Insurance: Mathematics and Economics 49 (2011) 496–500

Contents lists available at SciVerse ScienceDirect

Insurance: Mathematics and Economics

journal homepage: www.elsevier.com/locate/ime

Modeling of claim exceedances over random thresholds for related insurance portfolios

Serkan Eryilmaz[a,*], Omer L. Gebizlioglu[b], Fatih Tank[c]

[a] Atilim University, Department of Industrial Engineering, 06836, Incek, Ankara, Turkey
[b] Kadir Has University, Faculty of Economics and Administrative Sciences, 34083, Istanbul, Turkey
[c] Ankara University, Faculty of Science, Department of Statistics, 06100 Tandogan, Ankara, Turkey

ARTICLE INFO

Article history:
Received May 2010
Received in revised form
May 2011
Accepted 25 August 2011

Keywords:
Largest claim size
Order statistics
Exceedances
Renewal process
Copulas

ABSTRACT

Large claims in an actuarial risk process are of special importance for the actuarial decision making about several issues like pricing of risks, determination of retention treaties and capital requirements for solvency. This paper presents a model about claim occurrences in an insurance portfolio that exceed the largest claim of another portfolio providing the same sort of insurance coverages. Two cases are taken into consideration: independent and identically distributed claims and exchangeable dependent claims in each of the portfolios. Copulas are used to model the dependence situations. Several theorems and examples are presented for the distributional properties and expected values of the critical quantities under concern.

1. Introduction

A devastating adversity for insurance companies is the occurrence of exceedances of losses over a high threshold due to large claims. Unexpectedly large claim severities are the main cause of these subversive situations which can become worse if the number of exceedance events is also large. As a powerful risk modeling tool in this regard, models of exceedance events over fixed or random thresholds have been developed. Some examples of such models in the actuarial sciences can be found in the works of Embrechts et al. (2001), Boutsikas and Koutras (2002), Hashorva (2003) and Chavez-Demoulin and Embrechts (2004).

In the last two decades, analytical works for the exceedance modeling have proliferated in many scientific areas. Among these, we refer to the works of Davison and Smith (1990), Leadbetter (1995), Smith et al. (1997), Wesolowski and Ahsanullah (1998), Dupuis (1999), Bairamov and Kotz (2001), Bairamov and Tanil (2007) and Bairamov and Eryilmaz (2009) from the viewpoint of this paper.

Actuarial risk theory involves the threshold exceedance problems in the subject areas of risk measures, ordering of risks, premium principals, credibility, solvency and reinsurance (Kaas et al., 2008; Melnikov, 2004). Number and size of claims are the key components of all these subjects. This paper presents some new results about the process of the number of claims in a portfolio with respect to the largest claim size of another related but independent portfolio. The portfolios concerned here are to be comparable with respect to insurance branch, time scope and benefit coverages but they are assumed to be subdivided into sectors according to some factors like geographical regions, underwriting policies, insurance legislation and state regulations.

Let $X_1, X_2, \ldots$ be successive claim amounts arising from Portfolio I and $N_1(t)$, independent of X_i's, denote the number of claims in this portfolio that may occur during a specific time period $(0, t]$. Let $Y_1, Y_2, \ldots$ be claim amounts arising from Portfolio II, which is related to but assumed to be independent of Portfolio I, and $N_2(t)$, independent of Y_i's, is the number of claims that may occur during the same time period $(0, t]$. Let $X_{1:N_1(t)} \leq X_{2:N_1(t)} \leq \cdots \leq X_{N_1(t):N_1(t)}$ be the ordered values corresponding to the claim amounts $X_1, X_2, \ldots$ that occur in the time period $(0, t]$. Define

$$M(t) = \sum_{i=1}^{N_2(t)} I(Y_i > X_{N_1(t):N_1(t)}), \tag{1}$$

where $I(A) = 1$ if event A occurs, and $I(A) = 0$ otherwise, and $X_{n:n}$ denotes the largest order statistic among $X_1, \ldots, X_n$. The process defined by $M(t)$ shows the number of claims in Portfolio II which exceed the largest claim amount in Portfolio I during $(0, t]$.

* Corresponding author.
E-mail address: seryilmaz@atilim.edu.tr (S. Eryilmaz).

0167-6687/$ – see front matter © 2011 Elsevier B.V. All rights reserved.
doi:10.1016/j.insmatheco.2011.08.009

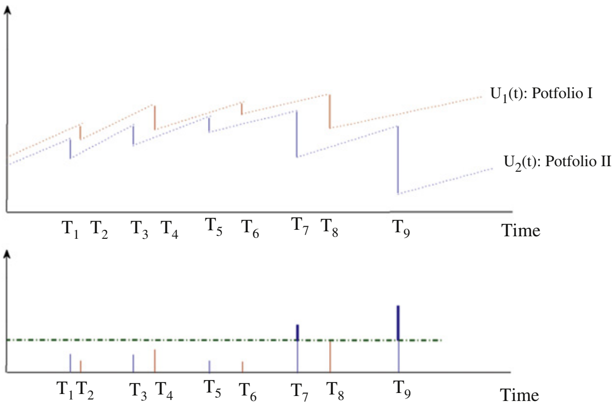

Fig. 1. Exceedance events for Portfolio II with respect to Portfolio I.

The largest claim in Portfolio I is actually a random threshold for exceedance events that are observable in Portfolio II. Note that, in this way, each exceedance is associated with a specific event.

Insurance companies can analyze the risk behavior of their subdivided portfolios by the $M(t)$ values. The most notable implementation of the process $M(t)$ can be realized in comparing risks and risk ordering of portfolios as mentioned in the last section of the paper. The process $M(t)$ can also be used for comparing the distributions of the claim sizes of two portfolios in the context of a nonparametric two sample problem. Obviously, the latter problem needs to derive the distribution of $M(t)$ for making the decision about the corresponding hypothesis testing procedure. The acceptance of the null hypothesis of equal population distributions implies that two portfolios are similar in terms of the claim sizes.

Fig. 1 depicts the exceedance events represented by $M(t)$ in terms of the surplus processes of two related portfolios. It is seen in the figure that there are realizations of an $M(t)$ process, in a $(0, t]$ time interval, at the time points T_7 and T_9.

Here, T_i's are random times and the surplus values are realizations of the so called surplus renewal processes $U_1(t)$ for Portfolio I and $U_2(t)$ for Portfolio II, under the collective risk modeling, such that

$$U_1(t) = U_{0,1} + c_1(t) - S_1(t)$$
$$U_2(t) = U_{0,2} + c_2(t) - S_2(t)$$

where $U_{0,1}$ and $U_{0,2}$ are the initial reserves, $c_1(t)$ and $c_2(t)$ are the premium income rates and $S_1(t) = \sum_{i=1}^{N_1(t)} X_i$ and $S_2(t) = \sum_{i=1}^{N_2(t)} Y_i$ are the aggregate claim amounts.

In this setup; the distribution and expected values of $M(t)$ and some extensions of these will be determined for two cases:

i. The claim sizes in each portfolio are independent and identically distributed (i.i.d.), and
ii. the claim sizes in each portfolio are dependent and the dependence is modeled by copulas.

2. Modeling under independent claims

Let X_i, $i = 1, 2, \ldots$ and Y_i, $i = 1, 2, \ldots$ be independent random claim amounts with common continuous cumulative distribution functions (c.d.f.) F_1 and F_2, respectively.

Theorem 2.1. *For $k = 0, 1, \ldots$*

$$P\{M(t) = k\} = \sum_{n_1}\sum_{n_2}\binom{n_2}{k} E\left(\bar{F}_2^k(X_{n_1:n_1})F_2^{n_2-k}(X_{n_1:n_1})\right) \times P\{N_1(t) = n_1\} P\{N_2(t) = n_2\}.$$

Proof. Conditioning on $N_1(t)$ and $N_2(t)$ we have

$$P\{M(t) = k\} = \sum_{n_1}\sum_{n_2} P\left\{\sum_{i=1}^{n_2} I(Y_i > X_{n_1:n_1}) = k\right\} \times P\{N_1(t) = n_1\} P\{N_2(t) = n_2\}. \quad (2)$$

It is clear that the random indicators $I(Y_i > X_{n_1:n_1})$, $i = 1, \ldots, n_2$ are exchangeable. Thus conditioning on $X_{n_1:n_1}$ one obtains

$$P\left\{\sum_{i=1}^{n_2} I(Y_i > X_{n_1:n_1}) = k\right\} = \binom{n_2}{k}\int_0^\infty \bar{F}_2^k(x)F_2^{n_2-k}(x)\mathrm{d}F_{n_1:n_1}(x) = \binom{n_2}{k} E\left(\bar{F}_2^k(X_{n_1:n_1})F_2^{n_2-k}(X_{n_1:n_1})\right), \quad (3)$$

where $F_{n_1:n_1}(x)$ is the c.d.f. of $X_{n_1:n_1}$. The proof follows using (3) in (2). □

Corollary 2.1. *If $F_1 = F_2$, then*

$$P\{M(t) = k\} = \sum_{n_1}\sum_{n_2}\frac{\binom{n_1+n_2-k-1}{n_2-k}}{\binom{n_1+n_2}{n_1}} P\{N_1(t) = n_1\} \times P\{N_2(t) = n_2\}.$$

Proof. Because $F_{n_1:n_1}(x) = F_1^{n_1}(x)$, for $F_1 = F_2$

$$P\left\{\sum_{i=1}^{n_2} I(Y_i > X_{n_1:n_1}) = k\right\} = \binom{n_2}{k} n_1 \int_0^1 u^{n_1+n_2-k-1}(1-u)^k \mathrm{d}u = \frac{\binom{n_1+n_2-k-1}{n_2-k}}{\binom{n_1+n_2}{n_1}}.$$

Thus the proof is completed. □

Proposition 2.1.

$$E(M(t)) = E(N_2(t))\sum_{n_1} E\left(\bar{F}_2(X_{n_1:n_1})\right) P\{N_1(t) = n_1\}.$$

Proof. Conditioning on $N_1(t)$ and $N_2(t)$ we have

$$E(M(t)) = \sum_{n_1}\sum_{n_2} E\left(\sum_{i=1}^{n_2} I(Y_i > X_{n_1:n_1})\right) P\{N_1(t) = n_1\} \times P\{N_2(t) = n_2\} = \sum_{n_1}\sum_{n_2} n_2 P\left\{Y_1 > X_{n_1:n_1}\right\} P\{N_1(t) = n_1\} \times P\{N_2(t) = n_2\}.$$

The proof follows noting that $P\left\{Y_1 > X_{n_1:n_1}\right\} = E\left(\bar{F}_2(X_{n_1:n_1})\right)$. □

The following result can be immediately obtained from Proposition 2.1.

Corollary 2.2. *If $F_1 = F_2$, then*

$$E(M(t)) = E\left(\frac{N_2(t)}{N_1(t)+1}\right).$$

Example 2.1. Let $N_1(t)$ and $N_2(t)$ be two independent homogeneous Poisson processes having intensities λ_1 and λ_2, respectively.

If $F_1 = F_2$, then it is easy to compute

$$E(M(t)) = \frac{\lambda_2}{\lambda_1}(1 - e^{-\lambda_1 t}),$$

for $t \geq 0$.

Example 2.2. Let the claim sizes in each portfolio follow a Pareto distribution with $F_1(x) = 1 - x^{-\theta_1}$, and $F_2(x) = 1 - x^{-\theta_2}$, $x \geq 1$. Then

$$\begin{aligned} E\left(\bar{F}_2(X_{n_1:n_1})\right) &= P\left\{Y_1 > X_{n_1:n_1}\right\} \\ &= n_1 \int_1^\infty \bar{F}_2(x) F_1^{n_1-1}(x) \mathrm{d}F_1(x) \\ &= n_1 B(n_1, \alpha + 1), \end{aligned}$$

where $\alpha = \theta_2/\theta_1$ and $B(a, b)$ is a Beta function. Thus we have

$$E(M(t)) = E(N_2(t)) \sum_{n_1} n_1 B(n_1, \alpha + 1) P\{N_1(t) = n_1\}.$$

In actuarial risk theory, the processes $N_1(t)$ and $N_2(t)$ are taken usually as renewal counting processes (Rolski et al., 1999). That is,

$$N(t) = \sup\left\{j : \sum_{i=1}^j T_i \leq t\right\}$$

for $t \geq 0$, where T_i, $i \geq 1$ are i.i.d. positive random variables with $E(T_i) = \mu < \infty$. The random variables $T_1, T_2, \ldots$ representing the arrival times in a renewal process should be seen as the occurrence times of claims in a portfolio. It is well known that if T_is are exponentially distributed, then $N(t)$ is a homogeneous Poisson process.

Proposition 2.2. *Let $N_1(t)$ and $N_2(t)$ be two independent renewal processes with arrival times $\{T_i, i \geq 1\}$ and $\{Z_i, i \geq 1\}$ with $E(T_i) = \mu_1 < \infty$ and $E(Z_i) = \mu_2 < \infty$. If $F_1 = F_2$, then*

$$E(M(t)) \to \frac{\mu_1}{\mu_2} \quad \text{as } t \to \infty.$$

Proof. Using Corollary 2.2

$$E(M(t)) = E\left(\frac{N_2(t)}{t}\right) E\left(\frac{t}{N_1(t) + 1}\right). \tag{4}$$

From the elementary renewal theorem (see, e.g. Rolski et al., 1999) we have

$$E\left(\frac{N_2(t)}{t}\right) \to \frac{1}{\mu_2} \quad \text{as } t \to \infty. \tag{5}$$

It is also known that with probability 1,

$$\frac{N_1(t)}{t} \to \frac{1}{\mu_1} \quad \text{as } t \to \infty. \tag{6}$$

Thus the proof follows using (5) and (6) in (4). □

If $N_1(t)$ and $N_2(t)$ are homogeneous Poisson processes with intensities λ_1 and λ_2, then

$$E(M(t)) \to \frac{\lambda_2}{\lambda_1} \quad \text{as } t \to \infty,$$

which can also be verified from Example 2.1.

3. Modeling under dependent claims

The claim amounts within each of the subdivided portfolios may be dependent. Consider, for instance, a home insurance case with several portfolios subdivided by the geographical regions; storms over all the regions would cause similar damage to the properties and generate comparable claim sizes within each region while the frequency and severity particulars of the damages in each might be different from the others due to some regional conditions. In such a case it is appropriate to model claim sizes for each subdivided portfolio as a sequence of exchangeable dependent random variables. Mena and Nieto-Barajas (2010)'s work is a recent research example for exchangeable claim sizes in a compound Poisson-type process.

Copulas are useful tools for modeling dependence among random variables. They have been successfully used in finance and actuarial science for the problems involving multivariate outcomes and dependence (Frees and Valdez, 1998; Pfeifer and Neslehova, 2003; Denuit et al., 2005).

The distribution and expectation of $M(t)$ are attained below by assuming that the claims in each of the portfolios are exchangeable dependent. The dependence is constructed by the copula modeling.

For any m, define

$$\begin{aligned} P\left\{X_{i_1} \leq x_1, \ldots, X_{i_m} \leq x_m\right\} &= F_1(x_1, \ldots, x_m) \\ &= C_1(F_1(x_1), \ldots, F_1(x_m)), \end{aligned}$$

and

$$\begin{aligned} P\left\{Y_{i_1} \leq x_1, \ldots, Y_{i_m} \leq x_m\right\} &= F_2(x_1, \ldots, x_m) \\ &= C_2(F_2(x_1), \ldots, F_2(x_m)), \end{aligned}$$

where $i_1, i_2, \ldots, i_m$ is a permutation of $1, 2, \ldots, m$ and C_1 and C_2 are copula functions corresponding to Portfolio I and Portfolio II, respectively.

Theorem 3.1. *For $k = 0, 1, \ldots$*

$$\begin{aligned} P\{M(t) \leq k\} = &\sum_{n_1}\sum_{n_2}\sum_{j=n_2-k}^{n_2} (-1)^{j-n_2+k} \binom{j-1}{n_2-k-1}\binom{n_2}{j} \\ &\times E(C_2(\underbrace{F_2(X_{n_1:n_1}), \ldots, F_2(X_{n_1:n_1})}_{j})) \\ &\times P\{N_1(t) = n_1\} P\{N_2(t) = n_2\}. \end{aligned}$$

Proof.

$$\begin{aligned} P\{M(t) \leq k\} = &\sum_{n_1}\sum_{n_2} P\left\{Y_{n_2-k:n_2} \leq X_{n_1:n_1}\right\} P\{N_1(t) = n_1\} \\ &\times P\{N_2(t) = n_2\}, \end{aligned}$$

where $Y_{i:n_2}$ is the ith smallest among $Y_1, \ldots, Y_{n_2}$.

$$P\left\{Y_{n_2-k:n_2} \leq X_{n_1:n_1}\right\} = \int_0^\infty P\left\{Y_{n_2-k:n_2} \leq x\right\} g_{n_1:n_1}(x)\mathrm{d}x, \tag{7}$$

where $g_{n_1:n_1}(x)$ is the p.d.f. of $X_{n_1:n_1}$ and is given by

$$g_{n_1:n_1}(x) = \frac{\mathrm{d}}{\mathrm{d}x} C_1(F_1(x), \ldots, F_1(x)).$$

On the other hand, for a sequence of exchangeable random claim size variables

$$\begin{aligned} &P\left\{Y_{n_2-k:n_2} \leq x\right\} \\ &= \sum_{j=n_2-k}^{n_2} (-1)^{j-n_2+k} \binom{j-1}{n_2-k-1}\binom{n_2}{j} P\left\{Y_{j:j} \leq x\right\}, \end{aligned} \tag{8}$$

(see, e.g. David and Nagaraja, 2003, p. 46). Using (8) in (7) one obtains

$$\begin{aligned} P\left\{Y_{n_2-k:n_2} \leq X_{n_1:n_1}\right\} = &\sum_{j=n_2-k}^{n_2} (-1)^{j-n_2+k} \binom{j-1}{n_2-k-1}\binom{n_2}{j} \\ &\times \int_0^\infty C_2(\underbrace{F_2(x), \ldots, F_2(x)}_{j}) g_{n_1:n_1}(x)\mathrm{d}x \end{aligned}$$

$$= \sum_{j=n_2-k}^{n_2} (-1)^{j-n_2+k} \binom{j-1}{n_2-k-1} \binom{n_2}{j} \times E(C_2(\underbrace{F_2(X_{n_1:n_1}), \ldots, F_2(X_{n_1:n_1})}_{j})).$$

Thus the proof is completed. □

Proposition 2.1 holds true under the assumptions that the claims are exchangeable dependent and the expected value $E\left(\bar{F}_2(X_{n_1:n_1})\right)$ can be computed as

$$\begin{aligned} E\left(\bar{F}_2(X_{n_1:n_1})\right) &= P\left\{Y_1 > X_{n_1:n_1}\right\} \\ &= P\left\{X_1 < Y_1, \ldots, X_{n_1} < Y_1\right\} \\ &= \int_0^{\infty} C_1(F_1(x), \ldots, F_1(x))\mathrm{d}F_2(x) \\ &= E(C_1(\underbrace{F_1(Y_1), \ldots, F_1(Y_1)}_{n_1})). \end{aligned}$$

Thus the expected value of the process $M(t)$ can be computed from

$$E(M(t)) = E(N_2(t)) \sum_{n_1} E(C_1(\underbrace{F_1(Y_1), \ldots, F_1(Y_1)}_{n_1}))P\left\{N_1(t) = n_1\right\}. \quad (9)$$

As it can be seen from (9), $E(M(t))$ depends on the distribution of the claim sizes of Portfolio II only through the marginal distributions of the claim size variables of Portfolio II and independent of the corresponding copula C_2. Below we illustrate the computation of the quantity $E(C_1(\underbrace{F_1(Y_1), \ldots, F_1(Y_1)}_{n_1}))$ for a particular copula function.

Example 3.1. Let

$$C_1(u_1, \ldots, u_{n_1}) = \prod_{i=1}^{n_1} u_i \left\{1 + \alpha_{n_1} \sum_{1 \le j < k \le n_1} (1-u_j)(1-u_k)\right\},$$

where $-\frac{1}{\binom{n_1}{2}} \le \alpha_{n_1} \le \frac{1}{\left[\frac{n_1}{2}\right]}$ and $[x]$ denotes the integer part of x. This model is known to be a simple Farlie–Gumbel–Morgenstern copula (see, e.g. Mari and Kotz, 2001, p. 144). For this model we obtain

$$C_1(F_1(x), \ldots, F_1(x)) = F_1^{n_1}(x) \left\{1 + \alpha_{n_1} \binom{n_1}{2} (1-F_1(x))^2\right\}.$$

Therefore

$$E(C_1(\underbrace{F_1(Y_1), \ldots, F_1(Y_1)}_{n_1})) = \int_0^{\infty} \left[F_1^{n_1}(x) \left\{1 + \alpha_{n_1} \binom{n_1}{2} (1-F_1(x))^2\right\}\right] \mathrm{d}F_2(x).$$

Let the marginal distribution functions be $F_1(x) = 1 - x^{-\theta_1}$, and $F_2(x) = 1 - x^{-\theta_2}$, $x \ge 1$. Then simple manipulations yield

$$E(C_1(\underbrace{F_1(Y_1), \ldots, F_1(Y_1)}_{n_1})) = \frac{\theta_2}{\theta_1} B\left(n_1+1, \frac{\theta_2}{\theta_1}\right) + \alpha_{n_1} \binom{n_1}{2} \frac{\theta_2}{\theta_1} B\left(n_1+1, \frac{\theta_2}{\theta_1}+2\right). \quad \square$$

The type or choice of marginal distributions $F_1(x_i)$ and $F_2(x_i)$ of the claim amounts in each of the subdivided portfolios are not constrained by the copula construction. So, any suitable set of copula functions C_1 and C_2 can be adopted from appropriate copula families for the modeling purposes here.

4. Implications of the models and conclusion

The preceding two sections produce the probability distributions and expected value expressions for the $M(t)$ process under the cases of independence and dependence of the claim size variables. Insurance risk managers can utilize these for the purposes of comparison and risk ordering of some independent but related insurance portfolios in connection with a portfolio that they have claim number and claim size experience as a benchmark. Comparing and ordering of risks is a long-established subject area of the actuarial science that is fundamental to many methods of actuarial modeling and analysis (Goovaerts et al., 1990). In this context, this section apprises stop-loss risk ordering of portfolios with respect to the $M(t)$ process.

Stochastic dominance and stop-loss ordering of risks embedded in some related portfolios can be performed by the use of $M(t)$ process and excessive claim amounts. The excessive claim amounts that may be subject for reinsurance considerations can be defined as

$$M_Y^*(t) = \sum_{i=1}^{N_2(t)} I(Y_i > X_{N_1(t):N_1(t)})Y_i.$$

The process $M_Y^*(t)$ represents the total claim size in Portfolio II that are in excess of the largest claim size in the Portfolio I. Similarly, excessive claim amounts can be defined for some other portfolios that are related to Portfolio I with respect to the largest claim size of it. Let $Z_1, Z_2, \ldots$ be the successive claim amounts of another subdivided portfolio, say Portfolio III, with $N_3(t)$ being the number of claims arising from it during the $(0, t]$ time interval. $N_3(t)$ is also assumed to be independent of Z_is. Denote the $M(t)$ values for Portfolio II and Portfolio III by $M_Y(t)$ and $M_Z(t)$, respectively, and express the amount for the reinsurance considerations in Portfolio III by $M_Z^*(t)$, which is defined similar to $M_Y^*(t)$.

Following Denuit et al. (2005), it is said that risk Y stochastically dominates risk Z, written $Z \le_{\mathrm{st}} Y$ when $E(g(Z)) \le E(g(Y))$ for all real valued and non-decreasing functions g. Stochastic dominance and the order between the distribution functions F_Y and F_Z of Z and Y imply each other, written $Z \le_{\mathrm{st}} Y$ iff $F_Z(z) \ge F_Y(z)$ for all z.

The stochastic dominance ordering of the risks Z and Y compares only the size of these risks. Another stochastic ordering modality that combines the size of the risks and their variability is the stop-loss ordering (Denuit et al., 2005). Z is said to precede Y in stop-loss order if $\Pi_Z(R) <_{\mathrm{sl}} \Pi_Y(R)$ for all real R values where $\Pi_Z(R) = E\left[(Z-R)_+\right]$, $\Pi_Y(R) = E\left[(Y-R)_+\right]$ are the well known stop-loss transform functions with R standing for the reinsurance retention limit. Note that the right hand derivatives $\Pi_Z'(R) = F_Z(R) - 1$ and $\Pi_Y'(R) = F_Y(R) - 1$ lead to the characterization of the probability distributions F_Y and F_Z from the risk ordering point of view.

Stop-loss ordering can be set in terms of the $M(t)$ values for fixed t, too. It is said that $M_Z(t)$ is more risky than $M_Y(t)$ in stop-loss ordering, $M_Z(t) <_{\mathrm{sl}} M_Y(t)$, if any of the following conditions are satisfied (Denuit et al., 2005): $E\left[(M_Z(t)-k)_+\right] \le E\left[(M_Y(t)-k)_+\right]$ for all $k \in \mathbb{N}$, $E(g(M_Z(t))) \le E(g(M_Y(t)))$ for all real valued g functions such that $\Delta g(k) \ge 0$ and $\Delta^2 g(k) \ge 0$ for all k, given that the expectations exist.

Furthermore, actuarial policies about portfolio size, premium ratings, risk reserves, retention levels and the similar other strategic matters for the subdivided portfolios can be dealt with in the light of the $M(t)$ process and its risk ordering properties.

Acknowledgment

The authors would like to thank the anonymous referee for the thorough reading and useful comments which helped to improve this paper.

The third author acknowledges the financial support of The Scientific and Technological Research Council of Turkey (TUBITAK) during his post doctoral research at Katholieke Universiteit Leuven, Belgium.

References

Bairamov, I., Eryilmaz, S., 2009. Waiting times of exceedances in random threshold models. Statistics and Probability Letters 79, 676–683.

Bairamov, I., Kotz, S., 2001. On distributions of exceedances associated with order statistics and record values for arbitrary distributions. Statistical Papers 42, 171–185.

Bairamov, I., Tanil, H., 2007. On distributions of exceedances based on generalized order statistics. Communications in Statistics—Theory and Methods 36 (7), 1479–1491.

Boutsikas, M.V., Koutras, M.V., 2002. Modeling claim exceedances over thresholds. Insurance: Mathematics and Economics 30, 67–83.

Chavez-Demoulin, V., Embrechts, P., 2004. Smooth extremal models in finance and insurance. The Journal of Risk and Insurance 71, 183–199.

David, H.A., Nagaraja, H.N., 2003. Order Statistics. In: Wiley Series in Probability and Statistics.

Davison, A.C., Smith, R.L., 1990. Models for exceedances over high thresholds. Journal of the Royal Statistical Society. Series B 52, 393–442.

Denuit, M., Dhaene, J., Goovaerts, M., Kaas, R., 2005. Actuarial Theory for Dependent Risks-Measure, Orders and Models. Wiley.

Dupuis, D.J., 1999. Exceedances over high thresholds: a guide to threshold selection. Extremes 1, 251–261.

Embrechts, P., Klüppelberg, C., Mikosch, T., 2001. Modelling Extremal Events for Insurance and Finance (Third printing). Springer-Verlag.

Frees, E.W., Valdez, E.A., 1998. Understanding relationships using copulas. North American Actuarial Journal 2, 1–25.

Goovaerts, M.J., Kaas, R., Van Heerwaarden, A.E., Bauwelinkx, T., 1990. Effective Actuarial Methods. North Holland.

Hashorva, E., 2003. On the number of near-maximum insurance claim under dependence. Insurance: Mathematics and Economics 32, 37–49.

Kaas, R., Goovaerts, M., Dhaene, J., Denuit, M., 2008. Modern Actuarial Risk Theory Using R, 2nd ed. Springer.

Leadbetter, M.R., 1995. On high level exceedance modeling and tail inference. Journal of Statistical Planning and Inference 45, 247–260.

Mari, D.D., Kotz, S., 2001. Correlation and Dependence. Imperial College Press.

Melnikov, A., 2004. Risk Analysis in Finance and Insurance. Chapman and Hall, CRC.

Mena, R.H., Nieto-Barajas, L.E., 2010. Exchangeable claim sizes in a compound Poisson-type process. Applied Stochastic Models in Business and Industry 26, 737–750.

Pfeifer, D., Neslehova, J., 2003. Modeling dependence in finance and insurance: the copula approach. Blätter der DGVFM 26, 177–191.

Rolski, T., Schmidli, H., Schmidt, V., Teugels, J., 1999. Stochastic Processes for Insurance and Finance. Wiley.

Smith, R.L., Tawn, J.A., Coles, S.G., 1997. Markov chain models for threshold exceedances. Biometrika 84, 249–268.

Wesolowski, I., Ahsanullah, M., 1998. Distributional properties of exceedance statistics. Annals of the Institute of Statistical Mathematics 50, 543–565.