

On preserving connectivity of autonomous mobile robots

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Abstract—The connectivity of the autonomous mobile robots is considered in this paper. The group navigation is provided using simple local steering rules and without any explicit communication. Sub-optimal solutions are invoked to avoid computational cost. We show that the connectivity of the group is preserved during the whole motion, in spite of bounded measurement errors on angles and distances. Some special cases of group topology are also discussed.

I. INTRODUCTION

A group of robots, rather than a single robot, are expected to be successful in performing some tasks that require cooperation. Indeed, there are several species in nature which exhibit very beautiful examples of collective working. Schooling fish, flocking birds, ant and bee colonies are quite well-known for their collective working behaviors.

Several works are available in the literature which utilize the cooperation of autonomous agents. One of the first efforts to model species which work collectively was given for flocking birds in 1987 [1]. In that work, it is asserted that large group behaviors arise as a result of simple principles about the motion of each member of the group. An important application of this idea in discrete-time was given in 1995 [2]. Since then, the concept of cooperative motion has greatly evolved. The formation of robot groups [3],[4] as well as the utilization of potential functions and artificial forces to accomplish a group behavior [5]-[7] has been widely studied. Some methodologies rely on limited communication between robots for desired group task [8],[9]. A more general discussion of the initial studies conducted in this area and concept development can be found in [10] and the references therein.

In this paper, we develop a methodology for the navigation of autonomous robots groups, preserving the group connectivity. We begin by defining the connectivity of robot groups. Then, a local steering strategy is built, which guarantees the connectivity of the group for the whole time of interest. We assume that the robots have no communication capability and their position sensors are of limited range with bounded measurement errors. These assumptions make our work more realistic, as we form an analogy between the mobile robots and aggregating organisms in nature. There are other works in the literature too, which make use of graph theory and connected group definitions, such as [11]-[13], and especially

in [14] where Hamiltonian graphs and potential fields are used. However, as far as we know, no work is available yet that guarantees the connectivity of the robot group without using communication. In [11] and [12], it is assumed that a state vector consisting of position and velocity is measurable and every robot can sense any other robot in the group without any restrictions. These works and also the study in [15] assume group connectivity during the motion as a pre-requisite for the success of their methods. However, the methodology proposed in [16] results in the navigation of a robot group having dynamic topology using only limited-range position sensors with guaranteed connectivity. Here we extend the work in [16] by including measurement errors and providing solution to a possible deadlock problem.

II. PROBLEM FORMULATION

The robots in this study are assumed to be identical in all physical properties. Each robot, represented by R , has the capability to move in all directions (*i.e.* the robots are *omni-directional*), and is equipped with limited-range position sensors. In this work, the position sensors are considered to be such sensors that measure both the distances and the angles between the measuring robot and the robots within its measurement range. The working space can be either 2D or 3D. These sensors have a known degree of accuracy. We assume that the sensing capability is continuous and available equally in all directions, but the sensor results bear both angular and longitudinal measurement errors. These errors are bounded by positive scalars $\Delta\theta$ and Δr , respectively. This is depicted in Fig. 1 for a 2D case. It is important that sensing other robots means obtaining the position information of the robots in the neighborhood via its position sensors. This establishes a *link* between the sensing robot and each robot in its sensing range. Hence, the robots can stay *connected* with their neighbours without using explicit communication. Also note that sensing other robots does not imply recognizing a specific robot. The robots have no ID's. Moreover, each robot does not even know its own label.

A number of such mobile robots constitute a group. First, we state some basic terms from the graph theory.

Definition 1: A **group** \mathcal{G} is a set of autonomous mobile robots $\{R_i, i = 1, \dots, N\}$, that can be connected by links.

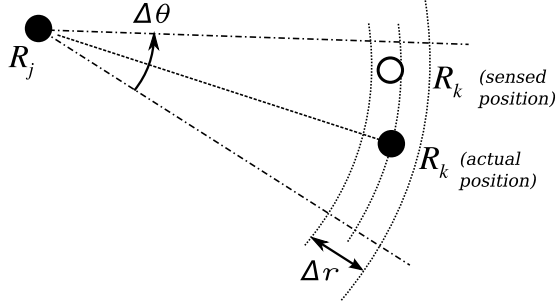


Fig. 1. Illustration of sensing errors on angle and distance in \mathbb{R}^2

Definition 2: The group \mathcal{G} is **connected** if there is a path from any robot to any other robot in the group through links. A group which has at least one pair of robots having no path inbetween is **disconnected**.

Since we assume that the position sensing range of the robots is limited and the total number of robots in the group can be large, a robot may not sense all other robots in the group. Based on this, we define subgroups as follows.

Definition 3: A **subgroup** \mathcal{S}_i is a group of robots sensed by the robot R_i .

Since there are N robots in the group, by definition, there are N subgroups, i.e. one for each robot. \mathcal{S}_i has a spherical shape with R_i lying at its center. We will denote the radius of \mathcal{S}_i as d_{max} . In other words, d_{max} is the maximum sensing distance for each robot. If R_i cannot sense any other robots in the group, then \mathcal{S}_i is an empty set. On the other hand, letting the largest distance between the robots in \mathcal{G} be denoted as D_{max} , \mathcal{G} will be connected according to Definition 2, if $d_{max} \geq D_{max}$. However, one faces nontrivial and more interesting cases whenever $d_{max} \ll D_{max}$, which corresponds to robot groups of a relatively large number of individuals having limited sensing ranges.

Fig. 2 depicts a group consisting of three robots. It is seen that $R_2 \in \mathcal{S}_1$, and $R_1 \in \mathcal{S}_2$. This establishes the links C_{12} and C_{21} , respectively. The links between R_2 and R_3 are formed likewise. Note that the robot R_2 has the position information of both R_1 and R_3 , but R_1 and R_3 cannot sense each other, as the distance between R_2 and R_3 is larger than d_{max} .

The group is required to navigate from a starting location to a target location. We assume that the location of the target is unknown to all group members except one robot. This robot is assigned as the *leader* of the group and denoted as R_N . The leader has the same physical properties and capabilities as the other robots. The only difference is that the direction of the target is given to R_N so that it can determine its movement using this target information. However, in this study, the leadership is *hidden*. None of the robots recognize the leader as a distinguished group member. In other words, if R_N is sensed by a robot R_j , i.e. $R_N \in \mathcal{S}_j$, R_j can only see it as one of its neighbors and the leadership of R_N does not affect the local steering strategy of R_j . In the remaining part of the paper, we will consider the group of N robots as one leader, R_N , and $N - 1$ as *followers*, R_j , where $j = 1, \dots, N - 1$.

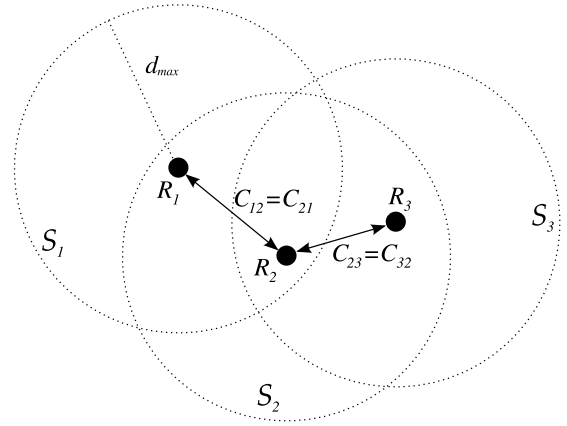


Fig. 2. Illustration of a robot group with three robots and subgroups

Note that the indexing of the robots is totally irrelevant as far as the problem we discuss here and the solution we propose. Yet we shall consider the number of the robots as above for the sake of notational simplicity.

Using these definitions and assuming that a set of robots initially form a connected group; our objective now is to develop a decentralized steering methodology that yields efficient navigation of the group while preserving its connectivity in the sense of Definition 2.

III. AUTONOMOUS MOTION

Our goal is to develop a methodology for simple autonomous agents, such that, a large group of such simple agents could navigate as a whole connected group, similar to those of some animal species in the nature. At any time t , let $t + \Delta t$ be the next sampling time, where $\Delta t > 0$ is the interval at which a robot senses the positions of other robots in its range. To take measurement errors on the distance into account, we define a positive scalar d_m as

$$d_m \stackrel{\text{def}}{=} d_{max} - \Delta r \quad (1)$$

where $\Delta r > 0$ is the bound of the distance measurement error. We will denote the position of a robot R_i at time t , as $X_i(t)$, $i = 1, \dots, N$. Since all robots in the group steer autonomously, we will set up local moving rules for each robot. Here, we propose a local steering strategy that is inspired by the preliminary study of Reynolds in [1]. At each time t , while the leader R_N aims in a given global target direction, each follower robot R_j , $j = 1, \dots, N - 1$, acquires the positions of other robots in its sensor range, i.e. in the subgroup \mathcal{S}_j , and determines a local target location for itself. This is most conveniently described in terms of the local coordinates of R_j , with R_j being at the origin. Let us denote the position vector in local coordinates as $x(t)$. We will use a notation such that the superscripts in x relate the coordinate frame to a robot, and the subscripts in x indicate which robot's position it is. For example, x_k^j represents the position vector of R_k in the coordinate frame of R_j . For the robots in \mathcal{S}_i ,

$i = 1, \dots, N$, we have

$$x_k^i(t) \stackrel{def}{=} X_k(t) - X_i(t), \quad k = 1, \dots, M \quad (2)$$

where M is the number of robots in \mathcal{S}_i . Then, we propose the following local steering strategy.

Local Steering Strategy: At each sampling time t , each robot in the group computes a target location for the time $t + \Delta t$ and moves towards that location, such that,

- 1) for each follower robot R_j , $j = 1 \dots, N - 1$, the target location $x_j^j(t + \Delta t)$ minimizes a cost function, $J(x_j^j(t + \Delta t))$, derived from the positions of the robots in \mathcal{S}_j ,
- 2) for the leader R_N , the target location is always towards the given global target,

and for both R_N and the follower robots R_j , the movement is subject to

$$\|x_i^i(t + \Delta t)\| \leq \frac{1}{2}(d_m - \max_k \|x_k^i(t)\|) \quad (3)$$

where $i = 1, \dots, N$ and $x_k^i(t)$ is as defined in (2).

Note that both $x_k^i(t)$ and $x_i^i(t + \Delta t)$ are positions in coordinates of the local reference frame attached to R_i at time t . The position that R_i is aiming at, i.e. $x_i^i(t + \Delta t)$, is restricted by an upper bound in (3). This upper bound is incorporated to guarantee the connectivity of the group.

Several types of cost functions can be used in implementing the local steering strategy. One example of these cost functions may be given for each follower robot R_j as

$$J(x_j^j(t + \Delta t)) = \max_k \|x_j^j(t + \Delta t) - x_k^j(t)\|,$$

which makes each robot try to decrease the distance to the farthest robot that it senses. Another possible approach could be to force the robots to keep their distances with the robots in their subgroups as close to a desired distance as possible. Let us denote the desired distance as d_0 ($d_0 \leq d_m$). Then we can define a suitable cost function for each follower robot R_j , $j = 1, \dots, N - 1$, as

$$J(x_j^j(t + \Delta t)) = \sum_{k=1}^M \left(\|x_j^j(t + \Delta t) - x_k^j(t)\| - d_0 \right)^2. \quad (4)$$

Regarding the connectivity of the group we can state the following theorem.

Theorem 1: Consider a group \mathcal{G} of N autonomous mobile robots which are connected at $t = 0$ as defined in Definition 2. If the robots in the group move according to the Local Steering Strategy defined above, the group preserves its connectivity for $t > 0$.

Proof: Let R_a and R_b be any two robots within their mutual sensing range, that is, $R_a \in \mathcal{S}_b$ and $R_b \in \mathcal{S}_a$ at time t . Let M_a and M_b be the number of robots in \mathcal{S}_a and \mathcal{S}_b , respectively. From (3), we have

$$2\|x_a^a(t + \Delta t)\| + \max_k \|x_k^a(t)\| \leq d_m, \quad k = 1, \dots, M_a \quad (5)$$

and

$$2\|x_b^b(t + \Delta t)\| + \max_l \|x_l^b(t)\| \leq d_m, \quad l = 1, \dots, M_b. \quad (6)$$

Noting that $\max_k \|x_k^a(t)\| \geq \|x_b^a(t)\|$, $\max_l \|x_l^b(t)\| \geq \|x_a^b(t)\|$, and $\|x_b^a(t)\| = \|x_a^b(t)\|$, it follows from (5) and (6),

$$\|x_a^a(t + \Delta t)\| + \|x_b^b(t + \Delta t)\| + \|x_b^a(t)\| \leq d_m. \quad (7)$$

Further, by triangular inequality, we get

$$\|x_a^a(t + \Delta t) - (x_b^a(t) + x_b^b(t + \Delta t))\| \leq d_m. \quad (8)$$

Note that the term $x_b^a(t) + x_b^b(t + \Delta t)$ is the position of R_b at time $t + \Delta t$ as expressed in the local coordinate frame attached to R_a at time t . Therefore, (8) shows that the distance between the robots R_a and R_b will not be larger than d_m . This holds for any robots R_a and R_b providing $R_a \in \mathcal{S}_b$ and $R_b \in \mathcal{S}_a$. Hence, any two robots sensing each other at time t will still be linked when they moved to their locations at $t + \Delta t$; that is, they will *stay connected*. Since group connectivity is formed through those robots which sense each other, and the group is connected at $t = 0$, it will also be connected for $t > 0$. ■

Remark 1: As long as the robots move according to the restriction stated in Theorem 1, each subgroup \mathcal{S}_i will keep the robots that are initially in \mathcal{S}_i . So, the number of robots in \mathcal{S}_i will not decrease; nevertheless, it may grow as new robots appear in the sensing range of R_i and hence, become a member of \mathcal{S}_i .

IV. SOLUTION TO A POSSIBLE DEADLOCK PROBLEM

The upper bound for the motion given in (3) may cause, in a very special case, all the robots in the group go into a *deadlock*. An example of such a case can occur if the initial distances between *all* robots are d_m exactly, which is in fact a pathological case. Note that Theorem 1 is still valid in this case, i.e. connectivity is preserved; nevertheless, the robots cannot move.

Considering the magnitude constraint given in (3); if one of the robots in \mathcal{S}_j is at a point that is as far as d_m to R_j , then the robot R_j is not allowed to move at that sampling instant. Note that although R_j stays stationary, the other robots in \mathcal{S}_j keeps moving and their movements possibly resolves the lock on the movement of R_j . But the deadlock case occurs if all the robots in the group are as distant to one another as at least d_m . To avoid such a deadlock, we will relax the magnitude constraint in (3) for the robots that are at the *corners* of the subgroups.

Definition 4: The **corner robots** are the robots which are in the set spanning the convex hull of all the robots in the group.

Fig. 3 depicts an example of a deadlock situation in \mathbb{R}^2 , where all the distances indicated by dotted lines are as large as d_m . In Fig. 3, the robots R_1, R_2, R_4, R_5 and R_7 are the corner robots. The movement constraint given in (3) is too restrictive for the corner robots, as the directions obtained by the minimization of the cost function given by (4) in fact push them towards the other robots in their subgroups. As it is seen, the displacements of the corner robots will not be so as to break up the connectivity; moreover, they will get even closer to the other members of the group. Hence, it is necessary for a robot to determine whether it is a corner robot or not. If so,

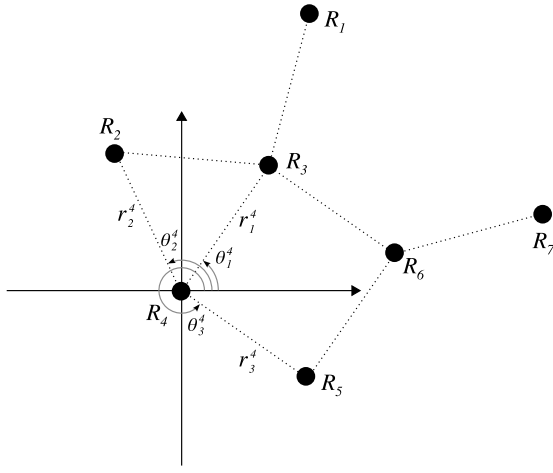


Fig. 3. An example of the deadlock situation in the local coordinate frame of R_4 in \mathbb{R}^2

it will omit the constraint in (3) as long as it is at the corner of the subgroup.

Whether a robot is at the corner of the subgroup can be determined most easily in polar coordinates. If the working space is \mathbb{R}^2 , location of each robot in \mathcal{S}_j , i.e. $x_k^j(t)$, is described by the pair (r_k^j, θ_k^j) , $k = 1, \dots, M$, where r_k^j and θ_k^j are the distance from the origin and the angle of the k^{th} -robot in \mathcal{S}_j , respectively, in the local coordinate frame of R_j . As an example, Fig. 3 includes the local polar coordinate frame of R_4 where the locations of the robots sensed by R_4 , namely R_3 , R_2 and R_5 , are indicated by the distances r_1^4 , r_2^4 , r_3^4 , and the angles θ_1^4 , θ_2^4 , θ_3^4 , respectively. In \mathbb{R}^3 , $x_k^j(t)$ is given by the triple $(r_k^j, \phi_k^j, \theta_k^j)$, where r_k^j is the distance from the origin, ϕ_k^j is the azimuth angle and θ_k^j is the zenith angle. For both \mathbb{R}^2 and \mathbb{R}^3 , the condition for a robot R_j to be a corner robot is

$$\max_k \theta_k^j(t) - \min_k \theta_k^j(t) < 180^\circ - \Delta\theta, \quad k = 1, \dots, M \quad (9)$$

where $\Delta\theta > 0$ is the maximum angular measurement error.

Note that if there is only one robot in \mathcal{S}_j and $r_1^j = d_m$, a small possibility is that $\theta_1^j = 180^\circ$ initially. But this does not give rise to a difficulty since the local coordinate frame of R_j could be rotated in such a case so that $\theta_1^j < 180^\circ$. More generally, before checking the condition in (9), a rotation of the local coordinate frame may be applied if necessary so that $\theta_1^j = 0^\circ$ is satisfied always. Also note that if the position measurements are available in Cartesian coordinates, they can always be transformed to polar coordinates easily. We can now revise the constraint in the second condition of the Local Steering Strategy, such that; for both R_N and the follower robots R_j , the movement is subject to

$$\|x_i^j(t + \Delta t)\| \leq \frac{1}{2}(d_m - \max_k r_k^i(t)), \quad (10)$$

$$\text{if } (\max_k \theta_k^i(t) - \min_k \theta_k^i(t)) \geq 180^\circ - \Delta\theta$$

Next, we will propose an easily computable method for the autonomous motion of each robot.

V. SUB-OPTIMAL SOLUTION TO LOCAL STEERING PROBLEM

The robots in this study are supposed to be quite simple and limited devices especially from computational point of view. Our purpose is to provide a decentralized control methodology which can be applied to such simple robots yet leading to a satisfactorily good group navigation. Below, we propose an iterative method to reduce the computational burden in the implementation of the Local Steering Strategy.

The minimum points of the cost function given in (4) are the locations where each follower robot R_j aims to reach at each sampling time. The minimization of (4) requires higher computation power as the number of the robots in \mathcal{S}_j increases.

When there is only one robot, say R_m , in a subgroup \mathcal{S}_j , the solution is a circle in \mathbb{R}^2 or a sphere in \mathbb{R}^3 . In this case, the solution set has infinite number of points which minimize J , but the robot R_j selects the nearest point in the solution set. This leads to the movement of R_j in the line which connects R_j to R_m . The movement direction is either towards R_m if the distance between R_j and R_m is larger than d_0 , or away from R_m if the distance is smaller than d_0 . Obviously, no movement is required if R_j is already lying on an optimal point at that time.

When there are only two robots, say R_m and R_n , in \mathcal{S}_j , one may consider different cases as far as the number of the optimal points is concerned. If the distance between R_m and R_n are larger than or equal to $2d_0$, the optimal point is unique both in \mathbb{R}^2 and \mathbb{R}^3 , and it is at the center of the line segment which connects R_m and R_n . Otherwise, in \mathbb{R}^2 , there are two optimal points which lie symmetrically at each side of the line connecting R_m and R_n . The robot R_j selects the nearest point as the local target. In \mathbb{R}^3 , number of the optimal points are infinite and they lie in a circle whose center is the center of the line segment connecting R_m to R_n . This line segment is also normal to the circle of solution points. Again, the robot R_j aims to the point on this circle which is nearest to itself.

As it is seen, the computation task is relatively easy when only one or two robots are sensed by each follower robot at a time. However, whenever three or more robots are in a subgroup \mathcal{S}_j , the minimization of J given by (4) is relatively more difficult. At the extremum points, we have

$$\frac{\partial J}{\partial x_j^j} = \sum_{k=1}^M \frac{\partial}{\partial x_j^j} \left(\|x_j^j(t + \Delta t) - x_k^j(t)\| - d_0 \right)^2 = 0 \quad (11)$$

which results in a nonlinear set of equations. The solution of the system in (11) may not be unique. After solving this system, the solution points must then be tested for being minimum. If more than one minimum are present, we find the global minimum by evaluating the cost function given in (4) at these points.

It should be noted that, the optimal points are computed for each follower robot R_j at every sampling time. The location of the optimal points depends on the positions of the robots in the subgroup \mathcal{S}_j . Since the sensed robots in \mathcal{S}_j also move autonomously, the location of the computed optimal points

will change at each time. This change will very likely happen before they are reached by R_j . Hence, the solution will provide only a direction to the optimal points, because the solution will change before R_j gets to that location.

This fact gives us the chance to decrease the computational burden in the local steering strategy. Rather than solving for the minimum points of the cost function, each robot R_j can move in the direction of negative gradient of the cost function, evaluated at the position of R_j for each time. That is,

$$x_j^j(t + \Delta t) = x_j^j(t) - \gamma \left. \frac{\partial J(x_j^j(t + \Delta t))}{\partial x_j^j(t + \Delta t)} \right|_{x_j^j(t + \Delta t) = x_j^j(t)} \quad (12)$$

where $\gamma > 0$ is a positive gain, and x_j^j is the position vector of R_j in its local coordinates. From (4) it follows that

$$\begin{aligned} \frac{\partial J(x_j^j(t + \Delta t))}{\partial x_j^j(t + \Delta t)} &= 2 \sum_{k=1}^M \left(\|x_j^j(t + \Delta t) - x_k^j(t)\| - d_0 \right) \\ &\quad \times \frac{x_j^j(t + \Delta t) - x_k^j(t)}{\|x_j^j(t + \Delta t) - x_k^j(t)\|}. \end{aligned} \quad (13)$$

Further, since $x_j^j(t) = 0$, from (12) and (13) we obtain

$$x_j^j(t + \Delta t) = 2\gamma \sum_{k=1}^M \left(\|x_k^j(t)\| - d_0 \right) \frac{x_k^j(t)}{\|x_k^j(t)\|}. \quad (14)$$

The gain γ in (14) is adjusted to assure the constraint given by (10). The application of (14) is much simpler than solving the system in (11). It gives the direction of the next movement and the movement in this direction is realized only if it satisfies the inequality in (10). In the next section, we will test the proposed methodology with the local steering of robots in the direction of negative gradient, as given in (12).

VI. SIMULATION RESULTS

We verify the theoretical results of the previous sections by computer simulations. The simulations are performed in MATLAB and the working space is a section of xy -plane in \mathbb{R}^2 . The sensor range (d_{max}) is 13 units, and desired distance (d_0) is 8.5 units. The bounds on the measurement errors are $\Delta\theta = 12^\circ$ for angle and $\Delta r = 0.01d_{max}$ for distance measurement. The following scenario is applied: A group of 32 robots start navigation under the Local Steering Strategy. The local steering is achieved by the use of (14) with the gain $\gamma = 0.07$. Total simulation length is 4633 iterations.

In Fig. 4, first part of the simulation is seen. The initial positions are displayed in gray. Initially, we have a group of scattered but connected robots. As soon as the simulation starts, the Local Steering Strategy forces each robot to form a more compact group and rapidly increases the group connectivity. The remaining part of the simulation is seen in Fig. 5. The trajectory of the leader is seen as dashed lines in these figures. The sharp turns in the trajectory of the leader and the reflections from the walls are important, as they perturbate the shape of the group. However, the group preserves the connectivity throughout the navigation.

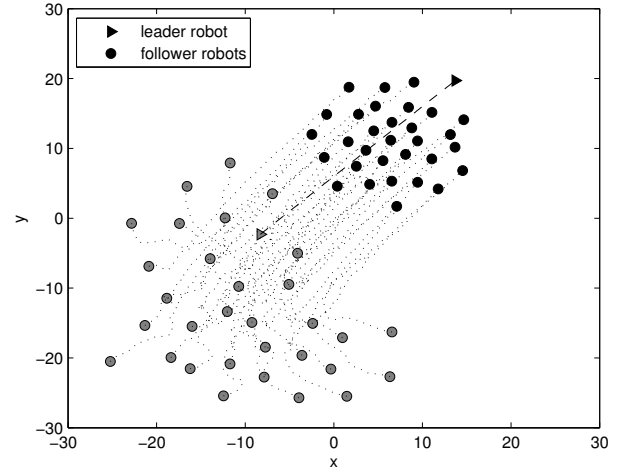


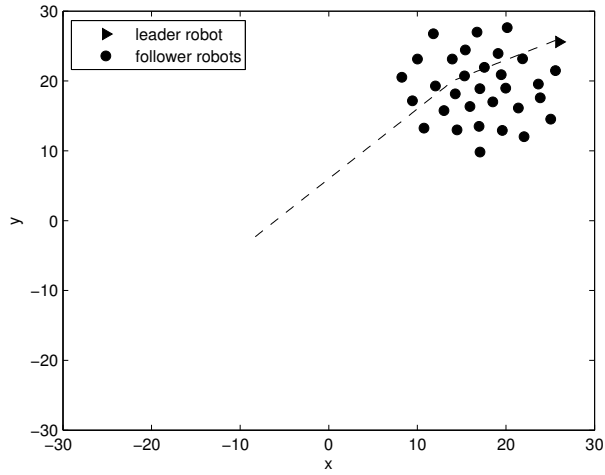
Fig. 4. Navigation of 32 robots (the grayed markers show initial positions)

Fig. 6 displays the connectivity of the group during the navigation. Note that for the case of 32 robots, the maximum possible number of links in the group is $32 \cdot (32 - 1) / 2 = 496$. Initially, the group starts navigation with a connectivity formed of 167 links. The number of links increases quickly as the navigation continues. After the 130th-iteration, the number of total links in the group rises over 400. At the end of the simulation, there are 433 total links in the group. It is clearly seen that the connectivity is both preserved and increased during the navigation with the proposed scheme.

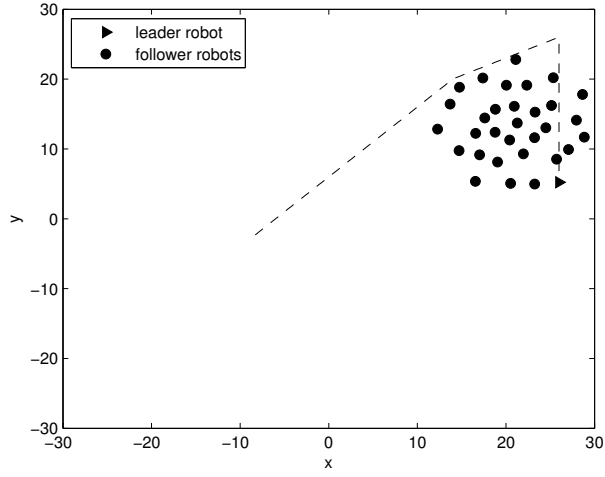
VII. CONCLUSIONS

The methodology presented in [16] which guarantees the connectivity of mobile robot groups has been extended in this study. The limited-range sensor models are assumed to have angular and longitudinal position measurement errors, in order to be more realistic. These measurement errors were included as extra restrictions on both angle and distance measurements. Also, a solution to some pathological cases in which a deadlock situation might occur has been proposed.

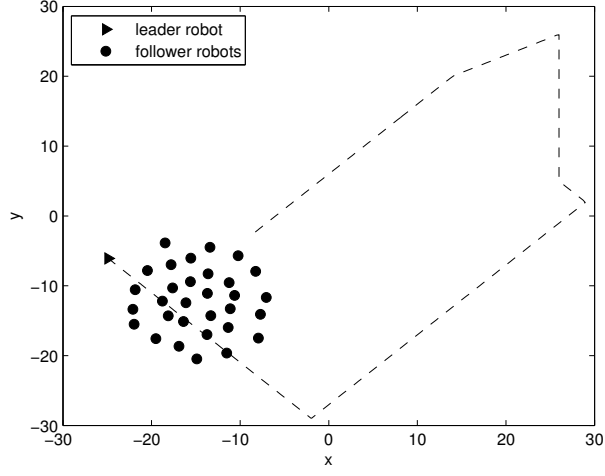
The fact that no communication or hierarchy among the robots is required allows new members to be accepted to the group very easily. Similarly, the departure of some members from the group does not cause any problems for the rest of the group as long as they do not break the overall connectivity. The simulations have verified the success of the methodology. The groups have not only preserved but also increased connectivity during navigation.



(a)



(b)



(c)

Fig. 5. Navigation of 32 robots (continued)

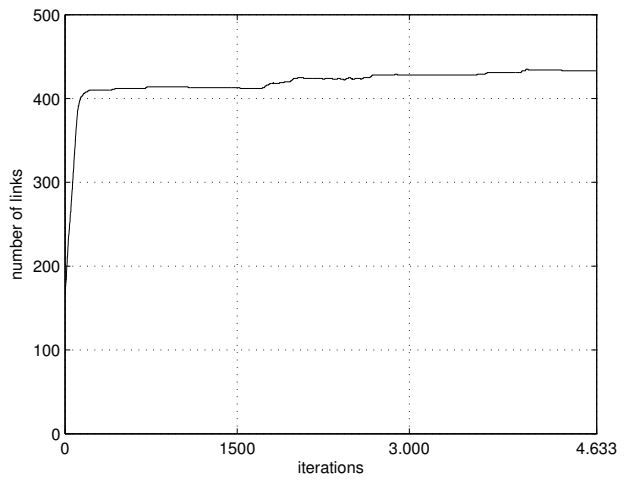


Fig. 6. Connectivity of the group of 32 robots

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