Partial Power and Rate Adaptation for MQAM/OFDM Systems under CFO

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***Abstract*—In this paper, a new *partial* power and rate adaptation scheme for orthogonal frequency division multiplexing (OFDM) systems is proposed in the presence of carrier frequency offset (CFO). The conventional adaptive scheme is shown to be a special case of the partially adaptive scheme technique which enables the resulting non-convex optimization problem, solved in a feasible way. It leads to a solution for optimal power adaptation that maximizes the spectral efficiency of an OFDM system using M-ary quadrature amplitude modulation (MQAM) under average power and instantaneous BER constraints. Closed-form expressions for the average spectral efficiency (ASE) of adaptive OFDM systems are derived. The theoretical results and computer simulations show that the range of the partial adaptation becomes narrow and the performance of constant power and continuous rate is very close to that of the partially adaptive power and continuous rate for higher CFO or high signal noise ratio (SNR) values.**

I. Introduction

OFDM has been shown to be an effective technique to overcome the inter-symbol interference (ISI) caused by frequency-selective fading with a simple transceiver structure. It has emerged as the leading transmission technique for a wide range of wireless communication standards [1]. However, OFDM is more sensitive to frequency synchronization errors than the single-carrier systems. Carrier frequency offset (CFO) in OFDM systems results in a loss of subchannel orthogonality, which leads to inter-channel interference (ICI). ICI will degrade the spectral efficiency of the system [2].

Adaptive modulation is a promising technique to improve the performance by increasing the data rate that can be reliably transmitted over fading channels. Transmit power, data rate, instantaneous bit error rate (BER) and channel code rate in each subchannel can be adapted relative to the channel [3]. On the other hand there are only a few works existing in the literature on OFDM based adaptive modulation schemes with CFO [4], [5], [6], [7]. In [4], the instantaneous and average effective SNR and average BER of OFDM systems are derived in the presence of CFO for Rayleigh fading channel. In [5] and [6], the authors have studied the power and rate adaptive scheme for MQAM/OFDM under very fast fading channel with perfect and imperfect CSI as opposed to the current work which mainly deals with power and rate adaptation in the presence of carrier frequency offset. However, these papers have not considered the optimization problem, taking into account the largest transmit power, the power distribution and the bit adaptation, jointly. Therefore, the scheme investigated in these works does not yield an optimal solution. In [7], a feasible solution is proposed by assuming the ICI power of the subcarrier to be optimized is approximated by its own power. However, as will be explained shortly, the BER constraints cannot be be met, in this case, when the transmit powers of the other subcarriers are larger than that of the subcarrier being adapted.

This paper is concerned with continuous power and rate adaptation of OFDM systems with CFO, under average power and instantaneous BER constraints. A new BER model for OFDM systems under CFO is proposed, resulting in a BER performance meeting the BER constraints easily. Based on the new BER model, closed form analytical expressions are obtained for optimal power distribution, maximum bit load and average spectral efficiency (ASE). Based on the expressions derived for optimal power distribution and maximum bit load, a new adaptive scheme called the *partially adaptive scheme* is proposed, which also leads to two different special cases. Namely, the *complete adaptive scheme* and the *uniform power distribution*. A new constraint, namely *variable largest transmit power*, included in the formulation of the problem, makes the resulting optimization nonconvex. Using the linear-fractional programming, it is solved by transforming the nonconvex optimization into a convex one and proved that the resulting solution is optimal.

II. System Model

We consider an OFDM system with K subcarriers. At the transmitter, N out of K subcarriers are actively employed

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to transmit data symbols and nothing is transmitted from the remaining $K-N$ carriers. Each active subcarrier is modulated by a data symbol $X[k]$, where k represents the OFDM subcarrier index. After taking a K-point inverse Fourier transform (IFFT) of the data sequence and adding a cyclic prefix (CP) of duration T_{CP} before transmission, to avoid inter-symbol interference (ISI), for a given frequency offset ν, the received OFDM symbol at the input of the discrete Fourier transform (DFT) can be expressed as [4], [7]

$$y[n] = \tfrac{1}{\sqrt{K}} \sum_{k=0}^{K-1} H[k]X[k]e^{j2\pi n(k+\nu)/K} + w[n], \qquad (1)$$

where $H[k]$ denotes the discrete frequency response of the channel at kth subcarrier and $w[n]$ is the additive zero-mean complex Gaussian noise with variance σ_w^2. The kth subcarrier output of DFT during one OFDM symbol can be expressed as

$$\begin{aligned} Y[k] &= \frac{1}{\sqrt{K}} \sum_{n=0}^{K-1} y[n]e^{-j2\pi nk/N} + W[k] \\ &= H_k^{\nu} X[k] + I_k + W[k], \end{aligned} \qquad (2)$$

where $W[k]$ is a frequency domain additive Gaussian noise on the kth subcarrier; I_k is the ICI caused by the frequency offset which is given by [2], [4], [8]

$$\begin{aligned} I_k = &\sum_{m\in\kappa, m\neq k} H[m]\, X[m] \left(\sin \pi\nu \cdot e^{j(\pi\nu(k-1)/K)}\right. \\ &\left. / K \sin\left(\pi\left(k-m+\nu\right)/K\right)\right) e^{-j(\pi\nu(k-m)/K)}, \end{aligned} \qquad (3)$$

$\kappa = \{0, 1, \cdots, K-1\}$ and H_k^{ν} denotes the distorted channel response, which is written as

$$H_k^{\nu} = \tfrac{H[k]\sin\pi\nu}{K\sin(\pi\nu/K)} e^{j(\pi\nu(K-1)/K)}. \qquad (4)$$

From (3), ICI power of the kth OFDM subcarrier can be obtained as

$$P_{ICI}^{(k)} = E\{\mid I_k\mid^2\} = \sum_{m\in\kappa, m\neq k} E\left\{|X[m]|^2\right\}\rho_{k,m}, \qquad (5)$$

where $E\{.\}$ denotes expectation and

$$\rho_{k,m} = \left(\frac{\sin\pi\nu}{K\sin\left(\pi\left(m-k+\nu\right)/K\right)}\right)^2.$$

Since data on each subcarrier are uncorrelated, it follows easily from (5) that the normalized ICI power σ_ν^2, adopted in this paper, is $\sigma_\nu^2 = P_{ICI}^K / E\left\{|X[m]|^2\right\} \approx (\pi\nu)^2/3$, [4], [7].

III. Adaptive OFDM systems under CFO

Let $\overline{S}$ denote the average transmitted signal power on each OFDM subcarrier, that is $E\{|X[k]|^2\} = \overline{S}$. With appropriate scaling of $\overline{S}$, we can assume that the average channel power gain on each subcarrier is unity. That is $E\{|H[k]|^2\} = 1$. For fixed power allocation (constant transmit power on each subcarrier), the instantaneous effective signal-to-noise (SNR), for fixed power allocation, is shown to be

$$\gamma_e(k) = \frac{\overline{S}|H[k]|^2}{\overline{S}\sigma_\nu^2 + \sigma_w^2} = \frac{\gamma[k]}{\overline{\gamma}\sigma_\nu^2 + 1}, \qquad (6)$$

where $\gamma[k] = \overline{\gamma}|H[k]|^2$ is called the instantaneous received SNR of the kth subcarrier and $\overline{\gamma} = \overline{S}/\sigma_w^2$ is the average SNR when there is no ICI.

Similarly, for different power levels allocated to each OFDM subchannels, which are a function of $\gamma[k]$, denoted by $s(\gamma[k])$, the effective SNR, in the kth subchannel, can be shown to be expressed as

$$\widetilde{\gamma}_e\big(s(\gamma[k])\big) = \frac{s(\gamma[k])|H[k]|^2}{\overline{S}P_N^{(k)} + \sigma_w^2} = \frac{\gamma[k]\frac{s(\gamma[k])}{\overline{S}}}{\overline{\gamma}P_N^{(k)} + 1}, \qquad (7)$$

where $P_N^{(k)}$ is the normalized ICI power (variance) of the kth subcarrier. From (5), it follows that

$$P_N^{(k)} = \sum_{m\in\kappa, m\neq k} \tfrac{s(\gamma[m])}{\overline{S}} \rho_{k,m}. \qquad (8)$$

The ASE of an OFDM transmission scheme is defined as $C_{ASE} = E_{(\gamma[k])} \sum_{k\in K} \beta(\gamma[k]) / \left(BT_{SYM}\right)$, where B is the total bandwidth, T_{SYM} denotes the duration of an OFDM symbol and $\beta(\gamma[k])$ is the bit load size for the kth subcarrier. This results in a subchannel spacing of $\Delta f = B/K$ and $T_{SYM} = 1/\Delta f = K/B$. If we neglect the impact of CP on ASE, the ASE becomes $C_{ASE} = E_{(\gamma[k])}\left(1/K \sum_{k\in K} \beta\left(\gamma[k]\right)\right)$. When the discrete frequency response of the channel is independent and identically distributed (iid) random variable with probability density function (pdf) $p(.)$, [7], [9], ASE can be expressed as

$$C_{ASE} = \int_0^\infty \beta(\gamma[k]) p(\gamma[k]) d\gamma[k] \quad \text{bits/sec/Hz}. \qquad (9)$$

We also assume an average transmit power constraint given by

$$E_{\gamma[k]}\{s(\gamma[k])\} = \int_0^\infty s(\gamma[k]) p(\gamma[k]) d\gamma[k] = \overline{S}. \qquad (10)$$

The rate adaptation, $\beta(\gamma[k])$, is typically parameterized by the received power $\overline{S}$ and the bit error rate (BER) of the modulation technique. A tight approximate BER expression for the square MQAM with Gray mapping in AWGN as a function of $s(\gamma[k])/\overline{S}$ is as follows [7].

$$BER(\gamma[k]) \approx 0.3\exp\left(\tfrac{-1.5\varphi\ \widetilde{\gamma}_e(s(\gamma[k]))}{2^{\beta(\gamma[k])}-1}\right), \qquad (11)$$

where $\widetilde{\gamma}_e(s(\gamma[k]))$ is given by (7) and $\varphi = \left(\sin(\pi\nu) / \left(K\sin\left(\pi\nu/K\right)\right)\right)^2 \approx \left(\sin(\pi\nu)/(\pi\nu)\right)^2$ [2].

We now derive the optimal continuous rate and power adaptation to maximize spectral efficiency (9) subject to the average power constraint (10) and an instantaneous BER constraint $BER(\gamma[k]) \leq \epsilon$. Taking into account $\widetilde{\gamma}_e\big(s(\gamma[k])\big)$ in (7) and inverting (11), the bit load size $\beta(\gamma[k])$ for each subchannel can be expressed as a function of the variable power on each subcarrier $s(\gamma[k])$ and the fixed bit error rate ϵ ($BER(\gamma[k]) = \epsilon$) as,

$$\beta\left(\gamma\left[k\right]\right) = \log_2\left(1 + \tfrac{(-1.5\varphi/\ln(\varepsilon/0.3))\gamma[k]\frac{s(\gamma[k])}{\overline{S}}}{\overline{\gamma}P_N^{(k)}+1}\right). \qquad (12)$$

It can be easily seen from (8) that maximizing the ASE of the kth subchannel (9) in an adaptive OFDM system, needs the knowledge of the other subcarrier powers $s(\gamma[m])/\overline{S},\ m = 0,1,\cdots,K-1$, which makes the solution of the optimization problem mathematically intractable. Note that it is very difficult to determine the exact ICI power, $P_N^{(k)}$, in (8) when instantaneous transmit powers carried by OFDM subcarriers are not the same during the power adaptation. The work in [7] assumes that the normalized ICI power of the kth subchannel can be determined by its own power, that is, $P_N^{(k)} \approx \left(s\left(\gamma\left[k\right]\right)/\overline{S}\right)\sigma_\nu^2$. However, the BER constraint, $(BER(\gamma[k]) \leq \epsilon$ for $k = 0,1,\cdots,K-1)$, in this case, cannot be met at all times when the transmit powers of other subcarriers are larger than that of the kth subcarrier. This is mainly due to the fact that the normalized ICI power of the kth subchannel $P_N^{(k)}$ is underestimated when it is determined by its own power. On the other hand, in our work, we propose an upper bound for $P_N^{(k)}$ in terms of the largest transmit normalized power $s_{\max}= \max\limits_{m\in\kappa}\left\{s\left(\gamma\left[m\right]\right)/\overline{S}\right\}$. It can be easily seen in this case that the BER constraint is always met since the powers transmitted by other subcarriers is less than or equal to $s_{\max}$. Consequently, employing the normalized ICI power, $P_N^{(k)}$ in (8), determined by $s_{\max}$, the maximum bit load size $\beta(\gamma[k])$ for each subchannel can be expressed from (12) as follows.

$$\beta\left(\gamma\left[k\right]\right) = \log_2\left(1 + \frac{a\gamma[k]s(\gamma[k])}{s_{\max}b+1}\right). \tag{13}$$

where $a = \frac{-1.5\varphi}{\overline{S}\ln(\varepsilon/0.3)},\ \ b = \sigma_\nu^2\overline{\gamma}$.

Based on the above developments, we now consider the following constrained optimization problem to solve the power and rate adaptation problem:

$$\max_{s(\gamma[k]),s_{max}} \int \beta\left(\gamma\left[k\right]\right) p_{\gamma[k]}\left(\gamma\left[k\right]\right) d\gamma\left[k\right] \tag{14a}$$

subject to

$$E_{\gamma[k]}\left\{s\left(\gamma\left[k\right]\right)\right\} = \overline{S} \qquad \forall\ k\in\kappa \tag{14b}$$

$$0 \leq s\left(\gamma\left[k\right]\right) \leq s_{\max}\overline{S} \qquad \forall\ k\in\kappa \tag{14c}$$

$$BER\left(\gamma(\left[k\right])\right) \leq \varepsilon \qquad \forall\ k\in\kappa. \tag{14d}$$

The above constraint optimization problem is not convex. However, it is in the form of a log linear-fractional model [10], [11]. In Appendix A, using the linear-fractional programming, the problem is solved by transforming it into a convex optimization, resulting in the following optimal power adaptation

$$\frac{s\left(\gamma\left[k\right]\right)}{\overline{S}} = \begin{cases} \frac{s_{\max}b+1}{\ln(2)\lambda(z)\overline{S}} - \frac{s_{\max}b+1}{a\gamma[k]\overline{S}} & \gamma_0 \leq \gamma[k] \leq \gamma_1; \\ s_{\max} & \gamma[k] \geq \gamma_1. \end{cases} \tag{15}$$

where $z = (s_{\max}b+1)^{-1},\ \ \lambda(z)$ is the Lagrange multiplier

$$\gamma_0 = \ln(2)\lambda(z)/a, \tag{16}$$

$$\gamma_1 = \frac{1}{a\left(1/(\ln(2)\lambda(z)) - (s_{\max}/(s_{\max}b+1))\overline{S}\right)}. \tag{17}$$

The values of s_{max} and the Lagrangian $\lambda(z)$ are found numerically in such a way that the average power and BER constraint (14b) and (14d) are satisfied. The details are described in Appendix A.

We call our method *partially adaptive scheme* since the transmission power is adapted relative to channel variations in $[\gamma_0,\gamma_1]$ and becomes constant in $(\gamma_1,\infty]$. γ_0 and γ_1, defined in (16), (17), are optimized thresholds for $\gamma[k]$ below which the channel is not used and above which the channel is used with a constant transmit power, respectively. On the other hand, the scheme proposed in [3], [12] is a *complete adaptive scheme*, since transmission power is adapted relative to channel variations over the whole range of the channel conditions.

It follows from (15) that $\lim_{\gamma[k]\to\infty}(\frac{s(\gamma[k])}{\overline{S}}) = \frac{s_{\max}b+1}{\ln(2)\lambda(z)\overline{S}}$. Consequently, if there is no limit for the largest power, the power adaption is realized by the complete adaptive scheme. However, since ICI is decided by the largest power $s_{\max}$, there is a tradeoff between the adaptive scheme and the ICI. That is,

a) If $s_{\max} > \frac{s_{\max}b+1}{\ln(2)\lambda(z)\overline{S}}$, the ICI is overestimated, and the ASE decreases as the $s_{\max}$ increases.

b) If $1 < s_{\max} \leq \frac{s_{\max}b+1}{\ln(2)\lambda(z)\overline{S}}$, the optimal $s_{\max}$ equals to $(1/z-1)/b$ as shown in Appendix A.

c) $s_{\max} = 1$, which means uniform power distribution.

The proposed scheme includes two special cases:

a) When $\gamma_0 = \gamma_1 = 0$, (15) becomes $s(\gamma[k])/\overline{S} = 1$ for $\gamma[k] \geq 0$, which means a uniform power distribution;

b) When $b = 0$ $(CFO = 0)$, it can be shown from (17) that $\gamma_1 = \infty$ and, thus, (15) reduces to $\frac{s(\gamma[k])}{\overline{S}} = \frac{1}{\ln(2)\lambda(z)\overline{S}} - \frac{1}{a\gamma[k]\overline{S}}$, which has been investigated in [3], [12]. Since there is no ICI, consequently, our adaptive scheme turns out to be a complete adaptive scheme.

From (12) and (15), the optimal bit rate adaptation on each subcarrier can be obtained as follows:

$$\beta\left(\gamma\left[k\right]\right) = \begin{cases} \log_2\left(\frac{a\gamma[k]}{\lambda(z)\ln(2)}\right) & \gamma_0 \leq \gamma[k] \leq \gamma_1 \\ \log_2\left(1 + \frac{s_{\max}a\overline{S}\gamma[k]}{1+s_{\max}b}\right) & \gamma[k] \geq \gamma_1. \end{cases} \tag{18}$$

A closed-form expression for the optimal ASE can be obtained from (9) and (18), and by using the exponential pdf for $\gamma[k]$ with mean $\overline{S}$. The final result is given below.

$$\begin{aligned} C_{\text{ASE}} = & \tfrac{1}{\ln(2)}\exp(-\gamma_0/\overline{S})\left(\ln(\gamma_0 a/(\ln(2)\lambda(z)))\right. \\ & \left. + Ei(1,\gamma_0/\overline{S})\exp(\gamma_0/\overline{S})\right) \\ & - \tfrac{1}{\ln(2)}\exp(-\gamma_1/\overline{S})\left(\ln(\gamma_1 a/(\ln(2)\lambda(z)))\right. \\ & \left. + Ei(1,\gamma_1/\overline{S})\exp(\gamma_1/\overline{S})\right) \\ & + \tfrac{\exp(-\gamma_1/\overline{S})}{\ln(2)}\left(\log(1+C\gamma_1)\right. \\ & \left. + Ei(1,\tfrac{1+C\gamma_1}{C\overline{S}})\exp(1+\tfrac{1+C\gamma_1}{C\overline{S}})\right), \end{aligned} \tag{19}$$

where $C = a(1-z)\overline{S}/b$, z and $\lambda(z)$ can be calculated as explained in Appendix A. $Ei(1,x) = \int_x^\infty \frac{e^{-t}}{t}dt$ denotes the exponential integral.

IV. Numerical Results And Discussions

We now investigate the performance of the power and rate adaptation schemes proposed in this paper numerically as well as by computer simulations.

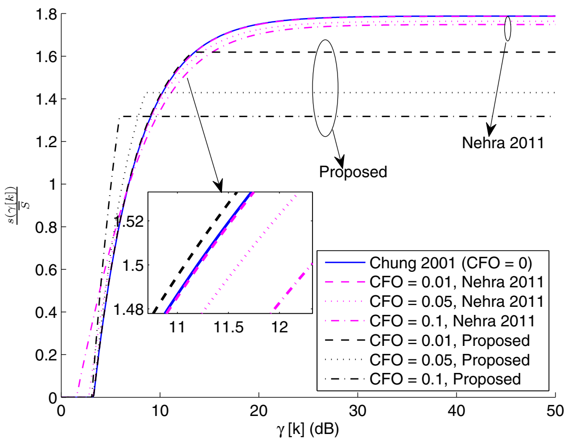

Fig. 1. $\frac{s(\gamma[k])}{S}$ for different adaptive schemes ($\overline{\text{BER}} = 10^{-3}$, $\overline{\gamma[k]} = 10dB$) at different CFO values

In Fig. 1, optimal power adaptation for different adaptive schemes are presented for different CFO values. As can be seen in Fig. 1, the curves of [7] are similar to the one of [3], especially when CFO is small. However, the scheme in [7] cannot meet the instantaneous BER constraint, since for mathematical tractability, the authors assume that the ICI power is determined by its own power only. However, the ICI power depends on and is determined by the other subchannel powers. Fig. 1 shows that the curves of the proposed partial power adaptation consists of two parts, representing the complete adaptive scheme and the constant adaptation, respectively. We also observe that the partial power adaptation scheme has the largest power $s_{\max} = \lim_{\gamma[k]\to\infty} \frac{s(\gamma[k])}{S}$ for all subcarriers. Therefore, the proposed scheme can meet the instantaneous BER constraint, and the higher CFO values result in a smaller range of $[\gamma_0, \gamma_1]$, implying that the optimal power adaptation adapts the constant power distribution over the high CFO values. However, $s_{\max}$ decreases as the CFO increases which is equivalent to stating that the optimal power adaptation is close to the constant transmit power threshold $\gamma_0 \approx \gamma_1$ in [3] for large values of the CFO. Finally we remark that our scheme also contains two special cases, namely, $s_{\max} = 1$ and CFO = 0, as discussed in [3], [12].

In Fig. 2, the bit rate adaptation, $\beta(\gamma[k])$ are plotted as a function of $\overline{\gamma[k]}$, for different adaptive schemes and for $\overline{\text{BER}} = 10^{-3}$, $\overline{\gamma[k]} = 10$ dB, for different CFO values. Fig. 2 shows that the smaller CFO values yield larger bit rates than its higher CFO value counterparts when $\gamma[k]$ is fixed. The curves of [7] and of those proposed in this paper, are close to the one of [3], especially when CFO is small. The Fig. 2 also shows that the bit rate curves consist of two parts. The first

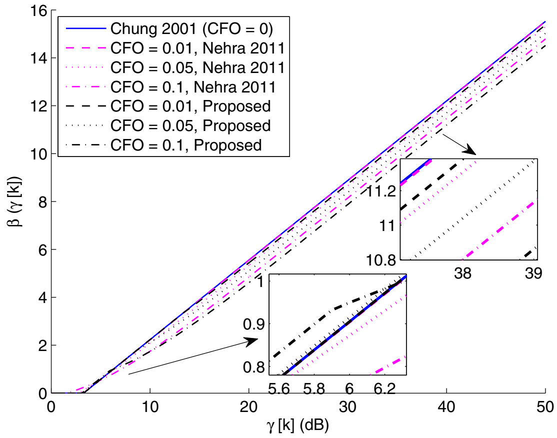

Fig. 2. $\beta(\gamma[k])$ for different adaptive schemes ($\overline{\text{BER}} = 10^{-3}$, $\overline{\gamma[k]} = 10dB$) at different CFO values

part has higher slope than the second part. From the figure, we can see that the difference of slopes increases with the CFO.

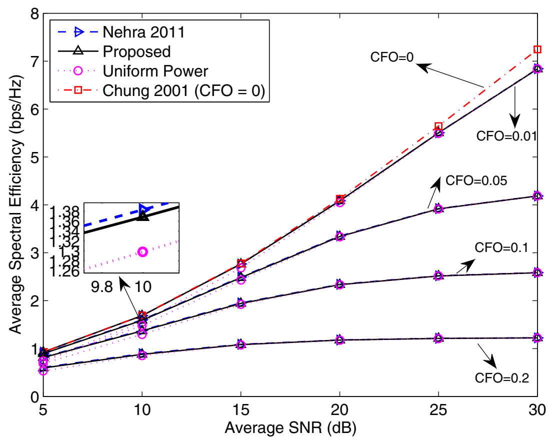

Fig. 3. The ASE vs average SNR curves for different adaptive schemes under different CFO values

In Fig. 3, the optimal ASE of the adaptive MQAM-OFDM system is plotted as a function of the average SNR, for the BER bound $\epsilon = 10^{-3}$ and for different CFO values. The figure also shows the results of Nehra *et al.* [7] and Chung *et al.* [3] with the same simulation parameters. As can be seen from these plots, the ASE curves obtained by [7] are very close to ours, while the BER constraint can not be satisfied in [7], it is met for every subcarrier in our work. We observe in Fig. 3 that the spectral efficiency of the system cannot be improved beyond a certain level, determined by the CFO values. This is mainly due to the contribution of the average SNR to the ICI power in the expression of effective SNR (7). In addition to this fact, we observe that the ASE performance of the uniform power and rate adaptation is very close to that of the optimal power and rate adaptation when SNR is high or for large CFO values. Therefore, improving the performance using the power and rate adaptation under high SNR or large CFO conditions is limited compared to the uniform power and rate adaptation.

V. Conclusion

In this paper, the power and rate adaptation has been investigated for MQAM/OFDM systems under CFO. Under an instantaneous BER and average power constraints, optimal power and rate adaption have been determined analytically to maximize the ASE for the system. Considering a target BER constraint for each subcarrier, a lower bound was derived and a closed form expressions were obtained under channel estimation errors. The simulation results show that the ASE performance of the proposed scheme is valid because the performance almost achieves the upper bound of the method given by [7] and concluded that the average spectral efficiency is seriously degraded under high CFO and the performance cannot be improved beyond a certain level. The results also show that improving the performance using power and rate adaptation under high SNR is limited compared with using uniform power and rate adaptation.

Appendix A
Solution of (14a-14d)

The optimization problem in (14) is not a convex program. It is rather a log-linear-fractional programming [10], [13]. However, we can transform it to convex optimization problem by means of a linear-fractional programming as follows. (14) can be expressed as

$$\text{maximize } J(\mathbf{x}) = \int \log_2\left(\frac{\mathbf{c}^T\mathbf{x}+1}{\mathbf{e}^T\mathbf{x}+1}\right) p_{\gamma[k]}\left(\gamma[k]\right) d\gamma[k] \quad (20a)$$

subject to

$$E_{\gamma[k]}\left\{\mathbf{g}^T\mathbf{x}\right\} = \overline{S}, \quad (20b)$$

$$\mathbf{g}^T\mathbf{x} \geq 0, \quad (20c)$$

$$\mathbf{d}^T\mathbf{x} \leq 0, \quad (20d)$$

$$BER(\gamma[k]) \leq \varepsilon, \quad (20e)$$

where $s_{\max} \geq 1$, $\mathbf{x} = \left[s(\gamma[k]), s_{\max}\right]^T$, $\mathbf{c} = \left[a\gamma[k], b\right]^T$, $\mathbf{e} = \left[0, b\right]^T$, $\mathbf{g} = \left[1, 0\right]^T$, $\mathbf{d} = \left[1, -\overline{S}\right]^T$.

If the feasible set $\left\{\mathbf{x}|\mathbf{g}^T\mathbf{x} \geq 0,\ \mathbf{d}_T\mathbf{x} \leq 0,\ E_{\gamma[k]}\left\{\mathbf{g}^T\mathbf{x}\right\} = \overline{S},\ BER\left([k]\right) \leq \varepsilon, \mathbf{e}^T\mathbf{x}+1 > 0\right\}$ is nonempty, (20a-20d) can be transformed to an equivalent problem as follows [10], [11]

$$\text{maximize} \quad \int \log_2\left(\mathbf{c}^T\mathbf{y} + z\right) p_{\gamma[k]}\left(\gamma[k]\right) d\gamma[k] \quad (21a)$$

subject to

$$E_{\gamma[k]}\left\{\mathbf{g}^T\mathbf{y}\right\} = z\overline{S} \quad , (21b)$$

$$\mathbf{g}^T\mathbf{y} \geq 0, \quad (21c)$$

$$\mathbf{d}^T\mathbf{y} \leq 0, \quad (21d)$$

$$\mathbf{e}^T\mathbf{y} + z = 1, \quad (21e)$$

$$z \geq 0, \quad (21f)$$

$$BER(\gamma[k]) \leq \varepsilon, \quad (21g)$$

with variables $\mathbf{y}$, z, where $\mathbf{y} = \frac{\mathbf{x}}{\mathbf{e}^T\mathbf{x}+1}$, $z = \frac{1}{\mathbf{e}^T\mathbf{x}+1} = \frac{1}{s_{\max}b+1}$.

Proposition : If ($\mathbf{y}$,z) is feasible in (20a-20e), with $z \neq 0$, then $\mathbf{x} = \mathbf{y}/z$ is feasible in (21a-21g), with the same objective value [10], [11].

Consequently, the log linear-fractional programming stated in (20) is transformed to an equivalent log linear program in the form of (21) which is clearly a convex optimization problems [10].

(21a-21g) can be simplified as

$$\max_{\{y_1(\gamma[k]),z\}} \quad \int \log_2\left(1 + a\gamma[k]y_1(\gamma[k])\right) p_{\gamma[k]}\left(\gamma[k]\right) d\gamma[k] \quad (22a)$$

subject to

$$E_{\gamma[k]}\left\{y_1(\gamma[k])\right\} = z\overline{S}, \qquad \forall k \in \kappa \quad (22b)$$

$$y_1(\gamma[k]) \geq 0, \qquad \forall k \in \kappa \quad (22c)$$

$$y_1(\gamma[k]) \leq (1-z)\overline{S}/b, \qquad \forall k \in \kappa \quad (22d)$$

$$BER(\gamma[k]) \leq \varepsilon, \qquad \forall k \in \kappa \quad (22e)$$

$\mathbf{y} = \left[y_1(\gamma[k]), y_2\right]^T = \left[\frac{s(\gamma[k])}{\mathbf{e}^T\mathbf{x}+1}, \frac{s_{max}}{\mathbf{e}^T\mathbf{x}+1}\right]^T$, $z = \frac{1}{\mathbf{e}^T\mathbf{x}+1} = \frac{1}{s_{\max}b+1} > 0$. There is no gap between (21) and (22) when $z \neq 0$.

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