Design of Practical Broadband Matching Networks With Lumped Elements

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***Abstract*—It is always preferable to use commercially available software tools to design broadband matching networks for microwave communication systems. However, for these tools, the matching network topology and element values must be selected properly. Therefore, in this paper, a practical method is presented to generate matching networks with good initial element values. Eventually, the performance of the designed matching network is optimized by employing the commercially available computer-aided design (CAD) tools. An example is given to illustrate the utilization of the proposed method. It is shown that the proposed method provides very good initials for CAD tools.**

***Index Terms*—Broadband matching, lossless networks, matching network, real frequency techniques.**

I. Introduction

FOR MICROWAVE engineers, broadband matching network design has been considered a vital problem [1]. In this regard, the broadband matching analytic theory [2], [3] and computer-aided design (CAD) tools are available for engineers [4]–[6]. It is well known that the analytic theory is quite difficult to use. Therefore, it is always common practice to employ CAD tools to design broadband matching networks. All CAD tools optimize the performance of the matched system. At the end of this process, the element values of the broadband matching network are obtained. Here, it should be emphasized that performance optimization is highly nonlinear with respect to element values and needs very good initials. In this regard, initial element value selection is important for successful optimization. Therefore, in this paper, a well-established initialization process is introduced for broadband matching network designs.

The matching problem is considered as the design of a lossless two-port network between a generator and complex load impedance in such a way that power transfer from the source to the load is maximized over an interested frequency band. The power transfer capability of the lossless matching network is best measured by means of the transducer power gain, which is defined as the ratio of power delivered to the load to the available power from the generator.

The matching problem can be classified basically as single and double matching problems. In classical single matching problems, a purely resistive generator is matched to an arbitrary complex load. In contrast, the power transfer problem from a complex generator to a complex load is referred to as a double matching problem.

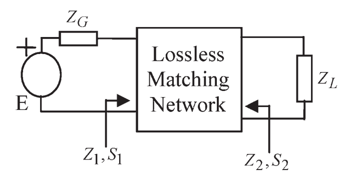

Fig. 1. Double matching arrangement.

Let us consider the classical double matching problem shown in Fig. 1. Transducer power gain (TPG) can be expressed in terms of the real and imaginary parts of the normalized load impedance $Z_L = R_L + jX_L$ and those of the normalized back-end impedance $Z_2 = R_2 + jX_2$ or in terms of the real and imaginary parts of the normalized generator impedance $Z_G = R_G + jX_G$ and those of the normalized front-end impedance $Z_1 = R_1 + jX_1$ of the matching network as follows:

$$TPG(\omega) = \frac{4R_\alpha R_\beta}{(R_\alpha + R_\beta)^2 + (X_\alpha + X_\beta)^2}. \qquad (1)$$

Here, if $\alpha = 1$, $\beta = G$, and if $\alpha = 2$, $\beta = L$.

The objective in broadband matching problems is to design the lossless matching network in such a way that the TPG given by (1) is maximized inside an interested frequency band. Thus, the matching problem in this formalism is reduced to the determination of a realizable impedance function Z_1 or Z_2. Once Z_1 or Z_2 is determined properly, the lossless matching network can be easily synthesized.

Carlin's real frequency line segment technique (RF-LST) is known as one of the best methods for determining a realizable data set for Z_2 [7], [8]. In this approach, Z_2 is realized as a minimum reactance function and its real part $R_2(\omega)$ is represented by line segments in such a way that $R_2(\omega) = \sum_{k=1}^{m} a_k(\omega) R_k$, passing through m selected pairs designated by $\{R_k, \omega_k; k = 1, 2, \ldots, m\}$. In this formalism, the break points (or break resistances) R_k are considered to be the unknowns of the matching problem. Then, these points are calculated via the nonlinear optimization of TPG.

The imaginary part $X_2(\omega) = \sum_{k=1}^{m} b_k(\omega) R_k$ of Z_2 is also written by means of the same break points R_k. It must be noted that coefficients $a_k(\omega)$ are known quantities and calculated in terms of the preselected break frequencies ω_k. The coefficients $b_k(\omega)$ are generated by means of the Hilbert transformation relation given for minimum reactance functions. Let $H\{o\}$ represent the Hilbert transformation operator. Then, $b_k(\omega) = H\{a_k(\omega)\}$.

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The disadvantages of RF-LST are the two independent approximation steps. This method can be extended to solve double matching problems; the computational efficiency applies only for single matching problems.

The basic principle of the direct computational technique (DCT) is similar to that of the RF-LST [9]. Here, the real part of the unknown matching network impedance R_2 is expressed as a real even rational function. Then, the unknown coefficients of this function are chosen to optimize the gain performance.

In DCT, R_2, which is a nonnegative even rational function, must be determined, guaranteeing the realizability of the resulting impedance function Z_2. For this purpose, an auxiliary polynomial is utilized for constructing R_2. As a result, although the realizability is guaranteed, the computational complexity and the nonlinearity of the transducer power gain with respect to the optimization parameters are increased.

In another method proposed by Fettweis, the parametric representation of the positive real back-end driving point impedance Z_2 is utilized [10]. More specifically, the positive real impedance Z_2 is written in a partial fraction expansion, and then, the poles of Z_2 are determined by optimizing the gain performance of the system in the interested frequency band.

The parametric approach can be used for solving single matching problems. The difficulty is to initialize the locations of the poles, which are critical.

In the methods explained briefly earlier, the lossless matching network is expressed in terms of a set of free parameters by means of driving point impedance Z_2. However, the matching problem can also be described by using any other set of parameters. In the real frequency scattering approach, which is referred to as the simplified real frequency technique (SRFT), the canonic polynomial representation of the scattering matrix is used to describe the lossless matching network [11], [12].

In another method proposed in [13], the back-end impedance of the matching network Z_2 is modeled as a minimum reactance function, and then, a Foster impedance is connected in series.

As seen in the aforementioned explanation, the aim is to express the back-end impedance Z_2 of the matching network in terms of any set of free parameters. Then, the gain performance of the matching network is optimized via (1). However, it is not necessary to make the back-end impedance expression so complicated. There is a very simple and obvious way to calculate the back-end impedance Z_2 or front-end impedance Z_1 of the matching network. This is the crux of the proposed method.

In the proposed method, these impedances (Z_1 or Z_2) are determined by using three parameters; the scattering parameters of the lossless matching network, source reflection coefficient and load reflection coefficient. In the next section, the canonic polynomial representation of the scattering parameters is briefly summarized, and then, the rationale of the proposed method is described.

II. Canonic Polynomial Representation of Scattering Matrix

By referring to the double matching configuration shown in Fig. 1, the scattering parameters of the lossless matching network can be written in terms of three real polynomials by using the well-known Belevitch representation as follows:

$$S_{11}(p) = \frac{h(p)}{g(p)} \quad S_{12}(p) = \frac{\mu f(-p)}{g(p)}$$

$$S_{21}(p) = \frac{f(p)}{g(p)} \quad S_{22}(p) = -\mu h(-p)/g(p) \tag{2}$$

where $p = \sigma + j\omega$ is the classical complex frequency variable, g is a strictly Hurwitz polynomial, f is a real monic polynomial, and μ is a unimodular constant ($\mu = \pm 1$). If the two-port network is reciprocal, then the polynomial f is either even or odd and $\mu = f(-p)/f(p)$.

The polynomials $\{f, g, h\}$ are related by the Feldtkeller equation [14]

$$g(p)g(-p) = h(p)h(-p) + f(p)f(-p). \tag{3}$$

It is clear from (3) that the Hurwitz polynomial $g(p)$ is a function of $h(p)$ and $f(p)$. If the polynomials $f(p)$ and $h(p)$ are specified, then the scattering parameters of the two-port network, and then, the network itself can be completely defined.

In almost all practical applications, the designer has an idea about the transmission zero locations of the matching network. Hence, the polynomial $f(p)$ which is constructed on the transmission zeros is usually defined by the designer. For practical problems, the designer may use the following form of $f(p)$:

$$f(p) = p^{m_1} \prod_{i=0}^{m_2} \left(p^2 + a_i^2\right) \tag{4}$$

where m_1 and m_2 are nonnegative integers and a_i's are arbitrary real coefficients. This form corresponds to ladder-type minimum phase structures, whose transmission zeros are on the imaginary axis of the complex p-plane.

III. Rationale of the Proposed Method

Consider the double matching arrangement shown in Fig. 1. The input reflection coefficient of the matching network when its output port is terminated in Z_L can be written in terms of the scattering parameters of the matching network as

$$S_1 = S_{11} + \frac{S_{12}S_{21}S_L}{1 - S_{22}S_L} \tag{5}$$

where S_L is the load reflection coefficient expressed as

$$S_L = \frac{Z_L - 1}{Z_L + 1}. \tag{6}$$

Similarly, the output reflection coefficient of the matching network when its input port is terminated in Z_G can be written in terms of the scattering parameters of the matching network as

$$S_2 = S_{22} + \frac{S_{12}S_{21}S_G}{1 - S_{22}S_G} \tag{7}$$

where S_G is the source reflection coefficient expressed as

$$S_G = \frac{Z_G - 1}{Z_G + 1}. \tag{8}$$

So, the input and output impedances of the matching network can be calculated via the following equations, respectively:

$$Z_1 = \frac{1 + S_1}{1 - S_1} \tag{9a}$$

$$Z_2 = \frac{1 + S_2}{1 - S_2}. \tag{9b}$$

As a result, the following algorithm can be proposed to solve both single and double broadband matching problems with lumped elements. However, the same algorithm can easily be adapted to design distributed or mixed-element broadband matching networks.

IV. Proposed Algorithm

Inputs:

- $Z_{L(\text{measured})} = R_{L(\text{measured})} + jX_{L(\text{measured})}$, $Z_{G(\text{measured})} = R_{G(\text{measured})} + jX_{G(\text{measured})}$: Measured load and generator impedance data, respectively.
- $\omega_{i(\text{measured})}$: Measurement frequencies, $\omega_{i(\text{measurement})} = 2\pi f_{i(\text{measurement})}$.
- f_{norm}: Normalization frequency.
- R_{norm}: Impedance normalization number in ohms.
- $h_0, h_1, h_2, \ldots, h_n$: Initial real coefficients of the polynomial $h(p)$. Here, n is the degree of the polynomial which is equal to the number of lossless lumped elements in the matching network. The coefficients can be initialized as ± 1 in an *ad hoc* manner or the approach explained in [15] can be followed.
- $f(p)$: A monic polynomial constructed on the transmission zeros of the matching network. A practical form is given in (4).
- δ_c: The stopping criteria of the sum of the square errors.

Outputs:

- Analytic form of the input reflection coefficient of the lossless matching network given in the Belevitch form of $S_{11}(p) = h(p)/g(p)$. It is noted that this algorithm determines the coefficients of the polynomials $h(p)$ and $g(p)$, which, in turn, optimizes system performance.
- Circuit topology of the lossless matching network with element values: The circuit topology and element values are obtained as results of the synthesis of $S_{11}(p)$. Synthesis is carried out in the Darlington sense, i.e., $S_{11}(p)$ is synthesized as a lossless two-port network which is the desired matching network [16]. Also, the synthesis process can be carried out by using impedance-based Foster or Cauer methods via $Z_{11}(p) = (1 + S_{11}(p))/(1 - S_{11}(p))$, as explained in [17].

Computational Steps:

Step 1: Normalize the measured frequencies with respect to f_{norm} and set all the normalized angular frequencies $\omega_i = f_{i(\text{measured})}/f_{\text{norm}}$.

TABLE I
Given Normalized Load and Generator Impedance Data

ω	R_L	X_L	R_G	X_G
0.0	1.0000	0.0000	1.0000	0.0000
0.1	0.8621	-0.3448	1.0000	0.1000
0.2	0.6098	-0.4878	1.0000	0.2000
0.3	0.4098	-0.4918	1.0000	0.3000
0.4	0.2809	-0.4494	1.0000	0.4000
0.5	0.2000	-0.4000	1.0000	0.5000
0.6	0.1479	-0.3550	1.0000	0.6000
0.7	0.1131	-0.3167	1.0000	0.7000
0.8	0.0890	-0.2847	1.0000	0.8000
0.9	0.0716	-0.2579	1.0000	0.9000
1.0	0.0588	-0.2353	1.0000	1.0000

Normalize the measured load and generator impedances with respect to the impedance normalization number R_{norm}. $R_L = R_{L(\text{measured})}/R_{\text{norm}}$, $X_L = X_{L(\text{measured})}/R_{\text{norm}}$, $R_G = R_{G(\text{measured})}/R_{\text{norm}}$, and $X_G = X_{G(\text{measured})}/R_{\text{norm}}$ over the entire frequency band.

Step 2: Obtain the strictly Hurwitz polynomial $g(p)$ from (3). Then, calculate the scattering parameters via (2).

Step 3: Calculate the load and source reflection coefficients S_L and S_G via (6) and (8), respectively.

Step 4: Calculate the input and output reflection coefficients S_1 and S_2 via (5) and (7), respectively.

Step 5: Calculate the input and output impedances Z_1 and Z_2 via (9a) and (9b), respectively.

Step 6: Calculate the transducer power gain via (1).

Step 7: Calculate the error via $\varepsilon(\omega) = 1 - TPG(\omega)$; then, $\delta = \sum |\varepsilon(\omega)|^2$.

Step 8: If δ is acceptable ($\delta \leq \delta_c$), stop the algorithm and synthesize $S_{11}(p)$. Otherwise, change the initialized coefficients of the polynomial $h_l(p)$ via any optimization routine and return to Step 2.

V. Example

In this section, a double matching example is presented for the design of a practical broadband matching network. The normalized load and generator impedance data are given in Table I. It should be noted that the given load data can easily be modeled as a capacitor $C_L = 4$ in parallel with a resistance $R_L = 1$ (i.e., $R_L//C_L$ type of impedance) and the generator data as an inductor $L_G = 1$ in series with a resistance $R_G = 1$ (i.e., $R + L$ type of impedance). Since the given impedance data are normalized, there is no need for a normalization step. The same example is solved here via SRFT.

The polynomial $h(p)$ is initialized as $h(p) = -p^5 + p^4 - p^3 + p^2 - p + 1$ in an *ad hoc* manner. Also, the polynomial $f(p)$ is selected as $f(p) = 1$, since a low-pass matching network is desired. In the example, α and β are selected as $\alpha = 1$ and $\beta = G$. Thus, front-end Z_1 and source Z_G impedances are used in the TPG expression in Step 6. Then, after running the proposed algorithm, the following scattering parameter of the matching network is obtained:

$$S_{11}(p) = \frac{h(p)}{g(p)}, \text{ where}$$

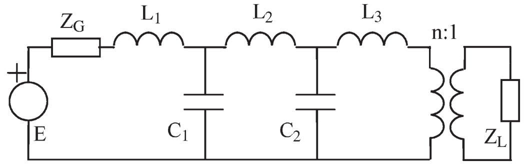

Fig. 2. Designed lumped-element double matching network. Proposed: $L_1 = 0.13233$, $L_2 = 1.9885$, $L_3 = 1.9043$, $C_1 = 1.4897$, $C_2 = 1.6979$, $n = 1.7135$. SRFT: $L_1 = 0.13216$, $L_2 = 1.988$, $L_3 = 1.8834$, $C_1 = 1.4898$, $C_2 = 1.7004$, $n = 1.7050$.

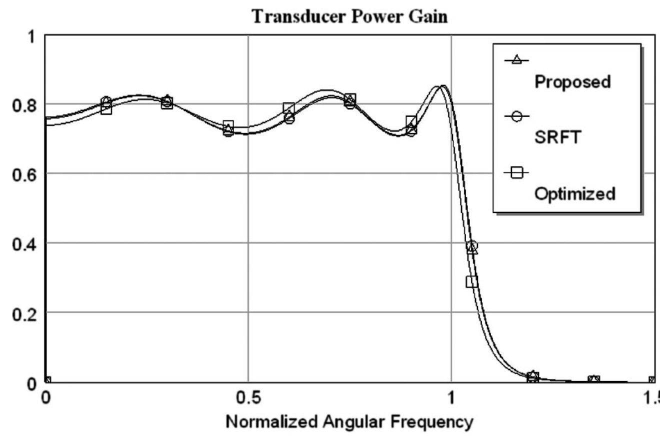

Fig. 3. Performance of the matched system designed with lumped elements.

$$h(p) = 0.3688p^5 - 2.2179p^4 - 2.0808p^3 + 0.6144p^2 - 1.5500p + 0.5616$$

$$g(p) = 0.3688p^5 + 3.3559p^4 + 6.5190p^3 + 5.8794p^2 + 3.8986p + 1.1469.$$

After synthesizing the obtained scattering parameter or the corresponding impedance function, the matching network seen in Fig. 2 is obtained.

As seen in Fig. 3, the initial performance of the matched system looks fairly good. However, it is further improved via optimization utilizing the commercially available design package called Microwave Office of Applied Wave Research Inc. [4]. Thus, the final normalized element values are given as $L_1 = 0.1322$, $L_2 = 2.004$, $L_3 = 2.017$, $C_1 = 1.515$, $C_2 = 1.692$, and $n = 1.759$. For comparison purposes, both the initial and optimized performances of the matched system and the performance obtained via SRFT are depicted in Fig. 3.

The algorithm is implemented via Matlab. The elapsed time for this example is 28.3161 s. It is 29.0417 s via SRFT.

In Fig. 4, the transducer power gain curves are zoomed. The curves obtained via the proposed method and SRFT are very close to each other and nearly the same. The ripple factor τ^2 for the curves in the passband can be calculated as

$$\tau^2_{\text{proposed}} = \frac{TPG_{\max} - TPG_{\min}}{TPG_{\min}} = \frac{0.849 - 0.7102}{0.7102} = 0.1954$$

$$\tau^2_{\text{SRFT}} = \frac{0.8542 - 0.7075}{0.7075} = 0.2073.$$

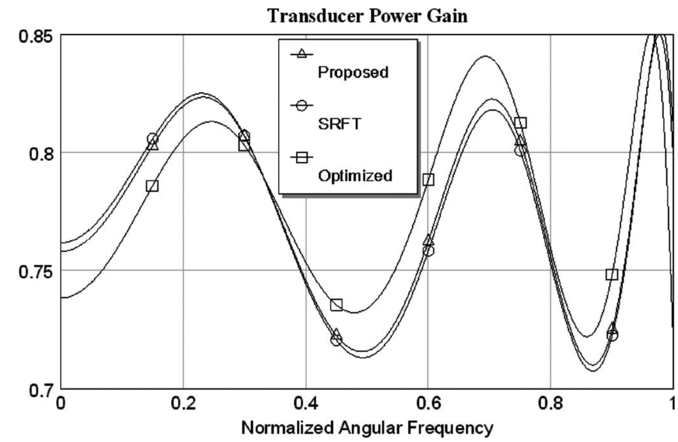

Fig. 4. Closer examination of the gain performances.

There is a very small difference. Consequently, it can be said that the proposed method and SRFT have nearly the same performance.

VI. Conclusion

The design of practical broadband matching networks is one of the biggest problems for microwave engineers. In this regard, commercially available CAD tools are utilized. Once the matching network topology is obtained, these packages are excellent tools to optimize system performance by working on the element values. In this way, initial element values become very vital, since system performance is highly nonlinear in terms of the element values of the matching network. Therefore, in this paper, an initialization method is proposed for the construction of lossless broadband matching networks.

In the proposed method, the back-end or front-end impedance of the matching network is determined in terms of the scattering parameters of the matching network, i.e., the source and load reflection coefficients. Then, this impedance and one of the termination impedances (Z_G or Z_L) are used to calculate the transducer power gain of the system. The scattering parameters of the matching network are optimized to be able to achieve maximum performance.

Finally, it is synthesized as a lossless two-port network yielding the desired matching network topology with the initial element values. Eventually, the actual performance of the matched system is improved by means of a commercially available CAD tool.

The advantages of the proposed method (and SRFT) can be explained as follows. The polynomial $f(p)$ is constructed by using the transmission zeros of the matching network, so they are under the control of the designer. Transducer power gain is quadratically dependent on the optimization parameters, so the problem is simply a quadratic optimization. As a result, the numerical convergence of the method is superb. Also, the proposed method is applicable to solving both single and double matching problems.

An example has been presented here to construct a broadband matching network with lumped elements. It has been shown that the proposed method generates very good initials to further improve the matched system performance by working on the element values. Therefore, it is expected that the proposed algorithm can be used as a front end for commercially available CAD tools to design practical broadband matching networks for microwave communication systems.

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