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Review

Relativistic antihydrogen production by pair production with positron capture

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ABSTRACT

Antihydrogen atoms may rarely be produced by the collision of antiprotons with ions. At relativistic velocities, the antiproton may pass around the Coulomb field of the nucleus and the electron–positron pairs can be produced electromagnetically. After this pair production, not so often, positron can be captured by the antiproton and as a result, antihydrogen atoms may be produced. In this work, we have calculated the antihydrogen production cross section in the framework of perturbation theory, by applying Monte-Carlo integration techniques. In order to compute the lowest-order Feynman diagrams amplitudes, we used Darwin wave functions for the bound states of the positrons and Sommerfeld–Maue wave functions for the continuum states of the electrons.

1. Formalism

The discovery of the antimatter goes to the work of Dirac in 1931. After that, electron–positron pairs are created from cosmic rays. Each particle has a corresponding antiparticle and they obey the CPT theorem that is the main principle of the relativistic quantum field theory. The CPT theorem is based on the basis of the product of the charge conjugation (C), parity (P) and time reversal (T) that symbolize the exact symmetry of nature. The CPT theorem implies that every particle has an antiparticle that corresponds it with equal mass, spin and lifetime but equal charge and magnetic moment with opposite sign. The hydrogen atom is a good example for this physical system and the antihydrogen is the best framework for the CPT studies at atomic interactions [1–3].

Antihydrogen atoms are the simplest bound state of antimatter. These atoms have been produced and observed for the first time in the LEAR (Low Energy Antiproton Ring) at CERN in 1995. In this experiment, the Xenon ($Z = 54$) has been used as a nucleus [4]. For the creation of the antihydrogen in electromagnetic way in a bound state, the two antiparticles, $\bar{p}$ and e^+ must be very near to each other in momentum and energy space to react and produce the antihydrogen $\bar{H}$ [1–3]. In this process, antiproton will pass near of the Coulomb field of the nucleus with charge Z and then electron–positron pair will be created. After that, the positron can be captured by the antiproton; therefore the antihydrogen can be created. The electron will go its way freely. Two types of this mechanism can be observed:

(i) $\gamma\gamma$ production of electron–positron pairs and then the antihydrogen production;

$$\bar{p} + Z \to \bar{p} + \gamma\gamma + Z \to \bar{p} + e^+e^- + Z \to \bar{H} + e^- + Z, \tag{1}$$

(ii) the e^+e^- pair may be converted from virtual time like Bremsstrahlung photons;

$$\bar{p} + Z \to \bar{p} + \gamma^* + Z \to \bar{p} + e^+e^- + Z \to \bar{H} + e^- + Z. \tag{2}$$

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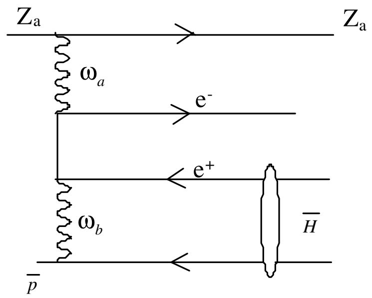

Fig. 1. The diagram of $\bar{H}$ production with the collision of $\bar{p}$ and nuclear target [7].

The calculations show that the second mechanism is two to three orders of magnitude lower than the first mechanism [5,6]. The first mechanism has a Z^2 and $\ln(\gamma)$ dependence and the Z^2 factors come from the Coulomb field of the nucleus with charge Z [5].

The $\bar{H}$ production diagram is shown in Fig. 1. Bound-free electron–positron pair production cross section calculations were done by equivalent photon approximation (EPA) for high energies. The $\bar{H}$ production experiments is mostly done at low energies and the calculation of the cross section of this process by using the EPA is questionable [7], which we will explain later in that paper.

The cross section calculations were done by Baur [6] in 1993, for $\gamma\gamma$ and Bremsstrahlung production of fast antihydrogen in $\bar{p}$-nucleus collisions. Baur concluded that the antihydrogen production cross section for the $\gamma\gamma$ mechanism rises slowly with energy ($\approx \ln\gamma$) and it has Z^2 dependence like

$$\sigma_{cap}^{\gamma\gamma} \cong \frac{33}{20} Z^2 \alpha^5 r_e^2 \cong 2.7 \times 10^{-3} Z^2 nb \tag{3}$$

and Baur used the equivalent photon method for this cross section calculations [6].

The process in Eq. (1) has been studied by Munger et al. [5] in 1994 and they used the equivalent photon approximation (EPA) in their antihydrogen production cross section calculations. The behaviour of the cross section for the production of anti-hydrogen can be written approximately as

$$\sigma\left(\bar{p} + Z \to \bar{H} + e^- + Z\right) \approx 4Z^2 pb, \tag{4}$$

when the momenta of the antiproton above $\approx$ 6 GeV/c. Similar cross section calculations have been done by Eichler [8] and he obtained the following results:

$$\sigma\left(\bar{p} + p \to \bar{H} + p + e^-\right) = 2.7 \ln(\gamma) pb. \tag{5}$$

Munger and his co-workers [5] have showed that their asymptotic cross section is,

$$\sigma\left(\bar{p} + Z \to \bar{H} + e^- + Z\right) = 2.8 \ln(\gamma) pb, \tag{6}$$

and this is close to Eichler's result.

Bertulani and Baur [7] have calculated the antihydrogen production in 1997, and they have written the cross section by using the plane-wave Born approximation (that is equal to the straight line semi-classical approximation). Both approximations give the same result for the antihydrogen production. They reached the cross section results that is $\sigma = (2.86pb)Z^2 \ln(\gamma)$. Munger's result [5] is close to this value only for $\gamma > 10^3$ energies. However, the equivalent photon approximation is valid only for low values of energy, ($\gamma < 10$).

Meier and his co-workers [4] in 1998, have calculated the production of relativistic antihydrogen atoms by bound-free pair production by using semi-classical approximation (SCA) or equivalently in the plane-wave Born approximation (PWBA) with the help of exact Dirac–Coulomb wave functions. They gave their results for different γ's for the Fermilab energies. They calculated the K-shell capture cross section ($Z_P = Z_T = 1$) results as $5.73 \times 10^{-1} pb$, $7.9 \times 10^{-1} pb$ and $1pb$ for the values of γ, equal to 5–7, respectively. They also calculated the contribution to the all higher shells for cross section results and they found that the contribution of the all higher shells approximately 20% of the K-shell capture [4].

In this study, we calculated the cross section of the $\bar{H}$ production by using the Monte-Carlo integration techniques. First, we computed the lowest-order Feynman diagrams amplitudes by using Darwin wave functions for the bound states of the positrons and Sommerfeld–Maue wave functions for the continuum states of the electrons [7].

In our calculations, we use semi-classical approximation and we assume that the antiproton goes on a straight way. For the electron we used Sommerfeld–Maue (plane-wave) wave function,

$$\Psi_k^{(-)} = N_+ \left(e^{i\mathbf{k}\cdot\mathbf{r}} \mathbf{u}_{\sigma_k}^{(-)}\right). \tag{7}$$

Table 1
Antihydrogen production cross sections results for $\bar{p}-\bar{p}$, $Au-\bar{p}$ and $Pb-\bar{p}$ collisions at RHIC and LHC collider facilities.

	$\bar{p}-\bar{p}$	$Au-\bar{p}$	$Pb-\bar{p}$
RHIC	$24.2\mu b$	$1.53\times10^{-2}b$	$1.49\times10^{-2}b$
LHC	$48.9\mu b$	$3.11\times10^{-2}b$	$3.02\times10^{-2}b$

In expression (7), moreover,

$$N_+ = e^{-\pi a_+/2}\,\Gamma(1+ia_+), \qquad a_+ = \frac{Ze^2}{v_+}, \tag{8}$$

is a normalization constant which accounts for the distortion of the wave function is acceptable for $Z\alpha \ll 1$ [9–11], and where $\alpha = e^2/\hbar c \cong 1/137$ is the fine structure constant and v_+ is the velocity of the electron in the rest frame of the ion.

After the pair production has occurred, the positron is captured by antiprotons and, thus, need to be described as a bound state. In a semi-relativistic approximation, these positron states are often represented by Darwin wave-function,

$$\Psi^{(+)} = \left[1+\frac{i}{2m}\alpha\cdot\nabla\right]\mathbf{u}\Psi_{non\text{-}rel}(r), \tag{9}$$

i.e. in terms of the non-relativistic (ground) state function

$$\Psi_{non\text{-}rel}(r) = \frac{1}{\sqrt{\pi}}\left(\frac{Z}{a_H}\right)^{3/2} e^{-Zr/a_H}, \tag{10}$$

of the hydrogen-like ion, and where $\mathbf{u}$ represents the spinor part of the captured positron and $a_H = 1/e^2$ the Bohr radius of atomic hydrogen.

Using the electron (Sommerfeld–Maue wave function) and the captured positron (Darwin wave function) states from above, the direct Feynman diagram for the second order perturbation calculation can be written as:

$$\left\langle\Psi^{(+)}\left|S_{ab}\right|\Psi_k^{(-)}\right\rangle = \frac{iN_+}{2\beta}\frac{1}{\sqrt{\pi}}\left(\frac{Z}{a_H}\right)^{3/2}\int\frac{d^2p_\perp}{(2\pi)^2}\,e^{i(\mathbf{p}_\perp-\frac{\mathbf{k}_\perp}{2})\cdot\mathbf{b}}\,\mathcal{F}(\mathbf{p}_\perp,\mathbf{k}_\perp,\beta), \tag{11}$$

where $\mathbf{b}$ is the impact parameter of the ion–ion collision, and the function $\mathcal{F}(\mathbf{p}_\perp,\mathbf{k}_\perp,\beta)$ consists the scalar part of the field associated with the ions a and b in momentum space and transverse momentum components. More information about these functions can be found in [12].

Having the amplitudes for the *direct* S_{ab} and *crossed* S_{ba} diagram, the cross section for the antihydrogen production in collisions of two heavy ions can be written down

$$\sigma = \int d^2b\sum_{k>0}\left|\left\langle\Psi^{(+)}\left|S\right|\Psi_k^{(-)}\right\rangle\right|^2, \tag{12}$$

where $S = S_{ab}+S_{ba}$ denotes the sum of the *direct* and *crossed* terms. Calculations have been performed for the total production cross sections of antihydrogen production by pair production with positron capture in relativistic collisions of ions. Theoretical cross sections are obtained especially for RHIC and LHC energies.

2. Conclusion

Fig. 2 displays the relativistic antihydrogen cross sections as function of the nuclear charge for RHIC and LHC energies. As seen in this figure, the maximum cross section value can be obtained, when nuclear charge is about 65 for both energy levels. The reason of this is, while the ions getting heavier, the effect of the square of the normalization constant in the antihydrogen cross section is getting higher inversely and the antihydrogen cross section values are decreasing.

Fig. 3 shows the relativistic antihydrogen cross sections for two different ions as functions of the Lorentz contraction factor. This figure displays that while γ is getting higher values, the probability of antihydrogen production is increasing. The antihydrogen cross section results of Au is higher than the antihydrogen cross section results of Pb. While Z is getting higher, the inverse effect of the square of the normalization constant makes the antihydrogen cross section values lower.

We calculate the antihydrogen production cross sections for $\bar{p}-\bar{p}$, $Au-\bar{p}$ and $Pb-\bar{p}$ collisions at RHIC and LHC energies. Our results can be seen in Table 1. Our cross section result for $\bar{p}-\bar{p}$ collisions is very small when it is compared with $Au-\bar{p}$ and $Pb-\bar{p}$ collisions for the antihydrogen production. The reason of that is the cross section includes Z^5 dependence for bound-free pair production, and an extra Z factor comes from the square of the normalization constant.

We have done our calculations for the high energies. For this reason, our results are much higher than the previous calculations mentioned above. In this antihydrogen cross section calculations, we did not add the effect of the distortion term due to the wave function of the free electron. Therefore in Fig. 2 and in Fig. 3, we have obtained incorrect values of the cross sections for large charges of the heavy ions. This clearly shows that the correction terms play important roles for

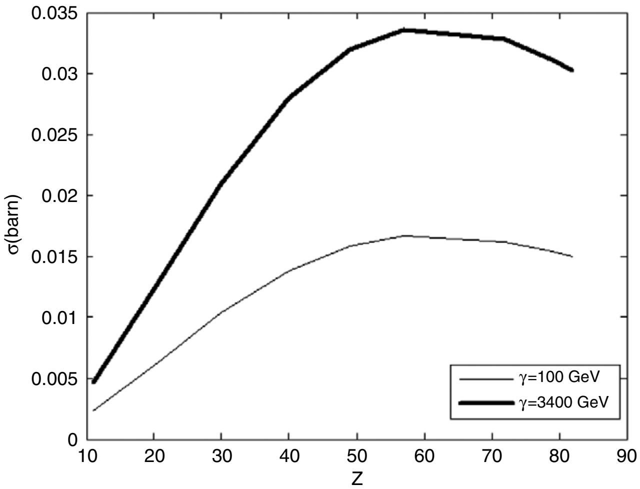

Fig. 2. Relativistic antihydrogen cross sections for two energy systems (at 100 GeV/nucleon-thin line and 3400 GeV/nucleon-thick line) as functions of the nuclear charge Z.

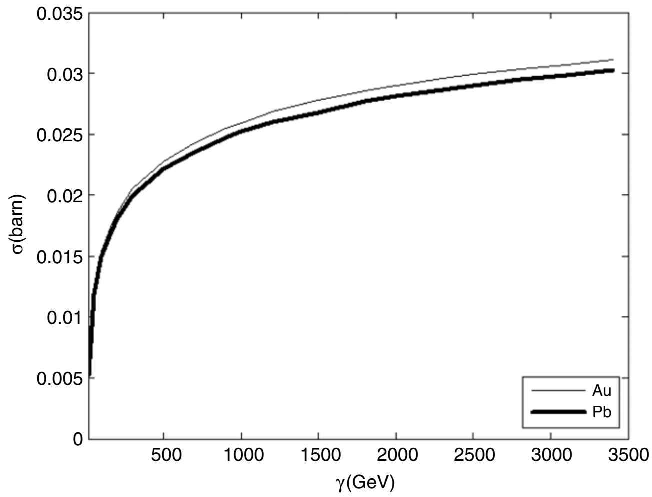

Fig. 3. Relativistic antihydrogen cross sections for two different ions (Au-thin line and Pb-thick line) as functions of γ that the magnitude of it is going from 10 to 3400.

large charges. In our future work, we are planning to calculate the effect of this correction term and add to the total cross section result. We predict that the effect of this term will decrease the value of the total cross section results.

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