Joint Data Detection and Channel Estimation for OFDM Systems in the Presence of Very High Mobility

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***Abstract*— This paper is concerned with the challenging and timely problem of joint channel estimation and data detection for orthogonal frequency division multiplexing (OFDM) systems in the presence of frequency selective and very rapidly time varying channels. The detection and estimation algorithm is based on the space alternating generalized expectation maximization (SAGE) technique which is particularly well-suited to multicarrier signal formats. In order to reduce the computational complexity of the algorithm, we apply the cosine orthogonal basis functions to describe the time-varying channel. It is shown that, depending on the normalized Doppler frequency, only a small number of expansion coefficients is sufficient to approximate the channel perfectly and there is no need to know the correlation function of the input signal. The proposed SAGE joint detection algorithm updates the data sequences in serial and the channel parameters are updated in parallel, leading to a receiver structure that also incorporates a partial interference cancelation of the interchannel interference. Computer simulations show that the cosine transformation represents the time-varying channel very effectively and the proposed algorithm has excellent symbol error rate and channel estimation performance even with a very small number of channel expansion coefficients employed in the algorithm, resulting in reduction of the computational complexity substantially.**

Index Terms: Joint data detection and channel estimation, OFDM systems, SAGE algorithm, high mobility channels

I. Introduction

Orthogonal frequency-division multiplexing (OFDM) with a cyclic prefix (CP) has been shown to be an effective method to overcome inter-symbol interference (ISI) effects due to frequency-selective fading with a simple transceiver structure. Consequently, it is becoming a key air interface of next-generation wireless communications systems such as the IEEE 802.11 and IEEE 802.16 families. IEEE 802.11 wireless local area networks (WLANs) have become very popular for providing data services to Internet users while its overall design and feature set are not well suited for outdoor broadband wireless access (BWA) applications. Therefore, IEEE 802.16m is being developed as a new standard for BWA applications [1] introducing high-mobility features to enable mobile broadband services at vehicular speeds beyond 120 km/h. OFDM eliminates ISI and simply uses a one-tap equalizer to compensate for multiplicative channel distortion in quasi-static channels. However, in fading channels with very high mobility, the time variation of the channel over an OFDM symbol period results in a loss of subchannel orthogonality which leads to inter-carrier interference (ICI). A considerable amount of research on OFDM receivers for quasi-static fading has been conducted, but a major hindrance to such receivers is the lack of mobility support [2]. Since mobility support is widely considered to be one of the key features in wireless communication systems, and in this case ICI degrades the performance of OFDM systems, OFDM transmission over very rapidly time varying multipath fading channels has been considered in a number of recent works [3], [4], [5], [6], [7], [8].

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To reduce the effects of ICI, a time-domain channel estimator was proposed in [3] which assumed that the channel impulse response (CIR) varies in a linear fashion within the symbol duration. However, this assumption no longer holds when the normalized Doppler frequency takes substantially higher values. In a rapidly time-varying channel, the time-domain channel estimation method proposed in [9] is a potential candidate for the channel estimator, in order to mitigate ICI. This technique estimates the fading channel by exploiting the time-varying nature of the channel as a provider of time diversity and reduces the computational complexity using the singular-value decomposition (SVD) method.

In [5], to handle rapid variation within an OFDM symbol, the pilot-based estimation scheme using channel interpolation was proposed. Moreover, coupled with the proposed channel estimation scheme, a simple Doppler frequency estimation scheme was proposed. In [6], to compensate for the ICI, a modified Kalman filter (MKF) channel estimator for OFDM systems in a fast and frequency-selective Rayleigh fading channel was proposed. The time-varying channel was modeled as an autoregressive (AR) process and the proposed MKF was used to estimate this AR parameter. In addition, a channel predictor using regression analysis and the minimum mean square error (MMSE) time-domain equalizer were also proposed to track the time-varying channel. The difference between the method described in [9] and the one proposed in [6] is that the former assumed the value of the fading parameter (or Doppler frequency) to be known in advance, whereas the latter estimated it by means of the MKF.

In [7], two methods to mitigate ICI in an OFDM system with coherent channel estimation were proposed. Both methods employed a piece-wise linear approximation to estimate channel time-variations in each OFDM symbol. The first method extracted channel time-variation information from the cyclic prefix while the second method estimated these variations using the next symbol. Moreover, a closed-form expression for the improvement in average signal-to-interference ratio (SIR) was derived for a narrowband time-varying channel.

A decision-directed channel predictor for OFDM signaling over time-varying channels was proposed [8], which is capable of yielding up-to-date channel state information even without regular transmission of pilot symbols. However, to avoid error propagation a certain minimum SNR is required.

In this paper, a computationally feasible space alternating generalized expectation maximization (SAGE) algorithm is proposed for the problem of joint multiuser data detection [10] and channel estimation for OFDM systems operating in highly mobile and frequency selective channels. The channel variations over the duration of a data block is upper bounded by the maximum Doppler bandwidth which is determined by the maximum speed of the users. We exploit the band-limited discrete-cosine orthogonal basis functions to represent the time-varying fading channel through a discrete cosine serial expansion of low dimensionality. In this way, the resulting reduced dimensional channel coefficients are estimated and the data symbols detected iteratively with tractable complexity. It is concluded that the resulting SAGE-based receiver scheme comprises a channel estimator, interference cancelation and a soft-input/hard-output serial data detector in each iteration. Computer simulations show that the cosine transformation represents the time-varying channel very effectively and the proposed algorithm has excellent symbol error rate (SER) and channel estimation performance even with a very small number of channel expansion coefficients employed in the algorithm, resulting in substantial reduction of the computational complexity.

II. Signal Model

We consider an OFDM system with N subcarriers. At the transmitter, K out of N subcarriers are actively employed to transmit data symbols and nothing is transmitted on the remaining $N-K$ carriers. The frequency-domain transmitted symbols are denoted as $s(n,k)$, where n is the OFDM symbol discrete-time index and $k \in \{0,1\cdots,K-1\}$ is the subcarrier discrete-frequency index. It is assumed that the transmitted signals has a constant envelope modulation format with $\mid s(n,k) \mid = 1$. A cyclic prefix of length L_c is then added. We assume a time-varying mobile radio channel with discrete-time impulse response $h(n,l), l=0,1,\cdots,L-1$ where L is the maximum channel length and it is assumed that $L \leq L_c$. The Fourier transform of the channel impulse response at time $n = 0,1,\cdots,$ is defined as $H(n,k) \triangleq \sum_{l=0}^{L-1} h(n,l)\exp(-j2\pi lk/N)$. For a classical OFDM system with cyclic prefix duration larger than the channel impulse response length, the received signal is not corrupted by previous symbols and therefore all OFDM symbols can be processed separately. If we focus on the detection of the kth data symbol transmitted during the mth OFDM timing slot, the final expression for the received signal before the discrete Fourier transform (DFT), after matched filtering, symbol-rate sampling and discarding the symbols falling in the cyclic prefix, can be expressed as [9]

$$\begin{aligned} r(mN_g+p) = \; & s(m,k)\frac{1}{N}\sum_{l=0}^{L-1} h(mN_g+p,l)\exp\left(j\frac{2\pi k(p-l)}{N}\right) \\ & + I_k(m,p) + w(mN_g+p), \end{aligned} \quad (1)$$

for $p = 0,1\cdots,N-1$ and $m = 0,1,\cdots,M-1$. Here, M denotes the length of one OFDM block consisting of M consecutive OFDM symbols and $N_g \triangleq N + L_c$ and, $w(\cdot)$ is zero-mean complex additive Gaussian noise with variance N_0. The term $I_k(m,p)$ in (1) represents the kth data symbol's ICI term caused by the time-varying nature of the channel and it can be expressed as

$$I_k(m,p) \triangleq \sum_{q=0,q\neq k}^{K-1} s(m,q)\frac{1}{N}\sum_{l=0}^{L-1} h(mN_g+p,l)\exp\left(j\frac{2\pi q(p-l)}{N}\right). \quad (2)$$

Equation (1) can be expressed in matrix form as follows:

$$\mathbf{r}(m) = s(m,k)\mathbf{V}_k\mathbf{h}(m) + \sum_{q=0,q\neq k} s(m,q)\mathbf{V}_q\mathbf{h}(m) + \mathbf{w}(m) \quad (3)$$

where

$$\mathbf{r}(m) = [r(mN_g), r(mN_g+1),\cdots,r(mN_g+N-1)]^T \in \mathcal{C}^N$$

and

$$\mathbf{w}(m) = [w(mN_g), w(mN_g+1),\cdots,w(mN_g+N-1)]^T \in \mathcal{C}^N.$$

The vector $\mathbf{h}(m)$ denotes the time-varying channel impulse response in the mth OFDM block:

$$\mathbf{h}(m) = [\mathbf{h}_0^T(m), \mathbf{h}_1^T(m),\cdots,\mathbf{h}_{L-1}^T(m)]^T \in \mathcal{C}^{NL} \quad (4)$$

where $\mathbf{h}_l(m) = [h(mN_g,l), h(mN_g+1,l),\cdots,h(mN_g+N-1,l]^T, l=0,1,\cdots,L-1$ represents $L-$path wide-sense stationary uncorrelated scattering (WSSUS) Rayleigh fading coefficients at the (mN_g+p)th discrete-time $p=0,1,\cdots,N-1$. Assuming the Jakes' model, the autocorrelation function of the channel is

$$\begin{aligned} & E\{h(mN_g+p,l)h^*(m'N_g+p',l')\} = \\ & \sigma_l^2 J_0\left(2\pi f_D T_s((m-m')N_g+(p-p'))\right)\delta(l-l') \end{aligned} \quad (5)$$

where $\sigma_l^2, l = 0,1\cdots,L-1$, represents the normalized power of the lth path of the channel satisfying $\sum_l \sigma_l^2 = 1$. Here, $J_0(\cdot)$ is the zeroth-order Bessel function of the first kind, f_D is the Doppler shift due to the vehicle motion and $\delta(\cdot)$ is the Kronecker delta. $T_s = T/N$ is the sampling duration with T being the OFDM symbol duration. Finally, $\mathbf{w}$ is the complex white Gaussian noise vector with zero-mean and $E[\mathbf{w}\mathbf{w}^\dagger] = N_0\mathbf{I}_K$, where $\mathbf{I}_K$ denotes a $K\times K$ identity matrix.

It can be shown that, for $q=0,1,\cdots,K-1$, the matrix $\mathbf{V}_q \in \mathcal{C}^{N\times NL}$ in (3) can be expressed as

$$\mathbf{V}_q = \mathbf{F}_L^T(q) \otimes \frac{1}{N}\text{diag}(\mathbf{F}_N^\dagger(q)) \quad (6)$$

where $\otimes$ denotes the Kronecker product and

$$\mathbf{F}_L(q) \triangleq \left[1, \exp(-j2\pi q/N),\cdots,\exp(-j2\pi q(L-1)/N)\right]^T \in \mathcal{C}^L.$$

The performance of the receiver depends critically on the estimate of the time-varying channel impulse response $\mathbf{h} = [\mathbf{h}^T(0), \mathbf{h}^T(1)\cdots,\mathbf{h}^T(M-1)]^T \in \mathcal{C}^{MNL}$ from the MN $(MN < MNL)$ dimensional received vector $\mathbf{r} = [\mathbf{r}^T(0), \mathbf{r}^T(1)\cdots,\mathbf{r}^T(M-1)]^T$. At fist glance, it might seem that the estimation of the $MNL\times 1$ channel vector $\mathbf{h}$ is impossible by means of $\mathbf{r}$ since there are more unknowns to be determined than known equations. However, the banded property of the channel matrix [11] enables us to reduce the number of parameters needed for channel estimation substantially, and consequently to reduce the computational complexity of channel estimation.

We first apply a suitable basis expansion that describes the time variations of the discrete-time channel impulse response $h(mN_g + p,l)$ over a data block consisting of M OFDM symbols. We do not make any assumption regarding the amount of time-variation (equivalently, Doppler frequency) in the channel. For notational simplicity, let $t \triangleq mN_g + p$. Then,

$$(m=0,\cdots,M-1 \text{ and } p=0,\cdots,N_g-1) \Leftrightarrow t=0,\cdots,MN_g-1.$$

For each channel path $l=0,1,\cdots,L-1$, the channel coefficients, $h(t,l)$, can be represented as a weighted sum of MN_g orthogonal basis functions $\{\psi_d(t)\}$ in the interval $[0, MN_gT_s]$:

$$h(t,l) = \sum_{d=0}^{MN_g-1} \psi_d(t)c(d,l), \qquad t=0,1,\cdots,MN_g-1, \quad (7)$$

where $\{c(d,l)\}$ represent the expansion coefficients. As $h(\cdot,l)$ is essentially a lowpass process whose bandwidth is determined by the Doppler frequency, it can be well approximated by the weighted sum of a substantially smaller number D $(<< MN_g)$ of suitable basis functions:

$$\widetilde{h}(t,l)=\sum_{d=0}^{D-1}c(d,l)\psi_d(t), \qquad t=0,1,\cdots,MN_g-1. \quad (8)$$

Similarly, using the orthogonality property of the basis functions, the expansion coefficients can be evaluated by the inverse transformation as

$$c(d,l)=\sum_{t=0}^{MN_g-1}h(t,l)\psi_d(t), \qquad d=0,1,\cdots,D-1. \quad (9)$$

In our work, we make use of the orthonormal Discrete Cosine Transform (DCT) basis functions, which are given by

$$\psi_d(t)=\begin{cases}\sqrt{(1/MN_g)} & \text{if } d=0,\\ \sqrt{2/MN_g}\cos\left[(\pi d/MN_g)(t+1/2)\right] & \text{if } d>0.\end{cases} \quad (10)$$

Hence, $c(d,l)$ is the dth DCT-coefficient of $h(t,l)$. The dimension D of the basis expansion fulfills $D^{'}\leq D\leq MN_g$. The lower bound $D^{'}$ is given by $D^{'}=\text{ceil}(2(f_D)_{max}M+1)$ [12], where ceil$(\cdot)$ rounds up to the closest integer and $(f_D)_{max}$ the maximum (one-sided) normalized Doppler bandwidth is defined by

$$(f_D)_{max}=\frac{v_{max}f_c}{c}T\ . \quad (11)$$

Note that v_{max},f_c and c denote the maximum supported velocity, the carrier frequency and the speed of light, respectively, and T is the OFDM symbol duration. Then, for each channel path l ($l=0,1,\cdots,L-1$), the channel and the expansion coefficients can be expressed in matrix form

$$\widetilde{\mathbf{h}}_l=\mathbf{\Psi}\mathbf{c}_l, \qquad \mathbf{c}_l=\mathbf{\Psi}^{\dagger}\widetilde{\mathbf{h}}_l \quad (12)$$

where

$$\begin{aligned}\widetilde{\mathbf{h}}_l &= [\widetilde{h}(0,l),\widetilde{h}(1,l),\cdots,\widetilde{h}(MN_g-1,l)]^T\in\mathcal{C}^{MN_g}\\ \mathbf{c}_l &= [c(0,l),c(1,l),\cdots,c(D-1,l)]^T\in\mathcal{C}^{D},\end{aligned}$$

and $\mathbf{\Psi}$ represents the DCT matrix expressed as

$$\mathbf{\Psi}=[\boldsymbol{\psi}(0),\boldsymbol{\psi}(1),\cdots,\boldsymbol{\psi}(MN_g-1)]^T\in\mathcal{R}^{MN_g\times D} \quad (13)$$

$$\boldsymbol{\psi}(t)=[\psi_0(t),\psi_1(t),\cdots,\psi_{D-1}(t)]^T,\ t=0,1,\cdots,MN_g-1.$$

Furthermore after removing the CP, it can be shown from (8) and (4) that the dimension of the channel vectors $\widetilde{\mathbf{h}}_l, l=0,1,\cdots,L-1$, in (12) reduces from MN_g to MN as follows:

$$\widetilde{\mathbf{h}}(m)=\mathbf{\Phi}(m)\mathbf{c} \quad (14)$$

where,

$$\begin{aligned}\widetilde{\mathbf{h}}(m) &= [\widetilde{\mathbf{h}}_0(m),\widetilde{\mathbf{h}}_1(m),\cdots,\widetilde{\mathbf{h}}_{L-1}(m)]^T\in\mathcal{C}^{NL}\\ \mathbf{c} &= [\mathbf{c}_0,\mathbf{c}_1,\cdots,\mathbf{c}_{L-1}]^T\in\mathcal{C}^{DL},\end{aligned}$$

and

$$\mathbf{\Phi}(m)\triangleq\text{diag}\underbrace{\left(\mathbf{\Psi}(m),\ \mathbf{\Psi}(m)\cdots,\mathbf{\Psi}(m)\right)}_{L}\in\mathcal{C}^{NL\times DL},$$

with

$$\mathbf{\Psi}(m)\triangleq[\boldsymbol{\psi}(mN_g),\boldsymbol{\psi}(mN_g+1),\cdots,\boldsymbol{\psi}(mN_g+N-1)]^T\in\mathcal{R}^{N\times D}.$$

Finally, substituting (14) into (3), the received signal is expressed in terms of the reduced dimensional channel vector $\mathbf{c}$ as follows:

$$\mathbf{r}(m)=s(m,k)\mathbf{A}_k(m)\mathbf{c}+\sum_{q=0,q\neq k}^{K-1}s(m,q)\mathbf{A}_q(m)\mathbf{c}+\mathbf{w}(m) \quad (15)$$

where $\mathbf{A}_q(m)\triangleq\mathbf{V}_q\mathbf{\Phi}(m)\in\mathcal{C}^{N\times DL}$.

For the later developments we also express (15) in a more compact matrix form as follows:

$$\mathbf{r}=\mathbf{Z_s}\mathbf{c}+\mathbf{w} \quad (16)$$

where

$$\begin{aligned}\mathbf{r} &= [\mathbf{r}^T(0),\mathbf{r}^T(1),\cdots,\mathbf{r}^T(M-1)]^T\in\mathcal{C}^{NM}\\ \mathbf{Z_s} &= \left[\mathbf{Z}_{\mathbf{s}}^T(0),\mathbf{Z}_{\mathbf{s}}^T(1),\cdots,\mathbf{Z}_{\mathbf{s}}^T(M-1)\right]^T\in\mathcal{C}^{NM\times DL}\\ \mathbf{Z_s}(m) &= \sum_{q=0}^{K-1}s(m,q)\mathbf{A}_q(m)\end{aligned}$$

III. Data Detection with SAGE Technique

The problem of interest is to derive an iterative algorithm based on the SAGE technique for data detection without complete channel state information, employing the signal model given by (2). Since the SAGE method has been studied and applied to a number of problems in communications over the years, the details of the algorithm will not be presented in this paper. The reader is referred [13] for a general exposition to SAGE algorithm and [14] for its application to the estimation problem related to the work herein. The suitable approach for applying the SAGE algorithm for the problem at hand is to decompose the received signal in (15) into the sum [15]

$$\mathbf{r}(m)=\mathbf{y}_k(m)+\overline{\mathbf{y}}_k(m) \quad (17)$$

where

$$\mathbf{y}_k(m) = s(m,k)\mathbf{A}_k(m)\mathbf{c}+\mathbf{w}(m), \quad (18)$$

$$\overline{\mathbf{y}}_k(m) = \sum_{q=0,q\neq k}^{K-1}s(m,q)\mathbf{A}_q(m)\mathbf{c}, \quad (19)$$

for $k=0,1\cdots,K-1$ and $m=0,1\cdots,M-1$.

We now derive the SAGE algorithm to detect the OFDM symbol vectors in the set $\mathbf{s}\triangleq\{\mathbf{s}_0,\mathbf{s}_1,\cdots,\mathbf{s}_{K-1}\}$, where $\mathbf{s}_k=[s(0,k),s(1,k),\cdots,s(M-1,k)]^T$ denotes the transmitted data symbols within an observed frame of M OFDM symbols, based on the received vector $\mathbf{r}$. To obtain a receiver architecture that iterates between soft-data and channel estimation, one might choose the parameter vector as $\{\mathbf{s}\}$. At each iteration (i), only the data symbol vector of one subchannel, say, k, i.e., $\{\mathbf{s}_k\}$, is updated, while the symbol vectors of other subchannels $\overline{\mathbf{s}}_k=\mathbf{s}\backslash\mathbf{s}_k$ are kept fixed.

In the SAGE algorithm, we view the observed data $\mathbf{r}$ as the *incomplete* data and since $\mathbf{c}$ is unknown, we incorporate $\mathbf{c}$ into the *admissible hidden* data set as $\chi_k=\{\mathbf{y}_k,\mathbf{c}\}$ to which the incomplete data $\mathbf{r}$ is related through a possibly nondeterministic mapping [13].

Expectation-Step (E-Step): In the *E-Step* computation of the average log-likelihood function, averaged over $\mathbf{c}$, is implemented. The conditional expectation is taken over $\boldsymbol{\chi}$ given the observation $\mathbf{r}$ and that $\mathbf{s}$ equals its estimate, calculated at ith iteration as

$$Q_k\left(\mathbf{s}_k|\mathbf{s}^{(i)}\right)=E\left\{\log p(\mathbf{y}_k|\mathbf{s}_k,\overline{\mathbf{s}}_k^{(i)}\mathbf{c})|\mathbf{r},\mathbf{s}^{(i)}\right\}. \quad (20)$$

Neglecting the terms independent of $\mathbf{s}$ and since a constant envelope signal modulation format is assumed, $\log p(\mathbf{y}_k|\mathbf{s},\mathbf{c})$ can be calculated from (18) as

$$\log p(\mathbf{y}_k|\mathbf{s},\mathbf{c})\sim\sum_{m=0}^{M-1}\Re\{s^*(m,k)\mathbf{c}^{\dagger}\mathbf{A}_k^{\dagger}(m)\mathbf{y}_k(m)\}, \quad (21)$$

where $\Re\{\cdot\}$ denotes the real part of its argument. Inserting (21) into (20), we have

$$Q_k(\mathbf{s}_k|\mathbf{s}^{(i)}) = \sum_{m=0}^{M-1} \Re\left\{s^*(m,k)E\left\{\mathbf{c}^\dagger \mathbf{A}_k^\dagger(m)\mathbf{y}_k(m)|\mathbf{r},\mathbf{s}^{(i)}\right\}\right\}. \tag{22}$$

Taking the expectation in (22) and after some algebra it follows that

$$Q_k(\mathbf{s}_k|\mathbf{s}^{(i)}) = \sum_{m=0}^{M-1} \Re\left\{s^*(m,k)\Upsilon_k^{(i)}(m)\right\} \tag{23}$$

where

$$\begin{aligned}\Upsilon_k^{(i)}(m) &= E\{\mathbf{c}^\dagger \mid \mathbf{r},\mathbf{s}^{(i)}\}\mathbf{A}_k^\dagger(m)\mathbf{r}(m) \\ &\quad - \sum_{q=0,q\neq k}^{K-1} s^{(i)}(m,q)E\{\mathbf{c}^\dagger\mathbf{\Gamma}_{k,q}\mathbf{c} \mid \mathbf{r},\mathbf{s}^{(i)}\}\end{aligned} \tag{24}$$

and $\mathbf{\Gamma}_{k,q} \triangleq \mathbf{A}_k^\dagger(m)\mathbf{A}_q(m)$.

In order to evaluate the expectations on the right hand side of (24), we first determine the conditional density of $\mathbf{c}$ given $\mathbf{r}$ and $\mathbf{s}^{(i)}$ as follows. The prior probability density function (pdf) of $\mathbf{c} = [\mathbf{c}_0^T, \mathbf{c}_1^T, \cdots, \mathbf{c}_{L-1}^T]^T$ is chosen as $\mathbf{c} \sim N(\mathbf{0}, \mathbf{\Sigma}_\mathbf{c}^{(0)})$. The covariance matrix of $\mathbf{c}$ can be determined from (14) as

$$\mathbf{\Sigma}_\mathbf{c}^{(0)} = \text{diag}\left(\mathbf{R}_\mathbf{c}(0), \mathbf{R}_\mathbf{c}(1), \cdots, \mathbf{R}_\mathbf{c}(L-1)\right) \in \mathcal{C}^{LD\times LD}$$

where

$$\mathbf{R}_\mathbf{c}(l) = \mathbf{\Psi}^T\mathbf{R}_\mathbf{h}(l)\mathbf{\Psi} \in \mathcal{C}^{D\times D}$$

and $\mathbf{R}_\mathbf{h}(l)$, the covariance matrix of $\mathbf{h}_l$, can be obtained from (5) as $\mathbf{R}_\mathbf{h}(l) = \sigma_l^2[r(j-i)]$, $i,j = 0,1,\cdots,MN_g-1$, with $r(k) = J_0(2\pi f_D kT_s)$. However, for sufficiently large block size M, it can be shown that $\mathbf{R}_\mathbf{c}(l)$, the covariance matrix of $\mathbf{c}$, for each channel path $l = 0,1,\cdots,L-1$ becomes diagonal as

$$\mathbf{R}_\mathbf{c}(l) = \sigma_l^2\text{diag}\left(\lambda(l,0), \lambda(l,1), \cdots, \lambda(l,D-1)\right) \tag{25}$$

where $\lambda(l,d) = S_h\left(l, d/2N_gMT_s\right), d = 0,1,\cdots,D-1$, and $S_h(\cdot,\cdot)$ is the channel's scattering function defined by the Fourier transform of $r(k)$. For Jake's Doppler profile this function is given by $S_h(f) = 1/\sqrt{f_d^2 - f^2}$ [16]. On the other hand, since $\mathbf{w} \sim N(\mathbf{0}, N_0\mathbf{I})$, using the observation equation for $\mathbf{r}$ (16), we can write the conditional pdf of $\mathbf{c}$ given $\mathbf{r}$ and $\mathbf{s}^{(i)}$ as $p(\mathbf{c}|\mathbf{r},\mathbf{s}^{(i)}) \sim p(\mathbf{r}|\mathbf{c},\mathbf{s}^{(i)})p(\mathbf{c})$. After some algebra it can be shown that [17]

$$p(\mathbf{c}|\mathbf{y},\mathbf{s}^{(i)}) \sim N(\boldsymbol{\mu}_\mathbf{c}^{(i)}, \mathbf{\Sigma}_\mathbf{c}^{(i)})$$

where

$$\boldsymbol{\mu}_\mathbf{c}^{(i)} = \frac{1}{N_0}\,\mathbf{\Sigma}_\mathbf{c}^{(i)}\,\mathbf{Z}_{\mathbf{s}^{(i)}}^\dagger\,\mathbf{r} \tag{26}$$

$$\text{and}\quad \mathbf{\Sigma}_\mathbf{c}^{(i)} = \left((\mathbf{\Sigma}_\mathbf{c}^{(0)})^{-1} + \frac{1}{N_0}\mathbf{Z}_{\mathbf{s}^{(i)}}^\dagger\mathbf{Z}_{\mathbf{s}^{(i)}}\right)^{-1}.$$

Now let us compute the terms on the right hand side of (24). The expectation in the first term on the right hand side of (24) can be expressed as

$$E\{\mathbf{c}^\dagger|\mathbf{r},\mathbf{s}^{(i)}\} = (\boldsymbol{\mu}_\mathbf{c}^{(i)})^\dagger. \tag{27}$$

We compute the last expectation in (24), $E\{\mathbf{c}^\dagger\mathbf{\Gamma}_{k,q}\mathbf{c}|\mathbf{r},\mathbf{s}^{(i)}\}$, as follows. By defining $\boldsymbol{\xi}_k \triangleq \mathbf{A}_k(m)\mathbf{c}$, it can be easily shown that

$$E\{\mathbf{c}^\dagger\mathbf{\Gamma}_{k,q}\mathbf{c}|\mathbf{r},\mathbf{s}^{(i)}\} = \text{tr}(\mathbf{\Xi}_{q,k}), \tag{28}$$

where $\text{tr}(\cdot)$ denotes the trace of a matrix, $\mathbf{\Gamma}_{k,q} = \mathbf{A}_k^\dagger(m)\mathbf{A}_q(m)$ and $\mathbf{\Xi}_{q,k} \triangleq E\{\boldsymbol{\xi}_q\boldsymbol{\xi}_k^\dagger\} = \mathbf{A}_q(m)\left(\mathbf{\Sigma}_\mathbf{c}^{(i)} + \boldsymbol{\mu}_\mathbf{c}^{(i)}(\boldsymbol{\mu}_\mathbf{c}^{(i)})^\dagger\right)\mathbf{A}_k^\dagger(m)$.

Maximization-Step (M-Step): In the M-step of the SAGE algorithm, the estimates of the data sequence are updated at the $(i+1)$th iteration according to

$$\mathbf{s}_k^{(i+1)} = \arg\max_{\mathbf{s}_k} Q_k(\mathbf{s}_k|\mathbf{s}^{(i)}),\;\; \bar{\mathbf{s}}_k^{(i+1)} = \bar{\mathbf{s}}_k^{(i)} \tag{29}$$

where $Q(\mathbf{s}_k|\mathbf{s}^i)$ is given by (23). Substituting (23) into (29) yields the following:

$$\mathbf{s}_k^{(i+1)} = \arg\max_{\mathbf{s}_k} \sum_{m=0}^{M-1} \Re\{s^*(m,k)\Upsilon_k^{(i)}(m)\},\;\; \bar{\mathbf{s}}_k^{(i+1)} = \bar{\mathbf{s}}_k^{(i)}. \tag{30}$$

Moreover, when no coding is used, it follows from (30) that each component of $s^{(i+1)}(m,k)$ can be separately obtained by maximizing the corresponding summation in the right-hand expression of (31) as

$$s^{(i+1)}(m,k) = \text{Quant}\left(\Upsilon_k^{(i)}(m)\right) \tag{31}$$

where Quant(.) denotes the quantization process that quantizes its argument to its nearest data symbol constellation point. Equation (30) can be interpreted as joint channel estimation and partial interference cancelation implemented in the time-domain, immediately following the analog-to-digital (A/D) conversion and cyclic prefix deletion processes at the OFDM receiver. We can think of the quantities $\Upsilon_k^{(i)}(m), k = 0,1,\cdots,N-1$, in (31) as the outputs of a successive interference canceler (SIC), generated at the ith iteration step of the SAGE algorithm.

Initialization of channel coefficients and data symbols: The initial channel estimate, $\mathbf{c}^{(0)}$, can be determined with the aid of the pilot symbols. Since the time-domain correlation plays the key role in channel estimation due to high-mobility, we consider all subcarriers in a given time slot dedicated to pilot symbols. Assuming that there are P pilot OFDM symbols located at time-slots $m_1, m_2, \cdots, m_P$ where $m_p \in \{0,1,\cdots,M-1\}$, they can be arranged in an $NP \times LD$ ($NP > LD$) pilot-symbol matrix $\mathbf{Z}_{\mathbf{s}_P} = \left[\mathbf{Z}_\mathbf{s}^T(m_1), \mathbf{Z}_\mathbf{s}^T(m_2), \cdots, \mathbf{Z}_\mathbf{s}^T(m_P)\right]^T$ where $\mathbf{Z}_\mathbf{s}(m_p) = \sum_{q=0}^{K-1} s(m_p,q)\mathbf{A}_q(m_p)$. Accordingly, from (16) the received vector corresponding to one OFDM frame of length M as

$$\mathbf{r}_P = \mathbf{Z}_{\mathbf{s}_P}\mathbf{c} + \mathbf{w}_P. \tag{32}$$

The MMSE estimate of the initial channel parameter $\mathbf{c}^{(0)}$ can be determined from (33) as

$$\mathbf{c}^{(0)} = \left(\mathbf{Z}_{\mathbf{s}_P}^\dagger\mathbf{Z}_{\mathbf{s}_P} + N_0\,(\mathbf{\Sigma}_\mathbf{c}^{(0)})^{-1}\right)^{-1}\mathbf{Z}_{\mathbf{s}_P}^\dagger\mathbf{r}_P. \tag{33}$$

Similarly, for each $m = 0,1,\cdots,M-1$, the MMSE estimate of the initial data symbol vector $\mathbf{s}^{(0)}(m)$ can be determined as

$$\mathbf{s}^{(0)}(m) = \mathbf{H}^{(0)\dagger}(m)\left(\mathbf{H}^{(0)}(m)\mathbf{H}^{(0)\dagger}(m) + N_0\mathbf{I}_N\right)^{-1}\mathbf{r}(m) \tag{34}$$

where $\mathbf{H}^{(0)}(m)$ represents, in the frequency domain, the time-varying channel coefficient matrix of the m^{th} OFDM block. Note that the banded property of the channel can be exploited to reduce the computational complexity of the matrix inversion in (34) by means of low complexity decompositions such as the Cholesky or the $\text{LL}^\dagger$ factorization of Hermitian banded matrices.

IV. Simulations

In this section, we present simulation results to assess the performance of OFDM systems based on the proposed receiver. The system operates on a 25 block-length OFDM frame with a 10 MHz bandwidth and 1024 subchannels using QPSK modulation. The normalized Doppler frequencies are $f_D T = 0.0853$ and $f_D T = 0.0427$ corresponding to a mobile terminal moving at speeds v of 360 km/h and 180 km/h, respectively, for a carrier frequency of 2.5 GHz. The wireless channel having an exponentially decaying power delay profile with the normalized powers, $\sigma_0^2 = 0.448$, $\sigma_1^2 = 0.321$, and $\sigma_2^2 = 0.230$ are chosen. For initialization of DCT coefficients and subsequently data symbols in the SAGE algorithm, one block in every 5(4) OFDM blocks is dedicated to pilot symbols, corresponding to the 180 km/h (360 km/h) velocity. In Fig. 1, the SER performance of the proposed algorithm is presented as a function of signal-to-noise ratio (SNR) for two different mobilities. The solid curves in Fig. 1 represent a lower bound for the SER assuming we have perfect channel state information (CSI) corresponding to the scenarios where $f_D T = 0.0853$ and $f_D T = 0.0427$. We conclude from these curves that even when the number of DCT coefficients is chosen as small as 8(9), the performance loss is not significant when CSI is not available. It was also observed that only 5 iterations are sufficient in order for the SAGE algorithm to converge. We further consider the average mean square error (MSE) performance of the channel estimation part of our algorithm. The average MSE curves are plotted as functions of several SNR values for the normalized Doppler frequencies $f_D T = 0.0853$ and $f_D T = 0.0427$. As can be seen in Fig 2, the algorithm achieves excellent MSE performance even when the number of DCT coefficients is truncated at 8(9) corresponding to different mobilities.

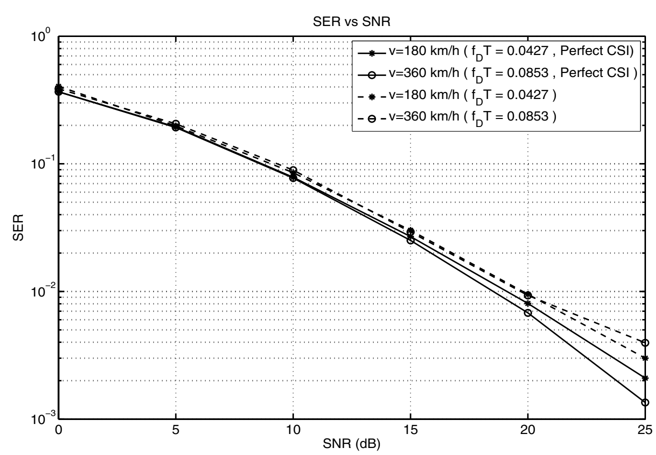

Fig. 1. SER vs. SNR simulation results of the proposed SAGE algorithm for joint channel estimation and data detection.

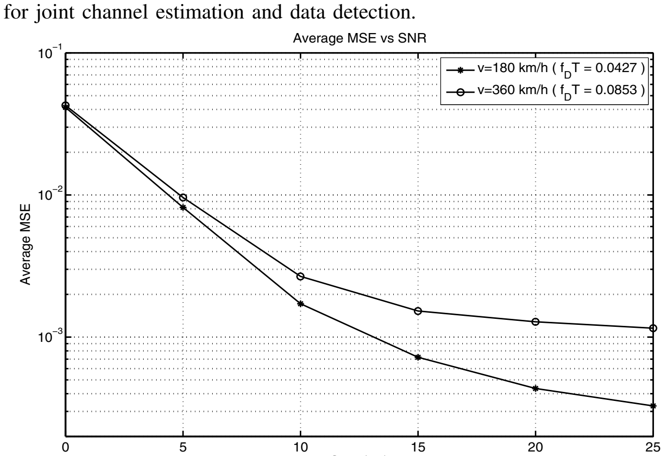

Fig. 2. Average MSE vs. SNR simulation results of the proposed SAGE algorithm for joint channel estimation and data detection.

V. Conclusions

The problem of joint data detection and channel estimation for uplink OFDM systems operating over frequency selective and very rapidly time-varying channels has been investigated in this work. We have presented an iterative approach based on the SAGE algorithm, and closed form expressions have been derived for data detection that incorporates the channel estimation as well as partial interference cancelation. The cosine orthogonal basis functions have been applied to describe the time-varying channel. It has been shown by computer simulation that, depending on the normalized Doppler frequency, only a small number of expansion coefficients is sufficient to approximate the channel perfectly, there is no need to know the statistics of the input signal, and the proposed algorithm has excellent symbol error rate and channel estimation performance even with a very small number of channel expansion coefficients, resulting in reduction of the computational complexity substantially.

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