Outage Scaling Laws and Diversity for Distributed Estimation Over Parallel Fading Channels

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***Abstract*—We consider scaling laws of the outage for distributed estimation problems over fading channels with respect to the total power and the number of sensors. Using a definition of diversity which involves a fixed number of sensors, we find tight upper and lower bounds on diversity which are shown to depend on the sensing (measurement) signal-to-noise ratios (SNRs) of the sensors. Our results indicate that the diversity order can be smaller than the number of sensors, and adding new sensors might not add to the diversity order depending on the sensing SNR of the added sensor. We treat a large class of envelope distributions for the wireless channel including those appropriate for line of sight scenarios. Finally, we consider fixed power per sensor with an asymptotically large number of sensors and show that the outage decays faster than exponentially in the number of sensors.**

***Index Terms*—Distributed estimation, diversity, sensor networks.**

I. Introduction

DISTRIBUTED estimation is an important task in wireless sensor networks (WSNs), where information collected by decentralized sensors is sent to a fusion center (FC) via wireless channels, after possible local processing. This contrasts sharply with the classical centralized sensor networks with the sensors wired to the FC, where all the signal processing occurs. Since the FC in decentralized networks only receives condensed or noisy information from the sensors, they exhibit a loss in performance compared with centralized systems.

Especially over the past few years, research on distributed estimation has been evolving very rapidly [1]. Universal decentralized estimators of a source observed in additive noise have been considered in [2] and [3]. Much of the literature has focused on finite-rate transmissions of quantized sensor observations [4]–[9]. The observations of the sensors can be delivered to the FC by analog or digital transmission methods. Amplify-and-forward is one analog option, whereas in digital transmission, observations are quantized, encoded and transmitted via digital modulation. The optimality of amplify and forward is described in [10]–[13]. In particular, [10] shows from an information theoretic point of view that a joint source-channel coding approach such as the amplify and forward scheme is superior to separate source-channel coding approaches such as decode and forward for estimating the sample mean of the sensor observations over AWGN channels. A type-based approach to estimating the histogram of the sensor observations is considered in [14] and [15], and the sensitivity of this approach to system nonidealities is addressed in [16]. In [13], an amplify-and-forward approach is employed over an orthogonal multiple access fading channel, where the concept of estimation diversity is introduced, and shown to be given by the number of sensors. This seminal result is obtained under the assumption of asymptotically large number of sensors, and large total transmission powers.

We consider a parallel multiple access fading channel model, and obtain expressions for the outage probability under different asymptotic regimes than those in [13]. In the first part of this work, we allow the number of sensors to be any finite value, and arrive at results which can be interpreted differently than those in [13], where infinitely many sensors were assumed. Toward this goal, in Section III we adopt a definition of diversity order commonly adopted in wireless communications, and consider finitely many sensors and large total transmit powers. In this setting, we show that, unlike the findings in [13], the diversity order need not be equal to the number of sensors, and depends on both the sensing signal-to-noise ratios (SNRs), and the threshold used to define the outage. Moreover, in contrast with [13] we consider statistically nonidentical sensors, and general fading distributions for the wireless channel. Our findings show that it is possible to add new sensors into the system without any diversity benefit, however, the outage performance can still improve with addition of new sensors even when the total power is fixed. We obtain results by finding bounds on the diversity order, which can be found in Theorems 1 and 2, which are respectively upper and lower bounds that are arbitrarily tight under certain conditions. We extend our analysis to the case of random sensing SNRs and found that the diversity order in this case is zero under general conditions on the sensing SNR distributions.

In Section IV, we consider the case where both the number of sensors, and the total transmit power are increased with their ratio (the power per sensor) remaining fixed. We characterize the scaling law of the outage in this regime in Theorems 4 and 5 which is seen to be a function decaying faster than an exponential in the number of sensors. In what follows, we introduce the underlying system model for this work.

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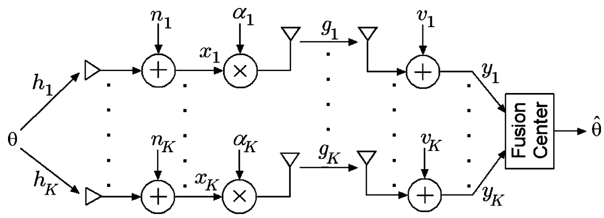

Fig. 1. Wireless sensor network with parallel channels.

II. System Model

Consider a distributed estimation problem in a WSN with parallel channels as shown in Fig. 1. We assume that there are K sensors and focus on a single time snapshot. The sensor measurements $\{x_k\}_{k=1}^K$ are related to the source parameter θ by

$$x_k = h_k\theta + n_k, \quad k = 1, \ldots, K \tag{1}$$

where $n_k \sim \mathcal{CN}(0, \sigma_{n_k}^2)$ is the sensing noise, and h_k is a parameter that controls the k^{th} sensing SNR given by $\gamma_k := |h_k|^2/\sigma_{n_k}^2$. The sensing SNRs $\{\gamma_k\}_{k=1}^K$ can be modelled as deterministic or random variables depending on the application. We mainly focus on deterministic $\gamma_k > 0$, but we also address the random case in the sequel. The sensors amplify and forward their measurements which are separately received by the FC over orthogonal channels:

$$y_k = \alpha_k g_k(h_k\theta + n_k) + v_k, \quad k = 1, \ldots, K \tag{2}$$

where $g_k \sim \mathcal{CN}(0, \sigma_{g_k}^2)$ is the k^{th} channel coefficient (we will relax this assumption when we consider line-of-sight channels), $v_k \sim \mathcal{CN}(0, \sigma_{v_k}^2)$ is the receiver noise, and α_k is the amplification coefficient which controls the power of the k^{th} sensor. We assume that n_k, v_k, h_k, and g_k are statistically independent of each other and across sensors. If the channel information is available at the sensor side, it is possible to optimize the transmit power as a function of the channel [13]. We will assume no channel information at the sensors, and consider equal power transmission in the sequel. Since each sensor's average power is given by

$$P_0 := E_{\theta, n_k}\left[|\alpha_k x_k|^2\right] = |\alpha_k|^2\left(|h_k|^2\sigma_\theta^2 + \sigma_{n_k}^2\right) \tag{3}$$

in order to ensure that the total power P_{tot} is equally distributed among all sensors, the per-sensor power is $P_0 := P_{\text{tot}}/K$, which implies

$$|\alpha_k|^2 = \frac{P_{\text{tot}}}{K\left(|h_k|^2\sigma_\theta^2 + \sigma_{n_k}^2\right)}. \tag{4}$$

We assume that the FC knows $\alpha_k, g_k, h_k, \forall k$, and the noise variances $\sigma_{n_k}^2, \sigma_{v_k}^2, \forall k$, and thus can employ maximal ratio combining before doing estimation of the source parameter θ. Combining the separately received signals y_k in (2) to get the maximum possible SNR at the output of the FC amounts to multiplying with the conjugate of the coefficient of θ when the noise variances are equal [17]. Since the k^{th} noise term in (2) is given by $w_k := \alpha_k g_k n_k + v_k$ with variance $\sigma_{w_k}^2 := \text{var}(w_k|g_k) = |\alpha_k|^2|g_k|^2\sigma_{n_k}^2 + \sigma_{v_k}^2$, we can normalize (2) with σ_{w_k} so that the k^{th} noise term has unit variance

$$\frac{y_k}{\sigma_{w_k}} = \frac{\alpha_k g_k h_k}{\sigma_{w_k}}\theta + \frac{w_k}{\sigma_{w_k}}. \tag{5}$$

The maximal ratio combining coefficients are given by $\alpha_k^* g_k^* h_k^*/\sigma_{w_k}$ [17]. We denote the resulting SNR at the output of the FC with the random variable $\texttt{snr}$ given by

$$\texttt{snr} = \sum_{k=1}^K \frac{|\alpha_k|^2|g_k|^2|h_k|^2}{|\alpha_k|^2|g_k|^2\sigma_{n_k}^2 + \sigma_{v_k}^2}. \tag{6}$$

Recalling that $\gamma_k := |h_k|^2/\sigma_{n_k}^2$, defining $\eta_k := |g_k|^2/\sigma_{v_k}^2$, and substituting for α_k in (4) into (6), we obtain

$$\texttt{snr} = \sum_{k=1}^K \frac{\eta_k\gamma_k}{\eta_k + \frac{K(\gamma_k\sigma_\theta^2+1)}{P_{\text{tot}}}}. \tag{7}$$

The $\texttt{snr}$ in (7) is random because the instantaneous SNR on the k^{th} channel, η_k, is random. Since $g_k \sim \mathcal{CN}(0, \sigma_{g_k}^2)$, the random variable $\eta_k := |g_k|^2/\sigma_{v_k}^2$ is exponentially distributed with mean $\zeta_k := E[\eta_k] = \sigma_{g_k}^2/\sigma_{v_k}^2$.

III. Outage and Diversity

In distributed estimation of θ, the variance of the best linear unbiased estimator (BLUE) is given by $\texttt{snr}^{-1}$ [13]. In this context, it was shown in [13] that the outage, defined as $\Pr[\texttt{snr} \leq z]$, satisfies

$$\log \Pr[\texttt{snr} \leq z] \sim -K \log P_{\text{tot}} \tag{8}$$

for large K and P_{tot}. To put it another way, [13] showed that

$$\lim_{K\to\infty} \frac{1}{K} \log \Pr[\texttt{snr} \leq z]$$

behaves like $\log P_{\text{tot}}$ for large P_{tot}. We emphasize that the result in [13] is established for an asymptotically large K, for a fixed (but large) total power P_{tot}. Since (8) can be seen to indicate that the outage behaves approximately like P_{tot}^{-K}, the exponent in the total power has been interpreted in [13] as an *estimation diversity* of order K. In what follows, we motivate a definition of estimation diversity which, unlike that of [13], applies for any finite number of sensors.

Traditionally, in wireless communications, diversity analysis is performed for a fixed set of system parameters (e.g., number of antennas in MIMO systems) for asymptotically large total transmit power corresponding to large signal to noise ratios. Therefore, it is of interest to fix the number of sensors and examine the behavior of outage for asymptotically large total powers. Interestingly, in our study of diversity for a fixed K, we find that the diversity order can be smaller than K, and depends on the sensing SNRs $\{\gamma_k\}_{k=1}^K$.

To define the diversity order formally, we recall that the outage for a fixed set of sensing SNRs $\{\gamma_k\}_{k=1}^K$ is the probability that $\texttt{snr}$ falls below a threshold z

$$P_{\text{out}} := \Pr\left[\sum_{k=1}^K \frac{\eta_k\gamma_k}{\eta_k + \frac{K\left(\gamma_k\sigma_\theta^2+1\right)}{P_{\text{tot}}}} \leq z\right] \tag{9}$$

where the randomness of $\texttt{snr}$ stems from the instantaneous channel SNRs η_k, and the sensing SNRs γ_k are assumed deterministic. Another way of viewing (9) is the outage conditioned on $\{\gamma_k\}_{k=1}^K$. Since $0 \leq \texttt{snr} < \sum_{k=1}^K \gamma_k$, we are interested in

TABLE I
DEFINITION OF VARIABLES

k^{th} sensing SNR	$\gamma_k := \|h_k\|^2/\sigma^2_{n_k}$
Instantaneous k^{th} channel SNR	$\eta_k := \|g_k\|^2/\sigma^2_{v_k}$
Average k^{th} channel SNR	$\zeta_k := \sigma^2_{g_k}/\sigma^2_{v_k}$
Instantaneous SNR at the output of FC	$\mathtt{snr}$
Outage probability	P_{out}

a threshold range of $0 < z < \sum_{k=1}^{K} \gamma_k$, because when $z \le 0$, $P_{\text{out}} = 0$, and when $z \ge \sum_{k=1}^{K} \gamma_k$, $P_{\text{out}} = 1$. Examining (9), we observe that if $z \in (0, \sum_{k=1}^{K} \gamma_k)$, then $P_{\text{out}} \to 0$ as $P_{\text{tot}} \to \infty$. We now quantify how fast the outage converges to zero as a function of the threshold z and the sensing SNRs $\gamma_1, \cdots, \gamma_K$ by investigating outage diversity order defined as

$$d = \lim_{P_{\text{tot}} \to \infty} -\frac{\log P_{\text{out}}}{\log P_{\text{tot}}}. \tag{10}$$

This definition of diversity is in perfect analogy to the definition of diversity for MIMO systems (see, e.g., [18, eq. 3]) where a large total transmit power is considered for a fixed set of system parameters. Table I summarizes some parameters that recur throughout the manuscript for convenience.

In what follows, we find upper and lower bounds for the diversity order for fixed sensing SNRs $\{\gamma_k\}_{k=1}^{K}$. Without loss of generality, we assume that $0 < \gamma_1 \le \cdots \le \gamma_K$. We first begin with the upper bound as a function of $\{\gamma_k\}_{k=1}^{K}$ and z.

Theorem 1:

1) If $\sum_{k=1}^{i} \gamma_k < z$ for some $i \in \{0, \ldots, K-1\}$[1] then $d \le K - i$. Clearly, the upper bound on d is most useful if we find the largest such i.
2) If $z < \gamma_k, \forall k$ then $d = K$.

Proof: See Appendix A. ■

Theorem 1 establishes that the diversity order is K if the threshold z is chosen to be sufficiently small: $z < \gamma_k, \forall k$. However, if z is increased (or the sensing SNRs $\{\gamma_k\}_{k=1}^{K}$ are decreased) the diversity order necessarily decreases, since its upper bound is given by $K - i$. This shows that the diversity order in distributed estimation problems with amplify and forward *can* be strictly less than the number of sensors K. However, with just an upper bound, it is not clear what the range of values the diversity order can take. For this reason, we state and prove the following lower bound on d:

Theorem 2: If $\sum_{k=1}^{i} \gamma_k \le z$ for some $i \in \{0, \ldots, K-1\}$, then

$$d \ge K - i - \frac{1}{\gamma_{i+1}} \left(z - \sum_{k=1}^{i} \gamma_k \right). \tag{11}$$

Proof: See Appendix B. ■

Combining the upper bound of Theorem 1 with the lower bound of Theorem 2, the diversity order is bounded as

$$K - i - \frac{1}{\gamma_{i+1}} \left(z - \sum_{k=1}^{i} \gamma_k \right) \le d \le K - i \tag{12}$$

[1]When $i = 0$, $\sum_{k=1}^{i} \gamma_k = 0$, by definition

provided that $\sum_{k=1}^{i} \gamma_k < z$ by the assumption of Theorem 1 part 1.

We now examine the tightness of the bounds. The threshold z falls in an interval of the form $\sum_{k=1}^{i} \gamma_k < z \le \sum_{k=1}^{i+1} \gamma_k$ for some $i \in \{0, \ldots, K-1\}$. In this case, the difference between the upper and lower bounds in (12) can be at most unity because

$$\frac{1}{\gamma_{i+1}} \left(z - \sum_{k=1}^{i} \gamma_k \right) \le \frac{1}{\gamma_{i+1}} \left(\sum_{k=1}^{i+1} \gamma_k - \sum_{k=1}^{i} \gamma_k \right) = 1. \tag{13}$$

Note also that the upper and lower bounds can be arbitrarily close to zero when the threshold z is sufficiently close to $\sum_{k=1}^{i} \gamma_k$. Therefore, with the upper and lower bounds, we can determine the diversity order anywhere within a difference of at most unity, to arbitrarily closely, depending on the exact value of the threshold z, and its relationship with the sensing SNRs $\{\gamma_k\}_{k=1}^{K}$.

Let us now examine a corollary of Theorem 1 and Theorem 2 for the case of equal sensing SNRs ($\gamma_k = \gamma, \forall k$) to get simpler expressions.

Corollary 1: If the sensing SNRs are equal, ($\gamma_k = \gamma, \forall k$), then we have the following simple upper and lower bounds on d whenever z/γ is not an integer

$$K - \frac{z}{\gamma} \le d \le K - \left\lfloor \frac{z}{\gamma} \right\rfloor. \tag{14}$$

If $z < \gamma$, we have the exact diversity order $d = K$.

Proof: Since $\gamma_k = \gamma, \forall k$, from part 1 of Theorem 1 we have the statement, for any integer i satisfying $i\gamma < z$ (or equivalently, $i < z/\gamma$) we have $d \le K - i$. Hence, we need to find the largest integer i which is strictly smaller than z/γ. Clearly, this integer is given by $i = \lfloor z/\gamma \rfloor$. Therefore, $d \le K - i = K - \lfloor z/\gamma \rfloor$. For the lower bound, using (11) with $\gamma_k = \gamma, \forall k$, we obtain $d \ge K - z/\gamma$. From part 2 of Theorem 1 we know that $d = K$ whenever $z < \gamma$. This establishes the corollary. ■

Note that in the case when z/γ is an integer, the same proof can be carried out with recognizing that the integer i can be chosen as $i = z/\gamma - 1$ which is less than z/γ. Using part 1 of Theorem 1 for this choice of i we obtain $d \le K - i = K - z/\gamma + 1$. Examining the tightness of the bounds in (14), we observe that, similar to the discussion in (13), the bounds can be apart at most by one. Fig. 2 illustrates the upper and lower bounds of the diversity order as a function of z/γ. It can be seen that $z \in (0, K\gamma)$. When $z < \gamma$, the diversity order is exactly K. When z/γ is an integer, the upper bound is $d \le K - z/\gamma + 1$ as per the discussion above, and it is exactly one more than the lower bound in (14). On the other hand, when z/γ is greater than, but sufficiently close to an integer, the difference between the upper and lower bounds becomes arbitrarily close to zero. In this setting where the sensing SNRs are equal, the upper and lower bounds in (14) show that for a fixed z and γ, when a new sensor is added into the system, the bounds both increase by one. In fact, the diversity order increases like $d = \mathcal{O}(K)$ for large K. We note, however, that the growth of the diversity order with K applies when $\{\gamma_k\}_{k=1}^{K}$ are equal, and does not necessarily hold when $\{\gamma_k\}_{k=1}^{K}$ are unequal. In fact, examining the statement of Theorem 1, we see that it is possible to add new sensors with very small γ_k's such that the upper bound in Theorem 1 does

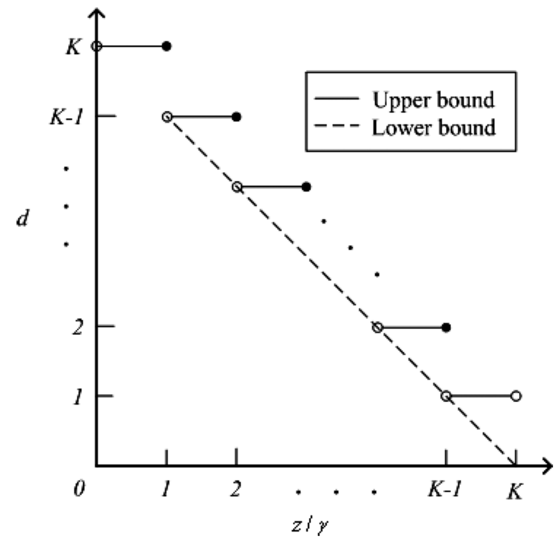

Fig. 2. Diversity order bounds when the sensing SNRs are equal.

not increase. To see this, suppose that the threshold z and set of sensing SNRs $\{\gamma_k\}_{k=1}^K$ are given. We add a new sensor whose sensing SNR is small enough to satisfy $\gamma_{\text{new}} < z - \sum_{k=1}^i \gamma_k$. This implies that we have $\sum_{k=1}^i \gamma_k + \gamma_{\text{new}} < z$. Using Theorem 1 with $i+1$ γ_k's, and $K+1$ sensors, we have $d \leq (K+1) - (i+1) = K - i$, the same diversity order as when we had K sensors. Therefore, it is possible to add new sensors into the system without getting any diversity benefit. Note that the new sensor that was introduced had to have a sensing quality (measured by γ_{new}) that was bad enough to not contribute to the diversity order. This example clearly illustrates that the diversity order depends on the sensing SNRs $\{\gamma_k\}_{k=1}^K$ and not just on the number of sensors.

The proofs of Theorems 1 and 2 which derive upper and lower bounds on the diversity order are expressed in terms of a general distribution for the instantaneous channel SNR on the k^{th} sensor η_k, and therefore can be easily extended to cases where η_k is not exponentially distributed. In the next section, we extend these bounds to cases that involve line-of-sight between the sensors and the FC.

A. Diversity with Line of Sight

So far, we have assumed that the channel g_k is zero-mean complex Gaussian implying Rayleigh fading (exponential η_k). However, in the presence of line of sight between some or all of the sensors and the FC, distributions other than the exponential might be suitable for η_k. We first begin by considering a Ricean amplitude (i.e., $\sqrt{\eta_k}$ is Ricean), which means that the density function of η_k in this case is given by

$$f_{\eta_k}(x) = \frac{(1+\kappa)}{\zeta_k} \exp(-\kappa) \exp\left(-\frac{(\kappa+1)}{\zeta_k} x\right) I_0 \times \left(2\sqrt{\frac{\kappa(\kappa+1)x}{\zeta_k}}\right) \quad (15)$$

where κ is the Ricean factor, and $\zeta_k := E[\eta_k]$. Recall that in the upper and lower bounds for the diversity order in the exponential case, we only used the fact that $f_{\eta_k}(0) \neq 0$, This also holds for the density function in (15) for any κ. Reconsidering the upper bound in (25) with (24) and the lower bound in (38), we conclude that the bounds on the diversity in the Ricean case remain the same as the Rayleigh case.

Another widely used distribution for the channel envelope $\sqrt{\eta_k}$ in the presence of line of sight is the Nakagami distribution. The corresponding density function for η_k is given by

$$f_{\eta_k}(x) = \frac{m^m x^{m-1}}{\Gamma(m)\zeta_k^m} \exp\left(-\frac{mx}{\zeta_k}\right), \quad m > 1 \quad (16)$$

where m is the Nakagami parameter, and $\zeta_k = E[\eta_k]$ as before. In this case, we now show that the bounds in (12) both scale by a factor of m.

Theorem 3: If $\sum_{k=1}^i \gamma_k \leq z \leq \sum_{k=1}^{i+1} \gamma_k$ and η_k are distributed as in (16) then

$$(K-i-1)m \leq d \leq (K-i)m. \quad (17)$$

Proof: The proof uses (25) and (38) for the upper and lower bounds, respectively. Both equations are expressed in terms of the density function of η_k and can straightforwardly applied to the density function in (16). The details are given in Appendix C. ■

Note that for the special case of $\gamma_k = \gamma, \ \forall k$, the bounds can be obtained by multiplying the upper and lower bounds in (14) by m.

B. Outage and Diversity when Sensing SNRs $\{\gamma_k\}_{k=1}^K$ are Random

In the analysis of outage defined in (9), the channel SNRs $\{\eta_k\}_{k=1}^K$ are the only source of randomness. It is also possible to consider scenarios where the sensing SNRs $\{\gamma_k\}_{k=1}^K$ are best modeled as random variables coming from a distribution. This might occur, for example, when the sensors undergo fast random motion in an underwater environment. When $\{\gamma_k\}_{k=1}^K$ are random, we will denote the outage by $\overline{P}_{\text{out}}$ defined as in (9) except the probability is calculated over the random variables $\{\eta_k\}_{k=1}^K$ *and* $\{\gamma_k\}_{k=1}^K$. We now show that the unconditional outage does not necessarily go to zero as $P_{\text{tot}} \to \infty$. To see why, consider removing the term $K(\gamma_k \sigma_\theta^2 + 1)/P_{\text{tot}}$ from the denominator in (9), which would make the sum larger, and the resulting probability smaller. Therefore, we have the following bound:

$$\overline{P}_{\text{out}} \geq \Pr\left[\sum_{k=1}^K \gamma_k \leq z\right] \quad (18)$$

which does not depend on P_{tot}. Therefore $\overline{P}_{\text{out}}$ is bounded away from zero when $\Pr[\sum_{k=1}^K \gamma_k \leq z]$ is positive, which holds for most probability distributions on γ_k. For example, when h_k in (1) is complex Gaussian, γ_k is exponential so that for any $z > 0$, $\Pr[\sum_{k=1}^K \gamma_k \leq z] > 0$. This implies that no matter how large the total power P_{tot} is, the outage calculated over the probability measures of η_k and γ_k is bounded away from zero. We define the diversity order in this case the same way as (10) except with $\overline{P}_{\text{out}}$. This diversity order is zero since $\overline{P}_{\text{out}}$ does not go to zero as $P_{\text{tot}} \to \infty$.

Clearly it is not necessary for γ_k to be exponential for this conclusion to hold. Any other choice for which $\sum_{k=1}^K \gamma_k$ can be smaller than z with positive probability would yield a diversity order of zero.

IV. Outage for Large K for Fixed Power Per-Sensor

We mentioned in Section III that in [13] the outage probability for large K but for a fixed $P_{\rm tot}$ was studied. In Section III we considered an approach that is more common in wireless communications when studying diversity, where K was fixed to any value, which highlighted the effect of γ_k on the diversity.

A third option of practical relevance is when the transmission power of each sensor is fixed to a certain value. In this context, an important question is how the outage performance scales with the number of sensors. Our analysis so far has considered large $P_{\rm tot}$ for K fixed. In what follows, we allow both $P_{\rm tot} \to \infty$ and $K \to \infty$ with a fixed per-sensor power of $P_{\rm tot}/K = P_0$. This is a natural scenario where each sensor that is deployed has a fixed individual power, and increasingly larger number of sensors are introduced. For simplicity, we assume $\gamma_k = \gamma, \forall k$ and $\zeta_k = E[\eta_k] = \zeta, \forall k$, so that $f_{\eta_k}(x) = \zeta^{-1}\exp(-x/\zeta)$. Since $P_{\rm tot}/K$ is fixed, the outage in (9) is given by

$$P_{\rm out} = \Pr\left[\sum_{k=1}^{K} \frac{\eta_k \gamma}{\eta_k + c} \leq z\right] \tag{19}$$

where $c := (\gamma\sigma_\theta^2 + 1)/P_0$. Recall that in Section III, where the number of sensors was fixed, the value of z could determine whether the outage in (9) converged to zero. In sharp contrast, if the number of sensors is allowed to increase with the total power, the sum in the outage expression in (19) grows without bound, and therefore the outage probability converges to zero regardless of the value of z. This hints that the behavior of the outage is markedly different in this scenario. In fact, we soon show that the outage behaves approximately like $\exp(-K\log K)$ in this regime. We have the following theorem.

Theorem 4: For any $\gamma > 0$ and $z > 0$, if $P_{\rm tot} = KP_0$, then

$$\lim_{K\to\infty} -\frac{\log P_{\rm out}}{K} = \infty. \tag{20}$$

Proof: See Appendix D. ■

Intuitively, Theorem 4 maintains that $P_{\rm out}$ goes to zero faster than $\exp(-C_1 K)$ for any fixed constant $C_1 > 0$. That is, the exponent of the outage grows faster than any linear function of K. In the next theorem, we show that the exponential rate cannot be faster than $K\log K$.

Theorem 5: For any $\gamma > 0$ and $z > 0$, if $P_{\rm tot} = KP_0$, then

1)

$$\lim_{K\to\infty} -\frac{\log P_{\rm out}}{K\log K} \leq 1 \tag{21}$$

2) and if $z < (\gamma P_0)/(\gamma\sigma_\theta^2 + 1)$, then

$$\lim_{K\to\infty} -\frac{\log P_{\rm out}}{K\log K} \geq 1 - \frac{\gamma\sigma_\theta^2 + 1}{P_0\gamma} z. \tag{22}$$

Proof: Please see Appendix E. ■

Part 2 of Theorem 5 shows that $P_{\rm out} < \exp[-(1 - (\gamma\sigma_\theta^2 + 1/P_0\gamma)z)K\log K]$, for sufficiently large K. This is useful only when $z < (\gamma P_0)/(\gamma\sigma_\theta^2 + 1)$, which guarantees that the right–hand side (RHS) of (22) is positive. This can be fulfilled if the per-sensor power P_0 is chosen sufficiently large.

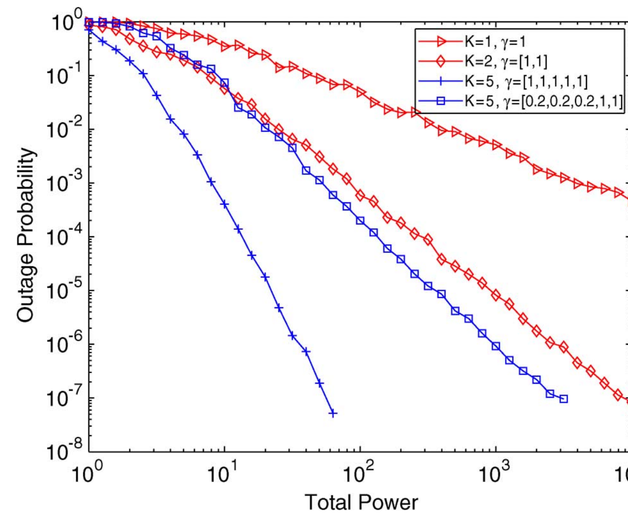

Fig. 3. Outage probability versus total power for different number of sensors ($z = 0.8$).

Combining the two results of Theorem 5, we can roughly state that $P_{\rm out} \sim \exp(-C_2 K\log K)$ where the constant satisfies $1 - z(\gamma\sigma_\theta^2 + 1)/(P_0\gamma) \leq C_2 \leq 1$. If the lower bound on C_2 is negative, which might happen when P_0 is not large enough, then we cannot guarantee $P_{\rm out} \sim \exp(-C_2 K\log K)$, for a positive C_2. However, we are still assured by Theorem 4 that $P_{\rm out} < \exp(-C_1 K)$ for any constant C_1 if K is sufficiently large.

Note that the results in Theorem 4 and Part 1 of Theorem 5 do not depend on γ. In other words, the outage probability for a fixed $\gamma > 0$ goes to zero at least as fast as exponentially in K, and not faster than $\exp(-K\log K)$, independently of the value of γ.

V. Simulations

In this section, we provide simulation results to verify and illustrate our findings in previous sections. We assume that the variance of the source parameter $\sigma_\theta^2 = 0.1$ and the instantaneous channel SNRs $\{\eta_k\}_{k=1}^K$ are i.i.d. exponential random variables with unit mean. The numerical results herein are obtained by generating over 10^9 runs, which is necessary since $P_{\rm out}$ is exceedingly small even for moderate values of K and $P_{\rm tot}$.

We first verify the outage and diversity for the case of fixed sensing SNRs $\{\gamma_k\}_{k=1}^K$, as in Theorems 1 and 2. Fig. 3 shows the outage probabilities as a function of the total power for four cases where the number of sensors K equals to 1, 2, 5, 5, respectively. The outage threshold z is set to 0.8. The first three cases are assumed to consist of all "good-observation" sensors which have the same large sensing SNR, i.e., $\gamma_k = 1 > z, \forall k$. Whereas, the fourth case has the same number of sensors as the third case ($K = 5$) but three of them are "bad-observation" sensors with small $\gamma_k = 0.2 < z$, $k \in \{1,2,3\}$. Similar to diversity in classic communication systems, we can see that a diversity gain is achieved as the number of sensors increases and the power saving is most substantial when going from no diversity to two-sensor diversity with diminishing returns as the

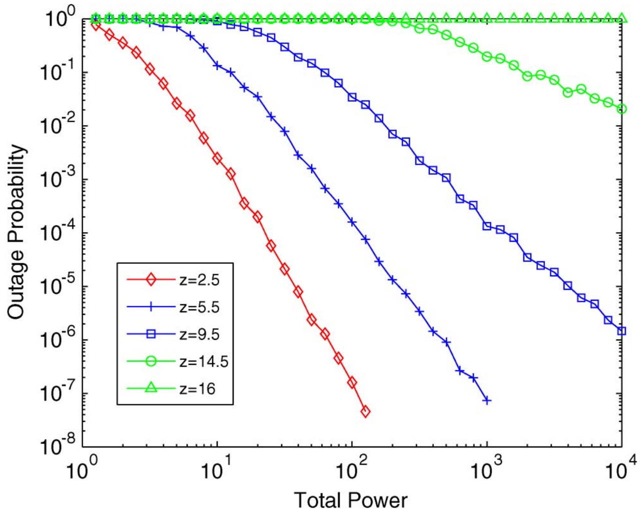

Fig. 4. Outage probability versus total power for a set of fixed and unequal sensing SNRs $\{\gamma_k\} = \{1, 2, 3, 4, 5\}$.

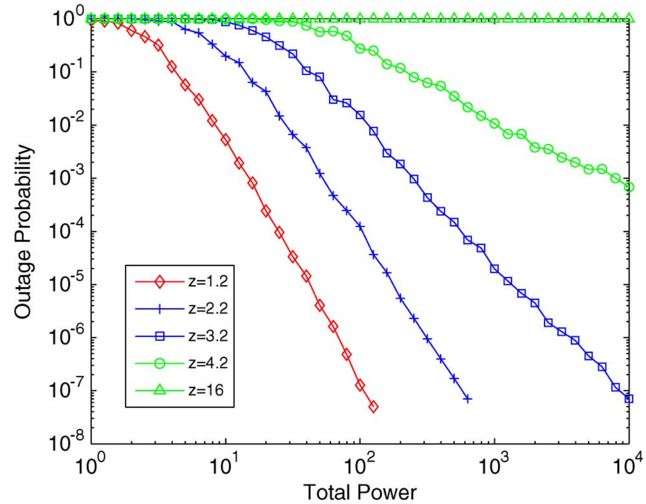

Fig. 5. Outage probability versus total power for fixed and equal sensing SNRs $\gamma_k = 1, \forall k$.

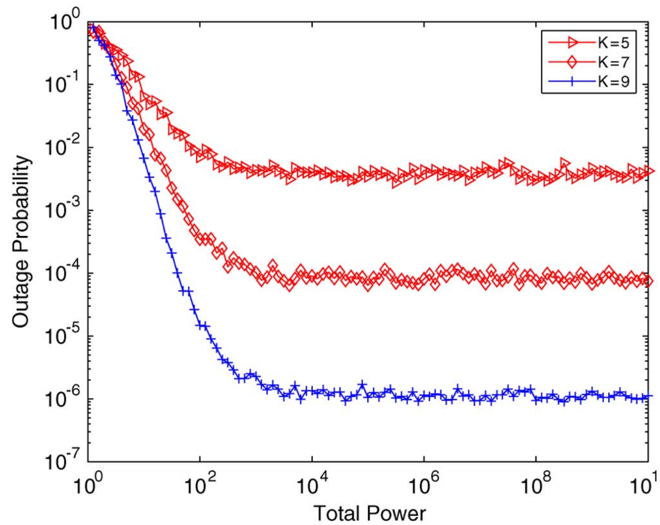

Fig. 6. Outage probability versus total power for random sensing SNRs (γ_k is i.i.d. exponential distributed with mean 1 and $z = 1$).

number of sensors increases. It is well known that the diversity order is equal to the number of independent channel paths in classic communication systems. Interestingly, this is not always true for distributed estimation systems. As seen in Fig. 3, the curve of the fourth case has a smaller slope than that of the third case although both have the same number of sensors, i.e., the same number of independent paths. Instead, the curve of the fourth case has a similar slope as that of the second case where $K = 2$. Clearly, the diversity order in distributed estimation systems does not only depend on the number of sensors. We continue with several case studies that further verify that the diversity order is given by the theorems we provided in previous sections.

We now consider a case where there are five sensors with different sensing SNRs: $\gamma_1 = 1$, $\gamma_2 = 2$, $\gamma_3 = 3$, $\gamma_4 = 4$, and $\gamma_5 = 5$. We simulate the outage probabilities as a function of the total power for different thresholds where $z \in \{2.5, 5.5, 9.5, 14.5, 16\}$. From Fig. 4 where the outage probability is plotted versus P_{tot}, we can see that the diversity order as seen from slopes decreases as the threshold z increases. When $z = 16$, the outage probability is always 1 since $z > \sum_{k=1}^{5} \gamma_k = 15$.

Fig. 5 shows the outage probability and the diversity order for the case of fixed and equal sensing SNRs where $\gamma_k = 1, \forall k$, with $z \in \{1.2, 2.2, 3.2, 4.2, 5.2\}$. With the number of sensors $K = 5$, the theoretical diversity order is given by (14). Again, we observe that our simulation results match with the theoretical results: as z increases, the diversity order decreases. More importantly, all the aforementioned figures show that, given the sensing SNRs, the diversity order of the outage probability depends on not only the number of active sensors K in the system but also the comparative values of the outage threshold z and the sensing SNRs $\{\gamma_k\}_{k=1}^{K}$.

We also study the case where the sensing SNRs are i.i.d. exponential random variables with unit mean. The outage probability is shown in Fig. 6, when the threshold is given as $z = 1$. In Fig. 6, it is seen that the outage probability is bounded away from zero for all cases of $K = 5, 7, 9$ no matter how large the total power is. Indeed, the outage probability converges to some nonzero value as $P_{\text{tot}} \to \infty$, which indicates the diversity order (measured by the slope) is always zero as shown in Fig. 6. Note that a zero diversity order does not necessarily mean that the outage probability does not improve with increasing K. Indeed, as shown in Fig. 6, the outage probability improves by several orders of magnitude as the number of sensors increases from 5 to 9.

We have illustrated and verified the outage diversity order when the number of sensors K is fixed and the total power P_{tot} is large. In the following, we consider the case where the power per sensor P_0 is fixed and show how the outage probability behaves as K increases. We assume that $P_0 = 10$ and $\gamma_k = \gamma$ is fixed and equal. Fig. 7 shows the values of $-\log P_{\text{out}}/K$ as a function of K for several cases with different choices of z and γ. From Fig. 7, we see that all curves continue to grow as K increases, which verifies Theorem 4. Fig. 8 plots $-\log P_{\text{out}}/(K \log K)$ versus K when $z = 4$ and $\gamma = 1$. The upper bound and the lower bound are given by the RHS of (21)

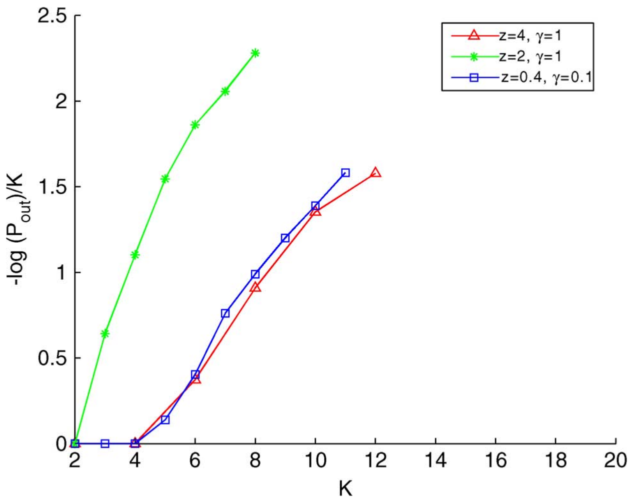

Fig. 7. Illustration of Theorem 4 ($\zeta = 1$ and $P_0 = 10$).

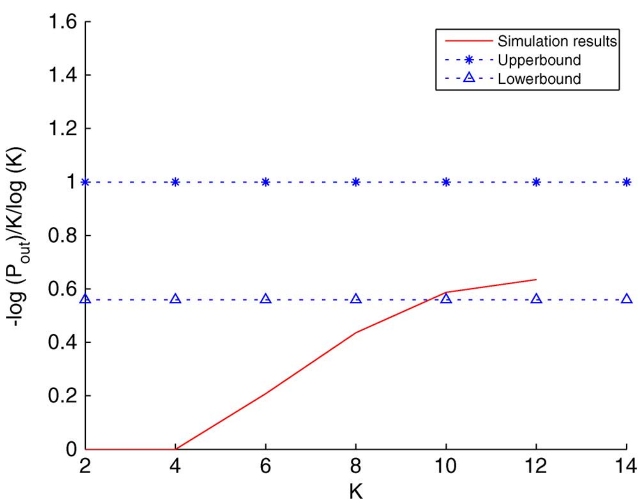

Fig. 8. Illustration of Theorem 5 ($z = 4, \gamma = 1, \zeta = 1$, and $P_0 = 10$).

and (22), respectively. As expected, the simulation results fall between the upper bound and the lower bound as K increases. This verifies that $K \log K$ is an asymptotical tight bound for $\log P_{\rm out}$ if $z < (\gamma P_0)/(\gamma\sigma_\theta^2 + 1)$.

VI. Conclusions and Future Work

In this paper, we considered the performance of distributed estimation algorithms, as quantified by the outage under different asymptotic regimes. We found upper and lower bounds on the diversity order, which are shown to coincide under certain conditions. Using these bounds, we showed that the estimation diversity order for a fixed K is not always given by K, and depends on the sensing SNRs, and the threshold used to define the outage. We observed that if the sensing SNR γ_k of a sensor is too small, that sensor does not contribute to the diversity order. We also showed generalizations to channel distributions such as Rice and Nakagami. In fact, any other channel distributions can be similarly treated. We extended our analysis to the case of random sensing SNRs and found that the diversity order in this case is zero for a wide range of sensing SNR distributions. This zero-diversity phenomenon occurs when the sum of the sensing SNRs can be arbitrarily small with positive probability. Since systems with such random sensing SNRs cannot be compared with using the diversity order, future research is needed for this scenario. Other areas of future work include, generalization of these results to the vector parameter case, maximum likelihood estimation of nonlinear observation models.

We also studied the natural scenario where the per-sensor power is fixed, with increasing number of sensors. In this regime, the outage is shown to go to zero more rapidly than exponentially in the number of sensors, and slower than $\exp(-K \log K)$.

Appendix A
Proof of Theorem 1

Proof: We begin with part 1. Let $Z_k := \eta_k\gamma_k/\eta_k + ac_k$ be the k^{th} term of the sum in (9), where $a := P_{\rm tot}^{-1}$ and $c_k := K(\gamma_k\sigma_\theta^2 + 1)$ for simplicity of notation. Recall that γ_k and c_k are deterministic, and η_k is exponentially distributed. Clearly $0 \le Z_k < \gamma_k$ with probability one.

Let $z_k, k = 1, \ldots, K$ be arbitrary strictly positive numbers that satisfy $\sum_{k=1}^K z_k = z$. We have

$$\begin{aligned} P_{\rm out} &= \Pr\left[\sum_{k=1}^K Z_k \le \sum_{k=1}^K z_k\right] \\ &\ge \Pr[Z_1 \le z_1, \ldots, Z_K \le z_K] \\ &= \prod_{k=1}^K \Pr[Z_k \le z_k] \end{aligned} \tag{23}$$

where in the inequality we used $Z_k \le z_k, \forall k \Rightarrow \sum_{k=1}^K Z_k \le \sum_{k=1}^K z_k$ and the second equality is due to independence. It is straightforward to verify that

$$\Pr[Z_k \le z_k] = F_{Z_k}(z_k) = F_{\eta_k}\left(\frac{az_kc_k}{\gamma_k - z_k}\right), \quad \gamma_k > z_k \tag{24}$$

where $F_{Z_k}(\cdot)$ and $F_{\eta_k}(\cdot)$ are the cumulative distribution functions of Z_k and η_k, respectively.

Substituting (23) in the definition of diversity in (10) we obtain

$$d \le \sum_{k=1}^K \lim_{P_{\rm tot}\to\infty} -\frac{\log\left(\Pr[Z_k \le z_k]\right)}{\log P_{\rm tot}} \tag{25}$$

$$= \sum_{k=1}^K \mathbf{I}(\gamma_k > z_k) \tag{26}$$

where (26) uses the indicator function to assert that the k^{th} term in the sum in (25) is one when $\gamma_k > z_k$, and zero otherwise. Too prove (26), consider first the case when $\gamma_k \le z_k$. Since $Z_k < \gamma_k$ almost surely, we have $Z_k < \gamma_k \le z_k$, and therefore $\Pr[Z_k \le z_k] = 1$, which makes the k^{th} term in (25) zero. Suppose now that $\gamma_k > z_k$ so that we may use (24) in (25), recall that $a = P_{\rm tot}^{-1}$, and use L'Hôspital's rule to obtain

$$\lim_{a\to 0} \frac{a}{F_{\eta_k}\left(\frac{az_kc_k}{\gamma_k - z_k}\right)} f_{\eta_k}\left(\frac{az_kc_k}{\gamma_k - z_k}\right)\left(\frac{z_kc_k}{\gamma_k - z_k}\right) \tag{27}$$

for the k^{th} term in the sum in (25) when $\gamma_k > z_k$. Using the fact that for the exponential distribution $f_{\eta_k}(0) \ne 0$, L'Hôspital's rule only needs to be used on the $a/F_{\eta_k}(az_kc_k/(\gamma_k - z_k))$

term in (27), and reveals that the k^{th} term in (25) is one when $\gamma_k > z_k$.

Since $\{z_k\}_{k=1}^K$ are free variables with the only requirement that they be positive and add up to z, we obtain the desired bound in Theorem 1 with the following choice: $z_k = \gamma_k, k = 1, \ldots, i$; arbitrary $z_k > 0$ for $k = i+1, \ldots, K$. Since $\sum_{k=1}^{i} \gamma_k < z$ by assumption, such positive z_k for $k = i+1, \ldots, K$ can be found. In (26) it is clear that the first i terms contribute zero, and the last $K - i$ terms each contribute to at most one. Therefore (26) is less than or equal to $K - i$ which is what we wanted to show for part 1.

To prove part 2, we begin by recalling that $d \leq K$ due to part 1. To show $d \geq K$, note that

$$\left\{Z_1, \ldots, Z_K : \sum_{k=1}^{K} Z_k \leq z\right\} \subset \{Z_1, \ldots, Z_K : Z_k \leq z\} \tag{28}$$

because $Z_k \geq 0$. Therefore, the probabilities of the events in (28) are related as

$$P_{\text{out}} \leq \Pr[Z_1 \leq z, \ldots, Z_K \leq z] = \prod_{k=1}^{K} \Pr[Z_k \leq z]. \tag{29}$$

Using (24) and taking the logarithms of both sides, (29) can be written as

$$\log P_{\text{out}} \leq \sum_{k=1}^{K} \log F_{\eta_k}\left(\frac{azc_k}{\gamma_k - z}\right) \tag{30}$$

where, we used $\gamma_k > z$ to write $\Pr[Z_k \leq z]$ in terms of $F_{\eta_k}(\cdot)$. Dividing through by $-\log P_{\text{tot}} = \log a$ and taking the limit as $P_{\text{tot}} \to \infty$ $(a \to 0)$ we obtain

$$d = \lim_{P_{\text{tot}} \to \infty} -\frac{\log P_{\text{out}}}{\log P_{\text{tot}}} \geq \sum_{k=1}^{K} \lim_{a \to 0} \frac{\log F_{\eta_k}\left(\frac{azc_k}{\gamma_k - z}\right)}{\log a}. \tag{31}$$

Using (27), it is straightforward that each limit on the RHS of (31) is given by 1 using L'Hôspital's rule, which proves that $d \geq K$, and completes the proof. ■

Appendix B
Proof of Theorem 2

Proof: Using the Chernoff bound on the outage in (9) we obtain

$$\begin{aligned} P_{\text{out}} &= \Pr\left[\sum_{k=1}^{K} \frac{\eta_k \gamma_k}{\eta_k + ac_k} \leq z\right] \\ &\leq \exp(\nu(a)z) \prod_{k=1}^{K} E\left[\exp\left(-\nu(a)\frac{\eta_k \gamma_k}{\eta_k + ac_k}\right)\right] \end{aligned} \tag{32}$$

where the expectation is with respect to η_k, and $\nu(a) > 0$ is an arbitrary but positive function of $a := P_{\text{tot}}^{-1}$, which we choose as $\nu(a) = -\beta \log a > 0$, for some constant $\beta > 0$, to be later specified, and for $a < 1$. Substituting $\nu(a)$ in (32), taking the logarithms of both sides, and expressing the expectation as an integral, we obtain

$$\log P_{\text{out}} \leq -z\beta \log a + \sum_{k=1}^{K} \log\left[\int_0^\infty f_{\eta_k}(x) a^{\frac{x\beta\gamma_k}{x+ac_k}} dx\right]. \tag{33}$$

Breaking up the integral in the k^{th} term of the sum for some function $g(a) > 0$, we have

$$\begin{aligned} \int_0^{g(a)} f_{\eta_k}(x) a^{\frac{x\beta\gamma_k}{x+ac_k}} dx + \int_{g(a)}^\infty f_{\eta_k}(x) a^{\frac{x\beta\gamma_k}{x+ac_k}} dx \\ \leq \int_0^{g(a)} f_{\eta_k}(x) dx + a^{\frac{g(a)\beta\gamma_k}{g(a)+ac_k}} \end{aligned} \tag{34}$$

where we obtained upper bounds on both terms on the left-hand side (LHS) of (34) by substituting the lower limits of both integrals for x in the exponent of a because $a^{x\beta\gamma_k/x+ac_k}$ is a monotonically decreasing function of x, and also used $\int_{g(a)}^\infty f_{\eta_k}(x)dx \leq 1$. Substituting $g(a) = a^{1-\delta}$, for some $0 < \delta < 1$, the exponent of the second term on the RHS of (34) can be written as

$$\frac{g(a)\beta\gamma_k}{g(a) + ac_k} = \beta\gamma_k - \frac{a^\delta \beta\gamma_k c_k}{1 + a^\delta c_k}. \tag{35}$$

Since the second term on the RHS of (35) is small for small a, $a^{g(a)\beta\gamma_k/g(a)+ac_k} \approx a^{\beta\gamma_k}$. More rigorously, for any $\epsilon > 0$, and any $0 < \delta < 1$, $a^{g(a)\beta\gamma_k/g(a)+ac_k} \leq a^{\beta\gamma_k}(1+\epsilon)$ for a sufficiently small. Using this result with (34), the k^{th} term in (33) can be bounded for a sufficiently small as

$$\log\left[\int_0^\infty f_{\eta_k}(x) a^{\frac{x\beta\gamma_k}{x+ac_k}} dx\right] \leq \log\left[\int_0^{a^{1-\delta}} f_{\eta_k}(x)dx + a^{\beta\gamma_k}(1+\epsilon)\right]. \tag{36}$$

Recalling the definition of the diversity order (10), and substituting (36) into (33), we have

$$d \geq -z\beta + \sum_{k=1}^{K} \lim_{a \to 0} \frac{\log\left[\int_0^{a^{1-\delta}} f_{\eta_k}(x) + a^{\beta\gamma_k}(1+\epsilon)\right]}{\log a}. \tag{37}$$

Using the L'Hospital's rule twice, we obtain (38), shown at bottom of the page. Substituting $f_{\eta_k}(a) = \zeta_k^{-1}\exp(-a/\zeta_k)$, we observe that $a^{1-\delta} f'_{\eta_k}(a^{1-\delta}) \to 0$ as $a \to 0$ and that $f_{\eta_k}(0) \neq 0$.

$$d \geq -z\beta + \sum_{k=1}^{K} \lim_{a \to 0} \frac{(1-\delta)^2 a^{1-\delta} f'_{\eta_k}(a^{1-\delta}) + (1-\delta)^2 f_{\eta_k}(a^{1-\delta}) + (\beta\gamma_k)^2 a^{\beta\gamma_k - 1 + \delta}(1+\epsilon)}{(1-\delta) f_{\eta_k}(a^{1-\delta}) + \beta\gamma_k a^{\beta\gamma_k - 1 + \delta}(1+\epsilon)}. \tag{38}$$

Examining (38), it is clear that the limit depends on whether $\beta\gamma_k > 1-\delta$ or not. Working out this limit, we have

$$d \geq -z\beta + \sum_{k=1}^{K} T_{1-\delta}(\beta\gamma_k) \tag{39}$$

where $T_{1-\delta}(\beta\gamma_k)$ is the limit in the k^{th} term in (38), and $T_y(x) = y$ if $x \geq y$, and $T_y(x) = x$ if $x \leq y$. Since the lower bound in (39) is most useful when it is larger, using the continuity of $T_y(x)$ with respect to y, we can take the supremum of the RHS of (39) over $0 < \delta < 1$ which is obtained as $\delta \to 0$, yielding

$$d \geq -z\beta + \sum_{k=1}^{K} T_1(\beta\gamma_k). \tag{40}$$

We can select any positive β for the Chernoff bound, which we choose as $\beta = 1/\gamma_{i+1}$ and substitute in (40) to obtain

$$\begin{aligned} d \geq & -\frac{z}{\gamma_{i+1}} + \sum_{k=1}^{i} T_1\left(\frac{\gamma_k}{\gamma_{i+1}}\right) + \sum_{k=i+1}^{K} T_1\left(\frac{\gamma_k}{\gamma_{i+1}}\right) \\ = & (K-i) - \frac{1}{\gamma_{i+1}}\left(z - \sum_{k=1}^{i}\gamma_k\right) \end{aligned} \tag{41}$$

where to get the RHS, we used $0 < \gamma_1 \leq \gamma_2 \leq \ldots \gamma_K$, and the definition of $T_1(\cdot)$. This is completes the proof. ■

Appendix C
Proof of Theorem 3

Proof: For the upper bound, (25) can be calculated for the density in (16), by evaluating the limit in (27). Unlike the exponentially distributed η_k case, $f_{\eta_k}(0) = 0$ in for the Nakagami distribution. However, by scaling (27) we obtain

$$\left[\lim_{a\to 0} \frac{a^m}{F_{\eta_k}\left(\frac{az_kc_k}{\gamma_k - z_k}\right)}\right]\left[\lim_{a\to 0} \frac{f_{\eta_k}\left(\frac{az_kc_k}{\gamma_k - z_k}\right)}{a^{m-1}}\right]\left(\frac{z_kc_k}{\gamma_k - z_k}\right). \tag{42}$$

The first limit in (42) can be related to the second limit by using L'Hospital's rule, and the second limit can be computed using $f_{\eta_k}(x) = x^{m-1} + o(x)$ as $x \to 0$ [19] for the Nakagami distribution to show that the bound is scaled by a factor of m.

The lower bound in (41) also can be shown to scale by a factor of m as follows. Unlike when η_k is exponential, $f_{\eta_k}(0) = 0$ in the Nakagami case, which yields an undefined ratio when $a = 0$ is substituted in (38). Since x corresponds to $a^{(1-\delta)}$ in (38), we are motivated to multiply the numerator and denominator with $a^{-(1-\delta)(m-1)}$ and then take the limit as $a \to 0$. Carrying out this calculation, we obtain

$$d \geq -z + \sum_{k=1}^{K} T_{m(1-\delta)}(\gamma_k) \tag{43}$$

where $T_{m(1-\delta)}(x) = m(1-\delta)$ if $x > m(1-\delta)$, and $T_{m(1-\delta)}(x) = x$ if $x < m(1-\delta)$. The remainder of the proof follows along the same lines as the exponential case (Appendix B), except in tightening the bound, the choice of ν is given by $\nu(a) = m\log a\gamma_{i+1}$. Combining with the lower bound we obtain

$$(K-i)m - \frac{m}{\gamma_{i+1}}\left(z - \sum_{k=1}^{i}\gamma_k\right) \leq d \leq (K-i)m. \tag{44}$$

■

Appendix D
Proof of Theorem 4

Proof: We start with deriving a general expression that is useful for both Theorem 4 and part 2 of Theorem 5. Taking the logarithm of the Chernoff bound of P_{out} in (19) we obtain

$$\log P_{\text{out}} \leq \nu z + K\log\left[\int_0^\infty f_{\eta_k}(x)\exp\left(-\frac{\nu\gamma x}{x+c}\right)dx\right] \tag{45}$$

where $c = (\gamma\sigma_\theta^2 + 1)/P_0$, and ν is any positive function of K which satisfies $\nu \to \infty$ as $K \to \infty$. Dividing through with ν, and splitting the integral into two pieces for an arbitrary $g := g(K) > 0$ we obtain

$$\begin{aligned} \frac{\log P_{\text{out}}}{\nu} \leq z + \frac{K}{\nu}\log\Bigg[&\int_0^g f_{\eta_k}(x)\exp\left(-\frac{\nu\gamma x}{x+c}\right)dx \\ &+ \int_g^\infty f_{\eta_k}(x)\exp\left(-\frac{\nu\gamma x}{x+c}\right)dx\Bigg]. \end{aligned} \tag{46}$$

Equation (46) can be further upper bounded if the lower limits of the integrals are substituted for x in the argument of the exponentials

$$\frac{\log P_{\text{out}}}{\nu} \leq z + \frac{K}{\nu}\log\left[\int_0^g f_{\eta_k}(x)dx + \exp\left(-\frac{\nu g\gamma}{g+c}\right)\right] \tag{47}$$

where we also used $\int_g^\infty f_{\eta_k}(x)dx \leq 1$. Recalling that both ν and g can be chosen as arbitrary positive functions of K, we focus on a choice that ensures that $g \to 0$, $\nu g \to \infty$, and $\nu g^2 \to 0$, as $K \to \infty$. Rewriting the exponential in (47) we get $\exp(-(\nu g\gamma/g+c)) = \exp(-(\nu g\gamma/c))\exp(\nu g^2\gamma/c(g+c))$. Since $\nu g^2 \to 0$ and $g \to 0$, the second term can be made arbitrarily close to 1 if K is sufficiently large. Therefore, for any $\epsilon > 0$, $\exp(-(\nu g\gamma/g+c)) \leq \exp(-(\nu g\gamma/c))(1+\epsilon)$ for sufficiently large K. Substituting into (47) we have a bound which is useful to prove both Theorem 4 and part 2 of Theorem 5

$$\frac{\log P_{\text{out}}}{\nu} \leq z + \frac{K}{\nu}\log\left[\int_0^g f_{\eta_k}(x)dx + \exp\left(-\frac{\nu g\gamma}{c}\right)(1+\epsilon)\right]. \tag{48}$$

For Theorem 4, we choose $\nu = K$ and $g = K^{-\delta}$ for $(1/2) < \delta < 1$. Substituting this choice, and taking the limit as $K \to \infty$, the RHS goes to $-\infty$ which implies that $-\log P_{\text{tot}}/K \to \infty$. This establishes the Theorem. ■

APPENDIX E
PROOF OF THEOREM 5

Proof: We begin with a lower bound on the outage in (19). Since $\eta_k/(\eta_k+c) \le \eta_k/c$,

$$P_{\rm out} = P\left[\sum_{k=1}^{K} \frac{\eta_k \gamma}{\eta_k + c} \le z\right] \ge P\left[\sum_{k=1}^{K} \eta_k \le \frac{zc}{\gamma}\right] = \frac{1}{\zeta^K \Gamma(K)} \int_0^{\frac{zc}{\gamma}} x^{K-1} e^{-x/\zeta} dx \tag{49}$$

where the RHS follows since $\sum_{k=1}^{K} \eta_k$ is a χ^2 random variable with $2K$ degrees of freedom. We further lower bound (49)

$$\frac{1}{\zeta^K \Gamma(K)} \int_0^{\frac{zc}{\gamma}} x^{K-1} e^{-x/\zeta} dx \ge \frac{1}{\zeta^K \Gamma(K)} e^{-\frac{zc}{\gamma\zeta}} \int_0^{\frac{zc}{\gamma}} x^{K-1} = \frac{1}{\zeta^K \Gamma(K)} e^{-\frac{zc}{\gamma\zeta}} \frac{1}{K} \left(\frac{zc}{\gamma}\right)^K \tag{50}$$

so that the RHS of (50) lower bounds $P_{\rm out}$. Using this, taking logarithms of both sides, and normalizing with $-K \log K$ we have

$$-\frac{\log P_{\rm out}}{K \log K} \le \frac{\log \Gamma(K)}{K \log K} + \frac{K \log \zeta}{K \log K} - \frac{-\log K - \frac{zc}{\gamma\zeta} + K \log \frac{zc}{\gamma}}{K \log K}. \tag{51}$$

Due to Stirling's formula, we can express the $\Gamma(\cdot)$ as

$$\Gamma(x) = \sqrt{\frac{2\pi}{x}} \left(\frac{x}{e}\right)^x \left(1 + \mathcal{O}\left(\frac{1}{x}\right)\right). \tag{52}$$

Taking the limit as $K \to \infty$, the first term on the RHS of the inequality in (51) converges to 1 due to (52). The second and third terms in (51) clearly go to zero. This completes the proof of (21).

For part 2, we choose $\nu = \beta K \log K$ and $g = K^{-1}$ in (48), where $\beta > 0$ is a constant that will be specified. With this choice, (48) becomes

$$\frac{\log P_{\rm out}}{K \log K} \le \beta z + \frac{1}{\log K} \log\left[\int_0^{K^{-1}} f_{\eta_k}(x) dx + K^{-\frac{\beta\gamma}{c}}(1+\epsilon)\right]. \tag{53}$$

Recalling $f_{\eta_k}(x) = \zeta^{-1} \exp(-x/\zeta)$, taking the limit as $K \to \infty$, and using L'Hospital's rule twice with $\beta > (c/\gamma)$ we obtain

$$\lim_{K\to\infty} -\frac{\log P_{\rm out}}{K \log K} \ge -\beta z + \lim_{K\to\infty} \frac{1 + \left(\frac{\beta\gamma}{c}\right)^2 K^{1-\frac{\beta\gamma}{c}}(1+\epsilon)}{1 + \left(\frac{\beta\gamma}{c}\right) K^{1-\frac{\beta\gamma}{c}}(1+\epsilon)} = -\beta z + 1. \tag{54}$$

Since $\beta > c/\gamma$, $-\beta z + 1$ can be made arbitrarily close to $-(c/\gamma)z + 1$. Since we would like this exponent to be positive, (22) is useful when $z < \gamma/c = (\gamma P_0)/(\gamma\sigma_\theta^2 + 1)$. This establishes part 2 and the Theorem. ∎

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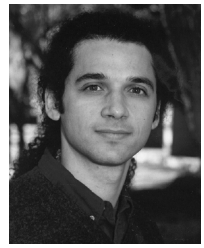

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