 under the null hypothesis of $\mu_1 = \mu_2 = \ldots = \mu_z = \mu$ in Eq. (2). The following proposition establishes the expected value and variance of δ_T^2 under the null hypothesis of no structural change (see Yazgan and Özkan (2014)).

Proposition 1. *Under* H_0, $\mathbb{E}\{\delta_T^2\} = \frac{\sigma^2}{2}$. $Var(\delta_T^2) \cong \frac{3\sigma^4}{T}$, *for large T.*

Let $s^2 = (1/T)\sum_{t=1}^{T}(y_t - \bar{y})^2$ be a consistent estimator of σ^2. Accordingly, we center δ_T^2 and work with $\delta_T^2 - \frac{1}{2}s^2$, which has zero expectation under H_0. The following proposition derives the variance of $\delta_T^2 - \frac{1}{2}s^2$ (see Yazgan and Özkan, 2014).

Proposition 2. *Under* H_0, $Var\left(\delta_T^2 - \frac{s^2}{2}\right) \cong \frac{\sigma^4}{4T}$ *for large T.*

Normalizing $\delta_T^2 - \frac{s^2}{2}$ by dividing it by its standard deviation, we propose the test statistic SB_W.

[9] The circular boundary treatment may be problematic for trending or nonstationary data processes; therefore, using data from the beginning of the sample to complete the analysis may introduce a superficial structural change due to the change in level between the beginning and the end of the sample. A longer filter will require more coefficients, and it is more likely that such an analysis will be affected by the boundary coefficients. However, longer filters may have better localization properties and may be more powerful for identifying the local structural variations in the data.

$$SB_W = \frac{\left(\delta_T^2 - \frac{s^2}{2}\right)}{\sqrt{s^4/4T}} = \sqrt{T}\left(2\frac{\delta_T^2}{s^2} - 1\right). \tag{5}$$

The asymptotic distribution of SB_W is given in the following proposition (see Yazgan and Özkan, 2014)[10]:

Proposition 3. *Under* $H_0, SB_W \xrightarrow{(d)} N(0,1)$ *as* $T \to \infty$ *where* $\xrightarrow{(d)}$ *denotes convergence in the distribution.*

3. Monte Carlo simulations

Our framework allows for single and multiple structural breaks in Monte Carlo simulations. Following Becker et al. (2004), Becker et al. (2006), and Ludlow and Enders (2000),[11] we allow smooth breaks. Abrupt structural breaks, random locations and sizes are introduced by extending their frameworks appropriately.

DGP1

Because a Fourier expansion is capable of approximating absolutely integrable functions to any desired degree of accuracy, smooth or abrupt breaks can be approximated by Fourier sinusoidals with an appropriate frequency mix. Therefore, the structural breaks in the regression coefficients of Eq. (1) are identified via the following function.[12]

$$\boldsymbol{\mu}(t) \cong \alpha \sum_{i=1}^{n} \left\{ (2i-1)^{-1} \sin\left[\frac{2\pi(2i-1)kt}{T}\right] \right\}, \tag{6}$$

where n is the number of frequencies included in the approximation, k represents a particular frequency and α indicate their size (amplitude). The frequency coefficient, k, alone determines the number of breaks and whether they are temporary or permanent. Moreover, if a single frequency is used ($n = 1$), then the transitions tend to be smooth, whereas higher $n$values facilitate the analysis of abrupt temporary or permanent breaks.

DGP2

There are two important limitations in the above framework in terms of the construction of structural breaks. First, the time length between two consecutive breaks are constant. Second, the sizes of breaks are constant. In the below formulation we remove these limitations by extending the framework to allow for stochastic locations and sizes of the break. For a pre-specified z,[13] we have

$$\begin{aligned}\boldsymbol{\mu}(t) \cong & \left\{ \sum_{j=0}^{z} \mathbb{1}_{\{\tau_i \leqslant t < \tau_{i+1}\}} (-1)^j \alpha_{i+1} \right\} \\ & \times \sum_{i=1}^{n} \left\{ (2i-1)^{-1} \sin\left[\frac{\pi(2i-1)\left(t - \sum_{i=0}^{z} \mathbb{1}_{\{\tau_i \leqslant t < \tau_{i+1}\}} \tau_i\right)}{\sum_{i=0}^{z} \mathbb{1}_{\{\tau_i \leqslant t < \tau_{i+1}\}} (\tau_{i+1} - \tau_i)} \right] \right\},\end{aligned} \tag{7}$$

where $\mathbb{1}_A$ is the indicator function taking value 1 when the statement A is true and 0 otherwise. We randomly select break times such that $\tau_i \sim Uniform(aT,\ T - aT)$ for all $i = 0, 1, \ldots, z+1$, where

[10] It can be shown that, under the alternative hypothesis, the value of the test statistic lives in $(-\infty; 0)$ (a left-tailed test), and increases its power with increases in T and break sizes as expected. Break locations also matter in the sense that the power of the test is higher when breaks are located close to the middle rather than the beginning or end of the sample. These results are included in the working paper version and available upon request.

[11] See also Ashley and Patterson (2010), Stengos and Yazgan (2014a), and Stengos and Yazgan (2014b).

[12] We approximate the discrete square function in Eq. (2) using the continuous function in Eq. (6).

[13] In our Monte Carlo experiments we want to control the number of breaks, hence we do not prefer to the number of breaks, z, to be random. However, as explained in the following footnote it is possible to have stochastic zs in our simulations.

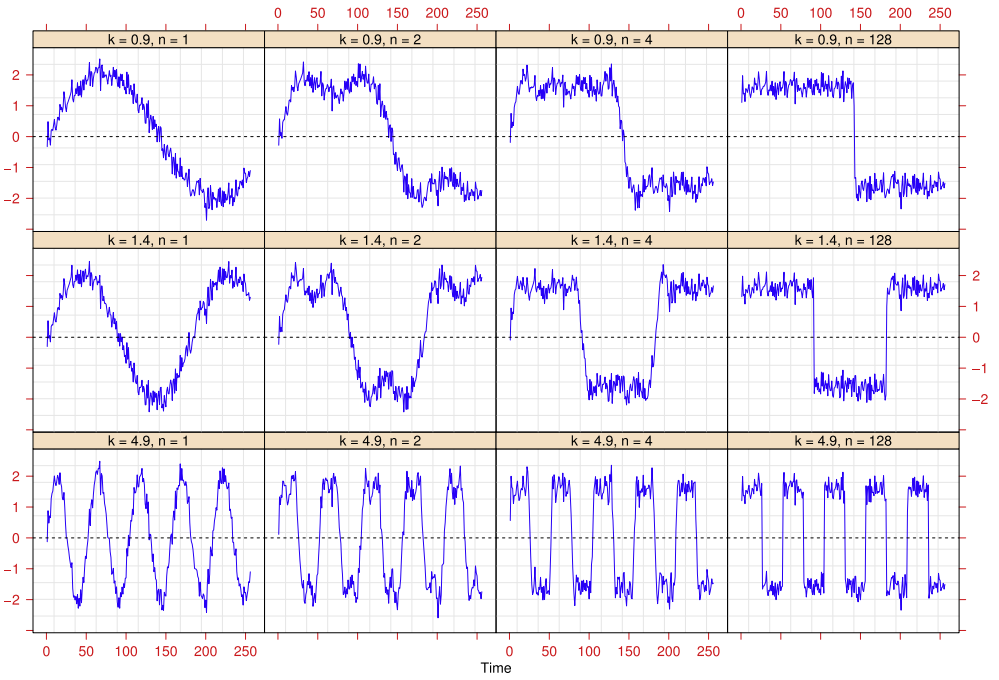

Fig. 1. Sample paths of smooth/abrupt and permanent/temporary breaks: DGP1. *Notes:* The above sample paths are generated using: ε_t is $iid \sim N(0, 0.25), \alpha = 1$, and $T = 256$.

$a \in \{1/T, 2/T, \ldots, 1/2\}$. Then, we sort the break times in an increasing order: $\tau_1 < \tau_2 < \cdots < \tau_z, \tau_i \neq \tau_j$ whenever $i \neq j, \tau_0 = 0, \tau_{z+1} = T$. [14,15]

Moreover we randomize the size of the breaks by assuming that $\{\alpha_i\} \sim i.i.d.N(h, \sigma_h)$ and α_i is independent from τ_j for all i and j.

Eq. (6) and (7) can be used to generate simulated data as $y_t = \boldsymbol{\mu}(t) + \varepsilon_t$. In Fig. 1, the sample paths of y_t are displayed for permanent and temporary breaks for DGP1. In the top panel where $k = 0.9$, we observe a single permanent structural break, which is permanent in the sense that the series do not return to their initial mean levels. In the second panel where $k = 1.4$, the series are subject to temporary single breaks. The remaining panel depicts multiple breaks. Moreover, Fig. 1 illustrates sample paths of y_t for higher $n = 1$ values where the breaks are more abrupt. When n assumes a value as high as 4 (the third column), the breaks become significantly more abrupt. For higher values of n such as 128, the structural breaks occur more suddenly. In Fig. 2 we repeat the same exercise for DGP2. The figures clearly illustrate how under this new setting the random break occurrences and sizes become possible.

[14] As illustrated in Eq. (1) the change in the mean of the process occurs right after the break time.

[15] In the formulation of Eq. (7), we control the number of breaks by pre-specifying z. However, also it is possible to make z, the number of breaks, be random by setting τ_is using the following formulation in Eq. (7):

$$\tau_i = \sum_{j=1}^{z} \rho_j, \ \rho_0 = 0, \ i = 1, 2, \ldots,$$

where $\{\rho_i\} \sim \exp(T/m)$. Obviously, $m$is the number of "expected breaks" in a T period of time. More particularly, the time difference between each break time is exponentially distributed and $\{\tau_i\}$ is an increasing series of stopping times such that it adds up exponential random variables $\tau_0 = 0, \tau_1 = \rho_1, \tau_2 = \rho_1 + \rho_2$, and so on. The number of breaks is determined as $z = \inf\left\{l : \sum_1^l \rho_i \geqslant T\right\}$, which is a random variable.

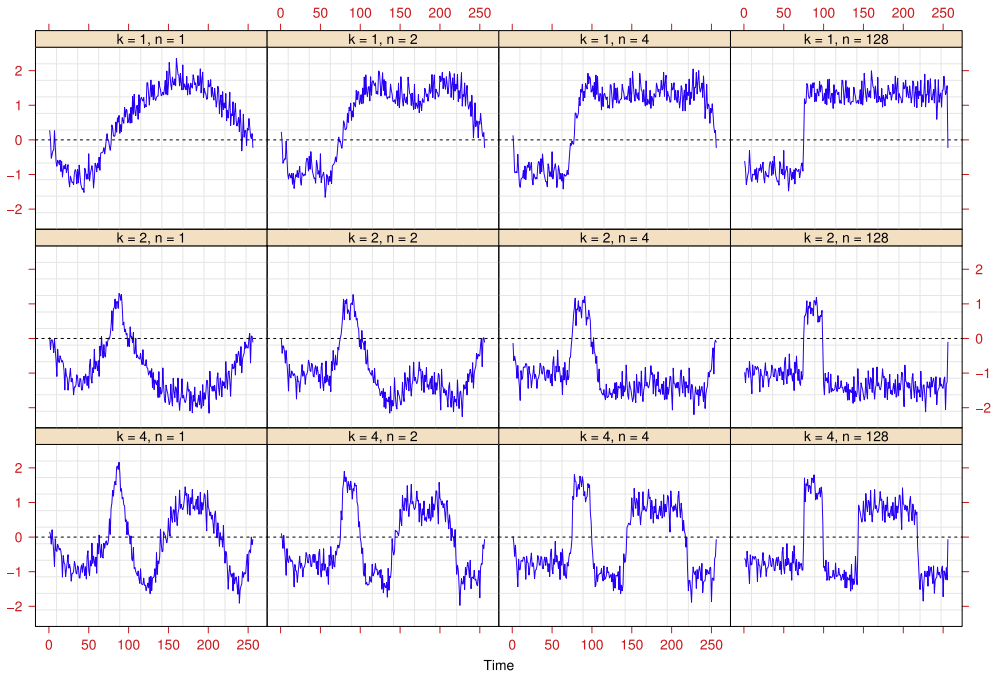

Fig. 2. Sample paths of smooth/abrupt and permanent/temporary breaks: DGP2. *Notes:* The above sample paths are generated using: $z = 3, h = 1, \sigma_h = 2.5, \sigma_\varepsilon = 0.25, T = 256$.

3.1. Results

We compare the performance of the SB_W test using three well known structural break tests statistics: the Sup-F, OLS-based cumulative CUSUM, and MOSUM tests. The Sup-F test was first proposed by Andrews (1993) and was generalized by Bai and Perron (1998). The OLS-based CUSUM and MOSUM test statistics are due to Ploberger and Kramer (1992) and Chu et al. (1995) respectively (see also Zeileis, 2001; Zeileis, 2005).

The empirical size and power properties of the wavelet coefficient test statistics, SB_W, and its alternatives, are calculated under various parameter sets of α, k, and T of Eq. (6). The experiment is run first for smooth structural breaks by setting $n = 1$, and it is then extended to abrupt breaks for higher values of n up to 1024. Because the SB_W test is a left-tailed test, we use one-sided p-values in our Monte Carlo simulations. The α values are allowed to vary in the range of $0.2, 0.4, \ldots, 2$. The k parameter varies between 0 and 5 by gradual increments of $\Delta k = 0.5$. In non-integer values of k we capture the possibility of temporary breaks. We consider cases where the data length, T, is 50, 100, and 200. In the case of DGP2, the α values are generated from a normal distribution with mean h whose values are allowed to vary in the range of $0.2, 0.4, \ldots, 2$ with variance $\sigma_h = 0.25$ and assume that $a = 0.15$. As in the case of DGP1, we allow up to 5 breaks and consider $z = 0, 1, 2, 3, 4, 5$. For each combination of α (or h), T, and k (or z), we run 10,000 replicates and use the same pseudo-random generated identically and independently distributed standard normal series to simulate ε_t. The rejection frequencies are calculated at the 1 and 5 percent levels of significance. We report empirical power calculations.[16]

For the case of smooth breaks ($n = 1$), the results of the Monte Carlo experiments are illustrated in Table 1. In Table 1, we provide the rejection frequencies of Sup-F, OLS-based CUSUM, OLS-based MOSUM, and SB_W test statistics for $T = 50, T = 100$ and $T = 200$. The values in the first panel, where

[16] For brevity, we report a subset of our Monte Carlo simulation results. Complete results are available upon request.

Table 1
Empirical powers of the SB_W, Sup-F, CUSUM, and MOSUM tests for smooth breaks (DGP1).

Test	SB_W						Sup-F						CUSUM						MOSUM					
T	50		100		200		50		100		200		50		100		200		50		100		200	
Sig.	1%	5%	1%	5%	1%	5%	1%	5%	1%	5%	1%	5%	1%	5%	1%	5%	1%	5%	1%	5%	1%	5%	1%	5%
0	0.01	0.04	0.01	0.04	0.01	0.05	0.01	0.05	0.01	0.05	0.01	0.05	0.00	0.02	0.01	0.03	0.01	0.04	0.00	0.01	0.00	0.02	0.01	0.03
k	$\alpha = 0.8$																							
0.5	0.04	0.16	0.08	0.23	0.15	0.35	0.07	0.21	0.18	0.41	0.49	0.78	0.01	0.06	0.04	0.19	0.19	0.54	0.00	0.04	0.06	0.22	0.29	0.59
1	0.54	0.79	0.87	0.96	0.99	1.00	0.94	0.98	1.00	1.00	1.00	1.00	0.9	0.98	1.00	1.00	1.00	1.00	0.07	0.37	0.87	0.99	1.00	1.00
1.5	0.47	0.73	0.81	0.94	0.99	1.00	0.17	0.43	0.6	0.92	0.99	1.00	0.05	0.26	0.4	0.85	0.99	1.00	0.12	0.42	0.9	0.99	1.00	1.00
2	0.51	0.76	0.87	0.96	0.99	1.00	0.28	0.55	0.75	0.92	0.99	1.00	0.04	0.26	0.42	0.79	0.96	1.00	0.03	0.23	0.72	0.96	1.00	1.00
2.5	0.48	0.75	0.84	0.95	0.99	1.00	0.06	0.21	0.25	0.57	0.79	0.96	0.00	0.03	0.02	0.18	0.27	0.75	0.03	0.21	0.65	0.94	1.00	1.00
3	0.48	0.74	0.86	0.95	0.99	1.00	0.08	0.23	0.3	0.59	0.82	0.95	0.01	0.05	0.05	0.24	0.36	0.76	0.01	0.1	0.3	0.73	0.98	1.00
3.5	0.44	0.72	0.84	0.95	0.99	1.00	0.02	0.09	0.06	0.21	0.29	0.62	0.00	0.01	0.00	0.04	0.03	0.23	0.01	0.06	0.18	0.56	0.92	1.00
4	0.42	0.71	0.84	0.96	0.99	1.00	0.02	0.08	0.05	0.19	0.22	0.5	0.00	0.02	0.01	0.09	0.09	0.34	0.00	0.02	0.04	0.22	0.42	0.84
4.5	0.38	0.68	0.84	0.95	0.99	1.00	0.00	0.03	0.01	0.04	0.02	0.1	0.00	0.01	0.00	0.02	0.01	0.08	0.00	0.01	0.02	0.11	0.2	0.59
5	0.34	0.66	0.83	0.95	0.99	1.00	0.01	0.03	0.01	0.06	0.04	0.16	0.00	0.01	0.01	0.04	0.04	0.16	0.00	0.01	0.00	0.03	0.02	0.13
k	$\alpha = 1.2$																							
0.5	0.15	0.35	0.31	0.57	0.59	0.81	0.18	0.41	0.49	0.8	0.94	0.99	0.02	0.13	0.13	0.47	0.66	0.95	0.01	0.07	0.2	0.51	0.78	0.96
1	0.97	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.19	0.72	1.00	1.00	1.00	1.00
1.5	0.94	0.99	1.00	1.00	1.00	1.00	0.33	0.74	0.97	1.00	1.00	1.00	0.09	0.5	0.88	1.00	1.00	1.00	0.41	0.83	1.00	1.00	1.00	1.00
2	0.96	0.99	1.00	1.00	1.00	1.00	0.56	0.84	0.98	1.00	1.00	1.00	0.1	0.51	0.85	0.99	1.00	1.00	0.07	0.48	1.00	1.00	1.00	1.00
2.5	0.94	0.99	1.00	1.00	1.00	1.00	0.09	0.32	0.53	0.88	0.99	1.00	0.00	0.03	0.04	0.37	0.74	0.99	0.08	0.46	0.99	1.00	1.00	1.00
3	0.95	0.99	1.00	1.00	1.00	1.00	0.13	0.38	0.62	0.88	0.99	1.00	0.00	0.05	0.1	0.46	0.79	0.98	0.01	0.15	0.67	0.99	1.00	1.00
3.5	0.93	0.99	1.00	1.00	1.00	1.00	0.02	0.1	0.11	0.37	0.66	0.93	0.00	0.00	0.00	0.04	0.06	0.49	0.01	0.09	0.46	0.92	1.00	1.00
4	0.92	0.99	1.00	1.00	1.00	1.00	0.02	0.08	0.08	0.28	0.47	0.81	0.00	0.01	0.01	0.12	0.18	0.61	0.00	0.02	0.06	0.34	0.82	1.00
4.5	0.9	0.98	1.00	1.00	1.00	1.00	0.00	0.01	0.00	0.02	0.02	0.11	0.00	0.00	0.00	0.01	0.01	0.1	0.00	0.01	0.02	0.15	0.46	0.92
5	0.88	0.98	1.00	1.00	1.00	1.00	0.00	0.02	0.01	0.05	0.06	0.25	0.00	0.01	0.00	0.04	0.05	0.24	0.00	0.00	0.00	0.01	0.02	0.15
k	$\alpha = 1.6$																							
0.5	0.36	0.63	0.7	0.88	0.95	0.99	0.34	0.65	0.83	0.97	1.00	1.00	0.03	0.23	0.32	0.81	0.97	1.00	0.02	0.15	0.46	0.82	0.99	1.00
1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.31	0.9	1.00	1.00	1.00	1.00
1.5	1.00	1.00	1.00	1.00	1.00	1.00	0.49	0.93	1.00	1.00	1.00	1.00	0.13	0.73	1.00	1.00	1.00	1.00	0.72	0.98	1.00	1.00	1.00	1.00
2	1.00	1.00	1.00	1.00	1.00	1.00	0.77	0.96	1.00	1.00	1.00	1.00	0.16	0.71	0.98	1.00	1.00	1.00	0.09	0.68	1.00	1.00	1.00	1.00
2.5	1.00	1.00	1.00	1.00	1.00	1.00	0.1	0.39	0.76	0.98	1.00	1.00	0.00	0.02	0.06	0.56	0.96	1.00	0.14	0.69	1.00	1.00	1.00	1.00
3	1.00	1.00	1.00	1.00	1.00	1.00	0.16	0.49	0.83	0.98	1.00	1.00	0.00	0.05	0.13	0.64	0.96	1.00	0.01	0.16	0.91	1.00	1.00	1.00
3.5	1.00	1.00	1.00	1.00	1.00	1.00	0.01	0.1	0.14	0.52	0.9	0.99	0.00	0.00	0.00	0.04	0.11	0.74	0.00	0.1	0.72	1.00	1.00	1.00
4	1.00	1.00	1.00	1.00	1.00	1.00	0.01	0.08	0.09	0.36	0.69	0.95	0.00	0.01	0.01	0.12	0.27	0.82	0.00	0.01	0.05	0.39	0.98	1.00
4.5	1.00	1.00	1.00	1.00	1.00	1.00	0.00	0.00	0.00	0.01	0.01	0.1	0.00	0.00	0.00	0.00	0.00	0.11	0.00	0.00	0.01	0.15	0.69	1.00
5	1.00	1.00	1.00	1.00	1.00	1.00	0.00	0.01	0.00	0.04	0.06	0.3	0.00	0.00	0.00	0.03	0.04	0.31	0.00	0.00	0.00	0.00	0.01	0.12

k	$\alpha = 2.0$																							
0.5	0.65	0.86	0.94	0.99	1.00	1.00	0.54	0.85	0.98	1.00	1.00	1.00	0.06	0.35	0.6	0.97	1.00	1.00	0.04	0.25	0.75	0.97	1.00	1.00
1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.4	0.97	1.00	1.00	1.00	1.00
1.5	1.00	1.00	1.00	1.00	1.00	1.00	0.62	0.99	1.00	1.00	1.00	1.00	0.15	0.88	1.00	1.00	1.00	1.00	0.9	1.00	1.00	1.00	1.00	1.00
2	1.00	1.00	1.00	1.00	1.00	1.00	0.89	0.99	1.00	1.00	1.00	1.00	0.21	0.85	1.00	1.00	1.00	1.00	0.1	0.8	1.00	1.00	1.00	1.00
2.5	1.00	1.00	1.00	1.00	1.00	1.00	0.1	0.45	0.89	1.00	1.00	1.00	0.00	0.01	0.07	0.71	1.00	1.00	0.18	0.85	1.00	1.00	1.00	1.00
3	1.00	1.00	1.00	1.00	1.00	1.00	0.17	0.58	0.94	1.00	1.00	1.00	0.00	0.04	0.15	0.76	1.00	1.00	0.00	0.15	0.98	1.00	1.00	1.00
3.5	1.00	1.00	1.00	1.00	1.00	1.00	0.01	0.08	0.17	0.64	0.98	1.00	0.00	0.00	0.00	0.02	0.15	0.9	0.00	0.09	0.89	1.00	1.00	1.00
4	1.00	1.00	1.00	1.00	1.00	1.00	0.01	0.06	0.09	0.42	0.84	0.99	0.00	0.00	0.00	0.11	0.34	0.93	0.00	0.00	0.03	0.4	1.00	1.00
4.5	1.00	1.00	1.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.09	0.00	0.00	0.00	0.00	0.00	0.13	0.00	0.00	0.01	0.13	0.83	1.00
5	1.00	1.00	1.00	1.00	1.00	1.00	0.00	0.00	0.00	0.02	0.05	0.34	0.00	0.00	0.00	0.02	0.03	0.34	0.00	0.00	0.00	0.00	0.00	0.08

Notes: The numbers represent the fraction of the cases that H_0 is rejected in 10,000 replications, for each test (size corrected powers). The DGP is $y_t = \boldsymbol{\mu}(t) + \varepsilon_t$, where $\boldsymbol{\mu}(t)$ is given in Eq. (6) with $n = 1$, and ε_t is $iid \sim N(0, 1)$. The bandwidth parameter is taken as 0.15 for Sup-F, CUSUM, and MOSUM tests. Sup-F test does not use the heteroskedasticity and autocorrelation consistent (HAC) kernel.

$k = 0$ corresponds to the case of a pure white-noise process, are the size values of the tests. The remaining values reported in the following panels are the rejection frequencies or empirical power of the tests statistics. Each panel corresponds to selected α values for different ks.

The information contained in the table clearly shows that, for all parameters in the region where $k > 1$, SB_W has more empirical power than the other tests. Conversely, for small values of αs and in a small region in which k is equal to 1, the Sup-F and CUSUM tests perform better than SB_W, although they lose power quickly as k increases. However when breaks become larger (as αs increase) the wavelet test has become as equally powerful as others, even in the case of single break ($k = 1$). This implies that the alternative tests do relatively better than our test in terms of power when there is a single and small break. Otherwise, our test has higher relative power than the other tests for higher k values, i.e., when there are multiple structural breaks. As the number of smooth structural breaks in the data increases, SB_W performs much better as a structural break test. To evaluate the performance of our test over a longer time horizon, we set $T = 100$ and $T = 200$ and we observe that all tests, as expected, gain power, however the wavelet test remain more powerful in the case of multiple breaks. Conversely, when multiple breaks spanned a longer time horizon and, especially, when breaks are larger in magnitude, the other tests do not lose their power as quickly as in the case of small T as k increases. Particularly, the MOSUM test keeps its high power up to a very large number of ks, i.e., for reasonably large magnitudes and numbers of breaks, which makes sense because both tests use local information.

We also consider the size and power properties of the tests when the breaks are abrupt in nature. In Table 2 we illustrate the results when $n = 128$. For abrupt breaks, our test not only preserves its empirical size but also increases its power compared to the smooth case in the case of a single break and for small values of α. Similar to the smooth break case, the other tests also become more powerful when T gets larger, and our test ceases to be the best performer except in cases with higher numbers of structural breaks.

Now, we turn attention to DGP2, where the mean breaks contains stochastic locations and sizes. In the case of single smooth breaks and when T is small (Table 3), Sup-F and CUSUM tests still perform better than the wavelet test. However, unlike the previous DGP, the wavelet test cannot improve its power significantly as the amplitude of the breaks increases. Hence, when there is a single break, the wavelet cannot become competitive with the Sup-F and CUSUM tests, not even in the large breaks. The wavelet test remain the best performer for multiple breaks except the case of 3 breaks where the performance of Sup-F test is better. When we have larger data we observe the similar patterns. Moreover, when $z = 3$, where the wavelet test appear to be weaker than the Sup-F test in the case of small T, its performance approaches to or even exceeds that of Sup-F test. On the other hand, the MOSUM test becomes more powerful or even the best performer, for large data. As illustrated in Table 4, abrupt breaks (i.e. $n = 128$) do not change the qualitative nature of these results.

Overall our wavelet tests have superior performance especially when the data register multiple structural breaks. The wavelet test deposits the structural breaks into the scaling coefficient variance in a local window. As the number of structural breaks increases, a larger percentage of the variance is allocated to the scaling coefficients and the contribution of the wavelet variance to the overall variance becomes marginal. The sum of the variances of the wavelet and scaling coefficients is equal to the overall variance. Because the scaling coefficient variance becomes the driver of the overall variance in the presence of multiple structural breaks, we use the marginalization of the wavelet coefficient variance as a test for the presence of structural breaks. The MOSUM and wavelet tests both operate in local windows. One possible drawback of the MOSUM test is that it assigns the same weight to each observation in its local window. The wavelet test, however, uses time–frequency optimized weights to sum the information.

Appendix A. Supplementary material

Supplementary data associated with this article can be found, in the online version, at http://dx.doi.org/10.1016/j.frl.2014.12.003.

Table 2
Empirical powers of the SB_W, Sup-F, CUSUM, and MOSUM tests for abrupt breaks (DGP1).

Test	SB_W						Sup-F						CUSUM						MOSUM					
T	50		100		200		50		100		200		50		100		200		50		100		200	
Sig.	1%	5%	1%	5%	1%	5%	1%	5%	1%	5%	1%	5%	1%	5%	1%	5%	1%	5%	1%	5%	1%	5%	1%	5%
0	0.01	0.04	0.01	0.04	0.01	0.05	0.01	0.05	0.01	0.05	0.01	0.05	0.00	0.02	0.01	0.03	0.01	0.04	0.00	0.01	0.00	0.02	0.01	0.03
k	$\alpha = 0.8$																							
0.5	0.01	0.04	0.01	0.04	0.01	0.05	0.02	0.05	0.01	0.05	0.01	0.05	0.00	0.02	0.01	0.03	0.01	0.04	0.00	0.01	0.00	0.02	0.01	0.03
1	0.63	0.85	0.94	0.99	1.00	1.00	0.99	1.00	1.00	1.00	1.00	1.00	0.98	1.00	1.00	1.00	1.00	1.00	0.03	0.2	0.67	0.96	1.00	1.00
1.5	0.49	0.76	0.87	0.96	1.00	1.00	0.21	0.52	0.76	0.97	1.00	1.00	0.07	0.35	0.59	0.95	1.00	1.00	0.07	0.34	0.9	0.99	1.00	1.00
2	0.52	0.78	0.92	0.98	1.00	1.00	0.36	0.64	0.88	0.97	1.00	1.00	0.07	0.34	0.63	0.91	1.00	1.00	0.01	0.14	0.56	0.92	1.00	1.00
2.5	0.48	0.75	0.89	0.98	1.00	1.00	0.07	0.23	0.33	0.67	0.91	0.99	0.00	0.03	0.04	0.26	0.49	0.91	0.03	0.19	0.66	0.95	1.00	1.00
3	0.43	0.71	0.89	0.97	1.00	1.00	0.1	0.29	0.41	0.71	0.95	0.99	0.01	0.07	0.06	0.29	0.6	0.91	0.01	0.09	0.38	0.82	1.00	1.00
3.5	0.36	0.65	0.86	0.96	1.00	1.00	0.03	0.11	0.1	0.31	0.44	0.78	0.00	0.01	0.01	0.06	0.06	0.37	0.01	0.09	0.27	0.7	0.99	1.00
4	0.33	0.64	0.86	0.96	1.00	1.00	0.02	0.1	0.05	0.19	0.26	0.56	0.00	0.02	0.02	0.11	0.14	0.47	0.00	0.02	0.04	0.2	0.42	0.85
4.5	0.28	0.58	0.83	0.95	1.00	1.00	0.00	0.02	0.00	0.04	0.02	0.1	0.00	0.01	0.00	0.02	0.01	0.1	0.00	0.01	0.01	0.08	0.15	0.5
5	0.27	0.59	0.85	0.96	1.00	1.00	0.01	0.03	0.01	0.07	0.06	0.21	0.00	0.02	0.01	0.05	0.05	0.21	0.00	0.01	0.00	0.02	0.01	0.09
k	$\alpha = 1.2$																							
0.5	0.01	0.04	0.01	0.05	0.01	0.05	0.01	0.05	0.01	0.05	0.01	0.05	0.00	0.02	0.01	0.03	0.01	0.04	0.00	0.01	0.00	0.02	0.01	0.03
1	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.03	0.3	0.97	1.00	1.00	1.00
1.5	0.94	0.99	1.00	1.00	1.00	1.00	0.45	0.88	1.00	1.00	1.00	1.00	0.16	0.71	0.99	1.00	1.00	1.00	0.21	0.71	1.00	1.00	1.00	1.00
2	0.96	0.99	1.00	1.00	1.00	1.00	0.7	0.92	1.00	1.00	1.00	1.00	0.18	0.68	0.97	1.00	1.00	1.00	0.02	0.22	0.93	1.00	1.00	1.00
2.5	0.94	0.99	1.00	1.00	1.00	1.00	0.11	0.36	0.69	0.95	1.00	1.00	0.00	0.04	0.09	0.57	0.95	1.00	0.06	0.39	0.99	1.00	1.00	1.00
3	0.9	0.98	1.00	1.00	1.00	1.00	0.18	0.48	0.78	0.96	1.00	1.00	0.01	0.09	0.12	0.58	0.96	1.00	0.01	0.14	0.81	1.00	1.00	1.00
3.5	0.84	0.96	1.00	1.00	1.00	1.00	0.03	0.14	0.2	0.56	0.86	0.99	0.00	0.00	0.00	0.07	0.18	0.79	0.01	0.16	0.7	0.99	1.00	1.00
4	0.82	0.96	1.00	1.00	1.00	1.00	0.02	0.12	0.07	0.28	0.52	0.85	0.00	0.02	0.02	0.16	0.33	0.82	0.00	0.02	0.04	0.28	0.8	1.00
4.5	0.73	0.93	1.00	1.00	1.00	1.00	0.00	0.01	0.00	0.02	0.01	0.1	0.00	0.00	0.00	0.01	0.01	0.17	0.00	0.01	0.01	0.08	0.27	0.8
5	0.8	0.95	1.00	1.00	1.00	1.00	0.00	0.02	0.01	0.06	0.1	0.34	0.00	0.01	0.01	0.05	0.07	0.36	0.00	0.00	0.00	0.01	0.00	0.06
k	$\alpha = 1.6$																							
0.5	0.01	0.04	0.01	0.04	0.01	0.05	0.01	0.05	0.01	0.05	0.01	0.05	0.00	0.02	0.01	0.03	0.01	0.04	0.00	0.01	0.00	0.02	0.01	0.03
1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.02	0.32	1.00	1.00	1.00	1.00
1.5	1.00	1.00	1.00	1.00	1.00	1.00	0.7	1.00	1.00	1.00	1.00	1.00	0.27	0.95	1.00	1.00	1.00	1.00	0.35	0.93	1.00	1.00	1.00	1.00
2	1.00	1.00	1.00	1.00	1.00	1.00	0.9	0.99	1.00	1.00	1.00	1.00	0.34	0.9	1.00	1.00	1.00	1.00	0.01	0.23	1.00	1.00	1.00	1.00
2.5	1.00	1.00	1.00	1.00	1.00	1.00	0.13	0.47	0.9	1.00	1.00	1.00	0.00	0.03	0.16	0.82	1.00	1.00	0.09	0.58	1.00	1.00	1.00	1.00
3	1.00	1.00	1.00	1.00	1.00	1.00	0.24	0.63	0.95	1.00	1.00	1.00	0.01	0.09	0.19	0.8	1.00	1.00	0.00	0.15	0.98	1.00	1.00	1.00
3.5	0.98	1.00	1.00	1.00	1.00	1.00	0.02	0.16	0.31	0.76	0.99	1.00	0.00	0.00	0.00	0.08	0.38	0.97	0.01	0.23	0.96	1.00	1.00	1.00
4	0.98	1.00	1.00	1.00	1.00	1.00	0.02	0.12	0.07	0.34	0.73	0.97	0.00	0.01	0.01	0.19	0.52	0.97	0.00	0.01	0.03	0.3	0.96	1.00
4.5	0.95	1.00	1.00	1.00	1.00	1.00	0.00	0.00	0.00	0.01	0.01	0.08	0.00	0.00	0.00	0.00	0.01	0.26	0.00	0.00	0.00	0.06	0.35	0.94
5	0.99	1.00	1.00	1.00	1.00	1.00	0.00	0.01	0.00	0.05	0.11	0.44	0.00	0.00	0.00	0.04	0.08	0.49	0.00	0.00	0.00	0.00	0.00	0.02

(*continued on next page*)

Table 2 (*continued*)

Test	SB_W						Sup-F						CUSUM						MOSUM					
T	50		100		200		50		100		200		50		100		200		50		100		200	
Sig.	1%	5%	1%	5%	1%	5%	1%	5%	1%	5%	1%	5%	1%	5%	1%	5%	1%	5%	1%	5%	1%	5%	1%	5%
k	$\alpha = 2.0$																							
0.5	0.01	0.04	0.01	0.04	0.01	0.05	0.02	0.05	0.01	0.05	0.01	0.05	0.00	0.02	0.01	0.03	0.01	0.04	0.00	0.01	0.00	0.02	0.01	0.03
1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.01	0.27	1.00	1.00	1.00	1.00
1.5	1.00	1.00	1.00	1.00	1.00	1.00	0.9	1.00	1.00	1.00	1.00	1.00	0.37	1.00	1.00	1.00	1.00	1.00	0.47	0.99	1.00	1.00	1.00	1.00
2	1.00	1.00	1.00	1.00	1.00	1.00	0.97	1.00	1.00	1.00	1.00	1.00	0.49	0.97	1.00	1.00	1.00	1.00	0.00	0.2	1.00	1.00	1.00	1.00
2.5	1.00	1.00	1.00	1.00	1.00	1.00	0.13	0.56	0.98	1.00	1.00	1.00	0.00	0.03	0.22	0.94	1.00	1.00	0.1	0.72	1.00	1.00	1.00	1.00
3	1.00	1.00	1.00	1.00	1.00	1.00	0.28	0.74	0.99	1.00	1.00	1.00	0.00	0.08	0.24	0.92	1.00	1.00	0.00	0.12	1.00	1.00	1.00	1.00
3.5	1.00	1.00	1.00	1.00	1.00	1.00	0.02	0.16	0.39	0.88	1.00	1.00	0.00	0.00	0.00	0.08	0.57	1.00	0.01	0.27	1.00	1.00	1.00	1.00
4	1.00	1.00	1.00	1.00	1.00	1.00	0.01	0.12	0.06	0.38	0.86	1.00	0.00	0.00	0.01	0.2	0.67	1.00	0.00	0.00	0.01	0.27	0.99	1.00
4.5	1.00	1.00	1.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.06	0.00	0.00	0.00	0.00	0.00	0.34	0.00	0.00	0.00	0.03	0.38	0.98
5	1.00	1.00	1.00	1.00	1.00	1.00	0.00	0.00	0.00	0.03	0.11	0.52	0.00	0.00	0.00	0.02	0.07	0.58	0.00	0.00	0.00	0.00	0.00	0.00

Notes: The numbers represent the fraction of the cases that H_0 is rejected in 10,000 replications, for each test (size corrected powers). The DGP is $y_t = \boldsymbol{\mu}(t) + \varepsilon_t$, where $\boldsymbol{\mu}(t)$ is given in Eq. (6) with $n = 128$, and ε_t is $iid \sim N(0, 1)$. The bandwidth parameter is taken as 0.15 for Sup-F, CUSUM, and MOSUM tests. Sup-F test does not use the heteroskedasticity and autocorrelation consistent (HAC) kernel.

Table 3
Empirical powers of the SB_W, Sup-F, CUSUM, and MOSUM tests for smooth breaks (DGP2).

Test	SB_W						Sup-F						CUSUM						MOSUM					
T	50		100		200		50		100		200		50		100		200		50		100		200	
Sig.	1%	5%	1%	5%	1%	5%	1%	5%	1%	5%	1%	5%	1%	5%	1%	5%	1%	5%	1%	5%	1%	5%	1%	5%
$z = 0$	0.01	0.04	0.01	0.05	0.01	0.05	0.01	0.05	0.01	0.05	0.01	0.05	0.00	0.03	0.01	0.03	0.01	0.04	0.00	0.01	0.00	0.02	0.01	0.03
z	$h = 0.8$																							
1	0.73	0.88	0.92	0.97	0.98	1.00	0.96	0.99	1.00	1.00	1.00	1.00	0.93	0.98	1.00	1.00	1.00	1.00	0.24	0.59	0.95	0.99	1.00	1.00
2	0.47	0.66	0.76	0.87	0.93	0.97	0.2	0.37	0.67	0.81	0.91	0.96	0.11	0.3	0.58	0.78	0.89	0.95	0.26	0.51	0.84	0.94	0.99	1.00
3	0.67	0.82	0.89	0.95	0.98	1.00	0.77	0.88	0.93	0.98	0.99	1.00	0.62	0.82	0.85	0.95	0.97	1.00	0.18	0.48	0.89	0.97	1.00	1.00
4	0.53	0.7	0.81	0.9	0.96	0.98	0.17	0.34	0.58	0.76	0.89	0.96	0.07	0.23	0.43	0.68	0.85	0.95	0.14	0.37	0.78	0.92	0.99	1.00
5	0.62	0.77	0.86	0.92	0.97	0.99	0.48	0.64	0.81	0.91	0.94	0.98	0.3	0.51	0.68	0.84	0.89	0.97	0.12	0.35	0.76	0.91	0.99	1.00
z	$h = 1.2$																							
1	0.74	0.88	0.92	0.97	0.99	1.00	0.97	0.99	1.00	1.00	1.00	1.00	0.94	0.99	1.00	1.00	1.00	1.00	0.21	0.58	0.97	1.00	1.00	1.00
2	0.47	0.65	0.77	0.88	0.93	0.97	0.12	0.27	0.61	0.77	0.89	0.95	0.05	0.2	0.53	0.75	0.87	0.94	0.24	0.5	0.84	0.93	0.98	0.99
3	0.69	0.83	0.89	0.95	0.99	1.00	0.77	0.89	0.94	0.98	0.99	1.00	0.64	0.82	0.86	0.95	0.97	1.00	0.13	0.43	0.88	0.98	1.00	1.00
4	0.53	0.7	0.81	0.9	0.96	0.98	0.12	0.28	0.52	0.72	0.87	0.95	0.05	0.17	0.38	0.64	0.83	0.93	0.12	0.33	0.76	0.91	0.99	1.00
5	0.63	0.77	0.86	0.93	0.98	0.99	0.47	0.65	0.82	0.92	0.94	0.99	0.3	0.51	0.69	0.85	0.89	0.97	0.09	0.29	0.74	0.91	0.99	1.00
z	$h = 1.6$																							
1	0.74	0.88	0.93	0.98	0.99	1.00	0.97	0.99	1.00	1.00	1.00	1.00	0.94	0.99	1.00	1.00	1.00	1.00	0.2	0.57	0.97	1.00	1.00	1.00
2	0.47	0.65	0.76	0.88	0.93	0.97	0.09	0.23	0.59	0.75	0.87	0.94	0.04	0.16	0.52	0.73	0.85	0.92	0.25	0.5	0.84	0.93	0.98	1.00
3	0.69	0.83	0.9	0.95	0.99	1.00	0.78	0.89	0.94	0.98	0.99	1.00	0.63	0.82	0.86	0.95	0.97	1.00	0.12	0.41	0.88	0.97	1.00	1.00
4	0.53	0.7	0.82	0.9	0.96	0.99	0.1	0.24	0.49	0.7	0.87	0.94	0.04	0.15	0.36	0.62	0.82	0.93	0.1	0.32	0.76	0.91	0.99	1.00
5	0.63	0.77	0.87	0.93	0.98	0.99	0.47	0.65	0.83	0.92	0.95	0.98	0.3	0.52	0.69	0.85	0.88	0.97	0.07	0.26	0.71	0.9	0.99	1.00
z	$h = 2.0$																							
1	0.75	0.89	0.93	0.98	0.99	1.00	0.97	0.99	1.00	1.00	1.00	1.00	0.94	0.99	1.00	1.00	1.00	1.00	0.19	0.57	0.97	1.00	1.00	1.00
2	0.47	0.65	0.76	0.87	0.93	0.97	0.06	0.2	0.55	0.72	0.86	0.93	0.03	0.13	0.52	0.73	0.85	0.93	0.24	0.5	0.83	0.93	0.98	1.00
3	0.69	0.83	0.9	0.95	0.99	1.00	0.78	0.89	0.95	0.98	0.99	1.00	0.64	0.82	0.86	0.95	0.97	1.00	0.11	0.4	0.87	0.97	1.00	1.00
4	0.53	0.7	0.81	0.9	0.96	0.98	0.1	0.23	0.47	0.69	0.87	0.94	0.04	0.13	0.35	0.61	0.82	0.92	0.09	0.31	0.77	0.91	0.99	1.00
5	0.63	0.78	0.87	0.93	0.98	0.99	0.46	0.64	0.83	0.93	0.95	0.99	0.31	0.52	0.69	0.85	0.89	0.97	0.06	0.25	0.71	0.9	0.99	1.00

Table 4
Empirical powers of the SB_W, Sup-F, CUSUM, and MOSUM tests for abrupt breaks (DGP2).

Test	SB_W						Sup-F						CUSUM						MOSUM					
T	50		100		200		50		100		200		50		100		200		50		100		200	
Sig.	1%	5%	1%	5%	1%	5%	1%	5%	1%	5%	1%	5%	1%	5%	1%	5%	1%	5%	1%	5%	1%	5%	1%	5%
$z = 0$	0.01	0.04	0.01	0.05	0.01	0.05	0.01	0.05	0.01	0.05	0.01	0.05	0.00	0.03	0.01	0.03	0.01	0.04	0.00	0.01	0.00	0.02	0.01	0.03
z	$h = 0.8$																							
1	0.77	0.9	0.93	0.97	0.97	0.99	0.99	1.00	1.00	1.00	1.00	1.00	0.98	1.00	1.00	1.00	0.99	1.00	0.13	0.43	0.93	0.99	0.99	1.00
2	0.41	0.58	0.72	0.82	0.87	0.92	0.24	0.4	0.69	0.8	0.88	0.92	0.14	0.34	0.64	0.78	0.86	0.91	0.26	0.48	0.77	0.87	0.91	0.94
3	0.69	0.83	0.91	0.96	0.99	1.00	0.85	0.92	0.96	0.99	0.98	0.99	0.75	0.88	0.91	0.97	0.98	1.00	0.11	0.36	0.85	0.96	0.99	1.00
4	0.48	0.65	0.8	0.88	0.95	0.97	0.23	0.41	0.67	0.81	0.91	0.96	0.11	0.3	0.54	0.75	0.88	0.95	0.14	0.35	0.75	0.88	0.97	0.98
5	0.62	0.77	0.87	0.93	0.98	0.99	0.57	0.72	0.89	0.95	0.96	0.99	0.42	0.61	0.78	0.9	0.93	0.98	0.08	0.28	0.73	0.9	0.98	0.99
z	$h = 1.2$																							
1	0.78	0.91	0.94	0.98	0.97	0.99	0.99	1.00	1.00	1.00	1.00	1.00	0.98	1.00	1.00	1.00	1.00	1.00	0.12	0.42	0.95	1.00	0.99	1.00
2	0.4	0.57	0.72	0.81	0.87	0.91	0.14	0.3	0.63	0.76	0.84	0.9	0.07	0.23	0.59	0.74	0.83	0.89	0.28	0.49	0.77	0.86	0.91	0.94
3	0.7	0.84	0.91	0.96	0.99	0.99	0.86	0.93	0.97	0.99	0.99	1.00	0.76	0.88	0.92	0.97	0.98	1.00	0.08	0.32	0.84	0.96	0.99	1.00
4	0.49	0.66	0.8	0.88	0.95	0.97	0.17	0.34	0.61	0.78	0.9	0.95	0.08	0.23	0.5	0.72	0.86	0.94	0.13	0.33	0.74	0.88	0.96	0.98
5	0.63	0.77	0.88	0.94	0.98	0.99	0.59	0.73	0.9	0.95	0.96	0.99	0.42	0.6	0.79	0.91	0.93	0.98	0.06	0.23	0.7	0.9	0.98	0.99
z	$h = 1.6$																							
1	0.79	0.91	0.94	0.98	0.98	0.99	0.99	1.00	1.00	1.00	1.00	1.00	0.98	1.00	1.00	1.00	1.00	1.00	0.11	0.42	0.95	1.00	0.99	1.00
2	0.4	0.57	0.72	0.81	0.87	0.91	0.1	0.24	0.61	0.73	0.84	0.89	0.05	0.19	0.58	0.72	0.82	0.88	0.28	0.5	0.77	0.85	0.91	0.93
3	0.7	0.84	0.91	0.96	0.99	1.00	0.87	0.94	0.98	0.99	0.99	1.00	0.77	0.89	0.91	0.97	0.98	1.00	0.07	0.3	0.83	0.96	0.99	1.00
4	0.49	0.66	0.8	0.88	0.95	0.97	0.15	0.31	0.58	0.75	0.89	0.94	0.07	0.21	0.47	0.7	0.86	0.93	0.12	0.33	0.74	0.88	0.96	0.98
5	0.63	0.78	0.88	0.94	0.98	0.99	0.58	0.73	0.9	0.96	0.96	0.99	0.42	0.62	0.79	0.91	0.93	0.98	0.05	0.21	0.7	0.89	0.98	0.99
z	$h = 2.0$																							
1	0.79	0.92	0.94	0.98	0.97	0.99	0.99	1.00	1.00	1.00	1.00	1.00	0.98	1.00	1.00	1.00	1.00	1.00	0.1	0.41	0.94	1.00	0.99	1.00
2	0.4	0.56	0.72	0.82	0.86	0.9	0.09	0.21	0.59	0.72	0.82	0.88	0.04	0.16	0.56	0.71	0.8	0.87	0.28	0.5	0.77	0.86	0.9	0.92
3	0.7	0.84	0.92	0.96	0.99	1.00	0.87	0.94	0.97	0.99	0.99	1.00	0.78	0.89	0.92	0.97	0.98	1.00	0.07	0.29	0.82	0.96	0.99	1.00
4	0.49	0.65	0.8	0.88	0.95	0.97	0.14	0.29	0.57	0.74	0.88	0.93	0.07	0.19	0.47	0.69	0.86	0.93	0.12	0.32	0.73	0.87	0.96	0.98
5	0.63	0.78	0.89	0.94	0.98	0.99	0.58	0.73	0.9	0.95	0.97	0.99	0.42	0.61	0.79	0.91	0.93	0.98	0.05	0.2	0.68	0.89	0.98	0.99

Notes: The numbers represent the fraction of the cases that H_0 is rejected in 10,000 replications, for each test at %1 and %5 levels of significances. The DGP is $y_t = \boldsymbol{\mu}(t) + \varepsilon_t$, where $\boldsymbol{\mu}(t)$ is given in Eq. (7) with n = 1 (Table 3), n = 128 (Table 4), and ε_t is $iid \sim N(0,1)$. σ_h is set to 0.25 and the bandwidth parameter is taken as 0.15 for Sup-F, CUSUM, and MOSUM tests. Sup-F test does not use the heteroskedasticity and autocorrelation consistent (HAC) kernel.

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