Channel Estimation for SM Systems over Time-Varying Rayleigh Fading Channels

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***Abstract*—This paper is concerned with the challenging and timely problem of channel estimation for spatial modulated systems in the presence of time varying channels. Recently, estimation of channel state information for SM systems is investigated by the recursive least square (RLS) algorithm for slow fading channels. However, it is clear that the RLS based receiver will have a performance degradation for fast fading channels. Therefore, we developed iterative channel estimation based on detected symbols and curve fitting to track the channel variations for SM systems. Simulation results have demonstrated that the proposed iterative channel estimation offer substantial performance gains over the RLS channel estimation. In particular, a savings of about $4dB$ is obtained at $BER = 10^{-7}$, as compared with RLS based receiver at $150km/h$ for 2×4 single carrier SM systems.**

***Keywords*—Spatial Modulation, Channel Estimation, MIMO, Iterative Receiver.**

I. INTRODUCTION

Capacity and error performance advantages of Multiple-input multiple-output (MIMO) systems depend on some important parameters such as the distance between receiver and transmitter antennas [1], [2], inter channel interference (ICI) at the receiver and inter antenna synchronization (IAS) at the transmitter [3], [4]. Therefore, spatial modulation (SM) has been developed transmission technique that exploits multiple antenna at either the transmitter, the receiver to solve practical problems encountered in MIMO systems [5]. SM has a very flexible mechanism that provides high spectral efficiency with low complexity. To achieve these benefits of the SM, accurate channel state information (CSI) must be available at the receiver.

SM technique adds a third dimension to the two dimensional signal space which is the spatial dimension. Thus, three-dimensional signal space is obtained. In SM systems, the number of total transmitted information bits depends on the constellation diagram and the total number of transmitter antennas. Antenna index detection is a crucial part of the SM scheme since only one transmit antenna is active among the set of transmit antennas and both the data symbol transmitted by this antenna and its index should be decided at the receiver. In the literature, the antenna number and symbol detection are realized by means of optimal and non-optimal detection methods [5], [6]. In these detection process it has assumed that the CSI is known at the receiver while channel estimation is crucial in practice.

Recently, the effects of channel estimation to the SM systems has been investigated [7], [8]. Moreover, channel estimation for single carrier SM systems has been done by means of pilot-based recursive least square (RLS) method while assuming the wireless channel is static for one frame [9]. However, it is clear that the performance of RLS channel estimator will be degraded in cases where the total number of pilot symbols is not sufficient and the channel is time-varying. The main problem in SM systems is that we have only one active antenna during transmission so other channels could not be known at that time.

Iterative channel estimation are very attractive because of their superb performance [10]–[12]. It was shown that iterative receivers have clear advantages for time-varying channels and they need less pilot symbols as compare to non-iterative channel estimators [13]. Moreover, it was also shown that the iterative receivers may be employed to decrease the computational complexity of the receiver [14]. Therefore, in this paper, it is shown that the performance of RLS channel estimator decreases in time varying Rayleigh channels and the iterative receiver design has been done with the required signal model. Finally, it has been shown that the proposed receiver design has better performance than the conventional RLS based receiver.

Notation: Throughout the paper, the following notations and assumptions are used. Bold and capital letters '**A**' denote matrices. Bold and small letters '**a**' denote vectors. The notations, $(.)^*$, $(.)^T$, $(.)^\dagger$, $(.)^+$, $(.)^{-1}$ and $\|.\|_F$ denote conjugate, transpose, Hermitian, pseudoinverse, inverse and Frobenius norm of a matrix or a vector respectively.

II. SIGNAL MODEL

The SM system is assumed to have N_t transmit antennas and N_r receive antennas. The total number of bits that is transmitted by a M-ary SM system is $k = \log_2(N_t) + \log_2(M)$ where M represents the total number of bits per transmitted symbol. At the n^{th} symbol interval the SM mapper takes a random sequence of k bits and maps them into a N_t-dimensional signal vector as

$$\mathbf{x}(n) = [x_1(n), x_2(n), \cdots, x_{N_t}(n)]^T. \quad (1)$$

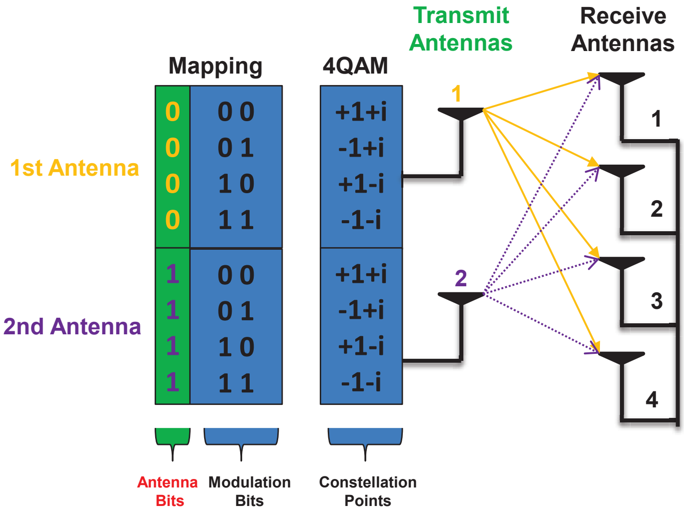

Fig. 1. Spatial Modulation Mapping: 3-bits transmission using 4-QAM, two transmit antennas and four receiver antennas

Only one of $x_j(n)$ that is active in $\mathbf{x}(n)$ is nonzero. Then, at the n^{th} symbol interval, the output of the SM system at the transmitter can be expressed as

$$\mathbf{x}_j(n) \triangleq [0 \cdots \underbrace{x_q(n)}_{j.\text{ transmitted antenna}} \cdots 0]^T \tag{2}$$

where j is the active antenna index and $x_q(n)$ is the q^{th} symbol from the M-ary constellation diagram. The other antennas remain silent over this symbol duration. The symbol $x_q(n)$ is transmitted from antenna j over an $N_r \times N_t$ MIMO channel. The observation model at receiver can be stated as

$$\begin{bmatrix} y_1(n) \\ \vdots \\ y_r(n) \\ \vdots \\ y_{N_r}(n) \end{bmatrix} = \begin{bmatrix} h_{11}(n) & h_{12}(n) & \cdots\cdots & h_{1N_t}(n) \\ h_{21}(n) & h_{22}(n) & \cdots\cdots & h_{2N_t}(n) \\ \vdots & \vdots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ h_{N_r1}(n) & h_{N_r2}(n) & \cdots\cdots & h_{N_rN_t}(n) \end{bmatrix} \begin{bmatrix} 0 \\ \vdots \\ x_q(n) \\ \vdots \\ 0 \end{bmatrix} + \begin{bmatrix} w_1(n) \\ \vdots \\ w_r(n) \\ \vdots \\ w_{N_r}(n) \end{bmatrix} \tag{3}$$

where $h_{r,j}(n)$ is the channel coefficient between j^{th} transmitter antenna and r^{th} receiver antenna, $w_r(n)$ is complex-valued, zero-mean white Gauss noise (AWGN) with variance σ^2. The observation model (3) can be written in the matrix form as follows:

$$\mathbf{y}(n) = \mathbf{H}(n)\mathbf{x}_j(n) + \mathbf{w}(n), \quad n = 1, 2, \cdots, N. \tag{4}$$

III. SIGNAL DETECTION

Since the desired information carried by the modulated signal and the transmit antenna index, the estimation of transmitter antenna number has a great importance. Optimal detector based on the maximum likelihood (ML) principle given in [6] as follows:

$$\left[\widehat{j}_{ML}(n), \widehat{q}_{ML}(n)\right] = \arg\max_{j,q} p_Y(\mathbf{y}(n) \mid \mathbf{x}_j(n), \mathbf{H}(n)) \tag{5}$$

where the vector $\mathbf{x}_j(n)$ varies for different q and j as indicated in (2). From (4), the probability density function (pdf) of $\mathbf{y}(n)$ conditioned on $\mathbf{x}_j(n)$ and $\mathbf{H}(n)$ can be written as:

$$p_Y(\mathbf{y}(n) \,|\, \mathbf{x}_j(n), \mathbf{H}(n)) = \pi^{-N_r} \exp(\|\mathbf{y}(n) - \mathbf{h}_j(n)x_q(n)\|_F^2) \tag{6}$$

where $\mathbf{h}_j(n)$ is j^{th} column vector of the matrix $\mathbf{H}(n)$. Using (6), optimal detector given in (5) can be expressed as

$$\left[\widehat{j}_{ML}(n), \widehat{q}_{ML}(n)\right] = \arg\max_{j,q} \|\mathbf{g}_{jq}(n)\|_F^2 - 2\Re e\{\mathbf{y}^\dagger(n)\mathbf{g}_{jq}(n)\} \tag{7}$$

where $\mathbf{g}_{jq}(n)$ is:

$$\mathbf{g}_{jq}(n) = \mathbf{h}_j(n)x_q(n), 1 \leq j \leq N_t, 1 \leq q \leq M. \tag{8}$$

If the receiver detects both $\widehat{j}_{ML}(n)$ and $\widehat{q}_{ML}(n)$ correctly, they can be easily de-mapped and combined to get back to the transmitted bits. However, it is clear that the receiver needs to know the full CSI, $\mathbf{H}$ where

$$\mathbf{H} = [\mathbf{H}(1), \mathbf{H}(2), \cdots, \mathbf{H}(n), \cdots \mathbf{H}(N)]\,. \tag{9}$$

IV. ITERATIVE-RLS CHANNEL ESTIMATION

In the SM system, the channel state information (CSI) is needed to detect modulated signal and transmit antenna number. In [9], the RLS channel estimation method and optimal detection is used for the SM system for quasi-static channels. However, most of wireless systems encounter with the time varying channel in practice. Therefore, it is obvious that the performance of RLS based SM receiver will be degraded over the time-varying channels. Therefore, iterative channel estimation method is used for the proposed receiver design. In this work, the initial values of proposed iterative channel estimation method is obtained by the RLS algorithm.

In the proposed iterative receiver, first, transmitted symbols are estimated by the initial channel estimation and then updated channel values corresponding to estimated symbols are obtained as shown in Figure 2. Unknown channel durations are estimated by interpolation where polynomial fitting is employed. In this case, channel variations may be tracked at the receiver. The channel matrix $\tilde{\mathbf{H}}$ can be writing as follows:

$$\tilde{\mathbf{H}} = \begin{bmatrix} h_{11}(1) & h_{11}(2) & \cdots & h_{11}(n) & \cdots & h_{11}(N) \\ h_{21}(1) & h_{21}(2) & \cdots & h_{21}(n) & \cdots & h_{21}(N) \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ h_{N_r1}(1) & h_{N_r1}(2) & \cdots & h_{N_r1}(n) & \cdots & h_{N_r1}(N) \\ h_{12}(1) & h_{12}(2) & \cdots & h_{12}(n) & \cdots & h_{12}(N) \\ h_{22}(1) & h_{22}(2) & \cdots & h_{22}(n) & \cdots & h_{22}(N) \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ h_{N_r2}(1) & h_{N_r2}(2) & \cdots & h_{N_r2}(n) & \cdots & h_{N_r2}(N) \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ h_{1N_t}(1) & h_{1N_t}(2) & \cdots & h_{1N_t}(n) & \cdots & h_{1N_t}(N) \\ h_{1N_t}(1) & h_{1N_t}(2) & \cdots & h_{1N_t}(n) & \cdots & h_{1N_t}(N) \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ h_{N_rN_t}(1) & h_{N_rN_t}(2) & \cdots & h_{N_rN_t}(n) & \vdots & h_{N_rN_t}(N) \end{bmatrix} \tag{10}$$

where

$$\mathbf{h}_{r,\tau} = [\ h_{r,\tau}(1), \quad \cdots, h_{r,\tau}(n), \quad \cdots, \quad h_{r,\tau}(N)\ ] \tag{11}$$

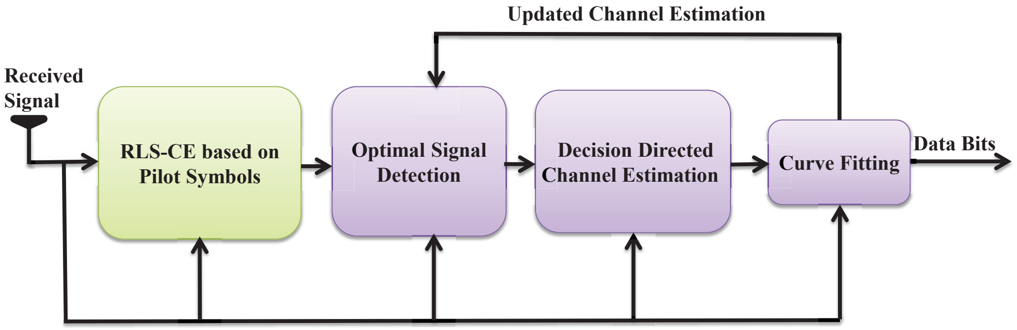

Fig. 2. Proposed Iterative Receiver Structure

represents r^{th} row vector of $\tilde{\mathbf{H}}$. If j^{th} transmitter antenna is assumed to be active ($\tau = j$), the received signal can be written as follows:

$$\mathbf{y}(n)=[h_{1j}(n)x_q(n)+w_1(n), \cdots h_{N_r j}(n)x_q(n)+w_{N_r}(n)] \quad (12)$$

Since only one transmitter antenna is active during the time slot n, the estimated channels between the receivers and the transmitter antennas can be written as:

$$\hat{\mathbf{h}}_{r,\tau} = \left[\ 0,\cdots ,\hat{h}^{DD}_{r,\tau=j}(n), \cdots ,0\ \right], \quad r = 1, \cdots , N_r \quad (13)$$

where $\hat{h}^{DD}_{r,\tau=j}(n)$ is the decision-directed (DD) channel estimates that can be determined as:

$$\hat{h}^{DD}_{r,\tau=j}(n) = y_r(n)/\hat{x}_q(n), \quad n \in \{1, 2, \cdots N\} \quad (14)$$

where $y_r(n)$ is the r^{th} component of $\mathbf{y}(n)$ and $\hat{x}_q(n)$ represents the detected symbol at the n^{th} symbol duration. Note that there are approximately $K = N/N_t$ symbols detected for each channel if we assume that all transmitted antennas are employed with equal probabilities. The detected symbols are then updated iteratively employing the last updated channel estimates for the next iteration as shown in Fig. 2.

By means of a polynomial curve fitting at discrete times $k = 1, 2, ..., K$, the wireless channels $\mathbf{h}_{r,\tau}$ between transmit and receive antennas can be modeled as an $(p-1)^{th}$ degree polynomial

$$h_{r,\tau}(t_k) = \theta^{(1)}_{r,\tau} + \theta^{(2)}_{r,\tau} t_k + ... + \theta^{(p)}_{r,\tau} t_k^{p-1} + u(t_k) \quad (15)$$

where $u(t_k)$ is an unobserved random error with mean zero conditioned on a scalar variables t_k. Then we have the following usual linear model:

$$\mathbf{h}_{r,\tau} = \mathbf{T}\mathbf{\Theta}_{r,\tau} + \mathbf{u} \quad (16)$$

where $\mathbf{h}_{r,\tau} = [h_{r,\tau}(t_1), h_{r,\tau}(t_2), ... h_{r,\tau}(t_K)]^T$, $\mathbf{\Theta}_{r,\tau} = \left[\theta^{(1)}_{r,\tau}\ \theta^{(2)}_{r,\tau} ... \ \theta^{(p)}_{r,\tau}\right]^T$ and $\mathbf{T} = \begin{bmatrix} 1 & t_1 & \cdots & t_1^{p-1} \\ 1 & t_2 & \cdots & t_2^{p-1} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & t_K & \cdots & t_K^{p-1} \end{bmatrix}$ where K is the total number of samples for curve fitting. The minimum variance unbias (MVU) estimator for $\mathbf{\Theta}_{r,\tau}$ is

$$\hat{\mathbf{\Theta}}_{r,\tau} = \left(\ \mathbf{T}^T\ \mathbf{T}\right)^{-1} \mathbf{T}^T \hat{\mathbf{h}}_{r,\tau}. \quad (17)$$

where $\hat{\mathbf{h}}_{r,\tau}$ is calculated in (13) and the observation matrix $\mathbf{T}$ has in the form of a Vandermonde matrix. Then the resulting curve fitting can be espressed as

$$\hat{h}^{CF}_{r,\tau}(n) = \sum_{i=1}^{p} \hat{\theta}^{(i)}_{r,\tau} t_n^{i-1}, \quad n = 1, 2, ..., N. \quad (18)$$

The channel estimation of time-varying channel can be performed by using (18) over the duration of one frame. After estimation of the time-varying channel matrix, the symbols are detected as shown in Fig. 2.

V. SIMULATION EXAMPLES

In this section, we provide computer simulation results to demonstrate the performance of the proposed iterative under a number of channel conditions. The simulated system has two transmit antennas and four receiver antennas as shown Fig. 1 and $4-QAM$ is used. SNR is defined as $\frac{E_s}{\sigma^2}$ where E_s is energy per symbol and σ^2 is noise power.

In all simulations, one iteration is employed for the proposed receiver. The channel between transmitter and receiver is modelled as time-varying Rayleigh fading channel with Doppler effect is considered. In Fig. 3, the RLS, and iterative channel estimation techniques have been compared with perfect CSI for $V = 30km/h$ over Rayleigh fading channel. Corresponding to pilot symbols, we employed the RLS estimate to obtain channel parameters as done in [9]. Iterative receiver uses RLS estimate as initial values and obtain enhanced channel estimation values as proposed in Fig.2. It is shown that the proposed iterative based channel estimator slightly outperforms RLS based channel estimator and has almost same performance as compare to perfect channel state information (P-CSI).

Mobile communication systems aim at providing high data rate under high speed scenarios. Therefore, we next investigate the performance difference for the high-mobility case. In Fig.4, it is observed that a savings of about $4.0dB$ is obtained at $BER = 10^{-7}$, as compared with RLS based receiver for $V = 150km/h$ over the Rayleigh fading channel. In order to show the potential advantages of our proposed scheme, $V = 180km/h$ over the Rayleigh fading channel is also considered and it is shown that the RLS based receiver exhibits an error floor at high SNRs and iterative receiver has similar BER performance to P-CSI case.

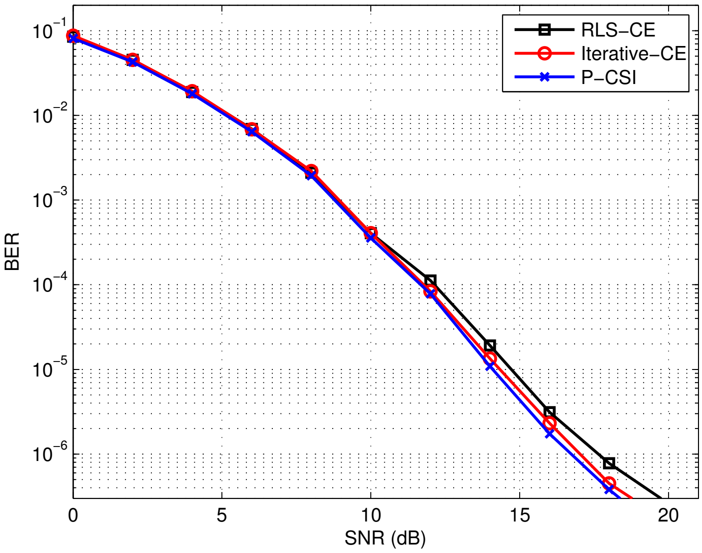

Fig. 3. The BER performance of RLS estimator and proposed iterative based channel estimator with $V_1 = 30km/h$

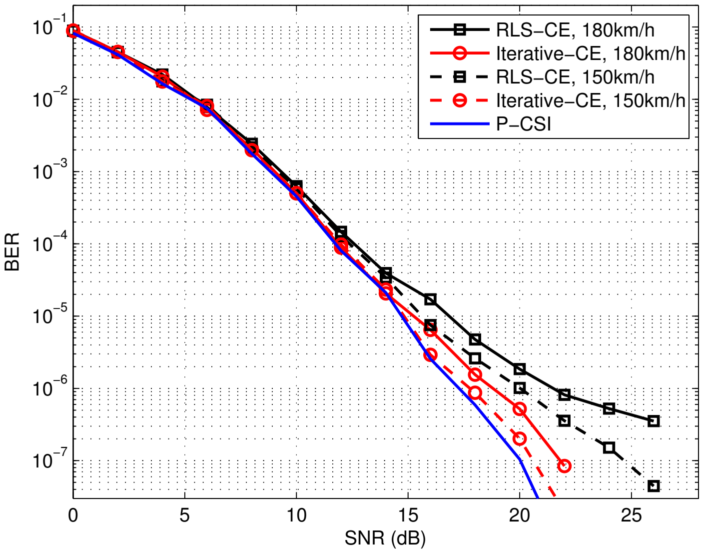

Fig. 4. The BER performance of RLS estimator and proposed iterative based channel estimator with $V_1 = 150km/h$ and $V_2 = 180km/h$

VI. CONCLUSIONS

Spatial modulation (SM) is a recently developed transmission technique that uses multiple antennas to solve practical problems encountered in MIMO systems. Channel estimation plays a crucial role in the performance of SM systems, since its knowledge is utilized to detect the data symbols. In this paper, we have developed iterative channel estimation for SM systems. We have studied the performance of the RLS-CE and proposed iterative channel estimation in the presence of time-varying channels. It is shown that the RLS channel estimation may reach an error floor for fast fading channels whereas the proposed iterative scheme works very well at high SNR values.

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