Space-Time Block Coded Spatial Modulation

Ertuğrul Başar, *Student Member, IEEE,* Ümit Aygölü, *Member, IEEE,* Erdal Panayırcı, *Fellow, IEEE,* and H. Vincent Poor, *Fellow, IEEE*

***Abstract*—A novel multiple-input multiple-output (MIMO) transmission scheme, called *space-time block coded spatial modulation (STBC-SM)*, is proposed. It combines spatial modulation (SM) and space-time block coding (STBC) to take advantage of the benefits of both while avoiding their drawbacks. In the STBC-SM scheme, the transmitted information symbols are expanded not only to the space and time domains but also to the spatial (antenna) domain which corresponds to the on/off status of the transmit antennas available at the space domain, and therefore both core STBC and antenna indices carry information. A general technique is presented for the design of the STBC-SM scheme for any number of transmit antennas. Besides the high spectral efficiency advantage provided by the antenna domain, the proposed scheme is also optimized by deriving its diversity and coding gains to exploit the diversity advantage of STBC. A low-complexity maximum likelihood (ML) decoder is given for the new scheme which profits from the orthogonality of the core STBC. The performance advantages of the STBC-SM over simple SM and over V-BLAST are shown by simulation results for various spectral efficiencies and are supported by the derivation of a closed form expression for the union bound on the bit error probability.**

***Index Terms*—Maximum likelihood decoding, MIMO systems, space-time block codes/coding, spatial modulation.**

I. INTRODUCTION

THE use of multiple antennas at both transmitter and receiver has been shown to be an effective way to improve capacity and reliability over those achievable with single antenna wireless systems [1]. Consequently, multiple-input multiple-output (MIMO) transmission techniques have been comprehensively studied over the past decade by numerous researchers, and two general MIMO transmission strategies, a space-time block coding[1] (STBC) and spatial multiplexing, have been proposed. The increasing demand for high data rates and, consequently, high spectral efficiencies has led to the development of spatial multiplexing systems such as V-BLAST (Vertical-Bell Lab Layered Space-Time) [2]. In V-BLAST systems, a high level of inter-channel interference (ICI) occurs at the receiver since all antennas transmit their own data streams at the same time. This further increases the complexity of an optimal decoder exponentially, while low-complexity suboptimum linear decoders, such as the minimum mean square error (MMSE) decoder, degrade the error performance of the system significantly. On the other hand, STBCs offer an excellent way to exploit the potential of MIMO systems because of their implementation simplicity as well as their low decoding complexity [3], [4]. A special class of STBCs, called orthogonal STBCs (OSTBCs), have attracted attention due to their single-symbol maximum likelihood (ML) receivers with linear decoding complexity. However it has been shown that the symbol rate of an OSTBC is upper bounded by $3/4$ symbols per channel use (pcu) for more than two transmit antennas [5]. Several high rate STBCs have been proposed in the past decade (see [6]-[8] and references therein), but their ML decoding complexity grows exponentially with the constellation size, which makes their implementation difficult and expensive for future wireless communication systems. Recently, a novel concept known as spatial modulation (SM) has been introduced by Mesleh *et al.* in [9] and [10] to remove the ICI completely between the transmit antennas of a MIMO link. The basic idea of SM is an extension of two dimensional signal constellations (such as M-ary phase shift keying (M-PSK) and M-ary quadrature amplitude modulation (M-QAM), where M is the constellation size) to a third dimension, which is the spatial (antenna) dimension. Therefore, the information is conveyed not only by the amplitude/phase modulation (APM) techniques, but also by the antenna indices. An optimal ML decoder for the SM scheme, which makes an exhaustive search over the aforementioned three dimensional space has been presented in [11]. It has been shown in [11] that the error performance of the SM scheme [9] can be improved approximately in the amount of 4 dB by the use of the optimal detector under conventional channel assumptions and that SM provides better error performance than V-BLAST and maximal ratio combining (MRC). More recently, Jeganathan *et al.* have introduced a so-called space shift keying (SSK) modulation scheme for MIMO channels in [12]. In SSK modulation, APM is eliminated and only antenna indices are used to transmit information, to obtain further simplification in system design and reduction in decoding complexity. However, SSK modulation does not provide any performance advantage compared to SM. In both of the SM and SSK modulation systems, only one transmit antenna is active during each transmission interval, and therefore ICI is totally eliminated. SSK modulation has been generalized in [13], where different combinations of the transmit antenna indices are used to

Paper approved by H. Leib, the Editor for Communication and Information Theory of the IEEE Communications Society. Manuscript received March 15, 2010; revised August 12, 2010.

This paper was presented in part at the IEEE International Symposium on Personal, Indoor and Mobile Radio Communications (PIMRC), Istanbul, Turkey, September 2010.

E. Başar and Ü. Aygölü are with Istanbul Technical University, Faculty of Electrical and Electronics Engineering, 34469, Maslak, Istanbul, Turkey (e-mail: {basarer, aygolu}@itu.edu.tr).

E. Panayırcı is with Kadir Has University, Department of Electronics Engineering, 34083, Cibali, Istanbul, Turkey (e-mail: eepanay@khas.edu.tr).

H. V. Poor is with the Department of Electrical Engineering, Princeton University, Princeton, NJ, 08544, USA (e-mail: poor@princeton.edu).

This work was supported in part by the U. S. National Science Foundation under Grant CNS-09-05398.

Digital Object Identifier 10.1109/TCOMM.2011.121410.100149

[1]The abbreviation "STBC(s)" stands for space-time block coding/code(s) depending on the context.

convey information for further design flexibility. Both the SM and SSK modulation systems have been concerned with exploiting the multiplexing gain of multiple transmit antennas, but the potential for transmit diversity of MIMO systems is not exploited by these two systems. This leads to the introduction here of *Space-Time Block Coded Spatial Modulation (STBC-SM)*, designed to take advantage of both SM and STBC.

The main contributions of this paper can be summarized as follows:

- A new MIMO transmission scheme, called STBC-SM, is proposed, in which information is conveyed with an STBC matrix that is transmitted from combinations of the transmit antennas of the corresponding MIMO system. The Alamouti code [3] is chosen as the target STBC to exploit. As a source of information, we consider not only the two complex information symbols embedded in Alamouti's STBC, but also the indices (positions) of the two transmit antennas employed for the transmission of the Alamouti STBC.
- A general technique is presented for constructing the STBC-SM scheme for any number of transmit antennas. Since our scheme relies on STBC, by considering the general STBC performance criteria proposed by Tarokh *et al.* [14], diversity and coding gain analyses are performed for the STBC-SM scheme to benefit the second order transmit diversity advantage of the Alamouti code.
- A low complexity ML decoder is derived for the proposed STBC-SM system, to decide on the transmitted symbols as well as on the indices of the two transmit antennas that are used in the STBC transmission.
- It is shown by computer simulations that the proposed STBC-SM scheme has significant performance advantages over the SM with an optimal decoder, due to its diversity advantage. A closed form expression for the union bound on the bit error probability of the STBC-SM scheme is also derived to support our results. The derived upper bound is shown to become very tight with increasing signal-to-noise (SNR) ratio.

The organization of the paper is as follows. In Section II, we introduce our STBC-SM transmission scheme via an example with four transmit antennas, give a general STBC-SM design technique for n_T transmit antennas, and formulate the optimal STBC-SM ML detector. In Section III, the performance analysis of the STBC-SM system is presented. Simulation results and performance comparisons are presented in Section IV. Finally, Section V includes the main conclusions of the paper.

Notation: Bold lowercase and capital letters are used for column vectors and matrices, respectively. $(.)^*$ and $(.)^H$ denote complex conjugation and Hermitian transposition, respectively. For a complex variable x, $\Re\{x\}$ denotes the real part of x. $\mathbf{0}_{m\times n}$ denotes the $m \times n$ matrix with all-zero elements. $\|\cdot\|$, $\mathrm{tr}(\cdot)$ and $\det(\cdot)$ stand for the Frobenius norm, trace and determinant of a matrix, respectively. The probability of an event is denoted by $P(\cdot)$ and $E\{\cdot\}$ represents expectation. The union of sets A_1 through A_n is written as $\bigcup_{i=1}^{n} A_i$. We use $\binom{n}{k}$, $\lfloor x \rfloor$, and $\lceil x \rceil$ for the binomial coefficient, the largest integer less than or equal to x, and the smallest integer larger than or equal to x, respectively. We use $\lfloor x \rfloor_{2^p}$ for the largest integer less than or equal to x, that is an integer power of 2. γ denotes a complex signal constellation of size M.

II. Space-Time Block Coded Spatial Modulation (STBC-SM)

In the STBC-SM scheme, both STBC symbols and the indices of the transmit antennas from which these symbols are transmitted, carry information. We choose Alamouti's STBC, which transmits one symbol pcu, as the core STBC due to its advantages in terms of spectral efficiency and simplified ML detection. In Alamouti's STBC, two complex information symbols (x_1 and x_2) drawn from an M-PSK or M-QAM constellation are transmitted from two transmit antennas in two symbol intervals in an orthogonal manner by the codeword

$$\mathbf{X} = \begin{pmatrix}\mathbf{x}_1 & \mathbf{x}_2\end{pmatrix} = \begin{pmatrix} x_1 & x_2 \\ -x_2^* & x_1^* \end{pmatrix} \tag{1}$$

where columns and rows correspond to the transmit antennas and the symbol intervals, respectively. For the STBC-SM scheme we extend the matrix in (1) to the antenna domain. Let us introduce the concept of STBC-SM via the following simple example.

Example (STBC-SM with four transmit antennas, BPSK modulation): Consider a MIMO system with four transmit antennas which transmits the Alamouti STBC using one of the following four codewords:

$$\begin{aligned} \chi_1 &= \{\mathbf{X}_{11}, \mathbf{X}_{12}\} = \left\{ \begin{pmatrix} x_1 & x_2 & 0 & 0 \\ -x_2^* & x_1^* & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & x_1 & x_2 \\ 0 & 0 & -x_2^* & x_1^* \end{pmatrix} \right\} \\ \chi_2 &= \{\mathbf{X}_{21}, \mathbf{X}_{22}\} = \left\{ \begin{pmatrix} 0 & x_1 & x_2 & 0 \\ 0 & -x_2^* & x_1^* & 0 \end{pmatrix}, \begin{pmatrix} x_2 & 0 & 0 & x_1 \\ x_1^* & 0 & 0 & -x_2^* \end{pmatrix} \right\} e^{j\theta} \end{aligned} \tag{2}$$

where $\chi_i, i = 1, 2$ are called the STBC-SM codebooks each containing two STBC-SM codewords $\mathbf{X}_{ij}, j = 1, 2$ which do not interfere to each other. The resulting STBC-SM code is $\chi = \bigcup_{i=1}^{2} \chi_i$. A non-interfering codeword group having a elements is defined as a group of codewords satisfying $\mathbf{X}_{ij}\mathbf{X}_{ik}^H = \mathbf{0}_{2\times 2}, j, k = 1, 2, \ldots, a, j \neq k$; that is they have no overlapping columns. In (2), θ is a rotation angle to be optimized for a given modulation format to ensure maximum diversity and coding gain at the expense of expansion of the signal constellation. However, if θ is not considered, overlapping columns of codeword pairs from different codebooks would reduce the transmit diversity order to one. Assume now that we have four information bits (u_1, u_2, u_3, u_4) to be transmitted in two consecutive symbol intervals by the STBC-SM technique. The mapping rule for 2 bits/s/Hz transmission is given by Table I for the codebooks of (2) and for binary phase-shift keying (BPSK) modulation, where a realization of any codeword is called a transmission matrix. In Table I, the first two information bits (u_1, u_2) are used to determine the antenna-pair position ℓ while the last two (u_3, u_4) determine the BPSK symbol pair. If we generalize this system to M-ary signaling, we have four different codewords each having M^2 different realizations. Consequently, the spectral efficiency of the STBC-SM scheme for four transmit antennas becomes $m = (1/2)\log_2 4M^2 = 1 + \log_2 M$ bits/s/Hz, where the factor $1/2$ normalizes for the two channel uses spanned by the matrices in (2). For STBCs using larger numbers of symbol

TABLE I
STBC-SM MAPPING RULE FOR 2 BITS/S/HZ TRANSMISSION USING BPSK, FOUR TRANSMIT ANTENNAS AND ALAMOUTI'S STBC

		Input Bits	Transmission Matrices			Input Bits	Transmission Matrices
χ_1	$\ell = 0$	0000	$\begin{pmatrix} 1 & 1 & 0 & 0 \\ -1 & 1 & 0 & 0 \end{pmatrix}$	χ_2	$\ell = 2$	1000	$\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & -1 & 1 & 0 \end{pmatrix} e^{j\theta}$
		0001	$\begin{pmatrix} 1 & -1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{pmatrix}$			1001	$\begin{pmatrix} 0 & 1 & -1 & 0 \\ 0 & 1 & 1 & 0 \end{pmatrix} e^{j\theta}$
		0010	$\begin{pmatrix} -1 & 1 & 0 & 0 \\ -1 & -1 & 0 & 0 \end{pmatrix}$			1010	$\begin{pmatrix} 0 & -1 & 1 & 0 \\ 0 & -1 & -1 & 0 \end{pmatrix} e^{j\theta}$
		0011	$\begin{pmatrix} -1 & -1 & 0 & 0 \\ 1 & -1 & 0 & 0 \end{pmatrix}$			1011	$\begin{pmatrix} 0 & -1 & -1 & 0 \\ 0 & 1 & -1 & 0 \end{pmatrix} e^{j\theta}$
	$\ell = 1$	0100	$\begin{pmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & -1 & 1 \end{pmatrix}$		$\ell = 3$	1100	$\begin{pmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & -1 \end{pmatrix} e^{j\theta}$
		0101	$\begin{pmatrix} 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & 1 \end{pmatrix}$			1101	$\begin{pmatrix} -1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \end{pmatrix} e^{j\theta}$
		0110	$\begin{pmatrix} 0 & 0 & -1 & 1 \\ 0 & 0 & -1 & -1 \end{pmatrix}$			1110	$\begin{pmatrix} 1 & 0 & 0 & -1 \\ -1 & 0 & 0 & -1 \end{pmatrix} e^{j\theta}$
		0111	$\begin{pmatrix} 0 & 0 & -1 & -1 \\ 0 & 0 & 1 & -1 \end{pmatrix}$			1111	$\begin{pmatrix} -1 & 0 & 0 & -1 \\ -1 & 0 & 0 & 1 \end{pmatrix} e^{j\theta}$

intervals such as the quasi-orthogonal STBC [15] for four transmit antennas which employs four symbol intervals, the spectral efficiency will be degraded substantially due to this normalization term since the number of bits carried by the antenna modulation ($\log_2 c$), (where c is the total number of antenna combinations) is normalized by the number of channel uses of the corresponding STBC.

A. STBC-SM System Design and Optimization

In this subsection, we generalize the STBC-SM scheme for MIMO systems using Alamouti's STBC to n_T transmit antennas by giving a general design technique. An important design parameter for quasi-static Rayleigh fading channels is the minimum coding gain distance (CGD) [15] between two STBC-SM codewords $\mathbf{X}_{ij}$ and $\hat{\mathbf{X}}_{ij}$, where $\mathbf{X}_{ij}$ is transmitted and $\hat{\mathbf{X}}_{ij}$ is erroneously detected, is defined as

$$\delta_{\min}(\mathbf{X}_{ij}, \hat{\mathbf{X}}_{ij}) = \min_{\mathbf{X}_{ij}, \hat{\mathbf{X}}_{ij}} \det(\mathbf{X}_{ij} - \hat{\mathbf{X}}_{ij})(\mathbf{X}_{ij} - \hat{\mathbf{X}}_{ij})^H. \tag{3}$$

The minimum CGD between two codebooks χ_i and χ_j is defined as

$$\delta_{\min}(\chi_i, \chi_j) = \min_{k,l} \delta_{\min}(\mathbf{X}_{ik}, \mathbf{X}_{jl}) \tag{4}$$

and the minimum CGD of an STBC-SM code is defined by

$$\delta_{\min}(\chi) = \min_{i,j,i \neq j} \delta_{\min}(\chi_i, \chi_j). \tag{5}$$

Note that, $\delta_{\min}(\chi)$ corresponds to the determinant criterion given in [14] since the minimum CGD between non-interfering codewords of the same codebook is always greater than or equal to the right hand side of (5).

Unlike in the SM scheme, the number of transmit antennas in the STBC-SM scheme need not be an integer power of 2, since the pairwise combinations are chosen from n_T available transmit antennas for STBC transmission. This provides design flexibility. However, the total number of codeword combinations considered should be an integer power of 2. In the following, we give an algorithm to design the STBC-SM scheme:

1) Given the total number of transmit antennas n_T, calculate the number of possible antenna combinations for the transmission of Alamouti's STBC, i.e., the total number of STBC-SM codewords from $c = \left\lfloor \binom{n_T}{2} \right\rfloor_{2^p}$, where p is a positive integer.
2) Calculate the number of codewords in each codebook $\chi_i, i = 1, 2, \ldots, n-1$ from $a = \lfloor n_T/2 \rfloor$ and the total number of codebooks from $n = \lceil c/a \rceil$. Note that the last codebook χ_n does not need to have a codewords, i.e, its cardinality is $a' = c - a(n-1)$.
3) Start with the construction of χ_1 which contains a non-interfering codewords as

$$\begin{aligned} \chi_1 = \big\{ & \big(\mathbf{X} \ \mathbf{0}_{2\times(n_T-2)}\big) \\ & \big(\mathbf{0}_{2\times 2} \ \mathbf{X} \ \mathbf{0}_{2\times(n_T-4)}\big) \\ & \big(\mathbf{0}_{2\times 4} \ \mathbf{X} \ \mathbf{0}_{2\times(n_T-6)}\big) \\ & \quad \vdots \\ & \big(\mathbf{0}_{2\times 2(a-1)} \ \mathbf{X} \ \mathbf{0}_{2\times(n_T-2a)}\big) \big\} \end{aligned} \tag{6}$$

   where $\mathbf{X}$ is defined in (1).
4) Using a similar approach, construct χ_i for $2 \leq i \leq n$ by considering the following two important facts:
   - Every codebook must contain non-interfering codewords chosen from pairwise combinations of n_T available transmit antennas.
   - Each codebook must be composed of codewords with antenna combinations that were never used in the construction of a previous codebook.
5) Determine the rotation angles θ_i for each χ_i, $2 \leq i \leq n$, that maximize $\delta_{\min}(\chi)$ in (5) for a given signal constellation and antenna configuration; that is $\boldsymbol{\theta}_{opt} = \arg\max_{\boldsymbol{\theta}} \delta_{\min}(\chi)$, where $\boldsymbol{\theta} = (\theta_2, \theta_3, \ldots, \theta_n)$.

As long as the STBC-SM codewords are generated by the algorithm described above, the choice of other antenna combinations is also possible but this would not improve the overall system performance for uncorrelated channels. Since we have c antenna combinations, the resulting spectral efficiency of the STBC-SM scheme can be calculated as

$$m = \frac{1}{2}\log_2 c + \log_2 M \ \text{[bits/s/Hz]}. \tag{7}$$

The block diagram of the STBC-SM transmitter is shown in Fig. 1. During each two consecutive symbol intervals, $2m$ bits $u = \left(u_1, u_2, \ldots, u_{\log_2 c}, u_{\log_2 c+1}, \ldots, u_{\log_2 c + 2\log_2 M}\right)$ enter the STBC-SM transmitter, where the first $\log_2 c$ bits determine the antenna-pair position $\ell = u_1 2^{\log_2 c-1} + u_2 2^{\log_2 c-2} + \cdots + u_{\log_2 c} 2^0$ that is associated with the corresponding antenna pair, while the last $2\log_2 M$ bits determine the symbol pair $(x_1, x_2) \in \gamma^2$. If we compare the spectral efficiency (7) of the STBC-SM scheme with that of Alamouti's scheme ($\log_2 M$ bits/s/Hz), we observe an increment of $1/2\log_2 c$ bits/s/Hz provided by the antenna modulation. We consider two different cases for the optimization of the STBC-SM scheme.

Case 1 - $n_T \leq 4$: We have, in this case, two codebooks χ_1 and χ_2 and only one non-zero angle, say θ, to be optimized. It can be seen that $\delta_{\min}(\chi_1, \chi_2)$ is equal to the minimum

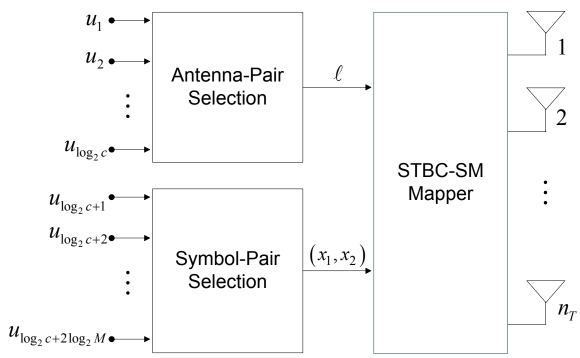


Fig. 1. Block diagram of the STBC-SM transmitter.

CGD between any two interfering codewords from χ_1 and χ_2. Without loss of generality, assume that the interfering codewords are chosen as

$$\begin{aligned}\mathbf{X}_{1k} &= \left(\mathbf{x}_1\ \mathbf{x}_2\ \mathbf{0}_{2\times(n_T-2)}\right)\\ \mathbf{X}_{2l} &= \left(\mathbf{0}_{2\times 1}\ \hat{\mathbf{x}}_1\ \hat{\mathbf{x}}_2\ \mathbf{0}_{2\times(n_T-3)}\right)e^{j\theta}\end{aligned} \tag{8}$$

where $\mathbf{X}_{1k} \in \chi_1$ is transmitted and $\hat{\mathbf{X}}_{1k} = \mathbf{X}_{2l} \in \chi_2$ is erroneously detected. We calculate the minimum CGD between $\mathbf{X}_{1k}$ and $\hat{\mathbf{X}}_{1k}$ from (3) as

$$\begin{aligned}&\delta_{\min}(\mathbf{X}_{1k}, \hat{\mathbf{X}}_{1k})\\ &= \min_{\mathbf{X}_{1k},\hat{\mathbf{X}}_{1k}} \det \begin{pmatrix} x_1 & x_2 - e^{j\theta}\hat{x}_1 & -e^{j\theta}\hat{x}_2 & \mathbf{0}_{1\times(n_T-3)} \\ -x_2^* & x_1^* + e^{j\theta}\hat{x}_2^* & -e^{j\theta}\hat{x}_1^* & \mathbf{0}_{1\times(n_T-3)} \end{pmatrix}\\ &\quad\times \begin{pmatrix} x_1^* & -x_2 \\ x_2^* - e^{-j\theta}\hat{x}_1^* & x_1 + e^{-j\theta}\hat{x}_2 \\ -e^{-j\theta}\hat{x}_2^* & -e^{-j\theta}\hat{x}_1 \\ \mathbf{0}_{(n_T-3)\times 1} & \mathbf{0}_{(n_T-3)\times 1} \end{pmatrix}\\ &= \min_{\mathbf{X}_{1k},\hat{\mathbf{X}}_{1k}} \Big\{ \left(\kappa - 2\Re\left\{\hat{x}_1^* x_2 e^{-j\theta}\right\}\right)\left(\kappa + 2\Re\left\{x_1\hat{x}_2^* e^{j\theta}\right\}\right)\\ &\quad -|x_1|^2|\hat{x}_1|^2 - |x_2|^2|\hat{x}_2|^2 + 2\Re\left\{x_1\hat{x}_1 x_2^*\hat{x}_2^* e^{j2\theta}\right\}\Big\}\end{aligned} \tag{9}$$

where $\kappa = \sum_{i=1}^{2}\left(|x_i|^2 + |\hat{x}_i|^2\right)$. Although maximization of $\delta_{\min}(\mathbf{X}_{1k}, \hat{\mathbf{X}}_{1k})$ with respect to θ is analytically possible for BPSK and quadrature phase-shift keying (QPSK) constellations, it becomes unmanageable for 16-QAM and 64-QAM which are essential modulation formats for the next generation wireless standards such as LTE-advanced and WiMAX. We compute $\delta_{\min}(\mathbf{X}_{1k}, \hat{\mathbf{X}}_{1k})$ as a function of $\theta \in [0, \pi/2]$ for BPSK, QPSK, 16-QAM and 64-QAM signal constellations via computer search and plot them in Fig. 2. These curves are denoted by $f_M(\theta)$ for $M = 2, 4, 16$ and 64, respectively. θ values maximizing these functions can be determined from Fig. 2 as follows:

$$\max_\theta \delta_{\min}(\chi) = \begin{cases} \max_\theta f_2(\theta) = 12, & \text{if } \theta = 1.57 \text{ rad} \\ \max_\theta f_4(\theta) = 11.45, & \text{if } \theta = 0.61 \text{ rad} \\ \max_\theta f_{16}(\theta) = 9.05, & \text{if } \theta = 0.75 \text{ rad} \\ \max_\theta f_{64}(\theta) = 8.23, & \text{if } \theta = 0.54 \text{ rad.} \end{cases}$$

Case 2 - $n_T > 4$: In this case, the number of codebooks, n, is greater than 2. Let the corresponding rotation angles to be optimized be denoted in ascending order by $\theta_1 = 0 < \theta_2 < \theta_3 < \cdots < \theta_n < p\pi/2$, where $p = 2$ for BPSK and $p = 1$ for QPSK. For BPSK and QPSK signaling, choosing

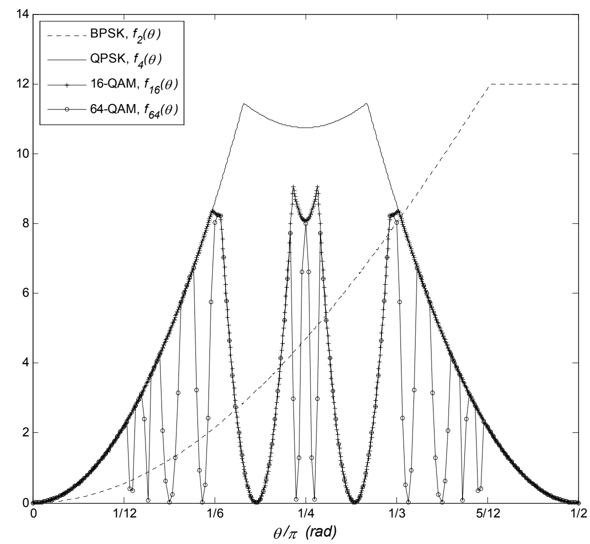


Fig. 2. Variation of $\delta_{\min}(\chi)$ given in (9) for BPSK, QPSK, 16-QAM and 64-QAM ($f_2(\theta)$, $f_4(\theta)$, $f_{16}(\theta)$ and $f_{64}(\theta)$).

$$\theta_k = \begin{cases} \frac{(k-1)\pi}{n}, & \text{for BPSK} \\ \frac{(k-1)\pi}{2n}, & \text{for QPSK} \end{cases} \tag{10}$$

for $1 \le k \le n$ guarantees the maximization of the minimum CGD for the STBC-SM scheme. This can be explained as follows. For any n, we have to maximize $\delta_{\min}(\chi)$ as

$$\begin{aligned}\max \delta_{\min}(\chi) &= \max \min_{i,j,i\neq j} \delta_{\min}(\chi_i, \chi_j)\\ &= \max \min_{i,j,i\neq j} f_M(\theta_j - \theta_i)\end{aligned} \tag{11}$$

where $\theta_j > \theta_i$, for $j > i$ and the minimum CGD between codebooks χ_i and χ_j is directly determined by the difference between their rotation angles. This can be easily verified from (9) by choosing the two interfering codewords as $\mathbf{X}_{ik} \in \chi_i$ and $\hat{\mathbf{X}}_{ik} = \mathbf{X}_{jl} \in \chi_j$ with the rotation angles θ_i and θ_j, respectively. Then, to maximize $\delta_{\min}(\chi)$, it is sufficient to maximize the minimum CGD between the consecutive codebooks χ_i and $\chi_{i+1}, i = 1, 2, \ldots, n-1$. For QPSK signaling, this is accomplished by dividing the interval $[0, \pi/2]$ into n equal sub-intervals and choosing, for $i = 1, 2, \ldots, n-1$,

$$\theta_{i+1} - \theta_i = \frac{\pi}{2n}. \tag{12}$$

The resulting maximum $\delta_{\min}(\chi)$ can be evaluated from (11) as

$$\begin{aligned}\max \delta_{\min}(\chi) &= \min\left\{f_4(\theta_2), f_4(\theta_3), \ldots, f_4(\theta_n)\right\}\\ &= f_4(\theta_2) = f_4\left(\frac{\pi}{2n}\right).\end{aligned} \tag{13}$$

Similar results are obtained for BPSK signaling except that $\pi/2n$ is replaced by π/n in (12) and (13). We obtain the corresponding maximum $\delta_{\min}(\chi)$ as $f_2(\theta_2) = f_2(\pi/n)$. On the other hand, for 16-QAM and 64-QAM signaling, the selection of $\{\theta_k\}$'s in integer multiples of $\pi/2n$ would not guarantee to maximize the minimum CGD for the STBC-SM scheme since the behavior of the functions $f_{16}(\theta)$ and $f_{64}(\theta)$

TABLE II
BASIC PARAMETERS OF THE STBC-SM SYSTEM FOR DIFFERENT NUMBER OF TRANSMIT ANTENNAS

n_T	c	a	n	$\delta_{\min}(\chi)$			m [bits/s/Hz]
				$M=2$	$M=4$	$M=16$	
3	2	1	2	12	11.45	9.05	$0.5+\log_2 M$
4	4	2	2	12	11.45	9.05	$1+\log_2 M$
5	8	2	4	4.69	4.87	4.87	$1.5+\log_2 M$
6	8	3	3	8.00	8.57	8.31	$1.5+\log_2 M$
7	16	3	6	2.14	2.18	2.18	$2+\log_2 M$
8	16	4	4	4.69	4.87	4.87	$2+\log_2 M$

is very non-linear, having several zeros in $[0,\pi/2]$. However, our extensive computer search has indicated that for 16-QAM with $n\leq 6$, the rotation angles chosen as $\theta_k=(k-1)\pi/2n$ for $1\leq k\leq n$ are still optimum. But for 16-QAM signaling with $n>6$ as well as for 64-QAM signaling with $n>2$, the optimal $\{\theta_k\}$'s must be determined by an exhaustive computer search.

In Table II, we summarize the basic parameters of the STBC-SM system for $3\leq n_T\leq 8$. We observe that increasing the number of transmit antennas results in an increasing number of antenna combinations and, consequently, increasing spectral efficiency achieved by the STBC-SM scheme. However, this requires a larger number of angles to be optimized and causes some reduction in the minimum CGD. On the other hand, when the same number of combinations can be supported by different numbers of transmit antennas, a higher number of transmit antennas requires fewer angles to be optimized resulting in higher minimum CGD (for an example, the cases $c=8, n_T=5$ and 6 in Table II).

We now give two examples for the codebook generation process of the STBC-SM design algorithm, presented above.

Design Example 1: From Table II, for $n_T=6$, we have $c=8, a=n=3$ and the optimized angles are $\theta_2=\pi/3, \theta_3=2\pi/3$ for BPSK and $\theta_2=\pi/6, \theta_3=\pi/3$ for QPSK and 16-QAM. The maximum of $\delta_{\min}(\chi)$ is calculated for BPSK, QPSK and 16-QAM constellations as

$$\max_{\boldsymbol{\theta}} \delta_{\min}(\chi)=\begin{cases} f_2(\pi/3)=8.00, & \text{for BPSK}\\ f_4(\pi/6)=8.57, & \text{for QPSK}\\ f_{16}(\pi/6)=8.31, & \text{for 16-QAM.}\end{cases}$$

According to the design algorithm, the codebooks can be constructed as below,

$$\chi_1=\left\{\left(\mathbf{x}_1\,\mathbf{x}_2\,\mathbf{0}\,\mathbf{0}\,\mathbf{0}\,\mathbf{0}\right),\left(\mathbf{0}\,\mathbf{0}\,\mathbf{x}_1\,\mathbf{x}_2\,\mathbf{0}\,\mathbf{0}\right),\left(\mathbf{0}\,\mathbf{0}\,\mathbf{0}\,\mathbf{0}\,\mathbf{x}_1\,\mathbf{x}_2\right)\right\}$$
$$\chi_2=\left\{\left(\mathbf{0}\,\mathbf{x}_1\,\mathbf{x}_2\,\mathbf{0}\,\mathbf{0}\,\mathbf{0}\right),\left(\mathbf{0}\,\mathbf{0}\,\mathbf{0}\,\mathbf{x}_1\,\mathbf{x}_2\,\mathbf{0}\right),\left(\mathbf{x}_2\,\mathbf{0}\,\mathbf{0}\,\mathbf{0}\,\mathbf{0}\,\mathbf{x}_1\right)\right\}e^{j\theta_2}$$
$$\chi_3=\left\{\left(\mathbf{x}_1\,\mathbf{0}\,\mathbf{x}_2\,\mathbf{0}\,\mathbf{0}\,\mathbf{0}\right),\left(\mathbf{0}\,\mathbf{x}_1\,\mathbf{0}\,\mathbf{x}_2\,\mathbf{0}\,\mathbf{0}\right)\right\}e^{j\theta_3}$$

where $\mathbf{0}$ denotes the 2×1 all-zero vector. Since there are $\binom{6}{2}=15$ possible antenna combinations, 7 of them are discarded to obtain 8 codewords. Note that the choice of other combinations does not affect $\delta_{\min}(\chi)$. In other words, the codebooks given above represent only one of the possible realizations of the STBC-SM scheme for six transmit antennas.

Design Example 2: From Table II, for $n_T=8$, we have $c=16, a=n=4$ and optimized angles are $\theta_2=\pi/4, \theta_3=\pi/2, \theta_4=3\pi/4$ for BPSK and $\theta_2=\pi/8, \theta_3=\pi/4, \theta_4=3\pi/8$ for QPSK and 16-QAM. Similarly, $\max\delta_{\min}(\chi)$ is calculated for BPSK, QPSK and 16-QAM constellations as

$$\max_{\boldsymbol{\theta}} \delta_{\min}(\chi)=\begin{cases} f_2(\pi/4)=4.69, & \text{for BPSK}\\ f_{4/16}(\pi/8)=4.87, & \text{for QPSK\&16-QAM.}\end{cases}$$

According to the design algorithm, the codebooks can be constructed as follows:

$$\begin{aligned}\chi_1=&\left\{\left(\mathbf{x}_1\,\mathbf{x}_2\,\mathbf{0}\,\mathbf{0}\,\mathbf{0}\,\mathbf{0}\,\mathbf{0}\,\mathbf{0}\right),\left(\mathbf{0}\,\mathbf{0}\,\mathbf{x}_1\,\mathbf{x}_2\,\mathbf{0}\,\mathbf{0}\,\mathbf{0}\,\mathbf{0}\right),\right.\\ &\left.\left(\mathbf{0}\,\mathbf{0}\,\mathbf{0}\,\mathbf{0}\,\mathbf{x}_1\,\mathbf{x}_2\,\mathbf{0}\,\mathbf{0}\right),\left(\mathbf{0}\,\mathbf{0}\,\mathbf{0}\,\mathbf{0}\,\mathbf{0}\,\mathbf{0}\,\mathbf{x}_1\,\mathbf{x}_2\right)\right\}\\ \chi_2=&\left\{\left(\mathbf{0}\,\mathbf{x}_1\,\mathbf{x}_2\,\mathbf{0}\,\mathbf{0}\,\mathbf{0}\,\mathbf{0}\,\mathbf{0}\right),\left(\mathbf{0}\,\mathbf{0}\,\mathbf{0}\,\mathbf{x}_1\,\mathbf{x}_2\,\mathbf{0}\,\mathbf{0}\,\mathbf{0}\right),\right.\\ &\left.\left(\mathbf{0}\,\mathbf{0}\,\mathbf{0}\,\mathbf{0}\,\mathbf{0}\,\mathbf{x}_1\,\mathbf{x}_2\,\mathbf{0}\right),\left(\mathbf{x}_2\,\mathbf{0}\,\mathbf{0}\,\mathbf{0}\,\mathbf{0}\,\mathbf{0}\,\mathbf{0}\,\mathbf{x}_1\right)\right\}e^{j\theta_2}\\ \chi_3=&\left\{\left(\mathbf{x}_1\,\mathbf{0}\,\mathbf{x}_2\,\mathbf{0}\,\mathbf{0}\,\mathbf{0}\,\mathbf{0}\,\mathbf{0}\right),\left(\mathbf{0}\,\mathbf{x}_1\,\mathbf{0}\,\mathbf{x}_2\,\mathbf{0}\,\mathbf{0}\,\mathbf{0}\,\mathbf{0}\right),\right.\\ &\left.\left(\mathbf{0}\,\mathbf{0}\,\mathbf{0}\,\mathbf{0}\,\mathbf{x}_1\,\mathbf{0}\,\mathbf{x}_2\,\mathbf{0}\right),\left(\mathbf{0}\,\mathbf{0}\,\mathbf{0}\,\mathbf{0}\,\mathbf{0}\,\mathbf{x}_1\,\mathbf{0}\,\mathbf{x}_2\right)\right\}e^{j\theta_3}\\ \chi_4=&\left\{\left(\mathbf{x}_1\,\mathbf{0}\,\mathbf{0}\,\mathbf{0}\,\mathbf{x}_2\,\mathbf{0}\,\mathbf{0}\,\mathbf{0}\right),\left(\mathbf{0}\,\mathbf{x}_1\,\mathbf{0}\,\mathbf{0}\,\mathbf{0}\,\mathbf{x}_2\,\mathbf{0}\,\mathbf{0}\right),\right.\\ &\left.\left(\mathbf{0}\,\mathbf{0}\,\mathbf{x}_1\,\mathbf{0}\,\mathbf{0}\,\mathbf{0}\,\mathbf{x}_2\,\mathbf{0}\right),\left(\mathbf{0}\,\mathbf{0}\,\mathbf{0}\,\mathbf{x}_1\,\mathbf{0}\,\mathbf{0}\,\mathbf{0}\,\mathbf{x}_2\right)\right\}e^{j\theta_4}.\end{aligned}$$

B. Optimal ML Decoder for the STBC-SM System

In this subsection, we formulate the ML decoder for the STBC-SM scheme. The system with n_T transmit and n_R receive antennas is considered in the presence of a quasi-static Rayleigh flat fading MIMO channel. The received $2\times n_R$ signal matrix $\mathbf{Y}$ can be expressed as

$$\mathbf{Y}=\sqrt{\frac{\rho}{\mu}}\mathbf{X}_\chi\mathbf{H}+\mathbf{N} \tag{14}$$

where $\mathbf{X}_\chi\in\chi$ is the $2\times n_T$ STBC-SM transmission matrix, transmitted over two channel uses and μ is a normalization factor to ensure that ρ is the average SNR at each receive antenna. $\mathbf{H}$ and $\mathbf{N}$ denote the $n_T\times n_R$ channel matrix and $2\times n_R$ noise matrix, respectively. The entries of $\mathbf{H}$ and $\mathbf{N}$ are assumed to be independent and identically distributed (i.i.d.) complex Gaussian random variables with zero means and unit variances. We assume that $\mathbf{H}$ remains constant during the transmission of a codeword and takes independent values from one codeword to another. We further assume that $\mathbf{H}$ is known at the receiver, but not at the transmitter.

Assuming n_T transmit antennas are employed, the STBC-SM code has c codewords, from which cM^2 different transmission matrices can be constructed. An ML decoder must make an exhaustive search over all possible cM^2 transmission matrices, and decides in favor of the matrix that minimizes the following metric:

$$\hat{\mathbf{X}}_\chi=\arg\min_{\mathbf{X}_\chi\in\chi}\left\|\mathbf{Y}-\sqrt{\frac{\rho}{\mu}}\mathbf{X}_\chi\mathbf{H}\right\|^2. \tag{15}$$

The minimization in (15) can be simplified due to the orthogonality of Alamouti's STBC as follows. The decoder can extract the embedded information symbol vector from (14), and obtain the following equivalent channel model:

$$\mathbf{y}=\sqrt{\frac{\rho}{\mu}}\mathcal{H}_\chi\begin{bmatrix}x_1\\x_2\end{bmatrix}+\mathbf{n} \tag{16}$$

where $\mathcal{H}_\chi$ is the $2n_R\times 2$ equivalent channel matrix [16] of the Alamouti coded SM scheme, which has c different realizations according to the STBC-SM codewords. In (16), $\mathbf{y}$ and $\mathbf{n}$ represent the $2n_R\times 1$ equivalent received signal and noise vectors, respectively. Due to the orthogonality of Alamouti's STBC, the columns of $\mathcal{H}_\chi$ are orthogonal to each other for all cases and, consequently, no ICI occurs in our scheme as in the case of SM. Consider the STBC-SM transmission model as described in Table I for four transmit antennas. Since there are $c = 4$ STBC-SM codewords, as seen from Table II, we have four different realizations for $\mathcal{H}_\chi$, which are given for n_R receive antennas as

$$\mathcal{H}_0 = \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{1,2}^* & -h_{1,1}^* \\ h_{2,1} & h_{2,2} \\ h_{2,2}^* & -h_{2,1}^* \\ \vdots & \vdots \\ h_{n_R,1} & h_{n_R,2} \\ h_{n_R,2}^* & -h_{n_R,1}^* \end{bmatrix}, \mathcal{H}_1 = \begin{bmatrix} h_{1,3} & h_{1,4} \\ h_{1,4}^* & -h_{1,3}^* \\ h_{2,3} & h_{2,4} \\ h_{2,4}^* & -h_{2,3}^* \\ \vdots & \vdots \\ h_{n_R,3} & h_{n_R,4} \\ h_{n_R,4}^* & -h_{n_R,3}^* \end{bmatrix},$$

$$\mathcal{H}_2 = \begin{bmatrix} h_{1,2}\varphi & h_{1,3}\varphi \\ h_{1,3}^*\varphi^* & -h_{1,2}^*\varphi^* \\ h_{2,2}\varphi & h_{2,3}\varphi \\ h_{2,3}^*\varphi^* & -h_{2,2}^*\varphi^* \\ \vdots & \vdots \\ h_{n_R,2}\varphi & h_{n_R,3}\varphi \\ h_{n_R,3}^*\varphi^* & -h_{n_R,2}^*\varphi^* \end{bmatrix}, \mathcal{H}_3 = \begin{bmatrix} h_{1,4}\varphi & h_{1,1}\varphi \\ h_{1,1}^*\varphi^* & -h_{1,4}^*\varphi^* \\ h_{2,4}\varphi & h_{2,1}\varphi \\ h_{2,1}^*\varphi^* & -h_{2,4}^*\varphi^* \\ \vdots & \vdots \\ h_{n_R,4}\varphi & h_{n_R,1}\varphi \\ h_{n_R,1}^*\varphi^* & -h_{n_R,4}^*\varphi^* \end{bmatrix} \tag{17}$$

where $h_{i,j}$ is the channel fading coefficient between transmit antenna j and receive antenna i and $\varphi = e^{j\theta}$. Generally, we have c equivalent channel matrices $\mathcal{H}_\ell, 0 \le \ell \le c-1$, and for the ℓth combination, the receiver determines the ML estimates of x_1 and x_2 using the decomposition as follows [17], resulting from the orthogonality of $\mathbf{h}_{\ell,1}$ and $\mathbf{h}_{\ell,2}$:

$$\begin{aligned} \hat{x}_{1,\ell} &= \arg\min_{x_1 \in \gamma} \left\| \mathbf{y} - \sqrt{\tfrac{\rho}{\mu}} \mathbf{h}_{\ell,1}\ x_1 \right\|^2 \\ \hat{x}_{2,\ell} &= \arg\min_{x_2 \in \gamma} \left\| \mathbf{y} - \sqrt{\tfrac{\rho}{\mu}} \mathbf{h}_{\ell,2}\ x_2 \right\|^2 \end{aligned} \tag{18}$$

where $\mathcal{H}_\ell = \left[\mathbf{h}_{\ell,1}\, \mathbf{h}_{\ell,2}\right]$, $0 \le \ell \le c-1$, and $\mathbf{h}_{\ell,j}, j = 1, 2$, is a $2n_R \times 1$ column vector. The associated minimum ML metrics $m_{1,\ell}$ and $m_{2,\ell}$ for x_1 and x_2 are

$$\begin{aligned} m_{1,\ell} &= \min_{x_1 \in \gamma} \left\| \mathbf{y} - \sqrt{\tfrac{\rho}{\mu}} \mathbf{h}_{\ell,1}\ x_1 \right\|^2 \\ m_{2,\ell} &= \min_{x_2 \in \gamma} \left\| \mathbf{y} - \sqrt{\tfrac{\rho}{\mu}} \mathbf{h}_{\ell,2}\ x_2 \right\|^2 \end{aligned} \tag{19}$$

respectively. Since $m_{1,\ell}$ and $m_{2,\ell}$ are calculated by the ML decoder for the ℓth combination, their summation $m_\ell = m_{1,\ell} + m_{2,\ell}, 0 \le \ell \le c-1$ gives the total ML metric for the ℓth combination. Finally, the receiver makes a decision by choosing the minimum antenna combination metric as $\hat{\ell} = \arg\min_{\ell} m_\ell$ for which $(\hat{x}_1, \hat{x}_2) = (\hat{x}_{1,\hat{\ell}}, \hat{x}_{2,\hat{\ell}})$. As a result, the total number of ML metric calculations in (15) is reduced from cM^2 to $2cM$, yielding a linear decoding complexity as is also true for the SM scheme, whose optimal decoder requires Mn_T metric calculations. Obviously, since $c \ge n_T$ for $n_T \ge 4$, there will be a linear increase in ML decoding complexity with STBC-SM as compared to the SM scheme. However, as we will show in the next section, this insignificant increase in decoding complexity is rewarded with significant performance improvement provided by the STBC-SM over SM. The last step of the decoding process is the demapping operation based on the look-up table used at the transmitter, to recover the input bits $\hat{u} = \left(\hat{u}_1, \ldots, \hat{u}_{\log_2 c}, \hat{u}_{\log_2 c+1}, \ldots, \hat{u}_{\log_2 c+2\log_2 M}\right)$ from the determined spatial position (combination) $\hat{\ell}$ and the information symbols $\hat{x}_1$ and $\hat{x}_2$. The block diagram of the ML decoder described above is given in Fig. 3.

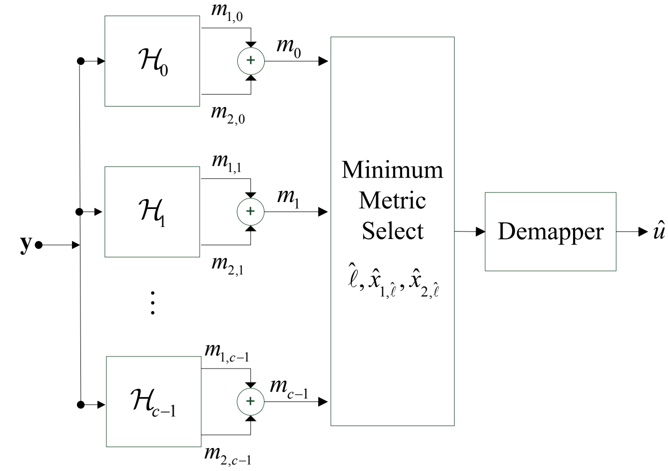


Fig. 3. Block diagram of the STBC-SM ML receiver.

III. Performance Analysis of the STBC-SM System

In this section, we analyze the error performance of the STBC-SM system, in which $2m$ bits are transmitted during two consecutive symbol intervals using one of the $cM^2 = 2^{2m}$ different STBC-SM transmission matrices, denoted by $\mathbf{X}_1, \mathbf{X}_2, \ldots, \mathbf{X}_{2^{2m}}$ here for convenience. An upper bound on the average bit error probability (BEP) is given by the well-known union bound [18]:

$$P_b \le \frac{1}{2^{2m}} \sum_{i=1}^{2^{2m}} \sum_{j=1}^{2^{2m}} \frac{P(\mathbf{X}_i \to \mathbf{X}_j) n_{i,j}}{2m} \tag{20}$$

where $P(\mathbf{X}_i \to \mathbf{X}_j)$ is the pairwise error probability (PEP) of deciding STBC-SM matrix $\mathbf{X}_j$ given that the STBC-SM matrix $\mathbf{X}_i$ is transmitted, and $n_{i,j}$ is the number of bits in error between the matrices $\mathbf{X}_i$ and $\mathbf{X}_j$. Under the normalization $\mu = 1$ and $E\left\{\operatorname{tr}\left(\mathbf{X}_\chi^H \mathbf{X}_\chi\right)\right\} = 2$ in (14), the conditional PEP of the STBC-SM system is calculated as

$$P(\mathbf{X}_i \to \mathbf{X}_j | \mathbf{H}) = Q\left(\sqrt{\frac{\rho}{2}} \left\| (\mathbf{X}_j - \mathbf{X}_i)\, \mathbf{H} \right\|\right) \tag{21}$$

where $Q(x) = (1/\sqrt{2\pi}) \int_x^\infty e^{-y^2/2} dy$. Averaging (21) over the channel matrix $\mathbf{H}$ and using the moment generating function (MGF) approach [18], the unconditional PEP is obtained as

$$P(\mathbf{X}_i \to \mathbf{X}_j) = \frac{1}{\pi} \int_0^{\pi/2} \left(\frac{1}{1 + \frac{\rho\lambda_{i,j,1}}{4\sin^2\phi}}\right)^{n_R} \left(\frac{1}{1 + \frac{\rho\lambda_{i,j,2}}{4\sin^2\phi}}\right)^{n_R} d\phi \tag{22}$$

where $\lambda_{i,j,1}$ and $\lambda_{i,j,2}$ are the eigenvalues of the distance matrix $(\mathbf{X}_i - \mathbf{X}_j)(\mathbf{X}_i - \mathbf{X}_j)^H$. If $\lambda_{i,j,1} = \lambda_{i,j,2} = \lambda_{i,j}$, (22) simplifies to

$$P(\mathbf{X}_i \to \mathbf{X}_j) = \frac{1}{\pi} \int_0^{\pi/2} \left(\frac{1}{1 + \frac{\rho\lambda_{i,j}}{4\sin^2\phi}}\right)^{2n_R} d\phi \tag{23}$$

which is the PEP of the conventional Alamouti STBC [15]. Closed form expressions can be obtained for the integrals in (22) and (23) using the general formulas given in Section 5 and Appendix A of [18].

In case of $c = an$, for $n_T = 3$ and for an even number of transmit antennas when $n_T \geq 4$, it is observed that all transmission matrices have the uniform error property due to the symmetry of STBC-SM codebooks, i.e., have the same PEP as that of $\mathbf{X}_1$. Thus, we obtain a BEP upper bound for STBC-SM as follows:

$$P_b \leq \sum\nolimits_{j=2}^{2^{2m}} \frac{P(\mathbf{X}_1 \to \mathbf{X}_j) n_{1,j}}{2m}. \quad (24)$$

Applying the natural mapping to transmission matrices, $n_{1,j}$ can be directly calculated as $n_{1,j} = w\left[(j-1)_2\right]$, where $w[x]$ and $(x)_2$ are the Hamming weight and the binary representation of x, respectively. Consequently, from (24), we obtain the union bound on the BEP as

$$P_b \leq \sum_{j=2}^{2^{2m}} \frac{w\left[(j-1)_2\right]}{2m\pi} \int_0^{\pi/2} \left(\frac{1}{1+\frac{\rho\lambda_{1,j,1}}{4\sin^2\phi}}\right)^{n_R} \left(\frac{1}{1+\frac{\rho\lambda_{1,j,2}}{4\sin^2\phi}}\right)^{n_R} d\phi, \quad (25)$$

which will be evaluated in the next section for different system parameters.

IV. Simulation Results and Comparisons

In this section, we present simulation results for the STBC-SM system with different numbers of transmit antennas and make comparisons with SM, V-BLAST, rate-$3/4$ OSTBC for four transmit antennas [15], Alamouti's STBC, the Golden Code [19] and double space-time transmit diversity (DSTTD) scheme [20]. The bit error rate (BER) performance of these systems was evaluated by Monte Carlo simulations for various spectral efficiencies as a function of the average SNR per receive antenna (ρ) and in all cases we assumed four receive antennas. All performance comparisons are made for a BER value of 10^{-5}. The SM system uses the optimal decoder derived in [11]. The V-BLAST system uses MMSE detection with ordered successive interference cancellation (SIC) decoding where the layer with the highest post detection SNR is detected first, then nulled and the process is repeated for all layers, iteratively [21]. We employ ML decoders for both the Golden code and the DSTTD scheme.

We first present the BER performance curves of the STBC-SM scheme with three and four transmit antennas for BPSK and QPSK constellations in Fig. 4. As a reference, the BEP upper bound curves of the STBC-SM scheme are also evaluated from (25) and depicted in the same figure. From Fig. 4 it follows that the derived upper bound becomes very tight with increasing SNR values for all cases and can be used as a helpful tool to estimate the error performance behavior of the STBC-SM scheme with different setups. Also note that the BER curves in Fig. 4 are shifted to the right while their slope remains unchanged and equal to $2n_R$, with increasing spectral efficiency.

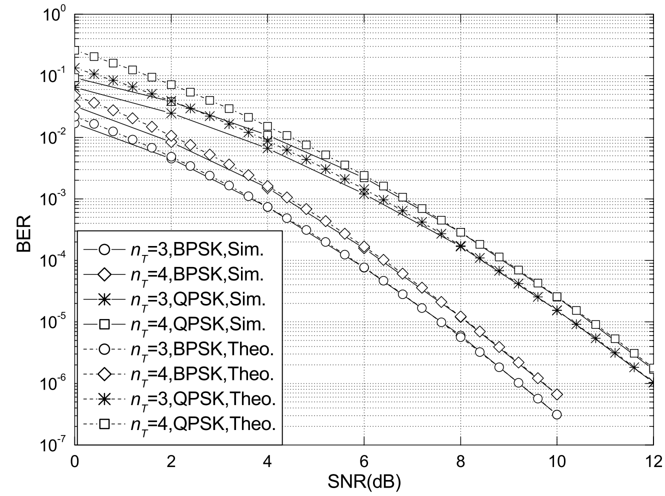


Fig. 4. BER performance of STBC-SM scheme for BPSK and QPSK compared with theoretical upper bounds.

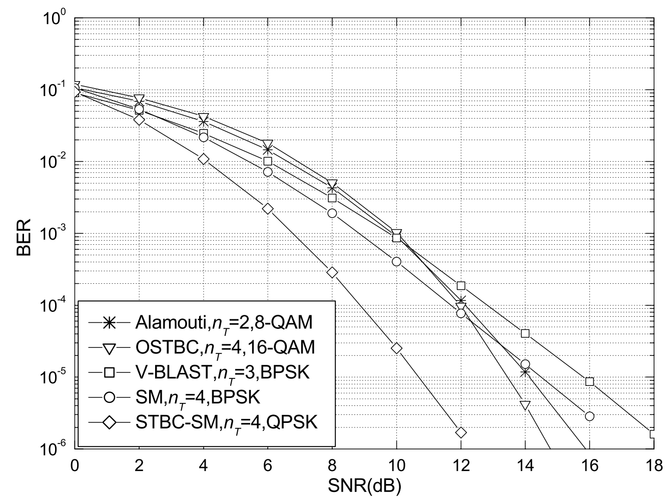


Fig. 5. BER performance at 3 bits/s/Hz for STBC-SM, SM, V-BLAST, OSTBC and Alamouti's STBC schemes.

A. *Comparisons with SM, V-BLAST, rate-*$3/4$ *OSTBC and Alamouti's STBC*

In Fig. 5, the BER curves of STBC-SM with $n_T = 4$ and QPSK, SM with $n_T = 4$ and BPSK, V-BLAST with $n_T = 3$ and BPSK, OSTBC with 16-QAM and Alamouti's STBC with 8-QAM are evaluated for 3 bits/s/Hz transmission. We observe that STBC-SM provides SNR gains of 3.8 dB, 5.1 dB, 2.8 dB and 3.4 dB over SM, V-BLAST, OSTBC and Alamouti's STBC, respectively.

In Fig. 6, we employ two different STBC-SM schemes with $n_T = 8$ and QPSK, and $n_T = 4$ and 8-QAM (for the case $n_T \leq 4$, the optimum rotation angle for rectangular 8-QAM is found from (9) as equal to 0.96 rad for which $\delta_{\min}(\chi) = 11.45$) for 4 bits/s/Hz, and make comparisons with the following schemes: SM with $n_T = 8$ and BPSK, V-BLAST with $n_T = 2$ and QPSK, OSTBC with 32-QAM, and Alamouti's STBC with 16-QAM. It is seen that STBC-SM with $n_T = 8$ and QPSK provides SNR gains of 3.5 dB, 5 dB, 4.7 dB and 4.4 dB over, SM, V-BLAST, OSTBC and Alamouti's STBC, respectively. On the other hand, we

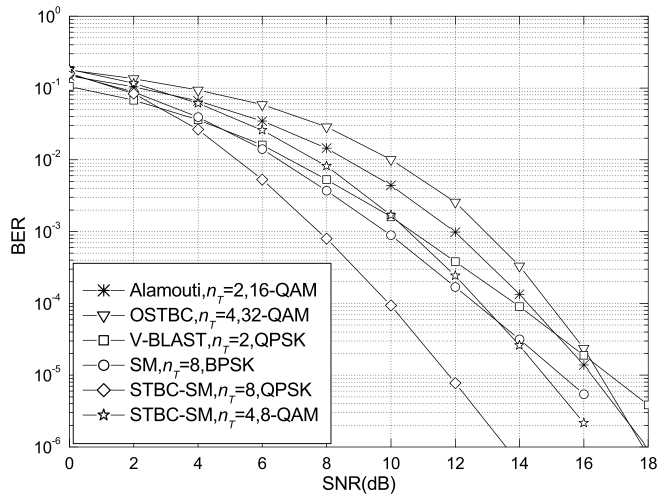


Fig. 6. BER performance at 4 bits/s/Hz for STBC-SM, SM, V-BLAST, OSTBC and Alamouti's STBC schemes.

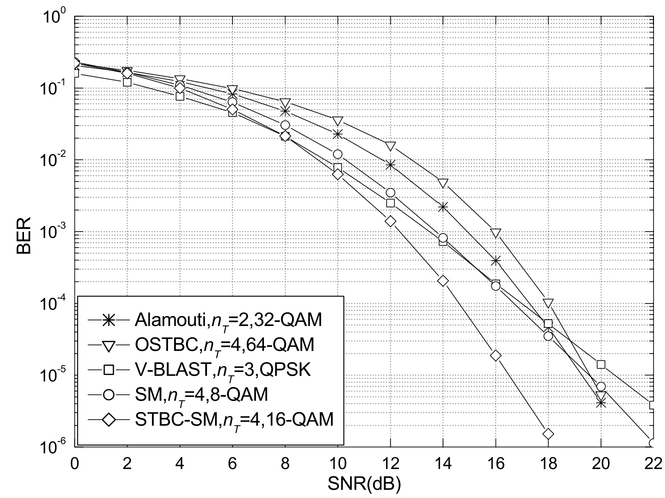


Fig. 7. BER performance at 5 bits/s/Hz for STBC-SM, SM, V-BLAST, OSTBC and Alamouti's STBC schemes.

observe 3 dB SNR gap between two STBC-SM schemes in favor of the one that uses a smaller constellation and relies more heaviy on the use of the spatial domain to achieve 4 bits/s/Hz. This gap is also verified by the difference between normalized minimum CGD values of these two schemes. We conclude from this result that one can optimize the error performance without expanding the signal constellation but expanding the spatial constellation to improve spectral efficiency. However the number of required metric calculations for ML decoding of the first STBC-SM scheme is equal to 128 while the other one's is equal to 64, which provides an interesting trade-off between complexity and performance. Based on these examples, we conclude that for a given spectral efficiency, as the modulation order M increases, the number of transmit antennas n_T should decrease, and consequently the SNR level needed for a fixed BER will increase while the overall decoding complexity will be reduced. On the other hand, as the modulation order M decreases, the number of transmit antennas n_T should increase, and as a result the SNR level needed for a fixed BER will decrease while the overall decoding complexity increases.

In Figs. 7 and 8, we extend our simulation studies to 5 and 6 bits/s/Hz transmission schemes, respectively. Since it is not possible to obtain 5 bits/s/Hz with V-BLAST, we depict the BER curve of V-BLAST for 6 bits/s/Hz in both figures. As seen from Fig. 7, STBC-SM with $n_T = 4$ and 16-QAM provides SNR gains of 3 dB, 4 dB, 3 dB and 2.8 dB over SM with $n_T = 4$ and 8-QAM, V-BLAST with $n_T = 3$ and QPSK, OSTBC with 64-QAM and Alamouti's STBC with 32-QAM, respectively. For 6 bits/s/Hz transmission we consider STBC-SM with $n_T = 8$ and 16-QAM, SM with $n_T = 8$ and 8-QAM, OSTBC with 256-QAM and Alamouti's STBC with 64-QAM. We observe that the new scheme provides 3.4 dB, 3.7 dB, 8.6 dB and 5.4 dB SNR gains compared to SM, V-BLAST, OSTBC and Alamouti's STBC, respectively.

By considering the BER curves in Figs. 5-8, we conclude that the BER performance gap between the STBC-SM and SM or V-BLAST systems increases for high SNR values due to the second order transmit diversity advantage of the STBC-SM scheme. We also observe that the BER performance of Alamouti's scheme can be greatly improved (approximately 3-5 dB depending on the transmission rate) with the use of the spatial domain. Note that although having a lower diversity order, STBC-SM outperforms rate-$3/4$ OSTBC, since this OSTBC uses higher constellations to reach the same spectral efficiency as STBC-SM. Finally, it is interesting to note that in some cases, SM and V-BLAST systems are outperformed by Alamouti's STBC for high SNR values even at a BER of 10^{-5}.

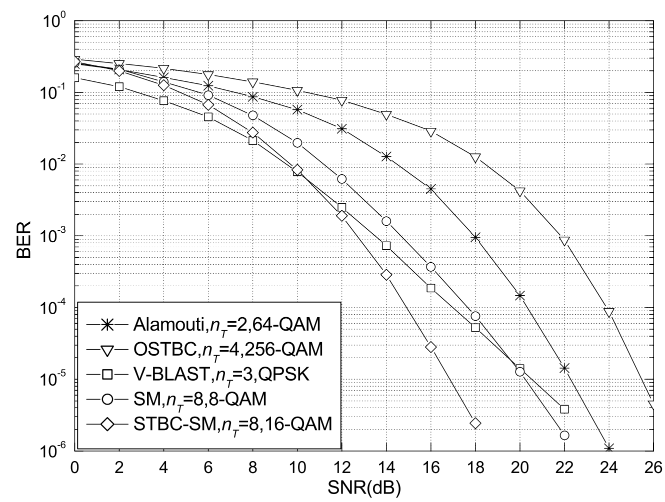


Fig. 8. BER performance at 6 bits/s/Hz for STBC-SM, SM, V-BLAST, OSTBC and Alamouti's STBC schemes.

B. Comparisons with the Golden code and DSTTD scheme

In Fig. 9, we compare the BER performance of the STBC-SM scheme with the Golden code and DSTTD scheme which are rate-2 (transmitting four symbols in two time intervals) STBCs for two and four transmit antennas, respectively, at 4 and 6 bits/s/Hz. Although both systems have a brute-force ML decoding complexity that is proportional to the fourth power of the constellation size, by using low complexity ML decoders recently proposed in the literature, their worst

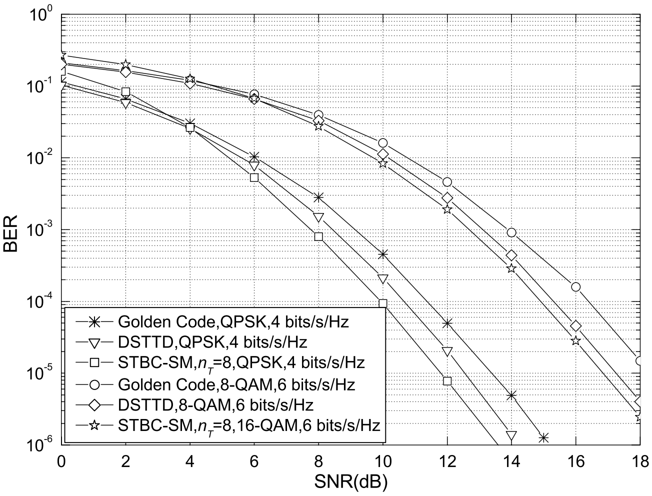


Fig. 9. BER performance for STBC-SM, the Golden code and DSTTD schemes at 4 and 6 bits/s/Hz spectral efficiencies.

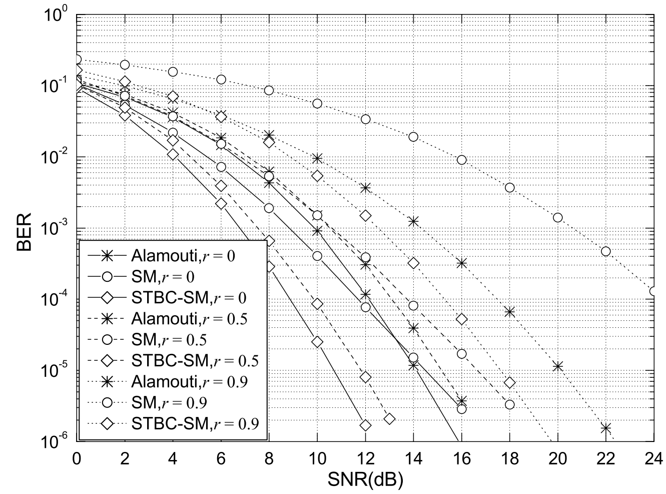


Fig. 10. BER performance at 3 bits/s/Hz for STBC-SM, SM, and Alamouti's STBC schemes for SC channel with $r = 0, 0.5$ and 0.9.

case ML decoding complexity can be reduced to $2M^3$ from M^4 for general M-QAM constellations, which we consider in our comparisons. MMSE decoding is widely used for the DSTTD scheme, however, we use an ML decoder to compare the pure performances of the considered schemes. From Fig. 9, we observe that STBC-SM offers SNR gains of 0.75 dB and 1.6 dB over the DSTTD scheme and the Golden code, respectively, at 4 bits/s/Hz, while having the same ML decoding complexity, which is equal to 128. On the other hand, STBC-SM offers SNR gains of 0.4 dB and 1.5 dB over the DSTTD scheme and the Golden code, respectively, at 6 bits/s/Hz, with 50% lower decoding complexity, which is equal to 512.

C. STBC-SM Under Correlated Channel Conditions

Inadequate antenna spacing and the presence of local scatterers lead to spatial correlation (SC) between transmit and receive antennas of a MIMO link, which can be modeled by a modified channel matrix [22] $\mathbf{H}_{corr} = \mathbf{R}_t^{1/2}\mathbf{H}\mathbf{R}_r^{1/2}$ where $\mathbf{R}_t = [r_{ij}]_{n_T \times n_T}$ and $\mathbf{R}_r = [r_{ij}]_{n_R \times n_R}$ are the SC matrices at the transmitter and the receiver, respectively. In our simulations, we assume that these matrices are obtained from the exponential correlation matrix model [23], i.e., their components are calculated as $r_{ij} = r_{ji}^* = r^{j-i}$ for $i \leq j$ where r is the correlation coefficient of the neighboring transmit and receive antennas' branches. This model provides a simple and efficient tool to evaluate the BER performance of our scheme under SC channel conditions. In Fig. 10, the BER curves for the STBC-SM with $n_T = 4$ and QPSK, the SM with $n_T = 4$ and BPSK, and the Alamouti's STBC with 8-QAM are shown for 3 bits/s/Hz spectral efficiency with $r = 0, 0.5$ and 0.9. As seen from Fig. 10, the BER performance of all schemes is degraded substantially by these correlations. However, we observe that while the degradation of Alamouti's STBC and our scheme are comparable, the degradation for SM is higher. Consequently, we conclude that our scheme is more robust against spatial correlation than pure SM.

V. Conclusions

In this paper, we have introduced a novel high-rate, low complexity MIMO transmission scheme, called STBC-SM, as an alternative to existing techniques such as SM and V-BLAST. The proposed new transmission scheme employs both APM techniques and antenna indices to convey information and exploits the transmit diversity potential of MIMO channels. A general technique has been presented for the construction of the STBC-SM scheme for any number of transmit antennas in which the STBC-SM system was optimized by deriving its diversity and coding gains to reach optimum performance. It has been shown via computer simulations and also supported by a theoretical upper bound analysis that the STBC-SM offers significant improvements in BER performance compared to SM and V-BLAST systems (approximately 3-5 dB depending on the spectral efficiency) with an acceptable linear increase in decoding complexity. From a practical implementation point of view, the RF (radio frequency) front-end of the system should be able to switch between different transmit antennas similar to the classical SM scheme. On the other hand, unlike V-BLAST in which all antennas are employed to transmit simultaneously, the number of required RF chains is only two in our scheme, and the synchronization of all transmit antennas would not be required. We conclude that the STBC-SM scheme can be useful for high-rate, low complexity, emerging wireless communication systems such as LTE and WiMAX. Our future work will be focused on the integration of trellis coding into the proposed STBC-SM scheme.

References

[1] E. Telatar, "Capacity of multi-antenna Gaussian channels," *European Trans. Telecommun.*, vol. 10, no. 6, pp. 558-595, Nov./Dec. 1999.

[2] P. Wolniansky, G. Foschini, G. Golden, and R. Valenzuela, "V-BLAST: an architecture for realizing very high data rates over the rich-scattering wireless channel," in *Proc. International Symp. Signals, Syst., Electron. (ISSSE'98)*, Pisa, Italy, pp. 295-300, Sep. 1998.

[3] S. M. Alamouti, "A simple transmit diversity technique for wireless communications," *IEEE J. Sel. Areas Commun.*, vol. 16, no. 8, pp. 1451-1458, Oct. 1998.

[4] V. Tarokh, H. Jafarkhani, and A. R. Calderbank, "Space-time block codes from orthogonal designs," *IEEE Trans. Inf. Theory*, vol. 45, no. 5, pp. 1456-1467, July 1999.

[5] X.-B. Liang, "Orthogonal designs with maximal rates," *IEEE Trans. Inf. Theory*, vol. 49, no. 10, pp. 2468-2503, Oct. 2003.

[6] E. Biglieri, Y. Hong, and E. Viterbo, "On fast-decodable space-time block codes," *IEEE Trans. Inf. Theory*, vol. 55, no. 2, pp. 524-530, Feb. 2009.

[7] E. Başar and Ü. Aygölü, "High-rate full-diversity space-time block codes for three and four transmit antennas," *IET Commun.*, vol. 3, no. 8, pp. 1371-1378, Aug. 2009.

[8] E. Başar and Ü. Aygölü, "Full-rate full-diversity STBCs for three and four transmit antennas," *Electron. Lett.*, vol. 44, no. 18, pp. 1076-1077, Aug. 2008.

[9] R. Mesleh, H. Haas, C. W. Ahn, and S. Yun, "Spatial modulation–a new low complexity spectral efficiency enhancing technique," in *Proc. Conf. Commun. Netw. China*, Beijing, China, pp. 1-5, Oct. 2006.

[10] R. Mesleh, H. Haas, S. Sinanovic, C. W. Ahn, and S. Yun, "Spatial modulation," *IEEE Trans. Veh. Technol.*, vol. 57, no. 4, pp. 2228-2241, July 2008.

[11] J. Jeganathan, A. Ghrayeb, and L. Szczecinski, "Spatial modulation: optimal detection and performance analysis," *IEEE Commun. Lett.*, vol. 12, no. 8, pp. 545-547, Aug. 2008.

[12] J. Jeganathan, A. Ghrayeb, L. Szczecinski, and A. Ceron, "Space-shift keying modulation for MIMO channels," *IEEE Trans. Wireless Commun.*, vol. 8, no. 7, pp. 3692-3703, July 2009.

[13] J. Jeganathan, A. Ghrayeb, and L. Szczecinski, "Generalized space shift keying modulation for MIMO channels," in *Proc. IEEE Symp. Pers. Indoor Mobile Radio Commun. (PIMRC)*, Cannes, France, Sep. 2008.

[14] V. Tarokh, N. Seshadri, and A. R. Calderbank, "Space-time codes for high data rate wireless communications: performance criterion and code construction," *IEEE Trans. Inf. Theory*, vol. 44, no. 2, pp. 744-765, Mar. 1998.

[15] H. Jafarkhani, *Space-Time Coding, Theory and Practice*. Cambridge University Press, 2005.

[16] B. Hassibi and B. M. Hochwald, "High-rate codes that are linear in space and time," *IEEE Trans. Inf. Theory*, vol. 48, no. 7, pp. 1804-1824, July 2002.

[17] X. Guo and X.-G. Xia, "On full diversity space-time block codes with partial interference cancellation group decoding," *IEEE Trans. Inf. Theory*, vol. 55, no. 10, pp. 4366-4385, Oct. 2009.

[18] M. S. Alaouni and M. K. Simon, *Digital Communications over Fading Channels*, 2nd edition. John & Wiley, 2005.

[19] J.-C. Belfiore, G. Rekaya, and E. Viterbo, "The Golden code: a 2×2 full-rate space-time code with non-vanishing determinants," *IEEE Trans. Inf. Theory*, vol. 51, no. 4, pp. 1432-1436, Apr. 2005.

[20] E. N. Onggosanusi, A. G. Dabak, and T. M. Schmidl, "High rate space-time block coded scheme: performance and improvement in correlated fading channels," in *Proc. IEEE Wireless Commun. Netw. Conf.*, Orlando, FL, pp. 194-199, Mar. 2002.

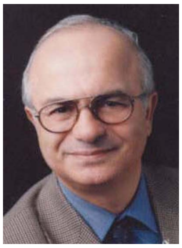
[21] R. Böhnke, D. Wübben, V. Kühn, and K. D. Kammeyer, "Reduced complexity MMSE detection for BLAST architectures," in *Proc. IEEE Global Commun. Conf.*, San Francisco, CA, USA, Dec. 2003.

[22] A. Paulraj, R. Nabar, and D. Gore, *Introduction to Space-Time Wireless Communications*. Cambridge University Press, 2003.

[23] S. L. Loyka, "Channel capacity of MIMO architecture using the exponential correlation matrix," *IEEE Commun. Lett.*, vol. 5, pp. 369-371, Sep. 2001.

Ertuğrul Başar (S'09) was born in Istanbul, Turkey, in 1985. He received the B.S. degree from Istanbul University, Istanbul, Turkey, in 2007, and the M.S. degree from the Istanbul Technical University, Istanbul, Turkey, in 2009. He is currently a research assistant at Istanbul Technical University while pursuing his Ph.D. degree at the same university. His primary research interests include MIMO systems, space-time coding and cooperative diversity.

Ümit Aygölü (M'90) received his B.S., M.S. and Ph.D. degrees, all in electrical engineering, from Istanbul Technical University, Istanbul, Turkey, in 1978, 1984 and 1989, respectively. He was a Research Assistant from 1980 to 1986 and a Lecturer from 1986 to 1989 at Yildiz Technical University, Istanbul, Turkey. In 1989, he became an Assistant Professor at Istanbul Technical University, where he became an Associate Professor and Professor, in 1992 and 1999, respectively. His current research interests include the theory and applications of combined channel coding and modulation techniques, MIMO systems, space-time coding and cooperative communication.



Erdal Panayırcı (S'73, M'80, SM'91, F'03) received the Diploma Engineering degree in Electrical Engineering from Istanbul Technical University, Istanbul, Turkey and the Ph.D. degree in Electrical Engineering and System Science from Michigan State University, USA. Until 1998 he has been with the Faculty of Electrical and Electronics Engineering at the Istanbul Technical University, where he was a Professor and Head of the Telecommunications Chair. Currently, he is Professor of Electrical Engineering and Head of the Electronics Engineering Department at Kadir Has University, Istanbul, Turkey. Dr. Panayırcı's recent research interests include communication theory, synchronization, advanced signal processing techniques and their applications to wireless communications, coded modulation and interference cancelation with array processing. He published extensively in leading scientific journals and international conferences. He has co-authored the book *Principles of Integrated Maritime Surveillance Systems* (Boston, Kluwer Academic Publishers, 2000).

Dr. Panayırcı spent the academic year 2008-2009, in the Department of Electrical Engineering, Princeton University, New Jersey, USA. He has been the principal coordinator of a 6th and 7th Frame European project called NEWCOM (Network of Excellent on Wireless Communications) and WIMAGIC Strep project representing Kadir Has University. Dr. Panayırcı was an Editor for IEEE TRANSACTIONS ON COMMUNICATIONS in the areas of Synchronization and Equalizations in 1995-1999. He served as a Member of IEEE Fellow Committee in 2005-2008. He was the Technical Program Chair of ICC-2006 and PIMRC-2010 both held in Istanbul, Turkey. Presently he is head of the Turkish Scientific Commission on Signals and Systems of URSI (International Union of Radio Science).

H. Vincent Poor (S'72, M'77, SM'82, F'87) received the Ph.D. degree in EECS from Princeton University in 1977. From 1977 until 1990, he was on the faculty of the University of Illinois at Urbana-Champaign. Since 1990 he has been on the faculty at Princeton, where he is the Michael Henry Strater University Professor, and Dean of the School of Engineering and Applied Science. Dr. Poor's research interests are in wireless networks and related fields. Among his publications in these areas are the books *MIMO Wireless Communications* (Cambridge University Press, 2007) and *Information Theoretic Security* (Now Publishers, 2009).

Dr. Poor is a member of the National Academy of Engineering, a Fellow of the American Academy of Arts and Sciences, and an International Fellow of the Royal Academy of Engineering of the U.K. He is also a Fellow of the Institute of Mathematical Statistics, the Optical Society of America, and other organizations. In 1990, he served as President of the IEEE Information Theory Society, and in 2004-07 he served as the Editor-in-Chief of the IEEE TRANSACTIONS ON INFORMATION THEORY. He was the recipient of the 2005 IEEE Education Medal. Recent recognition of his work includes the 2009 Edwin Howard Armstrong Award of the IEEE Communications Society, the 2010 IET Ambrose Fleming Medal for Achievement in Communications, and the 2011 IEEE Eric E. Sumner Award.