The Effect of Channel Models on Compressed Sensing Based UWB Channel Estimation

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***Abstract*—Ultra-wideband (UWB) multipath channels are assumed to have a sparse structure as the received consecutive pulses arrive with a considerable time delay and can be resolved individually at the receiver. Due to this sparse structure, there has been a significant amount of interest in applying the compressive sensing (CS) theory to UWB channel estimation. There are various implementations of the CS theory for the UWB channel estimation based on the assumption that the UWB channels are sparse. However, the sparsity of a UWB channel mainly depends on the channel environment. Motivated by this, in this study we investigate the effect of UWB channel environments on the CS based UWB channel estimation. Particularly, we consider the standardized IEEE 802.15.4a UWB channel models and study the channel estimation performance from a practical implementation point of view. The study shows that while UWB channel models for residential environments (e.g., CM1 and CM2) exhibit a sparse structure yielding a reasonable channel estimation performance, channel models for industrial environments (e.g., CM8) may not be treated as having a sparse structure due to multipaths arriving densely. The results of this study are important as it determines the suitability of different channel models to be used with the CS theory.**

I. Introduction

Ultra-wideband (UWB) impulse radio (IR) systems operate with low transmit power, have low-cost simple transceiver structures, and the received signal is rich in multipath diversity with fine time resolution [1]. These properties have made UWB-IRs suitable for accurate location-ranging applications and for sensor networks. Accordingly, they have been selected as the physical layer structure of the Wireless Personal Area Network (WPAN) standard IEEE 802.15.4a for location and ranging, and low data rate applications [2]. As for the channel estimation of UWB-IRs, the conventional maximum-likelihood (ML) channel estimation approach has been widely considered and adopted [3]. The main drawback of the implementation of an ML estimator is that very high sampling rates are required for accurate channel estimation due to the extremely wide bandwidth of the UWB-IRs (at least 500 MHz). This contradicts with the low-cost and low-power implementation purpose of UWB-IRs. Hence, a lower sampling rate would be preferred at the receiver for channel estimation purposes.

This work is supported in part by The Scientific and Technological Research Council of Turkey (TUBITAK) under grant no. 108E054.

Compressive sensing (CS) theory introduced in [4], [5] explains recovering a sparse signal of interest from fewer measurements. Accordingly, there has been a growing interest in applying the CS theory to sparse channel estimation [6], [7]. The recent literature on sparse channel estimation can be found in [6], [7] and in their references. As the UWB-IR signals have resolvable multipaths with a sparse structure at the receiver, the application of CS theory to UWB channel estimation has also found wide interest in the UWB community. For the CS based UWB channel estimation, the main goal has been to estimate the sparse channel with reduced number of observations [8]–[11]. That is equivalent to reducing the sampling rate at the receiver. In [8], a channel detection method based on the Matching Pursuit algorithm is proposed, where the path delays and gains are calculated iteratively. In [9], the authors combine the ML approach with the CS theory. In [10], a spread spectrum modulation structure is placed before the measurement matrix to enhance the estimation performance. In [11], a pre-filtering method is proposed so as to replace the measurement matrix. The common assumption of the studies in [8]–[11] is that the UWB channels are sparse. However, depending on the environment (e.g., an industrial environment may have dense multipaths), the sparsity assumption of the channels may not hold.

Motivated by this condition, we investigate the suitability of standardized UWB channel models, which are classified according to the measurement environments, to be used with the CS theory. For that, we particularly investigate the effect of the IEEE 802.15.4a UWB channel models [12] on the channel estimation performance from a practical implementation point of view. Accordingly, the channel estimation performance is determined in terms of the mean-square error (MSE) of the channel gain estimates, and the bit-error rate (BER) performance is investigated with the estimated channel parameters for various Rake receiver implementations. The MSE and BER performances are discussed considering the effects of system parameters. The results of this study are important for the practical implementation of the CS theory to UWB channel estimation.

II. CS for UWB Channel Estimation

Compressive sensing theory introduced in [4], [5] has shown that a sparse signal can be recovered with high probability from a set of small number of random linear projections. In the following, the overview of the CS theory and how it can be applied to sparse UWB channel estimation are presented.

Suppose that $\mathbf{y} \in \Re^N$ is a discrete-time signal that can be represented in an arbitrary basis $\mathbf{\Psi} \in \Re^{NxN}$ with the weighting coefficients $\mathbf{x} \in \Re^N$ as

$$\mathbf{y} = \mathbf{\Psi x}\,. \tag{1}$$

Let $\mathbf{x} = [x_1, x_2, \ldots, x_N]^T$ has M nonzero coefficients, where $M << N$. By projecting $\mathbf{y}$ onto a random measurement matrix $\mathbf{\Phi} \in \Re^{KxN}$, a set of measurements $\mathbf{z} \in \Re^K$ can be obtained as

$$\mathbf{z} = \mathbf{\Phi \Psi x} \tag{2}$$

where $K << N$. Instead of using the N-sample $\mathbf{y}$ to find the weighting coefficients $\mathbf{x}$, K-sample measurement vector $\mathbf{z}$ can be used. Accordingly, $\mathbf{x}$ can be estimated as

$$\hat{\mathbf{x}} = \min \|\mathbf{x}\|_1 \quad \text{subject to} \quad \mathbf{z} = \mathbf{\Phi \Psi x} \tag{3}$$

where ℓ_p-norm is defined as $\|\mathbf{x}\|_p = \left(\sum_{n=1}^{N} |x_n|^p\right)^{1/p}$. Note that, the advantage of estimating $\mathbf{x}$ from the vector $\mathbf{z}$ instead of $\mathbf{y}$ is that the former having much fewer samples corresponds to a much lower sampling rate at the receiver. We will now present how this concept can be used for UWB channel estimation.

The CS theory explained in (1)–(3) can be applied to UWB channel estimation. Suppose that $\mathbf{r} \in \Re^N$ is the discrete-time representation of the received signal given as

$$\mathbf{r} = \mathbf{Ph} + \mathbf{n} \tag{4}$$

where $\mathbf{P} \in \Re^{NxN}$ is a scalar matrix representing the time-shifted pulses, $\mathbf{h} = [\alpha_1, \alpha_2, \ldots, \alpha_N]^T$ are the channel gain coefficients, and $\mathbf{n}$ are the additive white Gaussian noise (AWGN) terms. Since the UWB channel structure is sparse, $\mathbf{h}$ has only M nonzero coefficients. Similar to (2), the received signal $\mathbf{r}$ can be projected onto a random measurement matrix $\mathbf{\Phi} \in \Re^{KxN}$ so as to obtain $\mathbf{z} \in \Re^K$ as

$$\begin{aligned} \mathbf{z} &= \mathbf{\Phi P h} + \mathbf{\Phi n} \\ &= \mathbf{Ah} + \mathbf{v}\,. \end{aligned} \tag{5}$$

Due to the presence of the noise term $\mathbf{v}$, the channel $\mathbf{h}$ can be estimated as

$$\hat{\mathbf{h}} = \min \|\mathbf{h}\|_1 \quad \text{subject to} \quad \|\mathbf{Ah} - \mathbf{z}\|_2 \leq \epsilon \tag{6}$$

where ϵ is related to the noise term as $\epsilon \geq \|\mathbf{v}\|_2$. Considering (6), the channel estimation performance depends on the sparsity of $\mathbf{h}$ (i.e., the value of M), as well as the number of observations K. It is therefore necessary to understand the discrete-time equivalent structure of $\mathbf{h}$ and the effects of standardized channel models.

III. Modeling the UWB Channel

In the following, we initially present the discrete-time equivalent channel $\mathbf{h}$ followed by the UWB channel models. In order to obtain $\mathbf{h}$, the general channel impulse response (CIR) should be presented first. Accordingly, the continuous-time channel $h(t)$ can be modeled as

$$h(t) = \sum_{m=1}^{L_r} h_m \delta(t - \tau_m) \tag{7}$$

where h_m is the mth multipath gain coefficient, τ_m is the delay of the mth multipath component, $\delta(\cdot)$ is the Dirac delta function and L_r is the number of resolvable multipaths.

The continuous-time CIR given in (7) assumes that the multipaths may arrive any time. This is referred to as the τ-spaced channel model [13]. Suppose that two consecutive multipaths with delays τ_k and τ_{k+1} arrive very close to each other. Further suppose that a pulse of duration T_s is to be transmitted through this channel. If $T_s > |\tau_{k+1} - \tau_k|$, then the pulse at the receiver cannot be resolved individually for each path, and experiences the combined channel response of the kth and $(k+1)$th paths. Let us define an approximate T_s-spaced channel model that combines multipaths arriving in the same time bin, $[(n-1)T_s, nT_s]$, $\forall n$. Accordingly, for $[(n-1)T_s, nT_s]$, $\forall n$, the delays $\{\tau_m | 1, 2, \ldots, L_r\}$ that arrive in the corresponding quantized time bins can be determined, and the associated $\{h_m | 1, 2, \ldots, L_r\}$ gains can be linearly combined to give the new channel coefficients $\{\alpha_n | 1, 2, \ldots, N\}$. Note that some of the $\{\alpha_n\}$ values may be zero due to no arrival during that time bin, hence, the number of nonzero coefficients M satisfies the condition $M \leq L_r \leq N$. The equivalent T_s-spaced channel model can be expressed as

$$h(t) = \sum_{n=1}^{N} \alpha_n \delta(t - nT_s) \tag{8}$$

where $T_c = NT_s$ is the channel length. Using (8), the discrete-time equivalent channel can be written as

$$\mathbf{h} = [\alpha_1, \alpha_2, \ldots, \alpha_N]^T \tag{9}$$

where the channel resolution is T_s. Then the discrete-time equivalent channel vector obtained above can be used in (4)–(6) in the context of CS theory. Next, we consider the UWB channel models to be used with the channel vector $\mathbf{h}$.

The CS based UWB channel estimation studies assume that the UWB channel vector $\mathbf{h}$ defined above is sparse. However, this is a vague assumption. In order to classify a channel as sparse, initially the channel environment should be examined. In [12], members of the IEEE 802.15.4a standardization committee have developed a comprehensive standardized model for UWB propagation channels. Accordingly, they have considered different environments and have conducted measurement campaigns in order to model the UWB channels for each environment. The channel environments that they have parameterized include indoor residential, indoor office,

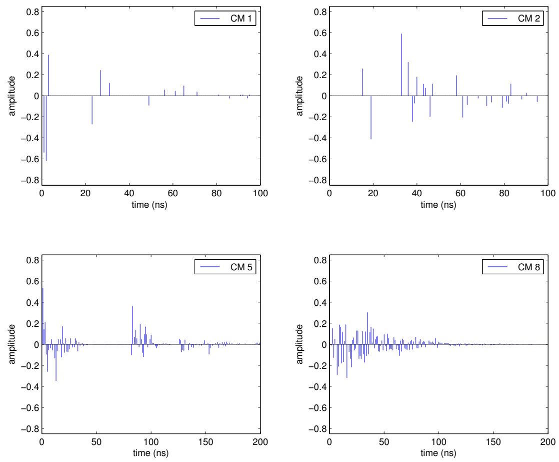

Fig. 1. Channel realizations for CM1, CM2, CM5, CM8 when $T_s = 1$ns.

outdoor, industrial environments, agricultural areas and body area networks. The details of the related channel models and their associated parameters can be found in [12]. We motivate our study with the selection of a variety of environments either having a line-of-sight (LOS) or a non-LOS (NLOS) transmitter-receiver connection. Accordingly, we select the CM1 (indoor residential LOS), CM2 (indoor residential NLOS), CM5 (outdoor LOS) and CM8 (industrial NLOS) channel models, which are widely used in UWB research. We now summarize the characteristics of channel models CM1, CM2, CM5 and CM8 in the following.

CM1: This is by-far the most commonly used channel model in order to assess the system performance. It models an LOS connection in an indoor residential environment. It is the most sparse channel model where few Rake fingers can collect considerable amount of signal energy.

CM2: This is a channel model with an NLOS connection in an indoor residential environment. It complements CM1. It is a sparse channel model but usually contains more multipaths compared to CM1.

CM5: This is a channel model with an LOS connection in an outdoor environment. Typically, the multipaths arrive in a few clusters.

CM8: This is a channel model with an NLOS connection in an industrial environment. The multipaths arrive densely so that the channel does not have a sparse structure.

Using the T_s-spaced channel model in (8) and the parameters for channel models CM1, CM2, CM5 and CM8 in [12], a realization for each channel model is plotted in Fig. 1 when the channel resolution is $T_s = 1$ns. It can be observed that the typical channel properties listed above can be observed in Fig. 1. Before assessing the channel estimation performance for different channel models, we present in Table I the sparsity ratio,[1] M/N, at various channel resolution values for different channel models obtained by averaging over 100 channel realizations when the channel length is fixed to $T_c = 100$ns. From the table, it can be deduced that the multipaths for CM5 and CM8 arrive very densely compared to CM1 and CM2, hence, even at the increased channel resolution (i.e., when T_s is decreased) the sparsity of these channels does not improve much.

TABLE I
THE SPARSITY RATIOS FOR DIFFERENT CHANNEL RESOLUTIONS

Channel Model	$T_s = 1$ns M/N	$T_s = 0.5$ns M/N	$T_s = 0.25$ns M/N
CM1	0.30	0.17	0.09
CM2	0.34	0.20	0.11
CM5	0.81	0.69	0.52
CM8	1.00	0.99	0.99

IV. SIMULATION RESULTS

In this section, we investigate the effects of IEEE 802.15.4a channel models on the channel estimation performance. For that, we evaluate the MSE of channel estimation[2] for various number of observations K and a fixed channel resolution[3] T_s with different channel models and various signal-to-noise (SNR) values. To remove the path loss effect and to treat each channel model fairly, we normalize the channel coefficients as $\sum_{n=1}^{N} \alpha_n^2 = 1$. For the generation of channels, the standardized IEEE 802.15.4a channel models [12] are used.

Initially, let us compare the channel models for the same set of parameters. In Figs. 2 and 3, the effect of number of observations K on the MSE performance at SNR=20dB is investigated when the channel length is $T_c = 250$ns for channel models CM1, CM2, CM5 and CM8. The channel resolutions (i.e., used pulse widths) in each figure are $T_s = 1$ns and $T_s = 0.25$ns resulting in $N = 250$ and $N = 1000$, respectively. Both figures can be compared to each other fairly based on the K/N ratio. It can be observed that for the same conditions the channel estimation is better for channel models in the order of CM1, CM2, CM5 and CM8 as expected. When the channel resolution is increased from $T_s = 1$ns to $T_s = 0.25$ns, it can be observed that the MSE performances of CM1, CM2 and CM5 are improved, while the performance of CM8 does not change. This can be explained by the dense multipaths arriving almost in each time bin although the resolution is increased as also shown in Table I. We can also observe that the MSE performances of CM1 and CM2 do not change much for the resolution $T_s = 0.25$ns when $400 < K < 500$. Hence, the number of observations can be

[1]We define the sparsity ratio as the ratio of the number of nonzero coefficients to the length of the discrete-time equivalent channel for the selected resolution.

[2]For the implementation of (6), the codes provided by Candes and Romberg publicly available at http://www.acm.caltech.edu/l1magic/ are used.

[3]The effect of channel resolution on the channel estimation performance has been investigated in detail in [14].

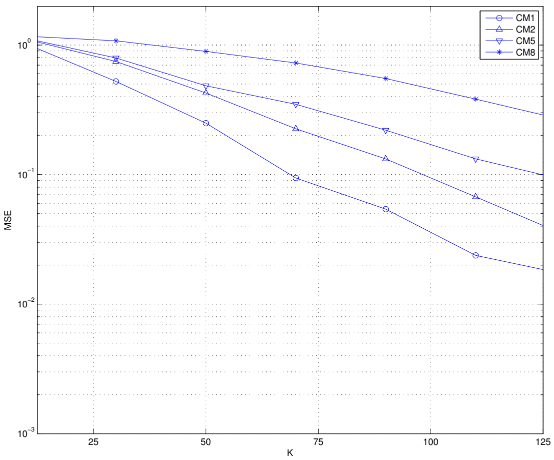

Fig. 2. The effect of number of observations K on the MSE performance at SNR=20dB when $T_c = 250$ns and $T_s = 1$ns for channel models CM1, CM2, CM5, CM8.

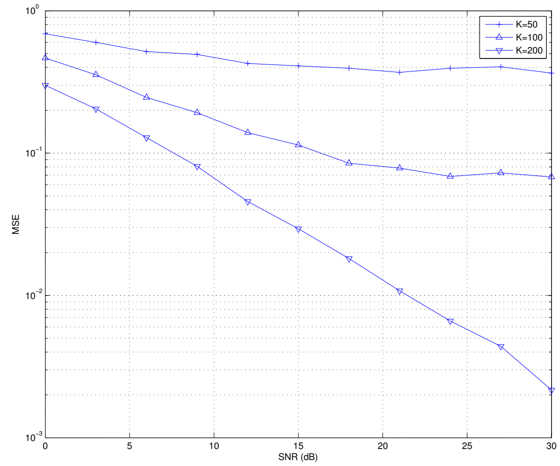

Fig. 4. The effect of number of observations K on the MSE performance when $T_c = 100$ns and $T_s = 0.25$ns for channel model CM1.

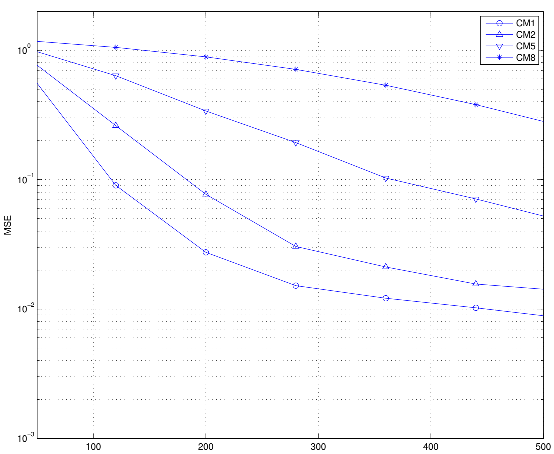

Fig. 3. The effect of number of observations K on the MSE performance at SNR=20dB when $T_c = 250$ns and $T_s = 0.25$ns for channel models CM1, CM2, CM5, CM8.

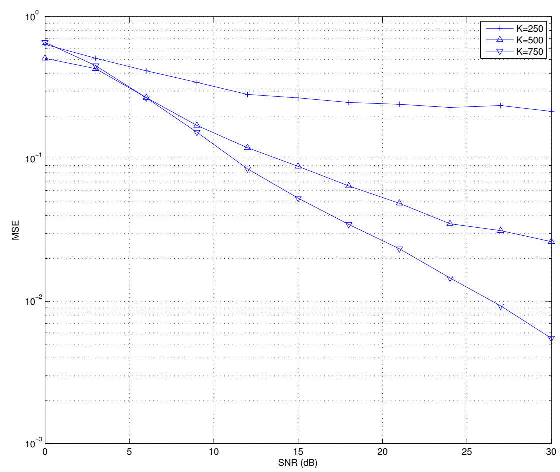

Fig. 5. The effect of number of observations K on the MSE performance when $T_c = 250$ns and $T_s = 0.25$ns for channel model CM5.

limited to $K \approx 400$, i.e., a lower sampling rate can be used for a similar MSE performance.

Next, we investigate the effects of number of observations for CM1 and CM5 for different SNR values. Channel models CM1 and CM5 are selected as the MSE performance of CM2 is similar to that of CM1, and the MSE performance of CM8 does not change much with the channel resolution. The channel resolution is fixed to $T_s = 0.25$ns for both cases. In Fig. 4 we investigate the effect of number of observations K on the MSE performance for CM1 when $T_c = 100$ns, where most of the multipaths arrive within the channel length. Since the channel resolution is $T_s = 0.25$ns, the discrete-time channel length is $N = T_c/T_s = 400$. Note that the number of nonzero coefficients M may vary for each channel realization generated by its equivalent probabilistic model. Here, K/N can be seen as the ratio of the compressed sampling rate to the conventional receiver sampling rate. As seen in Fig. 4, while $K = 50$ observations are not enough for channel estimation even at high SNR, $K = 200$ observations can achieve an MSE $\approx 10^{-2}$ at SNR=20dB for a fixed $T_s = 0.25$ns. That is, the sampling rate at the receiver can be reduced by $K/N = 50\%$ if a CS based channel estimation is used to achieve an MSE $\approx 10^{-2}$.

In Fig. 5 the effect of number of observations K on the MSE performance is investigated for CM5 when $T_c = 250$ns. The channel length is assumed to be longer as some clusters

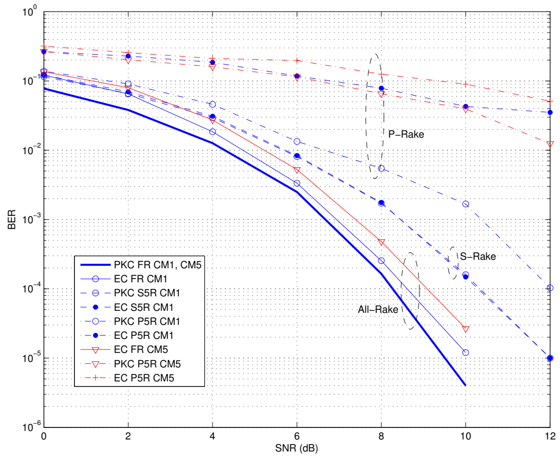

Fig. 6. BER performance of various Rake implementations (AR: All Rake, P5R: Partial 5-Rake, S5R: Selective 5-Rake) for perfectly known and estimated channels (PKC, EC).

may arrive beyond 200ns. Here, the equivalent discrete-time channel length is $N = T_c/T_s = 1000$. As can be observed, when $K = 250$ observations are used the MSE performance is poor (i.e., at the rate $K/N = 0.25$). On the other hand, increasing the observations to $K = \{500, 750\}$ improves the MSE at the expense of increasing the compressed sampling rate.

Finally in Fig. 6, we evaluate the BER performance with the estimated and perfectly known channels (EC and PKC) for various Rake receiver implementations when CM1 and CM5 are considered. The channel length and channel resolution are selected as $T_c = 250$ns and $T_s = 0.25$ns, respectively, and the sampling ratio, K/N, is fixed to 50%. As for the modulation, binary phase shift keying (BPSK) is used. When an all-Rake receiver is used, it can be observed that the BER performances are worse about 0.5 dB and 1 dB for CM1 and CM5, respectively. When a selective-Rake receiver with 5 fingers is used for CM1, the performances for the known and estimated channels are similar as the strongest paths are correctly determined by the CS based estimation. However, when a partial-Rake with 5 fingers is used for CM1, the BER performance for the estimated channel compared to the known channel has degraded much as the CS based estimation introduces non-zero components at low SNR, which are possibly selected as the fingers of a partial-Rake. Finally, it can be observed that for CM5, 5 fingers are not enough to collect significant energy for either known or estimated channels.

V. Conclusion

In this study, we investigated the effect of UWB channel environments on the CS based UWB channel estimation. We particularly considered the standardized IEEE 802.15.4a UWB channel models, which are classified according to the measurement environments, and studied the channel estimation performance from a practical implementation point of view. The channel estimation performance was determined in terms of the MSE of the channel gain estimates, and the BER performance was evaluated with estimated channel parameters for practical Rake implementations. It was shown that UWB channel models for residential environments exhibited a sparse structure yielding a reasonable channel estimation performance, whereas the channel models for industrial environments may not be treated as having a sparse structure due to multipaths arriving densely. It was also observed that the use of selective-Rake receivers after CS based sparse channel estimation yields a BER performance very close to the known channel case. The results of this study are important for the practical implementation of the CS theory to UWB channel estimation.

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