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2 authors:


Metin Sengul
Kadir Has University


S.B. Yarman
Istanbul University

PAPER

Broadband Equalizer Design with Commensurate Transmission Lines via Reflectance Modeling

Metin ŞENGÜL[†a)], *Nonmember and* Sıddık B. YARMAN[††], *Member*

SUMMARY In this paper, an alternative approach is presented, to design equalizers (or matching networks) with commensurate (or equal length) transmission lines. The new method automatically yields the matching network topology with characteristic impedances of the commensurate lines. In the implementation process of the new technique first, the driving point impedance data of the matching network is generated by tracing a pre-selected transducer power gain shape, without optimization. Then, it is modelled as a realizable bounded-real input reflection coefficient in Richard domain, which in turn yields the desired equalizer topology with line characteristic impedances. This process results in an excellent initial design for the commercially available computer aided design (CAD) packages to generate final circuit layout for fabrication. An example is given to illustrate the utilization of the new method. It is expected that the proposed design technique is employed as a front-end, to commercially available computer aided design (CAD) packages which generate the actual equalizer circuit layout with physical dimensions for mass production.
key words: *broadband matching, equalizer design, impedance modeling, fixed point iteration, commensurate transmission lines*

1. Introduction

In designing high frequency communication systems, as the wave length of the operation frequency becomes comparable with physical size of the lumped circuit elements, usage of distributed elements are inevitable.

Therefore, at Radio Frequencies (RF) or for wireless communication systems, design of wideband matching networks or so called equalizers with distributed elements or commensurate transmission lines have been considered as an important problem for engineers [1]. In this regard, analytic theory of broadband matching may be employed for simple problems [2], [3]. However, it is well known that beyond simple problems, this theory is inaccessible. Therefore, for practical applications, it is always preferable to employ CAD techniques, to design equalizers with transmission lines. In this approach, designers must supply a circuit topology and initial element values [4]–[6]. All the commercially available CAD techniques, optimizes the matched system performance. It should be mentioned that performance optimization is highly nonlinear with respect to line impedances and delay lengths, and therefore selection of initial values is crucial for successful optimization, since the convergence of the optimization depends on these initials. Therefore, in this paper, a well-established calculation methodology of good initial values is introduced, to design matching networks with equal length or commensurate transmission lines. These lines are also called Unit Elements (UE). The new method is based on the reflectance modeling via fixed point iteration. In the following sections first, the theoretical aspects of the new method is introduced. Then, the implementation algorithm is presented. Finally, utilization of the new algorithm is exhibited by designing a matching network for a passive load.

2. Generation of the Rough Estimate for the Driving Point Input Impedance of the Matching Network

Let us consider the single matching arrangement as shown in Fig. 1. It is well known that the matching network [N] can completely be specified by the Positive Real (PR) driving point impedance Z_2 or by the corresponding Bounded Real (BR) reflectance $S_{22} = \frac{Z_2-R_0}{Z_2+R_0}$; with R_0 being the standard normalization number of 50 ohms. If one generates $Z_2(\mathrm{j}\omega) = R_2(\omega) + \mathrm{j}X_2(\omega)$ as a proper data set to optimize the transducer power gain (TPG) of the matched system, then it can be modeled as a PR impedance which in turn yields the desired matching network via synthesis. In fact, Carlin's Real Frequency Line Segment Technique (RFLST) is known as the best method, to generate the proper or realizable data set for Z_2 [7], [8]. In Carlin's approach, Z_2 is assumed to be minimum reactance function, and its real part $R_2(\omega)$ is represented by line segments such that $R_2 = \sum_{k=1}^{m} a_k(\omega)R_k$, passing through m-selected pairs designated by $\{R_k, \omega_k; k = 1, 2, \ldots, m\}$. In this regard, break points (or break resistances) R_k are considered as the unknowns of the matching problems. Then, these points are determined via nonlinear optimization of TPG, expressed as

$$TPG = \frac{4R_2R_L}{(R_2 + R_L)^2 + (X_2 + X_L)^2}. \tag{1}$$

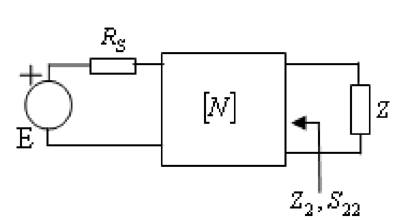

Fig. 1 Single matching arrangement.

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[†]The author is with the Faculty of Engineering, Kadir Has University, 34083 Cibali, Fatih, İstanbul, Turkey.
[††]The author is with the Faculty of Engineering, İstanbul University, Avcılar, İstanbul, Turkey.
a) E-mail: msengul@khas.edu.tr
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In (1), R_L and X_L are the real and the imaginary parts of the load data $Z_L(j\omega) = R_L(\omega) + jX_L(\omega)$ respectively, and the imaginary part $X_2(\omega) = \sum_{k=1}^{m} b_k(\omega)R_k$ of Z_2 is also expressed by means of the same break points R_k. In the above representations, $\omega = 2\pi f$ is the normalized angular frequency. It is noted that coefficients $a_k(\omega)$ are known quantities, and they are determined in terms of the pre-selected angular break frequencies (or in short break frequencies) ω_k which specify frequency location of the break points R_k. Similarly, for minimum reactance Z_2, coefficients $b_k(\omega)$ are also known, and generated by means of Hilbert Transformation Relation given for minimum reactance functions. In this case, let $H\{\circ\}$ designates the Hilbert transformation operator, then $b_k(\omega) = H\{a_k(\omega)\}$.

In the new technique proposed in this paper, the Real Frequency Line Segment Technique is simply omitted, and data for Z_2 are generated without optimization in a straight forward manner as follows.

For a desired shape of $TPG = T(\omega)$ which can even be specified as a set of data points, the ratio defined by $\alpha = R_2/R_L$ can directly be computed under the perfect cancellation condition of the imaginary parts (i.e., $X_2 = -X_L$). Actually, this assumption is a practical one, which maximizes TPG of the matched system over the band of operation.

On the other hand, it is well known that existence of the load network will lower the ideal flat gain from $T(\omega) = 1$, down to a level $T_{flat} < 1$ in the pass band. Furthermore, TPG must decrease monotonically out side of the band. In this case, one can always select a reasonable-realizable shape for TPG, such as Butterworth or Chebyshev forms, and then, generates the ratio specified by $\alpha = R_2/R_L$ under the perfect cancellation condition. Thus, the data set for the driving point impedance Z_2 given by

$$Z_2(j\omega_i) = R_2(\omega_i) + jX_2(\omega_i) \tag{2}$$

is computed over the angular frequencies ω_i of the load network.

Let us now derive the ratio $\alpha = R_2/R_L$ when perfect cancellation occurs on the imaginary parts. In this case, TPG is given by

$$TPG = \frac{4R_2R_L}{(R_2 + R_L)^2} \tag{3}$$

or

$$\alpha = \frac{R_2}{R_L} = \frac{2 - TPG + 2\mu\sqrt{1 - TPG}}{TPG} \tag{4}$$

where $\mu = \mp 1$ is a uni-modular constant and lands itself while taking the square-root of $(1 - TPG)$. Obviously, α is derived as a function of the transducer power gain. Hence, for a selected-suitable gain form, the impedance $Z_2 = R_2 + jX_2$ is approximated as

$$R_2 = \alpha R_L, \tag{5}$$

$$X_2 = -X_L. \tag{6}$$

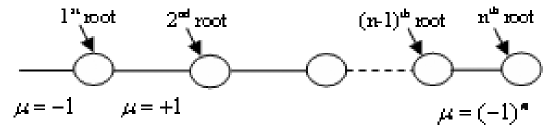

Fig. 2 Selection of the sign of μ.

At this point, it is crucial to choose the form for TPG, to describe the matched system performance. In this regard, it may be desirable, to have an equal ripple gain shape within the pass band as desired in many practical problems. Then, the following Chebyshev form may be used for TPG,

$$TPG = \frac{T_{max}}{1 + \epsilon^2 T_n^2(\omega)} \tag{7}$$

where ϵ is the ripple factor and T_n is the nth order Chebyshev polynomial. The degree n specifies the total number of elements in the equalizer topology. TPG takes its maximum value T_{max} at the zeros of the Chebyshev polynomial $T_n(\omega)$. It is minimum ($TPG = T_{min}$) when $T_n(\omega) = 1$. Obviously, ϵ is specified by

$$\epsilon^2 = \frac{T_{max} - T_{min}}{T_{min}} \tag{8}$$

and the average (T_{av}) gain level is determined as

$$T_{av} = \frac{T_{max} + T_{min}}{2} \tag{9}$$

Based on the gain-bandwidth theory [1]–[3], as the total number of elements (n) in the equalizer goes to infinity, TPG approaches to its ideal (or maximum) flat (or constant) value T_{flat} over the passband. In this case, it may be sufficient to approximate T_{flat} as

$$T_{flat} \approx T_{av}. \tag{10}$$

Beyond simple matching problems, it is almost impossible to determine the ideal value of T_{flat}. However, it may be assessed by trial and error as in (10).

It is interesting to note that selection of the sign of the uni-modular constant μ of (4) is important, to end up with realizable driving point impedance Z_2. In this regard, it is appropriate to flip the sign of μ along the frequency axis as transducer power gain fluctuates around the mean value T_{flat} within the pass band. For example, when working with Chebyshev forms of (7), it is known that TPG changes its direction of movement up and down at the roots of the Chebyshev polynomial $T_n(\omega^2)$. Starting with $\mu = -1$, the sign of μ is flipped as the frequency ω_i of $\alpha(\omega_i)$ moves between the roots of $T_n(\omega^2)$ of (7) as shown by Fig. 2.

Once, the data for the driving point impedance $Z_2(j\omega)$ is generated, then it is modeled employing the reflectance based fixed point iteration method presented in the following section. Finally, the reflectance model is synthesized yielding the desired equalizer topology with initial element values. Eventually, performance of the matched system is further optimized utilizing the commercially available CAD packages.

3. Reflectance Based Data Modeling

In this section, the reflectance data specified by $S_{22} = \frac{Z_2 - R_0}{Z_2 + R_0}$ are considered as the input reflection coefficient of a lossless equalizer [N], and it is modeled as a BR rational scattering coefficient in Belevitch form in terms of the so-called Richards variable $\lambda = \Sigma + j\Omega = \tanh(p\tau)$ where τ designates the equal or unit delay in seconds of the transmission lines, and $p = \sigma + j\omega$ is the classical complex frequency variable. If the equalizer topology is constructed with equal length transmission lines or called Unite Elements (UE), then the input reflection coefficient can be expressed as

$$S_{22}(\lambda) = \frac{h(\lambda)}{g(\lambda)} \tag{11}$$

or by setting $\Sigma = 0$,

$$S_{22}(j\Omega) = \frac{h(j\Omega)}{g(j\Omega)} = S_R(\Omega) + jS_X(\Omega) \tag{12}$$

where $\Omega = tan(\omega\tau)$.

On the transformed angular real frequency axis (or in short "frequency"), let the numerator polynomial be

$$h(j\Omega) = h_R(\Omega) + jh_X(\Omega) \tag{13}$$

and the denominator polynomial be

$$g(j\Omega) = g_R(\Omega) + jg_X(\Omega). \tag{14}$$

Then, at a selected frequency Ω_i, one can readily obtain h_R and h_X as

$$h_R = [g_R S_R - g_X S_X] \tag{15}$$

and

$$h_X = -[g_X S_R + g_R S_X]. \tag{16}$$

The above equations indicate that, if the denominator polynomial $g(j\Omega) = g_R(\Omega) + jg_X(\Omega)$ is known, then the numerator polynomial $h(j\Omega) = h_R(\Omega) + jh_X(\Omega)$ can readily be obtained. In fact, this way of thinking constitutes the crux of the fixed point iteration method in the following manner.

From (11), one can write

$$h(\lambda) = S_{22}(\lambda)g(\lambda) \tag{17}$$

or employing the concept of interpolation, at a given single frequency Ω_i, the following equation must be satisfied

$$h(j\Omega_i) = S_{22}(j\Omega_i)g(j\Omega_i). \tag{18}$$

Since $S_{22}(\lambda) = \frac{h(\lambda)}{g(\lambda)}$ belongs to a lossless-reciprocal two-port, which is specified by the given data, then rest of the scattering parameters of [N] are also represented in Belevitch form as

$$S_{12}(\lambda) = S_{21}(\lambda) = \frac{f(\lambda)}{g(\lambda)}, S_{11}(\lambda) = -\frac{f(\lambda)}{f(-\lambda)}\frac{h(-\lambda)}{g(\lambda)} \tag{19}$$

satisfying the losslessness condition of

$$gg_* = hh_* + ff_* \tag{20}$$

where "*" designates the complex conjugate (or para conjugate) of the given complex valued quantity. Thus, on the real frequency axis, $|S_{21}|^2$ is given by

$$|S_{21}|^2 = 1 - \rho^2 = \frac{|f|^2}{|g|^2} = \frac{|f|^2}{g_R^2 + g_X^2}. \tag{21}$$

It should be noted that the numerator polynomial $f(\lambda)$ of $S_{21}(\lambda)$ includes transmission zeros of the matching network to be designed. For example, if the equalizer topology consists of n_C number of cascaded UEs, then,

$$f(\lambda) = (1 - \lambda^2)^{n_C/2}. \tag{22}$$

This means that for a given BR reflection coefficient $S_{22} = S_R + jS_X$, one can readily compute the amplitude square of the denominator polynomial g, by selecting the form of f. Thus,

$$|g|^2 = g_R^2 + g_X^2 = \frac{|f|^2}{1 - \rho^2}. \tag{23}$$

Hence, (23) describes a known quantity over the specified frequencies with pre-selected f. In this case, the Hurwitz polynomial $g(\lambda)$ can be constructed by means of well established numerical methods [9].

Briefly, data points given by (23) for $\left|g(j\Omega)\right|^2$, describe an even polynomial such that

$$G(\Omega^2) = G_0 + G_1\Omega^2 + \ldots + G_n\Omega^{2n} > 0;\ \forall\Omega. \tag{24}$$

Coefficients $\{G_0, G_1, \ldots, G_n\}$ can easily be found by linear or nonlinear interpolation or curve fitting methods. Then, replacing Ω^2 by $-\lambda^2$, one can extract $g(\lambda)$ from $G(-\lambda^2) = g(\lambda)g(-\lambda)$ using explicit factorization techniques. At this point, the roots of $G(-\lambda^2)$ may be computed, and then $g(\lambda)$ is constructed on the left half-plane (LHP) roots of $G(-\lambda^2)$ as a strictly Hurwitz polynomial.

Once $g(\lambda)$ is generated, then $g_R = Re\{g(j\Omega)\}$ and $g_X = Im\{g(j\Omega)\}$ are computed which in turn yields the numerical pair of $\{h_R, h_X\}$ by means of (15) and (16). Let $h(\lambda) = \sum_{k=0}^{n} h_k\lambda^k$ designate the numerator polynomial of $S_{22}(\lambda) = \frac{h(\lambda)}{g(\lambda)}$. In this representation $\{h_k; k = 0, 1, 2, \ldots, n\}$ are the arbitrary real coefficients, and n specifies the total number of elements in the matching network.

Thus, data points corresponding to the real and the imaginary parts of $h(j\Omega)$ are given by

$$h_R(\Omega) = \sum_{0}^{m}(-1)^k h_{2k}\Omega^{2k} \tag{25}$$

where $m = \frac{n}{2}$ if n is even. $m = \frac{n-1}{2}$ if n is odd, and

$$h_X(\Omega) = \sum_{1}^{m}(-1)^{k-1}h_{2k-1}\Omega^{2k-1} \tag{26}$$

where $m = \frac{n}{2}$ if n is even. $m = \frac{n+1}{2}$ if n is odd.

Then, one can immediately determine the unknown real coefficients $\{h_k; k = 0, 1, 2, \ldots, n\}$ by means of straight linear interpolation over the selected frequencies.

At this point it is crucial to point out that polynomials $g(\lambda)$ and $h(\lambda)$ must satisfy the losslessness condition of $g(\lambda)g(-\lambda) = h(\lambda)h(-\lambda) + f(\lambda)f(-\lambda)$ rather than on the frequencies selected for interpolation. Therefore, herewith, an iterative approach which is named as the "Interpolation via Fixed Point Iteration" is introduced which yields the consistent triple of $\{h(\lambda), g(\lambda), f(\lambda)\}$ satisfying the losslessness condition.

3.1 Fixed Point Interpolation of $h(\lambda)$

In this section, let us first briefly review the fixed point iteration technique, as it is described in classical numerical analysis text books such as [10].

Zeros of a nonlinear function $\Re(X) = F(X) - X$ can be determined using the iterative loop described by

$$X_r = F(X_{r-1}). \tag{27}$$

It is straight forward to prove that for any initial guess X_0, (27) converges to one of the real root $X_{root} = lim_{r\to\infty}F(X_r)$ if and only if $\left|\frac{dF}{dX}\right| < 1;\ \forall X$.

For the problem under consideration, in fact, the numerator polynomial $h(\mathrm{j}\Omega)$ can be determined point by point by means of an iterative process which may be described employing (18) over the selected frequencies Ω_i such that

$$h_r(\mathrm{j}\Omega) = S_{22}(\mathrm{j}\Omega)g_{r-1}(\mathrm{j}\Omega). \tag{28}$$

In this case, one has to show that the term $S_{22} \cdot g$ describes a function $h = F(h)$ for which $\left|\frac{dF}{dh}\right| < 1;\ \forall h$.

In the following, first the iterative process of (28) is described, then its convergence is proven.

After selecting $f(\lambda)$, in (28), $g_0(\lambda)$ is generated solely in terms of the given data $S_{22}(\mathrm{j}\Omega)$ employing the explicit factorization of (24) as described above. Then, the first loop is started by computing $h_1(\mathrm{j}\Omega_\mathrm{i}) = h_{R1}(\Omega_\mathrm{i}) + \mathrm{j}h_{X1}(\Omega_\mathrm{i})$ over the chosen set of frequencies $\{\Omega_\mathrm{i}; \mathrm{i} = 0, 1, 2, \ldots, n\}$, and using (25) and (26), analytic form of $h_1(\lambda)$ is obtained by means of a linear interpolation algorithm.

Employing the losslessness equation

$$\begin{aligned} G_1(-\lambda^2) &= g_1(\lambda)g_1(-\lambda) \\ &= h_1(\lambda)h_1(-\lambda) + f(\lambda)f(-\lambda) \\ &= G_{10} - G_{11}\lambda^2 + \ldots + (-1)^n G_{1n}\lambda^{2n}, \end{aligned} \tag{29}$$

$g_1(\lambda)$ is generated on the LHP roots of $G_1(-\lambda^2)$. Hence, the second iteration loop starts on the computed $g_1(\mathrm{j}\Omega_\mathrm{i})$ which yields $h_2(\mathrm{j}\Omega_\mathrm{i})$. Then, g_2 is constructed yielding h_3 etc. Iterative loops continue until $\|h_r - h_{r-1}\| \le \delta$. Here, δ is selected as a negligibly small positive number to terminate the iterations.

The above process describes the interpolation of $h(\lambda)$ via fixed point iteration over the selected frequencies. As a matter of fact, the denominator polynomial g can be described in terms of the numerator polynomial h by using losslessness condition,

$$\begin{aligned} g(\mathrm{j}\Omega) &= h(\mathrm{j}\Omega)\frac{h(-\mathrm{j}\Omega)}{g(-\mathrm{j}\Omega)} + f(\mathrm{j}\Omega)\frac{f(-\mathrm{j}\Omega)}{g(-\mathrm{j}\Omega)} \\ &= h(\mathrm{j}\Omega)S_{22}(-\mathrm{j}\Omega) + f(\mathrm{j}\Omega)S_{21}(-\mathrm{j}\Omega). \end{aligned} \tag{30}$$

Using (29) and (30) in (28) one obtains

$$h(\mathrm{j}\Omega) = h(\mathrm{j}\Omega)\rho^2 + S_{22}(\mathrm{j}\Omega)f(\mathrm{j}\Omega)S_{21}(-\mathrm{j}\Omega) \tag{31}$$

where $\rho = \left|S_{22}(\mathrm{j}\Omega)\right|$, and it is specified by the given data. In short, right hand side of (31) describes a function F in h such that $F(h) = h\rho^2 + S_{22}fS_{21}^*$.

Therefore, $h = F(h)$ describes a convergent fixed point iteration process provided that $\left|\frac{dF}{dh}\right| < 1$. In fact, $S_{21}(\mathrm{j}\Omega)$ is also specified by means of $S_{22}(\mathrm{j}\Omega)$ and pre-selected $f(\mathrm{j}\Omega)$. Then, practically, $\frac{dF}{dh} = \rho^2 < 1$ over the entire frequencies by bounded realness; except at isolated points where ρ hits unity. Thus, for the given reflection coefficient data, the polynomial form of $h(\lambda)$ is readily obtained via fixed point iteration of $h = F(h)$ which in turn results in a realizable driving point reflectance $S_{22}(\lambda) = \frac{h(\lambda)}{g(\lambda)}$.

The above results can be collected under the following theorem, to generate the reflectance based circuit model.

Theorem: Modeling via Fixed Point Iteration

Referring to Fig. 1, let $S_{22}(\mathrm{j}\Omega_\mathrm{i}) = S_R(\Omega_\mathrm{i}) + \mathrm{j}S_X(\Omega_\mathrm{i})$ be the input reflectance coefficient data of the lossless matching network $[N]$ specified over the frequency points Ω_i such that $\left|S(\mathrm{j}\Omega)\right| < 1$ for all frequencies. Let $\left\{S_{ij};\ i, j = 1, 2\right\}$ be the real normalized bounded real scattering parameters of the lossless matching network $[N]$ described in Belevitch sense. Once, the polynomial $f(\lambda)$ of $S_{21}(\lambda) = \frac{f(\lambda)}{g(\lambda)}$ is selected properly, then, the iterative process $h_r = F(h_{r-1})$ described by (28) is always convergent and yields the numerator polynomial $h(\lambda)$ of $S_{22}(\lambda) = \frac{h(\lambda)}{g(\lambda)}$ satisfying the losslessness condition of $g(\lambda)g(-\lambda) = h(\lambda)h(-\lambda) + f(\lambda)f(-\lambda)$.

Obviously, proof of this theorem follows as in above.

Depending on the modeling problem under consideration, numerical implementation of the fixed point iteration method may require some care. Therefore, in the following section some practical issues are covered.

4. Practical Issues

4.1 Transmission Zeros

In order to end up with a successful equalizer design, the fit between the generated reflectance data and the model must be as good as possible. In this regard, selection of transmission zeros of the equalizer which is specified by the transfer scattering parameter $S_{21}(\lambda) = \frac{f(\lambda)}{g(\lambda)}$ is quite important.

For most practical problems, it is sufficient to select $f(\lambda)$ as

$$f(\lambda) = \lambda^k(1 - \lambda^2)^{n_c/2} \tag{32}$$

and the denominator polynomial $g(\lambda)$ is given by

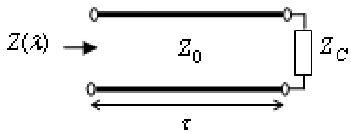

Fig. 3 A UE in complex termination.

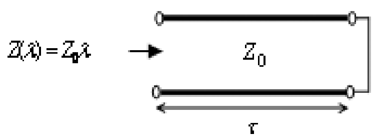

Fig. 4 Shorted UE stub.

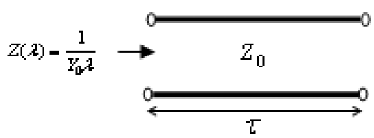

Fig. 5 Open-ended UE stub.

$$g(\lambda) = g_0 + g_1\lambda + \ldots + g_n\lambda^n. \tag{33}$$

In the above equations, integer k is the count of multiple transmission zeros introduced at DC (i.e., $\Omega = 0$) and n_c is the total number of commensurate transmission lines (or UE) in cascade connections, the integer $n_\infty = n - (n_c + k)$ is the count of multiple transmission zeros placed at infinity

4.2 Realization of Transmission Zeros and the Unit Elements in Cascade Connection

Real frequency transmission zeros of a structure consist of UEs may be realized by means of open and short circuited UE stubs as follows.

Referring to Fig. 3, the driving point input impedance of a UE which is terminated in arbitrary complex impedance Z_C is given by [8]

$$Z(\lambda) = Z_0\frac{Z_C + Z_0\tanh(p\tau)}{Z_0 + Z_C\tanh(p\tau)} = Z_0\frac{Z_C + Z_0\lambda}{Z_0 + Z_C\lambda}. \tag{34}$$

In (34), if $Z_C = 0$ (shorted UE) then, the stub $Z(\lambda) = Z_0\lambda$ acts as an inductance introducing a transmission zero at infinity in series configuration (Fig. 4). The same stub can be used in shunt configuration introducing a transmission zero at DC. On the other hand, if $Z_C = \infty$ (opened UE) then, $\left\{Z(\lambda) = \frac{1}{Y_0\lambda};\ with\ Y_0 = \frac{1}{Z_0}\right\}$ acts as a capacitor, introducing a transmission zero at DC or at infinity, in series or in shunt configuration, respectively, (Fig. 5).

4.3 Synthesis of the Reflection Coefficient $S_{22}(\lambda)$

Richards' Theorem provides the synthesis of the distributed structures made up of cascaded UE sections of different characteristic impedances in a sequential fashion [8], [11]. Referring to Fig. 6, consider the reflectance function and the corresponding input impedance at the beginning of the sequential synthesis process as $S_{22}(\lambda) = S_1^{(0)} = \frac{h(\lambda)}{g(\lambda)}$ *and* $z_1(\lambda) = \frac{1+S_1^{(0)}(\lambda)}{1-S_1^{(0)}(\lambda)}$, respectively.

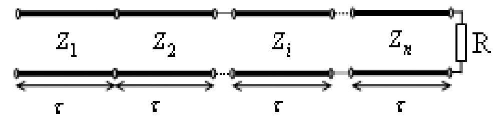

Fig. 6 Cascaded connection of UEs.

Then, at step (i), sequential cascaded extractions can be described by

$$S_{i+1}^{(i)}(\lambda) = \frac{S_i^{(i-1)}(\lambda) - S_i^{(i-1)}(1)}{1 - S_i^{(i-1)}(1)S_i^{(i-1)}(\lambda)}\frac{1+\lambda}{1-\lambda};\ i = n_c - 1 \tag{35}$$

$$z_{i+1}(\lambda) = z_i(1)\frac{z_i(\lambda) - \lambda z_i(1)}{z_i(1) - \lambda z_i(\lambda)};\ i = n_c - 1 \tag{36}$$

where $S_{i+1}^{(i)}(\lambda)$ and $z_{i+1}(\lambda)$ are reduced degree (degree of $n_c - i$) (BR) and Positive Real (PR) reflectance and impedance functions, respectively. (36) yields the normalized line characteristic impedances $r_i = z_i(1)$.

On the other hand, transmission zeros at DC and at infinity may easily be extracted by continuous fraction expansion of the PR impedance function when appropriate (i.e., usual long division of $z_{i+1}(\lambda)$).

Alternatively, Fettweis' decomposition method can be employed for the extraction of all the transmission zeros and cascaded UE [12]. Also the algorithm and formulae presented in [13] and [14] can be used to synthesize the cascade connected UEs, respectively.

4.4 Normalizations

In the course of design process, numerical stability is maintained by means of frequency and impedance normalizations. In other words, all the computations must be carried out in the normalized domain. Eventually, de-normalization is performed on the unit delay τ, and the line characteristic impedances of the matching network. In this regard, it may be appropriate, to normalize the frequencies at the upper edge of the frequency band. For the impedance normalization, standard $R_0 = 50$ *ohm* termination may be utilized. The normalized unit delay τ is fixed by means of

$$\omega_e\tau = \frac{2\pi}{\kappa} \tag{37}$$

where $\omega_e = 2\pi f_e$ is selected as the edge of the stop band in the normalized angular frequency domain. 2π corresponds the full wave length and κ designates the fraction of it, measured at the frequency f_e.

For example if the normalized angular cut-off frequency of the passband is fixed at $\omega_c = 1$ and if $\omega_e = 1.5\omega_c$ is chosen with the normalized line delay τ corresponding quarter wavelength (i.e., $\kappa = 4$ is chosen) at f_e then, it is found that normalized delay $\tau = \frac{\pi}{1.5\cdot 2} = 1.04719$. For de-normalization, if $f_e = 10\,\mathrm{GHz}$ then, actual line delay is $\tau = \frac{1}{4f_e} = 0.025\,\mathrm{nsec}$.

On the other hand, throughout the computations, one deals with the BR scattering parameters normalized with respect to standard normalization number $R_0 = 50$ ohms and eventually, actual line characteristic impedances are computed as $Z_{i+1} = R_0 \cdot z_{i+1}$.

4.5 Selected Forms of TPG

It has been experienced that utilization of monotone roll-off Chebyshev transfer functions are useful, to generate matching networks with initial element values. For low-pass prototypes, TPG is given by

$$TPG = \frac{T_{max}}{1 + \epsilon^2 \left[\cos(n \cos^{-1}(\omega))\right]^2}. \quad (38)$$

The above form results in an equal ripple monotone-roll-off transfer function over the frequency band $-1 \leq B \leq +1$. For bandpass problems described by $\omega_1 \leq B \leq \omega_2$, first, the frequency band dictated by (38) must be normalized, to yield the desired band width over $-\omega_c \leq B \leq \omega_c$ such that $\omega_c = \frac{B}{2} = \frac{\omega_2 - \omega_1}{2}$, and then it is shifted by an amount of $\frac{B}{2} + \omega_1$ to obtain the required shape of the TPG in the frequency interval specified by $\omega_1 \leq B \leq \omega_2$. This process replaces the frequency ω of (38) by

$$\omega \Rightarrow \left[\omega - \left(\frac{B}{2} + \omega_1\right)\right] / \omega_c. \quad (39)$$

It is important to note that the above mentioned Chebyshev forms are only utilized, to generate a meaningful trace for the ratio α of (4). However, the designer is free, to choose any shape for TPG which yields reasonable solutions for the equalizer under consideration.

For the sake of clear understanding, let us now summarize the details of the proposed design procedure in the following algorithm.

5. Algorithm: Construction of Lossless Matching Networks without Optimization

This algorithm outlines the procedure, to construct lossless equalizers for single matching problems without optimization.

Inputs:

- Measured load data in the form of impedance $\{R_L(\omega_i), X_L(\omega_i); i = 1, 2, \ldots, l\}$ where l designates the total sample points.
- Desired form of the transducer power gain $TPG = T(\omega)$ over the entire frequency band: It should be noted that this form can be input either in closed form as in (7) or as sample points. In this manner, monotone-roll-off Chebyshev forms is recommended as in above section.
- Realizable gain levels T_{max} and T_{min} over the pass band: In this regard, T_{max} and T_{min} are selected with practical considerations. For example, a lowpass matching network which is free of ideal transformer, demands $T_{max} = 1$. On the other hand, T_{min} may be selected as the allowable minimum gain level in the passband.
- Lower (f_{LE} or f_1) and the upper (f_{UE} or f_2) edges of the passband.
- Desired unit delay τ (in sec.).
- Normalization frequency f_{Norm} (or f_e).
- Impedance normalization number R_0 in ohms.
- n: Desired number of elements in the equalizer.
- n_c: Desired number of UE in the equalizer.
- k: Total number of transmission zeros introduced at DC.
- Selected form of the numerator polynomial $f(\lambda)$ of the transfer scattering parameter S_{21}.
- δ: Stopping criteria selected to terminate the fixed point iterations. Note that if the computations are run on PC, δ is usually selected as $10^{-5} \leq \delta \leq 10^{-3}$.

Outputs:

- Analytic form of the input reflection coefficient of the lossless equalizer given in Belevitch form of $S_{22}(\lambda) = \frac{h(\lambda)}{g(\lambda)} = \frac{h_0 + h_1\lambda + \ldots + h_n\lambda^n}{g_0 + g_1\lambda + \ldots + g_n\lambda^n}$. It is noted that this algorithm determines the coefficients $\{h_0, h_1, \ldots, h_n\}$ and $\{g_0, g_1, \ldots, g_n\}$.
- Circuit topology of the lossless equalizer with characteristic impedances of the UEs and stubs. The circuit topology is obtained as the result of the synthesis of $s_{22}(\lambda)$ using Richards' procedure [11] or decomposition technique of Fettweis [12]. Alternatively, the methods presented in [13] and [14] can be used.

Computational Steps:

Step 1:

(a) Normalize the measured frequencies with respect to f_{Norm} and set all the normalized angular frequencies $\omega_i = f_i / f_{Norm}$.

(b) Normalize the load impedance with respect to normalization number R_0 and $R_L = R_L/R_0$; $X_L = X_L/R_0$ over the entire frequency band.

Step 2: Employing T_{max} and T_{min}, compute the ripple factor $\epsilon^2 = \frac{T_{max} - T_{min}}{T_{min}}$ as in (8).

Step 3: Compute the real roots of the Chebyshev polynomial in ascending order $-\omega_{Rn} < \ldots < -\omega_{R1} < \ldots < \omega_{R1} < \ldots < \omega_{Rn}$ for the given degree n.

Step 4: Using the positive roots, constitute frequency intervals I_k such that $I_1 = \{\omega_1 \leq \omega < \omega_{R1}\}$, $I_2 = \{\omega_{R1} \leq \omega < \omega_{R2}\}$, $I_3 = \{\omega_{R2} \leq \omega < \omega_{R3}\}$, …, $I_{n+1} = \{\omega_{Rn} \leq \omega < \omega_2\}$.

Step 5: Compute α using (4). In the course of computations set $\mu = (-1)^i$ when $\omega \in I_i$.

Step 6: Compute the real part $R_2 = \alpha R_L$ point by point and extrapolate it beyond the given frequencies. At this step, it may be suitable to fix $R_2 = 1$ at DC (i.e., $\omega = 0$) and $R_2 \approx 0$ for $\omega > 1.5\omega_e$ for low pass designs (i.e. when $f(\lambda) = (1 - \lambda^2)^{n_c/2}$; where ω_e designates the upper edge of the stop band.

Step 7: Generate the reflection coefficient $S_{22} = \frac{(\alpha R_L - jX_L) - 1}{(\alpha R_L - jX_L) + 1} = S_R + jS_X$ over the frequencies ω_i.

Step 8: Employing the fixed point iteration method, model the reflection coefficient as $S_{22}(\lambda) = \frac{h_0 + h_1\lambda + \ldots + h_n\lambda^n}{g_0 + g_1\lambda + \ldots + g_n\lambda^n}$.

Stop the fixed point iteration process when $||h_r - h_{r-1}|| \leq \delta$.

Step 9: Synthesize the modeled reflectance function and obtain the desired equalizer.

Step 10: Draw the obtained and desired TPG curves. If the obtained one is not good enough, go to the next step. Otherwise, stop the algorithm.

Step 11: Compute the imaginary part of R_2 as $X_2 = H\{R_2\}$ over the frequencies ω_i, where $H\{\circ\}$ denotes Hilbert transformation. So $Z_2 = R_2 + jX_2$ will be minimum reactive impedance data.

Step 12: Generate the reflection coefficient $S_{22} = \frac{(R_2+jX_2)-1}{(R_2+jX_2)+1} = S_R + jS_X$ of the minimum reactive part of the equalizer over the frequencies ω_i.

Step 13: Employing the fixed point iteration method, model the reflection coefficient as $S_{22}(\lambda) = \frac{h_0+h_1\lambda+\ldots+h_n\lambda^n}{g_0+g_1\lambda+\ldots+g_n\lambda^n}$. Stop the fixed point iteration process when $||h_r - h_{r-1}|| \leq \delta$.

Step 14: Synthesize the obtained reflection function and form minimum reactive part of the equalizer.

Step 15: Compute the Foster data as $X_F = -(X_L + X_2)$ over the frequencies ω_i.

Step 16: Obtain the Foster function of X_F data by using the methods described in [9], [15].

Step 17: Synthesize the Foster function, and connect it between the minimum reactive part obtained in Step 14 and the load. Stop the algorithm.

Eventually, the above algorithm can be integrated with a commercially available CAD package, to further improve the performance of the matched system via optimization [4]–[6].

Let us now present an example, to design a practical equalizer for a passive load.

6. Example

In this section, a matching network design is presented. The normalized load impedance data are given in Table 1.

It should be noted that the above load data can easily be modeled as a capacitor $C_L = 4$ in parallel with a resistor $R_L = 1$ (i.e. $R_L//C_L$ type of load). In this case, using Fano's or Youla's relations [1]–[3], the ideal flat gain level T_{flat} is computed as

$$\begin{aligned} T_{flat} &= 1 - \exp(-2\pi/R_L C_L \omega_c) \\ &= 1 - \exp(-2\pi/1 \cdot 4 \cdot 1) = 0.7921. \end{aligned}$$

Let us design the equalizer over the normalized pass band of $0 \leq B \leq 1$. Thus, a low-pass Chebyshev transfer function of (7) can be utilized, to generate TPG trace for the matched system under consideration. In this manner, let us choose $T_{max} = 1$ and $T_{min} = 0.7921$, then the ripple factor ϵ^2 is found as

$$\begin{aligned} \epsilon^2 &= (T_{max} - T_{min})/T_{min} = (1 - 0.7921)/0.7921 \\ &= 0.2625. \end{aligned}$$

To ease the physical implementation, let us employ only four UEs in the minimum reactive part of the equalizer. Thus, selecting $n_c = n = 4$, $k = 0$, pre-selected form of TPG is found as $TPG = \frac{1}{1+\epsilon^2 T_4^2(\omega)}$ with $T_4(\omega) = 8\omega^4 - 8\omega^2 + 1$.

Using the above inputs, the proposed algorithm yields the trace of TPG, the ratio α, the real part R_2 of Z_2, the imaginary part $X_2 = H\{R_2\}$ and the Foster part X_F as listed in Table 2.

Eventually,

- choosing the normalized cutoff frequency at $\omega_c = 1$,
- upper edge frequency at $\omega_e = 2.4\omega_c$,
- equal line delays of UEs as $\tau = \frac{2\pi}{4\omega_e} = 0.6545$ yielding a quarter wave length $(\lambda_e/4)$ at ω_e,

the back-end reflectance $S_{22}(\lambda) = \frac{h(\lambda)}{g(\lambda)}$ of the minimum reactive part of the equalizer is determined via fixed point iteration algorithm as

$$\begin{aligned} h(\lambda) &= -46.6935\lambda^4 - 16.3815\lambda^3 - 17.2858\lambda^2 \\ &\quad - 3.0477\lambda - 0.6492, \\ g(\lambda) &= 46.7042\lambda^4 + 29.4148\lambda^3 + 23.6290\lambda^2 \\ &\quad + 6.8694\lambda + 1.1922. \end{aligned}$$

Then, the Foster data is modeled as a series shorted stub with $Z_F = 0.7787$, $\tau = 0.6545$.

Finally, $S_{22}(\lambda)$ is synthesized, and the Foster part is connected between the equalizer and the load yielding the complete equalizer topology with initial element values as shown in Fig. 7.

Table 1 Given normalized impedance data.

ω	R_L	X_L
0.0	1.00	0.0000
0.1	0.86	−0.3448
0.2	0.60	−0.4878
0.3	0.41	−0.4918
0.4	0.28	−0.4495
0.5	0.20	−0.4000
0.6	0.14	−0.3550
0.7	0.11	−0.3167
0.8	0.09	−0.2847
0.9	0.07	−0.2579
1.0	0.06	−0.2353

Table 2 Trace of TPG, α, R_2, X_2 and X_F.

ω	TPG	α	R_2	X_2	X_F
0.0	0.7921	0.3737	0.3737	0	0
0.1	0.8180	0.4019	0.3465	−0.0607	0.4056
0.2	0.8881	0.4987	0.3041	−0.0844	0.5722
0.3	0.9697	0.7036	0.2884	−0.0832	0.5750
0.4	0.9985	1.0801	0.3034	−0.0889	0.5384
0.5	0.9384	1.6601	0.3320	−0.1249	0.5249
0.6	0.8427	2.3144	0.3424	−0.1965	0.5515
0.7	0.7924	2.6743	0.3025	−0.2823	0.5990
0.8	0.8427	2.3144	0.2059	−0.3395	0.6242
0.9	0.9862	1.2666	0.0907	−0.3327	0.5905
1.0	0.7921	0.3737	0.0220	−0.2787	0.5139

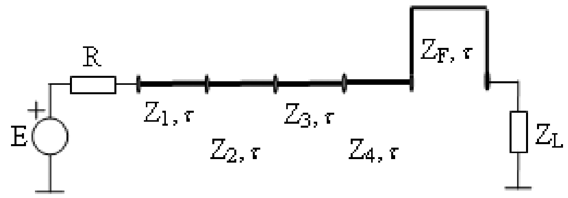

Fig. 7 Equalizer topology with line characteristic impedances: Z_1 = 0.7207, Z_2 = 0.1267, Z_3 = 1.1041, Z_4 = 0.1238, Z_F = 0.7787, τ = 0.6545, R_S = 0.2949.

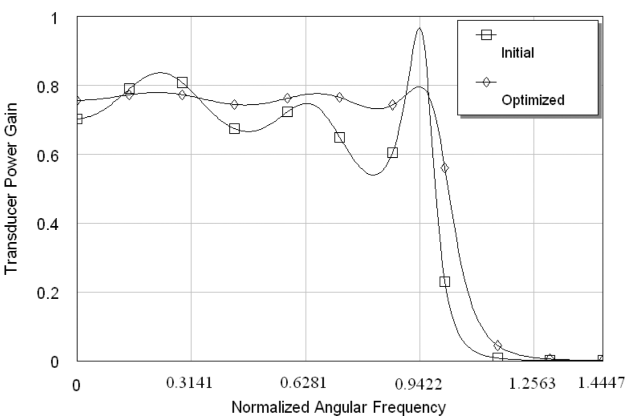

Fig. 8 Performance of the matched system.

Resulting transducer power gain of the matched system is depicted in Fig. 8. Close examination of this figure indicates that, current equalizer design yields a reasonable initial solution with minimum gain of $T_{min} \cong 0.54$ and maximum gain of $T_{max} \cong 0.96$ (or the average gain of T_{av} = 0.75 ± 0.21). This rough estimate is used to improve the performance of the matched system utilizing the commercially available design package called "Advanced Design System (ADS)" of Agilent Technologies via optimization [6]. Thus, the final normalized characteristic impedances of the UEs are found as Z_1 = 0.5204, Z_2 = 0.1657, Z_3 = 0.8837, Z_4 = 0.1307, Z_F = 0.8288 with τ = 0.6545, R_S = 0.3395 yielding $T_{min} \cong 0.73$ and $T_{max} \cong 0.79$ (or the average gain of $T_{av} = 0.76 \pm 0.03$). For comparison purpose, both initial and the final performances of the matched system are shown in Fig. 8.

As it is seen from above results, ADS's optimizer operates as a nice trimming tool on the characteristics impedances (initial Z_1 = 0.7207 vs. final Z_1 = 0.5204, initial Z_2 = 0.1267 vs. final Z_2 = 0.1657, initial Z_3 = 1.1041 vs. Z_3 = 0.8837, initial Z_4 = 0.1238 vs. final Z_4 = 0.1307, initial Z_F = 0.7787 vs. final Z_F = 0.8288), reducing the ripples of the transducer power gain in the passband as expected ($T_{av}(initial) = 0.75 \pm 0.21$ vs. $T_{av}(final) = 0.76 \pm 0.03$), preserving the flat gain level about $T_{flat} = 0.75$.

7. Conclusion

Actual design and realization of broadband equalizers (or matching networks) demands the utilization of distributed elements in the network topology. In this regard, commercially available computer aided design tools (CAD-Tools) are employed with properly chosen equalizer topology. Once the matching network topology is provided with good initials, these packages are excellent tools to optimize the system performance by working on the physical dimensions of the circuit components such as lengths and widths of the transmission lines. From the practical point of view, the width of a transmission line section is associated with line characteristic impedance (Z_i). The length of a section introduces a time delay in (τ_i) seconds. At this point, selection of the equalizer topology and initialization of (τ_i, Z_i) become very crucial, since the system performance is highly nonlinear in terms of these quantities or equivalently physical sizes. Therefore, in this paper, a new design procedure is proposed to construct lossless equalizers consist of commensurate transmission lines (or equal length lines) for matching problems. The new procedure includes three major steps. In the first step, for a pre-selected transducer power gain form, input reflectance data of the equalizer is generated over the real frequencies. Then, the computed data is modeled as a realizable-bounded real-reflectance function via fixed point iteration method. Finally, this function is synthesized as a lossless two-port as cascade connection of UEs, open and short UE stubs in resistive termination, yielding the desired equalizer topology with transmission line-characteristic impedances. Eventually, the actual performance of the matched system is improved utilizing any commercially available CAD tool which in turn results in the physical layout of the matching network to be manufactured as a microwave monolithic integrated circuit.

An example is presented to construct matching networks with UEs. It is exhibited that the proposed method provides very excellent initials for the commercially available CAD packages to further improve the matched system performance by trimming the values of (τ_i, Z_i) for each section. Therefore, it is expected that the proposed design procedure will be utilized as a front-end for the commercially available CAD packages to design practical matching networks for wireless or in general, microwave communication systems.

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Metin Şengül received B.Sc. and M.Sc. degrees in Electronics Engineering from İstanbul University, Turkey in 1996 and 1999, respectively. He completed his Ph.D. in 2006 at Işık University, İstanbul, Turkey. He worked as a technician at İstanbul University from 1990 to 1997. He was a circuit design engineer at R&D Labs of the Prime Ministry Office of Turkey between 1997 and 2000. Since 2000, he is a lecturer at Kadir Has University, İstanbul, Turkey. Currently he is working on microwave matching networks/amplifiers, data modeling and circuit design via modeling. Dr. Şengül was a visiting researcher at Institute for Information Technology, *Technische Universität Ilmenau*, Ilmenau, Germany in 2006 for six months.

Sıddık B. Yarman received B.Sc. in Electrical Engineering (EE), İstanbul Technical University (I.T.U.), İstanbul, Turkey, February 1974; M.E.E.E in Electro-Math Stevens Institute of Technology (S.I.T.) Hoboken, NJ., June 1977; Ph.D. in EE-Math Cornell University, Ithaca, NY, January 1982. Member of the Technical Staff (MTS) at Microwave Technology Centre, RCA David Sarnoff Research Center, Princeton, NJ (1982–1984). Associate Professor, Anadolu University, Eskiehir, Turkey, and Middle East Technical University, Ankara, Turkey (1985–1987). Visiting Professor and Research Fellow of Alexander Von Humboldt, Ruhr University, Bochum, Germany (1987–1994). Founding Technical Director and Vice President of STFA Defense Electronic Corp. İstanbul, Turkey (1986–1996). Full Professor, Chair of Div. of Electronics, Chair of Defense Electronics, Director of Technology and Science School, İstanbul University (1990–1996). Founding President of Işık University, İstanbul, Turkey (1996–2004). Chief Advisor in Charge of Electronic and Technical Security Affairs to the Prime Ministry Office of Turkey (1996–2000). Chairman of the Science Commission in charge of the development of the Turkish Rail Road Systems of Ministry of Transportation (2004–). Young Turkish Scientist Award, National Research Council of Turkey (NRCT) (1986). Technology Award of Husamettin Tugac Foundation of NRCT (1987). International Man of the Year in Science and Technology, Cambridge Biography Center of U.K. (1998). Member Academy of Science of New York (1994), Fellow of IEEE (2004). Four U.S. patents (1985–1986), More than 100 Technical papers, Technical Reports in the field of Design of Matching Networks and Microwave Amplifiers, Mathematical Models for any Systems, Speech and Biomedical Signal Processing (since 1982). Prof. Yarman has been back to Istanbul University since October 2004. Dr. Yarman has been spending his sabbatical year of 2006 at Tokyo Institute of Technology. He is the Fellow member IEEE.