Design of Practical Broadband Matching Networks With Mixed Lumped and Distributed Elements

Metin Şengül

***Abstract*—Computer-aided design (CAD) tools are always preferred for designing broadband matching networks. However, these tools give excellent results when the suitable matching network topology and initial element values are provided. Therefore, in this brief, a new initialization algorithm is proposed to get suitable network topology and element values for CAD tools. Then, the power transfer capability of the matching network can be improved by using any CAD tool. It is clear from the example studied that the new method generates excellent initials.**

***Index Terms*—Broadband matching, lossless networks, matching networks, real frequency techniques.**

I. Introduction

FOR numerous applications, networks containing lumped elements are preferred since their dimensions are small, although the unavoidable connections between lumped elements are detrimental to the performance of the network at microwave frequencies. However, it is possible to use these interconnections as circuit elements. In this way, it is clear that the networks will have mixed, lumped, and distributed elements.

In the characterization of mixed-element structures, transcendental or multivariable functions are used. In the first approach, which is based on the classical study of cascaded noncommensurate transmission lines [1], nonrational single-variable transcendental functions are utilized. The synthesis of a transcendental driving-point impedance function as a cascade of lumped lossless two-port networks and ideal uniform lossless transmission lines were studied in [2]. The other approach to describing mixed-, lumped-, and distributed-element two-port networks is based on Richards transformation ($\lambda = \tanh p\tau$; the transformation can be written as $\lambda = j\Omega = j\tan\omega\tau$ on the imaginary axis, where τ is the commensurate delay of the transmission lines), which converts the transcendental functions of a distributed network into rational functions [3]. Attempts to generalize this approach to mixed, lumped, and distributed networks led to multivariable synthesis procedures, in which the Richards variable $\lambda = \Sigma + j\Omega$ is used for distributed elements and the original frequency variable $p = \sigma + j\omega$ is used for lumped elements.

Unfortunately, a design theory for mixed, lumped, and distributed element networks still does not exist. That is, the problems associated with the approximation and synthesis of arbitrary mixed-element networks have yet to be solved.

In most existing studies, particular interest is devoted to a special and very useful network configuration. In addition to the lumped reactances, the lossless two-port network is allowed to contain cascaded ideal commensurate transmission lines [unit elements (UEs)]. In summary, the structure consists of cascaded lossless lumped-element two-port networks and UEs.

In this brief, a new algorithm used to design broadband matching networks with the configuration aforementioned has been proposed. In the succeeding section, the broadband matching problem is briefly described, followed by an explanation of the characterization of a mixed-element two-port network. After giving the algorithm that is an extension of the algorithm using only lumped elements [4] and only distributed elements [5], a mixed-element broadband matching network is designed in the example section.

II. Broadband Matching Problem

The broadband matching problem is to design a lossless power transfer network in such a way that the transferred power from source to load over the given frequency band is maximized. To measure the transferred power, the ratio of power delivered to the load to the available power from the source is used, which is known as transducer power gain (TPG) [6]–[8].

If the load impedance is complex, and the source impedance is purely resistive, it is a single-matching problem. If both source impedance and load impedance are complex, then it is a double-matching problem.

Let us take into account the double-matching arrangement shown in Fig. 1. The TPG of the system can be expressed in terms of the load impedance $Z_L = R_L + jX_L$ and the output impedance $Z_2 = R_2 + jX_2$, or in terms of the source impedance $Z_S = R_S + jX_S$ and the input impedance $Z_1 = R_1 + jX_1$ of the matching network as

$$\mathrm{TPG}(\omega) = \frac{4R_\alpha R_\beta}{(R_\alpha + R_\beta)^2 + (X_\alpha + X_\beta)^2}. \qquad (1)$$

Here, if $\alpha = 1$, then $\beta = S$, or if $\alpha = 2$, then $\beta = L$.

As said earlier, in broadband matching problems, a lossless two-port network is designed and it is desired to get maximum TPG given by (1) over the given frequency band. Therefore, in accordance with (1), the broadband matching problem can be reduced to obtain a realizable impedance function Z_1 or Z_2. If Z_1 or Z_2 are obtained, it means that the lossless broadband matching network is designed.

Manuscript received January 23, 2014; accepted August 28, 2014. Date of publication September 10, 2014; date of current version November 1, 2014. This brief was recommended by Associate Editor A. Demosthenous.

The author is with the Faculty of Engineering and Natural Sciences, Kadir Has University, 34083 Cibali-Fatih-Istanbul, Turkey (e-mail: msengul@khas.edu.tr).

Digital Object Identifier 10.1109/TCSII.2014.2356872

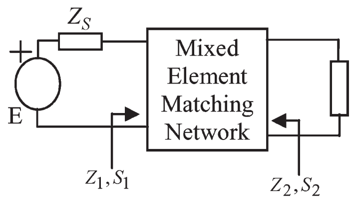

Fig. 1. Double-matching arrangement.

There are several techniques in the literature. In the real-frequency line-segment technique, Z_2 is realized as a minimum reactance function [6], [7]. In the direct computational technique, the real part of Z_2 is first expressed as a real even rational function and Z_2 is then formed via Gewertz procedure [8]. In the parametric approach, Z_2 is written as a partial fraction expansion [9]. In the simplified real-frequency technique (SRFT), the scattering matrix of the lossless matching network is used [10], [11]. In the method proposed in [12], Z_2 is modeled as a minimum reactance function.

In the proposed method, the impedance (Z_2 or Z_1) is obtained by using reflection coefficients at the source and load and the scattering parameters of the lossless matching network. In the following section, a two-variable representation of a scattering matrix is summarized and then the proposed method is explained.

III. Two-Variable Representation of a Scattering Matrix

For a two-port network containing mixed, lumped, and distributed elements, the two-variable scattering parameters can be expressed in terms of the two-variable polynomials g, h, f as follows [13], [14]:

$$S(p,\lambda) = \begin{bmatrix} S_{11}(p,\lambda) & S_{12}(p,\lambda) \\ S_{21}(p,\lambda) & S_{22}(p,\lambda) \end{bmatrix} = \frac{1}{g(p,\lambda)} \begin{bmatrix} h(p,\lambda) & \mu f(-p,-\lambda) \\ f(p,\lambda) & -\mu h(-p,-\lambda) \end{bmatrix} \quad (2)$$

where μ is a constant such that $|\mu| = 1$.

The variables of the polynomials in (2) are p and λ. $\lambda = \Sigma + j\Omega$ is the Richards variable related with commensurate transmission lines, and $p = \sigma + j\omega$ is the complex frequency variable related with lumped elements.

The degree of the scattering Hurwitz polynomial $g(p, \lambda)$ is $(n_p + n_\lambda)$th, and its coefficients are real. It can be defined as $g(p,\lambda) = \mathbf{P}^T \Lambda_g \boldsymbol{\lambda} = \boldsymbol{\lambda}^T \Lambda_g^T \mathbf{P}$ where

$$\Lambda_g = \begin{bmatrix} g_{00} & g_{01} & \cdots & g_{0n_\lambda} \\ g_{10} & g_{11} & \cdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ g_{n_p0} & \cdots & \cdots & g_{n_pn_\lambda} \end{bmatrix}, \quad \begin{matrix} \mathbf{P}^T = [1 \;\; p \;\; p^2 \;\; \ldots \;\; p^{n_p}] \\ \boldsymbol{\lambda}^T = [1 \;\; \lambda \;\; \lambda^2 \;\; \ldots \;\; \lambda^{n_\lambda}] \end{matrix}. \quad (3)$$

The partial degrees of two-variable polynomial $g(p, \lambda)$ are defined as the highest power of a variable whose coefficient is nonzero, i.e., $n_p = \deg_p g(p,\lambda)$, $n_\lambda = \deg_\lambda g(p,\lambda)$.

Similarly, $h(p, \lambda)$ is also a $(n_p + n_\lambda)$th degree polynomial with real coefficients such that $h(p,\lambda) = \mathbf{P}^T \Lambda_h \boldsymbol{\lambda} = \boldsymbol{\lambda}^T \Lambda_h^T \mathbf{P}$, where

$$\Lambda_h = \begin{bmatrix} h_{00} & h_{01} & \cdots & h_{0n_\lambda} \\ h_{10} & h_{11} & \cdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ h_{n_p0} & \cdots & \cdots & h_{n_pn_\lambda} \end{bmatrix}. \quad (4)$$

$f(p, \lambda)$ is a real polynomial that can be constructed by using all the transmission zeros of the two-port network. The polynomial $f(p, \lambda)$ can be written as

$$f(p,\lambda) = \prod_{i,j} f_i(p) f_j(\lambda); \qquad \begin{matrix} i = 1, 2, \ldots, n \\ j = 1, 2, \ldots, m \end{matrix} \quad (5)$$

where n is the number of transmission zeros of the lumped-element section, m is the number of transmission zeros of the distributed-element section, and the polynomials $f_i(p)$ and $f_j(\lambda)$ are constructed by using the transmission zeros of the lumped- and distributed-element sections, respectively. From (5), it is clear that $f(p, \lambda)$ of the mixed-element two-port network can be written as

$$f(p,\lambda) = f(p) f(\lambda). \quad (6)$$

If only the real-frequency transmission zeros related with lossless lumped-element section are considered, $f(p)$ will be either an even or an odd polynomial in p. Furthermore, if UEs are cascade connected, $f(\lambda)$ will be written as

$$f(\lambda) = (1 - \lambda^2)^{\frac{n_\lambda}{2}}. \quad (7)$$

If the finite imaginary axis zeros, except the zeros at dc, are disregarded, then a practical form of $f(p, \lambda)$ can be obtained as

$$f(p,\lambda) = p^k (1 - \lambda^2)^{\frac{n_\lambda}{2}} \quad (8)$$

where k denotes the number of transmission zeros at dc.

If $h(p, 0)$, $g(p, 0)$, and $f(p, 0)$ are written, the resulting boundary polynomials are single-variable polynomials that are describing the lumped-element section. Similarly, if $h(0, \lambda)$, $g(0, \lambda)$, and $f(0, \lambda)$ are written, the resulting boundary polynomials are single-variable polynomials that are describing the distributed-element section. These single-variable boundary polynomials completely describe the sections with corresponding type of elements.

Since it is desired to get a lossless matching network, then the losslessness condition requires that

$$S(p,\lambda) S^T(-p,-\lambda) = I \quad (9)$$

where I is the identity matrix. Equation (9) can be expressed in an open form as

$$g(p,\lambda) g(-p,-\lambda) = h(p,\lambda) h(-p,-\lambda) + f(p,\lambda) f(-p,-\lambda). \quad (10)$$

IV. Rationale of the Proposed Method

Let us take into account the double-matching problem depicted in Fig. 1 and then express the input reflection coefficient in terms of the scattering parameters of the two-port network when its output is terminated with the load impedance Z_L

$$S_1 = S_{11} + \frac{S_{12}S_{21}S_L}{1 - S_{22}S_L} \tag{11}$$

where S_L is the reflection coefficient at load and written as

$$S_L = \frac{Z_L - 1}{Z_L + 1}. \tag{12}$$

Similarly, let us express the output reflection coefficient in terms of the scattering parameters of the two-port network when its input is terminated with the source impedance Z_S as

$$S_2 = S_{22} + \frac{S_{12}S_{21}S_S}{1 - S_{22}S_S} \tag{13}$$

where S_S is the reflection coefficient at source and written as

$$S_S = \frac{Z_S - 1}{Z_S + 1}. \tag{14}$$

As a result, the input impedance and output impedance of the two-port network can be obtained by means of the following expressions, respectively:

$$Z_1 = \frac{1 + S_1}{1 - S_1} \tag{15a}$$

$$Z_2 = \frac{1 + S_2}{1 - S_2}. \tag{15b}$$

Therefore, the following algorithm is proposed to design broadband matching networks consisting of mixed, lumped, and distributed elements for single- and double-matching problems.

V. Proposed Broadband Matching Algorithm

Inputs:

- $Z_{S(\text{given})} = R_{S(\text{given})} + jX_{S(\text{given})}$, $Z_{L(\text{given})} = R_{L(\text{given})} + jX_{L(\text{given})}$: Given source and load impedance data, respectively.
- $\omega_{i(\text{given})}$: Given frequencies, $\omega_{i(\text{given})} = 2\pi f_{i(\text{given})}$.
- R_{Norm}: Normalization resistance.
- f_{Norm}: Normalization frequency.
- $h_{00}, h_{10}, h_{20}, \ldots, h_{n_p0}$: Initial real coefficients of the polynomial $h(p, 0)$. Here, n_p is the degree of the polynomial. Moreover, it is equal to the number of lossless lumped elements in the two-port network. They can be initialized as ± 1, or the method given in [15] can be applied.
- $h_{00}, h_{01}, h_{02}, \ldots, h_{0n_\lambda}$: Initial real coefficients of the polynomial $h(0, \lambda)$. Here, n_λ is the degree of the polynomial. Moreover, it is equal to the number of cascade-connected UEs in the two-port network. Similarly, they can be initialized as ± 1, or the method given in [15] can be applied.
- $f(\lambda)$: A polynomial formed by using the transmission zeros of the distributed-element section of the matching network. It must be noted that if UEs are cascade connected, then $f(\lambda)$ has the following form: $f(\lambda) = (1 - \lambda^2)^{n_\lambda/2}$, where n_λ is the number of UEs.
- $f(p)$: A polynomial formed by using the transmission zeros of the lumped-element section of the matching network.
- τ: Initial delay of the distributed elements.
- δ: The stopping criteria.

Outputs:

- Expression of the input reflection coefficient of the designed lossless matching network written in Belevitch form of $S_{11}(p, \lambda) = h(p, \lambda)/g(p, \lambda)$. The proposed algorithm determines the coefficients of the polynomials $h(p, \lambda)$ and $g(p, \lambda)$ by optimizing the system performance.
- Element values and matching network topology are obtained after synthesizing $S_{11}(p, \lambda)$ [16]–[19].

Computational Steps:

Step 1) Normalize the given frequencies via f_{Norm} as $\omega_i = f_{i(\text{given})}/f_{\text{Norm}}$.
Normalize the given load impedance and source impedance via R_{Norm} over the given frequency band as $R_L = R_{L(\text{given})}/R_{\text{Norm}}$, $X_L = X_{L(\text{given})}/R_{\text{Norm}}$, $R_S = R_{S(\text{given})}/R_{\text{Norm}}$, and $X_S = X_{S(\text{given})}/R_{\text{Norm}}$.

Step 2) Compute the corresponding Richards variable values via $\lambda_i = j\Omega_i = j\tan\omega_i\tau$.

Step 3) Obtain the strictly Hurwitz polynomials $g(p, 0)$ and $g(0, \lambda)$ via (10).

Step 4) Compute $h(p, \lambda)$ and $g(p, \lambda)$ via (10) by using $h(p, 0)$, $g(p, 0)$, $h(0, \lambda)$, $g(0, \lambda)$, and $f(p, \lambda) = f(p)f(\lambda)$. Then, compute the scattering parameters by using (2).

Step 5) Compute the load and source reflection coefficients S_L and S_S via (12) and (14), respectively.

Step 6) Compute the input and output reflection coefficients S_1 and S_2 via (11) and (13), respectively.

Step 7) Compute input and output impedances Z_1 and Z_2 via (15a) and (15b), respectively.

Step 8) Compute TPG(ω) via (1).

Step 9) Compute the error via $\varepsilon(\omega) = 1 - \text{TPG}(\omega)$, then $\delta = \sum |\varepsilon(\omega)|^2$.

Step 10) If δ is acceptable ($\delta \le \delta_c$), stop the algorithm and synthesize $S_{11}(p, \lambda)$. Otherwise, change the initialized delay and coefficients of the polynomials $h(p, 0)$ and $h(0, \lambda)$ by means of any optimization method and return to step 2.

VI. Example

As an example, a double-matching problem is solved. The normalized load and source impedance data are given in Table I. It should be noted that the given load data belong to a capacitor $C_L = 3$ in parallel with a resistance $R_L = 1$ (i.e., $R_L//C_L$ type of impedance), and the source data as an inductor $L_S = 1$ in series with a resistance $R_S = 1$ (i.e., $R + L$ type of

TABLE I
GIVEN NORMALIZED LOAD AND SOURCE IMPEDANCE DATA

ω	R_L	X_L	R_S	X_S
0.0	1.0000	0.0000	1.0000	0.0000
0.1	0.9174	-0.2752	1.0000	0.1000
0.2	0.7353	-0.4412	1.0000	0.2000
0.3	0.5525	-0.4972	1.0000	0.3000
0.4	0.4098	-0.4918	1.0000	0.4000
0.5	0.3077	-0.4615	1.0000	0.5000
0.6	0.2358	-0.4245	1.0000	0.6000
0.7	0.1848	-0.3882	1.0000	0.7000
0.8	0.1479	-0.3550	1.0000	0.8000
0.9	0.1206	-0.3257	1.0000	0.9000
1.0	0.1000	-0.3000	1.0000	1.0000

impedance). For comparison, the same example is also solved by using SRFT [10], [11].

In the design, the delay (τ) and the polynomials $h(0, \lambda)$ and $h(p, 0)$ are initialized as $\tau = 0.6$, $h(0, \lambda) = \lambda^2 + \lambda + 1$, and $h(p, 0) = p^2 + p + 1$ in an *ad hoc* manner, respectively. Moreover, the polynomial $f(\lambda)$ is selected as $f(\lambda) = f(p)f(\lambda) = (1 - \lambda^2)$. Therefore, this means that there will be two cascade-connected UEs and two low-pass-type lumped elements in the two-port network. In the example, α and β are selected as $\alpha = 2$ and $\beta = L$. Therefore, output impedance Z_2 and load impedance Z_L are used in the TPG expression in Step 8. As the result of the proposed algorithm, the following input reflection coefficient expression of the designed lossless two-port network is obtained:

$$S_{11}(p, \lambda) = \frac{h(p, \lambda)}{g(p, \lambda)}$$

where $h(p, \lambda) = \mathbf{P}^T \Lambda_h \boldsymbol{\lambda} = \boldsymbol{\lambda}^T \Lambda_h^T \mathbf{P}$ with

$$\Lambda_h = \begin{bmatrix} 0.4943 & 0.1108 & 0.0980 \\ -0.7174 & -1.2313 & -1.0428 \\ -0.4615 & -0.5824 & 0 \end{bmatrix}$$

and $g(p, \lambda) = \mathbf{P}^T \Lambda_g \boldsymbol{\lambda} = \boldsymbol{\lambda}^T \Lambda_g^T \mathbf{P}$ with

$$\Lambda_g = \begin{bmatrix} 1.1155 & 2.0389 & 1.0048 \\ 1.4144 & 2.1109 & 1.0428 \\ 0.4615 & 0.5824 & 0 \end{bmatrix}$$

where $\mathbf{P}^T = [1\ p\ p^2]$, $\boldsymbol{\lambda}^T = [1\ \lambda\ \lambda^2]$.

The proposed algorithm and SRFT are realized via MATLAB. The elapsed times via the proposed method and SRFT are 14.3184 and 19.4460 s, respectively.

Then, the obtained two-variable input reflection coefficient is synthesized and the mixed-element matching network shown in Fig. 2 is obtained. The TPG curve of the matching network is given in Fig. 3.

As shown in Fig. 3, initial performance of the matching network is very good and the power curves resulting from the proposed method and SRFT are nearly the same. However, it can be improved by using any commercially available design tool [20]. After optimization, the normalized elements values are obtained as $C = 1.302$, $Z_1 = 1.067$, $L = 0.7195$, $Z_2 = 4.499$, $\tau = 0.2078$, and $n = 1.569$. For comparison, both initial and optimized gain curves and the gain curve obtained via SRFT are given in Fig. 3.

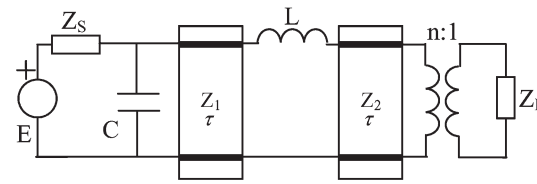

Fig. 2. Mixed-element matching network topology with initial element values: $C = 1.3242$, $Z_1 = 1.4069$, $L = 1.1221$, $Z_2 = 2.0537$, $\tau = 0.2078$, $n = 1.6098$.

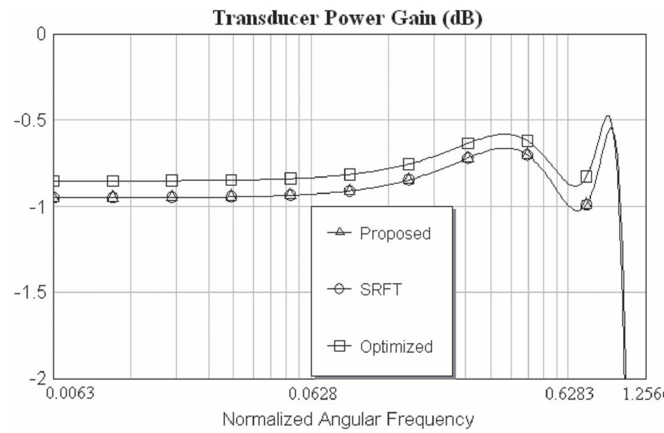

Fig. 3. Transducer power gain of the matched system with mixed elements.

VII. CONCLUSION

The design of broadband matching networks is an important problem for microwave engineers. In this regard, broadband matching analytic theory and computer-aided design (CAD) tools are indispensible for engineers. It is well known that analytic theory is quite difficult to use. Therefore, it is always common practice to employ commercially available CAD tools. However, it is necessary for these tools to provide suitable matching network topology and initial element values. In this way, element value initialization is extremely important since system performance is highly nonlinear in terms of the element values. Therefore, in this brief, a new initialization method is proposed for the construction of lossless broadband matching networks consisting of mixed, lumped, and distributed elements.

In the proposed algorithm, the input or output impedance of the mixed-element two-port network is obtained in terms of reflection coefficients at source and load and the scattering parameters of the two-port network. Then, this impedance and source or load impedance (Z_S or Z_L) are employed to compute the TPG. The scattering parameters of the two-port network are optimized to facilitate maximum performance.

Then, after synthesis, the designed lossless mixed-element matching network with initial element values is obtained. Finally, the performance of the matching network can be improved via any available CAD tool.

The proposed method has the following advantages. The polynomial $f(p, \lambda)$ is formed by using the transmission zeros of the matching network; therefore, they are under the control of the designer. Transducer power gain is quadratically dependent on the optimization parameters; therefore, it can be regarded as a quadratic optimization, yielding a superb numerical convergence. Moreover, the proposed new algorithm can be used to design broadband single or double matching networks.

From the given example, it is clear that the proposed algorithm provides suitable initial element values. Therefore, the proposed algorithm can be accepted as an initialization step for CAD tools.

REFERENCES

[1] B. K. Kinariwala, "Theory of cascaded structures: Lossless transmission lines," *Bell Syst. Tech. J.*, vol. 45, no. 4, pp. 631–649, Apr. 1966.

[2] L. A. Riederer and L. Weinberg, "Synthesis of lumped-distributed networks: Lossless cascades," *IEEE Trans. Circuits Syst.*, vol. 27, no. 10, pp. 943–956, Oct. 1980.

[3] P. I. Richards, "Resistor-transmission-line circuits," *Proc. Inst. Radio Eng.*, vol. 36, no. 2, pp. 217–220, Feb. 1948.

[4] M. Şengül, "Design of practical broadband matching networks with lumped elements," *IEEE Trans. CAS-II: Exp. Briefs*, vol. 60, no. 9, pp. 552–556, Sep. 2013.

[5] M. Şengül, "Design of practical broadband matching networks with distributed elements," *Int. J. Electron. Commun. (AEU)*, vol. 67, pp. 676–680, Aug. 2013.

[6] H. J. Carlin, "A new approach to gain-bandwidth problems," *IEEE Trans. CAS*, vol. 23, no. 4, pp. 170–175, Apr. 1977.

[7] H. J. Carlin and P. P. Civalleri, *Wideband Circuit Design*. Boca Raton, FL, USA: CRC Press, 1998, Library of Congress Card Number: 97 26966.

[8] H. J. Carlin and B. S. Yarman, "The double matching problem: Analytic and real frequency solutions," *IEEE Trans. Circuits Syst.*, vol. 30, no. 1, pp. 15–28, Jan. 1983.

[9] A. Fettweis, "Parametric representation of brune functions," *Int. J. Circuit Theory Appl.*, vol. 7, no. 1, pp. 113–119, 1979.

[10] B. S. Yarman, "A simplified real frequency technique for broadband matching complex generator to complex loads," *RCA Rev.*, vol. 43, pp. 529–541, Sep. 1982.

[11] B. S. Yarman and H. J. Carlin, "A simplified real frequency technique applied to broadband multi-stage microwave amplifiers," *IEEE Trans. Microw. Theory Tech.*, vol. 30, no. 12, pp. 2216–2222, Dec. 1982.

[12] B. S. Yarman, M. Şengül, and A. Kılınç, "Design of practical matching networks with lumped elements via modeling," *IEEE Trans. CAS I, Reg. Papers*, vol. 54, no. 8, pp. 1829–1837, Aug. 2007.

[13] S. Basu and A. Fettweis, "On synthesizable multidimensional lossless two-ports," *IEEE Trans. CAS*, vol. 35, no. 12, pp. 1478–1486, Dec. 1988.

[14] M. Şengül, "Construction of lossless ladder networks with simple lumped elements connected via commensurate transmission lines," *IEEE Trans. CAS II, Exp. Briefs*, vol. 56, no. 1, pp. 1–5, Jan. 2009.

[15] M. Şengül, B. S. Yarman, C. Volmer, and M. Hein, "Desing of distributed-element RF filters via reflectance data modeling," *Int. J. Electron. Commun.*, vol. 62, no. 7, pp. 483–489, Aug. 2008.

[16] A. Fettweis, "Factorization of transfer matrices of lossless two-ports," *IEEE Trans. Circuit Theory*, vol. 17, no. 1, pp. 86–94, Feb. 1970.

[17] A. Fettweis, "Cascade synthesis of lossless two ports by transfer matrix factorization," in *Network Theory*, R. Boite, Ed. New York, NY, USA: Gordon and Breach, 1972.

[18] H. J. Carlin, "Distributed circuit design with transmission line elements," *Proc. IEEE*, vol. 3, pp. 1059–1081, 1971.

[19] M. Şengül, "Synthesis of cascaded commensurate transmission lines," *IEEE Trans. CAS II, Exp. Briefs*, vol. 55, no. 1, pp. 89–91, Jan. 2008.

[20] AWR: Microwave Office of Applied Wave Research Inc. [Online]. Available: www.appwave.com