Performance Analysis of Transmit and Receive Antenna Selection over Flat Fading Channels

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***Abstract*— The paper considers two different antenna selection schemes for space-time coded systems over flat fading channels. First we explore antenna selection at the transmitter side based on the received signal to noise ratios. We then study the joint selection of receive and transmit antennas. Both schemes assume a slowly fading channel (i.e., quasi-static fading) and require some limited feedback from the receiver to the transmitter. By computing upper bounds on the pairwise error probabilities and conducting extensive simulations, we show that the space-time coded systems achieve full diversity even with antenna selection provided that the code is full rank. These results are extensions of earlier work on antenna selection for MIMO systems [1] which only considers receive antenna selection.**

***Index Terms*—Space-time coding, MIMO communications, antenna selection, transmit/receive antenna selection, spatial diversity, Rayleigh fading channels, pair-wise error probability.**

I. Introduction

THE use of multiple antennas have become popular as the channel capacity for multiple input multiple output (MIMO) systems over wireless links increases substantially [2], [3]. With the motivation of attaining high data rates and low error rates in these systems, space-time codes (STC) [4]–[6] which can achieve full spatial diversity can be employed. On the other hand, a major drawback in realizing MIMO systems is the cost of implementing multiple radio frequency (RF) circuits. As mobile devices are desired to be small, having multiple RF transceivers in a single unit has considerable realization issues such as proper isolation, increased price, etc. Furthermore, the computational complexity of signal processing required by MIMO transceivers, especially space-time decoders, increases exponentially with the number of transmit antennas. Because of these limiting factors, application of antenna selection [7]–[9] can be an effective technique to reduce the cost and the complexity of STC systems. With antenna selection, a limited number of all available antennas (both at the transmitter and the receiver) can be used with a reduced number of RF chains still providing full diversity benefits in MIMO communications.

In the literature, there has been considerable research on antenna selection recently. A general overview of the capacity and performance of MIMO systems with antenna selection is presented in [7], [10]. Receive antenna selection is studied extensively in [11]–[13]. In [1], the authors consider antenna selection at the receiver based on maximizing the signal-to-noise ratio (SNR) over quasi-static flat fading channels. The performance degradation of STC systems when the MIMO subchannels experience correlated fading [14] and the performance with fast fading [15] are also studied. The number of works on transmit antenna selection is also increasing [16], [17]. In [18], by simulations, the authors demonstrate that transmit antenna selection combined with space-time trellis codes can achieve full available diversity. However, they do not perform an analytical error-rate analysis. Two algorithms are presented to select the number and subset of active transmit antennas in a correlated multiple-input multiple-output (MIMO) multiple access channel in [19]. An adaptive transmit antenna selection based on the minimization of the conditional pairwise error probability is proposed in [20]. Transmit antenna selection in uncoded spatial multiplexing systems is also considered and several selection algorithms are proposed [21], [22]. In a practical system, it may be desirable to employ antenna selection both at the transmitter and receiver. Recently, selection algorithms based on capacity maximization for joint transmit/receive antenna selection are developed [23], [24]. However, to the best of our knowledge, no error probability results for STCs with joint transmit and receive antenna selection are available in the literature.

In this paper, we study the diversity gain that STCs can offer over flat fading channels when transmit or joint transmit/receive antenna selection is employed based on the largest SNR observed. We perform a pairwise error probability analysis for both cases. We show that if the space-time code used achieves full-rank over flat fading channels, then antenna selection does not degrade the diversity obtained compared to that of the full complexity system. Furthermore, we show that if the code does not achieve full diversity for the full-complexity system, then performing antenna selection results in a loss of diversity order. We note that the results are very general, and apply for different space-time codes, and even for concatenated coding schemes as they are only based on pairwise error probabilities.

The paper is organized as follows: Section II presents the system model. Section III discusses space-time coded systems with transmit antenna selection. Section IV considers space-time coded systems with joint transmit and receive antenna selection. We comment on antenna selection for rank deficient STCs in Section V. Finally, Section VI concludes the paper.

II. System Description

In this section, we provide the system model and the pairwise-error probability (PEP) for space-time coded MIMO

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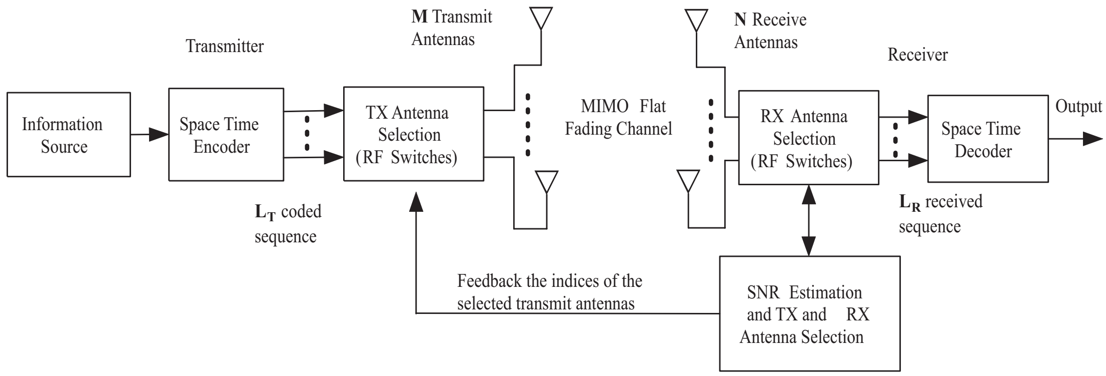

Fig. 1. Block diagram of space-time coded multiple antenna system with antenna selection over a flat fading channel.

systems over flat fading channel. Figure 1 shows the system block diagram with antenna selection. The channel is modelled as quasi-static flat Rayleigh fading where the channels for different transmit and receive antenna pairs fade independently and remain constant over the entire transmitted frame of symbols. In order to determine the antennas to be used, the pilot symbols can be transmitted from all available M transmit antennas (could be in a round-robin fashion), and then the SNR for each transmit antenna or joint transmit/receive antenna combinations can be obtained. Once the selection of transmit and receive antennas is done based on the largest of the received SNRs, the receiver feeds back the indices of the L_T transmit antennas to be used at each frame. The feedback information about the selected transmit antennas requires at most M bits in each frame, thus, it does not slow down the transmission rate significantly. After the selection of antennas is performed, the information sequence is encoded by the space-time encoder and then the coded sequence is divided by a serial-to-parallel converter into several data streams. The resulting data streams are then modulated and transmitted through the selected L_T antennas simultaneously. At the receiver, space-time decoding is performed using the demodulated signals from the selected L_R receive antennas.

For general STC-MIMO systems, the received signal at the receive antenna n at time k, can be written as

$$y_n(k) = \sqrt{\frac{\rho}{M}} \sum_{m=1}^{M} h_{m,n} s_m(k) + w_n(k), \tag{1}$$

where $h_{m,n}$ is the fading coefficient between transmit antenna m and receive antenna n, $s_m(k)$ is the transmitted symbol from antenna m at time k, N is the number of receive antennas, $w_n(k)$ is the noise sample at the receive antenna n at time k, $(k = 1, \cdots, K)$, and K is the frame length. $h_{m,n}$ and $w_n(k)$ are i.i.d. complex Gaussian random variables having zero mean and variance $1/2$ per dimension. ρ is the expected SNR at each receive antenna. The received signals at all antennas can be stacked in a matrix form as

$$\mathbf{Y} = \sqrt{\frac{\rho}{M}} \mathbf{HS} + \mathbf{W}, \tag{2}$$

where the $N \times M$ channel coefficients matrix is given by

$$\mathbf{H} = \begin{pmatrix} h_{1,1} & ... & h_{M,1} \\ \vdots & \ddots & \vdots \\ h_{1,N} & ... & h_{M,N} \end{pmatrix},$$

the $M \times K$ codeword matrix is

$$\mathbf{S} = \begin{pmatrix} s_1(1) & \dots & s_1(K) \\ \vdots & \vdots & \vdots \\ s_M(1) & \dots & s_M(K) \end{pmatrix}, \tag{3}$$

and the $N \times K$ noise matrix $\mathbf{W}$ contains the noise samples, $w_n(k)$.

When the channel state information (CSI) is known at the receiver, the PEP conditioned on the instantaneous CSI is the same as the one for the case of an AWGN channel. Given $\mathbf{H}$, the PEP of erroneously receiving $\widehat{\mathbf{S}}$, when $\mathbf{S}$ was transmitted, is given by [1],

$$P(\mathbf{S} \to \widehat{\mathbf{S}}|\mathbf{H}) = \frac{1}{2} erfc\left(\sqrt{\frac{\rho}{4M}} \|\mathbf{HB}\|\right), \tag{4}$$

which can be upper bounded by employing the Chernoff bound as

$$P(\mathbf{S} \to \widehat{\mathbf{S}}|\mathbf{H}) \leq \exp\left(-\frac{\rho}{4M} \|\mathbf{HB}\|^2\right), \tag{5}$$

where $\mathbf{B} = \mathbf{S} - \widehat{\mathbf{S}}$ is the codeword difference matrix. $\|.\|^2$ represents the sum of magnitude squares of all entries of a matrix (i.e., $\|\mathbf{V}\|^2 = \sum_{i=1}^{I} \sum_{j=1}^{J} |v_{ij}|^2$ is the Frobenius norm of the $I \times J$ matrix $\mathbf{V}$, where v_{ij} is the entry of $\mathbf{V}$ at the i^{th} row and the j^{th} column). To find the PEP over a MIMO fading channel, we simply average this quantity in (5) over the fading statistics [4], [25].

III. Transmit Antenna Selection

In this section, we investigate the diversity order of a STC with transmit antenna selection over quasi-static flat fading channels. We will derive an upper bound on the PEP for the case where an arbitrary number of transmit antennas are used. Since the more interesting case is the one where at least two antennas are selected, the channel codes in this case are space-time codes (e.g., space-time trellis codes or space-time block codes). The upper bound on the PEP expression provides information on the achieved diversity and coding gain which are useful in designing novel space-time codes with transmit antenna selection. We note that when only one transmit antenna is selected, any channel code for a single antenna system can be used and although the derivations are not provided here, full spatial diversity can still be achieved in a straightforward manner.

Let us denote L_T columns of the $N \times M$ channel coefficient matrix $\mathbf{H}$ having the largest norms by $\tilde{\mathbf{h}}_1, \cdots, \tilde{\mathbf{h}}_{L_T}$. The indices of these columns correspond to the indices of the selected transmit antennas. In order to derive an upper bound on the PEP, we first need to compute the joint probability density function (pdf) of the columns having the largest norms. Similar to the approach in [1], the joint pdf of the columns of $\mathbf{H}$ with the largest norms can be written as

$$f_{\tilde{\mathbf{H}}_1,\cdots,\tilde{\mathbf{H}}_{L_T}}(\mathbf{h}_1,\cdots,\mathbf{h}_{L_T}) = \kappa\Bigg(\sum_{l=1}^{L_T}\left[1-e^{-\|\mathbf{h}_l\|^2}\sum_{n=0}^{N-1}\frac{\|\mathbf{h}_l\|^{2n}}{n!}\right]^{M-L_T} I_{\mathcal{R}_l}(\mathbf{h}_1,\cdots,\mathbf{h}_{L_T})\Bigg)\frac{e^{-(\|\mathbf{h}_1\|^2+\cdots+\|\mathbf{h}_{L_T}\|^2)}}{\pi^{NL_T}}, \quad (6)$$

where $\kappa=\frac{M!}{(M-L_T)!L_T!}$, $I_{\mathcal{R}_l}(\mathbf{h}_1,\cdots,\mathbf{h}_{L_T})$ is the indicator function

$$I_{R_l}(\mathbf{h}_1,\cdots,\mathbf{h}_{L_T})=\begin{cases}1 & \text{if } (\mathbf{h}_1,\cdots,\mathbf{h}_{L_T})\in\mathcal{R}_l\\ 0 & \text{else}\end{cases}$$

which is nonzero in the region $\mathcal{R}_l$ where column l ($\mathbf{h}_l$) has the smallest norm among the selected L_T columns, i.e.,

$$\mathcal{R}_l=\left\{\mathbf{h}_1,\cdots,\mathbf{h}_{L_T}:\|\mathbf{h}_l\|<\|\mathbf{h}_k\|,\ \ k=1,\cdots,l-1,l+1,\cdots,L_T\right\}.$$

Using the new channel matrix $\hat{\mathbf{H}}$ with the selected columns of $\mathbf{H}$, the PEP can be upper bounded by averaging over the above joint pdf as,

$$P(\mathbf{S}\to\hat{\mathbf{S}}) \leq \sum_{l=1}^{L_T}\int_{\mathcal{R}_l} e^{-\frac{\rho}{4L_T}\|\hat{\mathbf{H}}\mathbf{B}\|^2}\kappa\left[1-e^{-\|\mathbf{h}_l\|^2}\sum_{n=0}^{N-1}\frac{\|\mathbf{h}_l\|^{2n}}{n!}\right]^{M-L_T}\frac{e^{-\sum_{i=1}^{L_T}\|\mathbf{h}_i\|^2}}{\pi^{NL_T}}d\mathbf{h}_1\cdots d\mathbf{h}_{L_T}. \quad (7)$$

We utilize the eigenvalue decomposition of $\mathbf{BB}^*=\mathbf{U\Lambda U}^*$ where $\mathbf{U}$ is a unitary matrix and $\mathbf{\Lambda}$ is a diagonal matrix with eigenvalues of $\mathbf{BB}^*$. We note that

$$\|\hat{\mathbf{H}}\mathbf{B}\|^2 = trace\left((\hat{\mathbf{H}}\mathbf{U})\mathbf{\Lambda}(\hat{\mathbf{H}}\mathbf{U})^*\right)=\sum_{i=1}^{L_T}\lambda_i\|\mathbf{c}_i\|^2, \quad (8)$$

where $\mathbf{c}_i$ is the i^{th} column of $\hat{\mathbf{H}}\mathbf{U}$, and

$$\begin{aligned}\sum_{i=1}^{L_T}\|\mathbf{c}_i\|^2 &= trace\left((\hat{\mathbf{H}}\mathbf{U})(\hat{\mathbf{H}}\mathbf{U})^*\right)\\ &= trace\left(\hat{\mathbf{H}}\mathbf{U}\mathbf{U}^*\hat{\mathbf{H}}^*\right)\\ &= trace\left(\hat{\mathbf{H}}\hat{\mathbf{H}}^*\right)\\ &= \sum_{i=1}^{L_T}\|\mathbf{h}_i\|^2. \end{aligned}\quad (9)$$

Let us now assume that we have a full-rank space-time code which means that the eigenvalues of the matrix $\mathbf{BB}^*$ are all positive (i.e., nonzero). Later, we will also consider the rank-deficient STCs (where some of the eigenvalues of $\mathbf{BB}^*$ are zeros) as well. In order to simplify the PEP bound further, we denote the minimum of $\lambda_1,...,\lambda_{L_T}$ by $\hat{\lambda}>0$ and note that

$$\begin{aligned}\sum_{i=1}^{L_T}\lambda_i\|\mathbf{c_i}\|^2 &\geq \sum_{i=1}^{L_T}\hat{\lambda}\|\mathbf{c_i}\|^2\\ &\geq \sum_{i=1}^{L_T}\hat{\lambda}\|\mathbf{h_i}\|^2.\end{aligned}\quad (10)$$

Hence, the expression can be further upper bounded as

$$P(\mathbf{S}\to\hat{\mathbf{S}}) \leq \sum_{l=1}^{L_T}\int_{\mathcal{R}_l} e^{-\frac{\rho}{4L_T}\sum_{i=1}^{L_T}\hat{\lambda}\|\mathbf{h}_i\|^2}\kappa\Bigg[1-e^{-\|\mathbf{h}_l\|^2}\sum_{n=0}^{N-1}\frac{\|\mathbf{h}_l\|^{2n}}{n!}\Bigg]^{M-L_T}\frac{e^{-\sum_{i=1}^{L_T}\|\mathbf{h}_i\|^2}}{\pi^{NL_T}}d\mathbf{h}_1\cdots d\mathbf{h}_{L_T}. \quad (11)$$

To simplify this expression further, we can use the following result (as in [1])

$$g(v)=1-e^{-v}\sum_{n=0}^{N-1}\frac{v^n}{n!}\leq\frac{v^N}{N!}, \quad (12)$$

for $v>0$, and write the l^{th} term of the right hand side of the PEP bound as,

$$\mathcal{I}_l\leq\kappa\int_{\mathcal{R}_l}e^{-\frac{\rho}{4L_T}\sum_{i=1}^{L_T}\hat{\lambda}\|\mathbf{h}_i\|^2}\left[\frac{\|\mathbf{h}_l\|^{2N}}{N!}\right]^{M-L_T}\frac{1}{\pi^{NL_T}}e^{-(\|\mathbf{h}_1\|^2+\cdots+\|\mathbf{h}_{L_T}\|^2)}d\mathbf{h}_1\cdots d\mathbf{h}_{L_T}. \quad (13)$$

Then, with the change of variables $h_{nl}=\sigma_{nl}e^{\theta_{nl}}$, $u_{nl}=\sigma_{nl}^2$ where $\|\mathbf{h}_l\|^2=\sum_{n=1}^{N}u_{nl}$ (with differential units $dh_{nl}=\sigma_{nl}d\sigma_{nl}d\theta_{nl}$, $du_{nl}=2\sigma_{nl}d\sigma_{nl}$) and after taking the integral with respect to $d\theta$ over $[0,2\pi]$, we obtain

$$\mathcal{I}_l\leq\kappa\int_0^\infty\cdots\int_0^\infty e^{-\frac{\rho\hat{\lambda}}{4L_T}\left((u_{11}+\cdots+u_{N1})+\cdots+(u_{1L_T}+\cdots+u_{NL_T})\right)}\left(\frac{(u_{1l}+\cdots+u_{Nl})^N}{N!}\right)^{M-L_T}e^{-(u_{11}+\cdots+u_{NL_T})}du_{11}\cdots du_{NL_T}. \quad (14)$$

Note that for analytical tractability, we evaluate the integral throughout the whole space which results in a looser upper bound. To obtain simpler expressions, we write the upper bound of $\mathcal{I}_l$ as $\mathcal{I}_l\leq\mathcal{I}_l^{(1)}\mathcal{I}_l^{(2)}$ with

$$\mathcal{I}_l^{(1)}=\kappa\int_0^\infty\cdots\int_0^\infty e^{-\frac{\rho}{4L_T}\sum_{i=1,i\neq l}^{L_T}\hat{\lambda}\left(\sum_{n=1}^{N}u_{ni}\right)}e^{-\left(\sum_{i=1,i\neq l}^{L_T}\sum_{n=1}^{N}u_{ni}\right)}\prod_{i=1,i\neq l}^{L_T}\prod_{n=1}^{N}du_{ni}$$

$$\mathcal{I}_l^{(2)}=\int_0^\infty\cdots\int_0^\infty e^{-\frac{\rho}{4L_T}\hat{\lambda}\sum_{n=1}^{N}u_{nl}}e^{-\sum_{n=1}^{N}u_{nl}}\left(\frac{\left(\sum_{n=1}^{N}u_{nl}\right)^N}{N!}\right)^{M-L_T}du_{1l}\cdots du_{Nl}.$$

Using $\int_0^\infty e^{-kx}dx=\frac{1}{k}$, we obtain

$$\mathcal{I}_l^{(1)}=\kappa\left(\frac{1}{\prod_{i=1,i\neq l}^{L_T}\left(1+\frac{\rho\hat{\lambda}}{4L_T}\right)}\right)^N. \quad (15)$$

For $\mathcal{I}_l^{(2)}$, we first use $v_n = u_{nl}$ and note that

$$\left(\sum_{n=1}^{N} v_n\right)^{NM-NL_T} = \sum_{n_1=1}^{N} \cdots \sum_{n_{NM-NL_T}=1}^{N} v_{n_1} \cdots v_{n_{NM-NL_T}}, \tag{16}$$

and $v_{n_1} \cdots v_{n_{NM-NL_T}} = \prod_{n=1}^{N} (v_n)^{l_n}$ such that $\sum_{n=1}^{N} l_n = NM - NL_T$. Then we obtain

$$\begin{aligned} \mathcal{I}_l^{(2)} = \left(\frac{1}{N!}\right)^{M-L_T} \int_0^\infty \cdots \int_0^\infty e^{-\sum_{n=1}^{N}(\frac{\rho\hat{\lambda}}{4L_T}+1)v_n} \\ \sum_{n_1=1}^{N} \cdots \sum_{n_{NM-NL_T}=1}^{N} \prod_{n=1}^{N} (v_n)^{l_n} dv_1 \cdots dv_N. \end{aligned} \tag{17}$$

Changing the order of summation and integration and using

$$\int_0^\infty x^m e^{-ax} dx = \frac{m!}{a^{m+1}}, \tag{18}$$

results in

$$\begin{aligned} \mathcal{I}_l^{(2)} = \left(\frac{1}{N!}\right)^{M-L_T} \sum_{n_1=1}^{N} \cdots \sum_{n_{NM-NL_T}=1}^{N} \\ \frac{l_1! \cdots l_N!}{(\frac{\rho\hat{\lambda}}{4L_T}+1)^{l_1+1} \cdots (\frac{\rho\hat{\lambda}}{4L_T}+1)^{l_N+1}}. \end{aligned} \tag{19}$$

At high SNRs, from $\mathcal{I}_l^{(1)}$ and $\mathcal{I}_l^{(2)}$ we obtain

$$\begin{aligned} P(\mathbf{S} \to \hat{\mathbf{S}}) \le \frac{M!}{(M-L_T)! L_T! (N!)^{M-L_T}} \left(\frac{1}{\hat{\lambda}^{NM}}\right) \\ \left(\sum_{n_1=1}^{N} \cdots \sum_{n_{NM-NL_T}=1}^{N} l_1! \cdots l_N!\right) \left(\frac{\rho}{4L_T}\right)^{-MN}. \end{aligned} \tag{20}$$

This result shows that the diversity order is MN which is the full diversity available in the system. The coding gain depends on the minimum of the eigenvalues of the square of the codeword difference matrix, $\mathbf{BB}^*$. Obviously, the coding gain with antenna selection will be lower than that of full-complexity system. If a full-rank STC is used, $\hat{\lambda}$ will be nonzero and one way to design new codes suitable for transmit antenna selection would be maximizing $\hat{\lambda}$ of all codes having full rank $\mathbf{BB}^*$. Although not shown here, the analysis for the simplest case of one transmit antenna selection is much simpler and agrees with the above PEP bound. That is, the diversity advantage of NM can be achieved even when only one transmit antenna is selected based on the instantaneous SNR at the receiver.

Figure 2 shows the plots of exact PEP from (4), PEP bound from the expression (5) and derived PEP bound from (20) (where averaging is done over selected fading channels using Monte Carlo simulations) for the system with M transmit and $N = 1$ receive antennas (to have small diversity orders of $MN = M$) when only $L_T = 2$ transmit antennas are used in actual transmission. Two codeword matrices with QPSK symbols

$$\mathbf{S} = \begin{pmatrix} 1 & j & -1 & -j \\ j & 1 & -j & -1 \end{pmatrix} \qquad \hat{\mathbf{S}} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix},$$

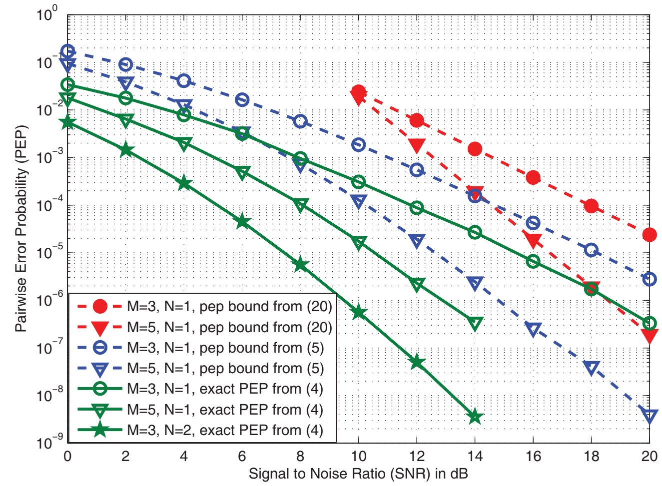

Fig. 2. PEP for a full rank code with transmit antenna selection, $L_T = 2$.

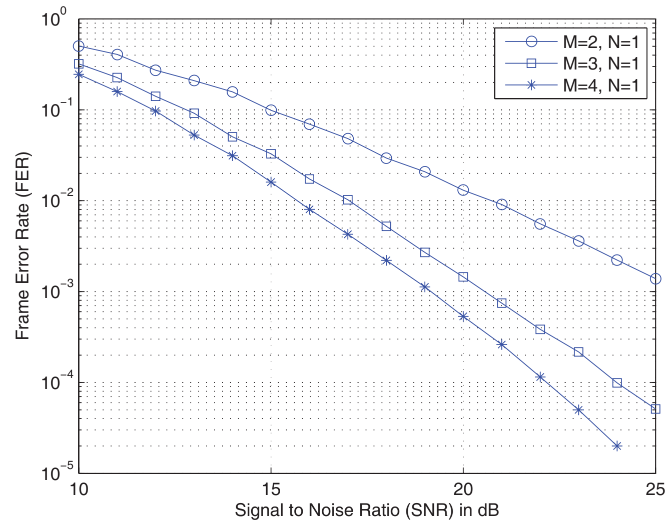

Fig. 3. FER for a full rank 4 state STTC from [4] with transmit antenna selection, $L_T = 2$.

where $j = \sqrt{-1}$ are used in the simulations. We observe that when $M = 3$ and $M = 5$, the diversity order is 3 and 5 respectively (regardless of the PEP bound). We also note that when $M = 3, N = 2$ diversity becomes 6. These results verify that full diversity can be achieved as expected theoretically when full rank codes are used. Figure 3 shows frame error rate (FER) plots for the M transmit and $N = 1$ receive antenna systems (with $L_T = 2$) when the 4-state space-time trellis code (STTC) from [4] with a frame length of 130 QPSK symbols is used. As seen from the plots, with no antenna selection, this STTC achieves full space diversity of order 2 when $M = 2$ and $N = 1$. When the number of available transmit antennas is increased to $M = 3$ and $M = 4$, while still using $L_T = 2$ of them for transmission, the diversity order becomes 3 and 4, respectively. We note that this difference in the diversity orders can only be observed for very high signal to noise ratios.

IV. Joint Transmit and Receive Antenna Selection

In this section, we investigate the diversity orders of STCs over flat fading channels with antenna selection both at the transmitter and the receiver. We first derive the PEP when only one antenna on both sides are selected. Then, we consider selection of more than one antenna, where we first select a receive antenna resulting in the maximum SNR and then select some of the transmit antennas corresponding to the selected receive antenna for ease of analytical tractability. We note that this simplified selection rule is more practical since it will result in a faster selection by eliminating the search of all possible antenna combinations. This type of selection can be suboptimal in obtaining the largest possible SNR for the entire system, i.e., the antennas resulting in the largest SNR may not be selected. However, we will see that this suboptimal selection still attains full diversity. We also consider multiple transmit and receive antenna selection schemes to study the setup in a more comprehensive manner.

A. Selection of Only One Transmit and One Receive Antenna

We will first study the simplest case in which only one antenna is selected at the transmitter and the receiver. In this case, unlike simplified selection rule as described above, the selection is implemented in only one step by finding the largest channel coefficient from the channel matrix. We also note that since there is only one transmit antenna selected, any channel code designed for single antenna system over flat fading can be used.

Our PEP analysis starts with the selection of the complex entry h, from $\mathbf{H}$, the channel gain (having the largest norm) corresponding to the selected transmit and receive antenna pair. Then, the system model can be written as

$$\mathbf{X} = \sqrt{\frac{\rho}{M}} h\mathbf{S} + \mathbf{W}, \tag{21}$$

where the received signal matrix $\mathbf{X}$ is of size $1 \times K$, transmitted signal matrix $\mathbf{S}$ is of size $1 \times K$. Then, the upper bound on the conditional PEP can be written as

$$P(\mathbf{S} \to \widehat{\mathbf{S}}|h) \leq \exp\left(-\frac{\rho}{4}\|h\mathbf{B}\|^2\right), \tag{22}$$

and taking average over all possible h

$$P(\mathbf{S} \to \widehat{\mathbf{S}}) \leq \int_{C^1} \exp\left(-\frac{\rho}{4}\|h\mathbf{B}\|^2\right) f_h(h)dh, \tag{23}$$

where C^1 is the 1 dimensional complex space and $f_h(h)$ denotes the pdf of h which is a complex Gaussian random variable and can be written as

$$f_h(h) = MN\left(1 - e^{-\|h\|^2}\right)^{(MN-1)} \frac{1}{\pi} e^{-\|h\|^2}. \tag{24}$$

We can utilize eigenvalue decomposition $\mathbf{BB}^* = \mathbf{U\Lambda U}^*$ where $\mathbf{U}$ is a unitary matrix and $\mathbf{\Lambda}$ is a diagonal matrix with eigenvalues of $\mathbf{BB}^*$. For the 1 transmit and 1 receive antenna selection case, $\mathbf{BB}^*$ is 1×1, and thus, $\mathbf{\Lambda} = \lambda$ and $\mathbf{U} = 1$. Then we can write

$$P(\mathbf{S} \to \widehat{\mathbf{S}}) \leq MN \int_{C^1} \exp\left(-\frac{\rho}{4}\lambda\|h\|^2\right)\left(1 - e^{-\|h\|^2}\right)^{(MN-1)} \frac{1}{\pi} e^{-\|h\|^2} dh. \tag{25}$$

Using $h = \sigma e^{j\theta}$, $dh = \sigma d\sigma d\theta$ and $\int_0^{2\pi} d\theta = 2\pi$, we obtain

$$P(\mathbf{S} \to \hat{\mathbf{S}}) \leq 2MN \int_0^\infty e^{-\frac{\rho}{4}\lambda\sigma^2}\left(1 - e^{-\sigma^2}\right)^{(MN-1)} e^{-\sigma^2}\sigma d\sigma. \tag{26}$$

For further simplification, we use the upper bound in (12) to obtain

$$P(\mathbf{S} \to \hat{\mathbf{S}}) \leq MN \int_0^\infty e^{-\left(\frac{\rho\lambda}{4}\lambda+1\right)v} v^{(MN-1)} dv. \tag{27}$$

Then, with (18), we easily arrive at

$$P(\mathbf{S} \to \hat{\mathbf{S}}) \leq MN \frac{(MN-1)!}{\left(\frac{\rho\lambda}{4}+1\right)^{MN}}. \tag{28}$$

In this final PEP expression, we observe that the diversity advantage of MN, as in the full-complexity system, can be achieved. We also note that the coding gain depends on the eigenvalue, λ, of the square of the codeword difference matrix, $\mathbf{BB}^*$. This result is quite useful since we can use just one transmit antenna and one receive antenna and utilize all the benefits of the MIMO systems. However, it is trivial as well since this is nothing but selection combining out of MN independent fading coefficients.

B. Selection of 2×1 Antennas from a 3×2 System

Having studied the simplest selection case, we will now obtain the PEP when more than one transmit antenna are selected in order to analyze the full-rank STCs with joint transmit and receive antenna selection over quasi-static flat fading channels. We now consider a special case where there are $M = 3$ transmit antennas and $N = 2$ receive antennas. Our goal is to select two transmit antennas ($L_T = 2$), and one receive antenna ($L_R = 1$).

Using the simplified selection rule, we start with the selection of one row, $\mathbf{r}$, of $\mathbf{H}$ with the largest norm (or SNR). The joint pdf of the selected row, $\mathbf{r}$, is

$$f_{\mathbf{R}}(\mathbf{r}) = N\left(1 - e^{-\|\mathbf{r}\|^2}\sum_{m=0}^{M-1}\frac{\|\mathbf{r}\|^{2m}}{m!}\right)^{N-1}\frac{1}{\pi^M}e^{-\|\mathbf{r}\|^2}, \tag{29}$$

where $\mathbf{R}$ is the random variable representation of row $\mathbf{r} = [c_1, c_2, c_3]$. With the definition $v_i = |c_i|^2$, $1 \leq i \leq 3$, the pdf can be written as

$$f_{\hat{C}_1,\hat{C}_2,\hat{C}_3}(c_1, c_2, c_3) = 2\left(1 - e^{-(v_1+v_2+v_3)}\left(1 + (v_1+v_2+v_3) + \frac{(v_1+v_2+v_3)^2}{2}\right)\right)\frac{1}{\pi^3}e^{-(v_1+v_2+v_3)}, \tag{30}$$

where $\hat{C}_i$ is the random variable with realization c_i. After selecting one row with 3 entries, we select 2 channel coefficients with the largest norms. Then the resulting pdf is

$$f_{\hat{C}_1,\hat{C}_2}(c_1, c_2) = f_{C_1,C_2}(c_1, c_2|G), \tag{31}$$

where the event G is defined as the first two elements, C_1, C_2, having the largest norms. Using Bayes' rule, we get

$$f_{\hat{C}_1,\hat{C}_2}(c_1, c_2) = \frac{P(G|C_1 = c_1, C_2 = c_2) f_{C_1,C_2}(c_1, c_2)}{P(G)}, \tag{32}$$

where $f_{C_1,C_2}(c_1,c_2)$ is the joint pdf of any two elements in the selected row and can be obtained as

$$f_{C_1,C_2}(c_1,c_2) = \int f_{C_1,C_2,C_3}(c_1,c_2,c_3)dc_3. \quad (33)$$

After using the pdf expression in (30) and taking the two dimensional integral with respect to the angle and magnitude of the complex number c_3, $f_{C_1,C_2}(c_1,c_2)$ becomes

$$f_{C_1,C_2}(c_1,c_2) = \left(\frac{1}{\pi^2}\right)\left(2e^{-(v_1+v_2)} - e^{-2(v_1+v_2)}\left(\frac{7}{4} + \frac{3}{2}(v_1+v_2) + \frac{1}{2}(v_1+v_2)^2\right)\right). \quad (34)$$

The other term in (32) can be written as

$$P(G|C_1 = c_1, C_2 = c_2) = P(|c_3|^2 < |c_m|^2), \quad (35)$$

where $|c_m|^2 = min(|c_1|^2, |c_2|^2)$. With further simplification,

$$P(G|C_1 = c_1, C_2 = c_2) = P(|c_3| < |c_m|) = P(\sigma_3 < \sigma_m), \quad (36)$$

where $\sigma_3 = |c_3|$ and $\sigma_m = |c_m|$ and finally

$$P(G|C_1 = c_1, C_2 = c_2) = \int_0^{2\pi}\int_0^{\sigma_m} f_{C_3|C_1,C_2}(c_3|c_1,c_2)dc_3, \quad (37)$$

where we use $dc_3 = \sigma_3 d\sigma_3 d\theta_3$ for complex integration and $f_{C_3|C_1,C_2}(c_3|c_1,c_2)$ is the pdf of the third entry of the selected row when the first two entries $C_1 = c_1, C_2 = c_2$ are known. Using Bayes' rule, the conditional pdf can be written as

$$f_{C_3|C_1,C_2}(c_3|c_1,c_2) = \frac{f_{C_3,C_1,C_2}(c_3,c_1,c_2)}{f_{C_1,C_2}(c_1,c_2)}. \quad (38)$$

Since $v_i = |c_i|^2$, for brevity of expressions, we note that

$$f_{\hat{C}_1,\hat{C}_2}(c_1,c_2) = \frac{1}{P(G)}\int_0^{v_m} f_{C_1,C_2,C_3}(v_1,v_2,v_3)dv_3. \quad (39)$$

Then, the final pdf of the selected two entries will be

$$f_{\hat{C}_1,\hat{C}_2}(c_1,c_2) = \left(-\frac{6}{\pi^2}\right)e^{-(v_1+v_2+v_m)} + \left(\frac{3}{\pi^2}\right)e^{-2(v_1+v_2+v_m)}\left(\frac{7}{4} + \frac{3}{2}(v_1+v_2+v_m) + \frac{1}{2}(v_1+v_2+v_m)^2\right) + \left(\frac{3}{\pi^2}\right)\left(2e^{-(v_1+v_2)} - e^{-2(v_1+v_2)}\left(\frac{7}{4} + \frac{3}{2}(v_1+v_2) + \frac{1}{2}(v_1+v_2)^2\right)\right). \quad (40)$$

Using this derived pdf of the selected channel coefficients, the PEP bound can be written as

$$P(\mathbf{S} \to \widehat{\mathbf{S}}) \le \int\int \exp\left(-\frac{\rho}{8}\|\hat{\mathbf{H}}\mathbf{B}\|^2\right) f_{\hat{C}_1,\hat{C}_2}(c_1,c_2)dc_1dc_2. \quad (41)$$

Similar to the derivations in the transmit antenna selection, this upper bound is not affected when $\|\hat{\mathbf{H}}\mathbf{B}\|^2$ is replaced with $\hat{\lambda}v_1 + \hat{\lambda}v_2$ where $\hat{\lambda} = min(\lambda_1, \lambda_2)$. This integration is quite lengthy and with the help of computing tools (e.g. symbolic integration in Matlab), the result can be obtained as

$$P(\mathbf{S} \to \widehat{\mathbf{S}}) \le \frac{512 + 752\alpha + 52\alpha^2 + 346\alpha^3}{(\alpha+4)^3(\alpha+2)^4(\alpha+1)^2} \approx 346 \times \alpha^{-6}, \quad (42)$$

where $\alpha = (\rho/8)\hat{\lambda}$ and the last approximation is obtained by considering large SNRs. From this PEP bound, we observe that the maximum diversity order which is the exponent of the SNR term, ρ, at high SNRs is $MN = 6$. i.e, full spatial diversity is achieved with full rank STCs as in the full-complexity case. We note that in the PEP derivation for this special case, we used computing tools and again showed that full diversity is achieved. Tighter bounds can be obtained using similar techniques to design better codes. However, when the number of antennas increases such tools do not help. Thus, we would like to derive other analytical PEP bounds for selection of arbitrary number of antennas.

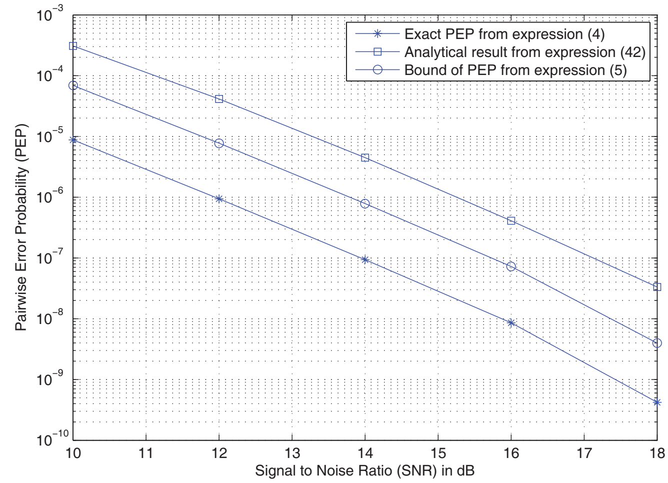

Fig. 4. Comparison of the exact PEP and the upper bounds on PEP for a full rank code with joint transmit/receive antenna selection for $\lambda_{min} = 4, \lambda_1 = 4, \lambda_2 = 12$, $M = 3$, $L_T = 2$, $N = 2$, $L_R = 1$.

To verify our analytical derivations, we have simulated several systems. However, due to space requirements, the results for only one case are provided. Figure 4 shows the PEP plots for the system where $M = 3$ transmit and $N = 2$ receive antennas are available, but only $L_T = 2$ transmit and $L_R = 1$ receive antennas resulting in the largest received SNRs are used. We use the same codeword pair as provided in the previous section. The exact PEP plot is obtained by averaging the PEP expression in (4) over the fading channel statistics after selection. Similarly, the PEP bound plot is obtained by averaging the upper bound on the PEP shown in expression (5) over the fading channel statistics after the selection. The analytical PEP bound is obtained from the derived upper bound in the expression (42) with $\lambda_1 = 4$, $\lambda_2 = 12$, and thus $\lambda_{min} = 4$ (eigenvalues of the square of the codeword difference matrix $\mathbf{BB}^*$). We note that the derived PEP bound shows that full diversity is achieved and it further upper bounds the exact PEP. Although not shown, when the eigenvalues are close to each other, then the difference between the derived upper bound and exact PEP bound is smaller.

C. Selection of $L_T \times 1$ Antennas from an $M \times N$ System

Having studied a special case, we now consider the selection problem with an arbitrary number of transmit and receive antennas. In the following PEP derivation, we focus on selecting L_T out of M transmit and one out of N receive antennas.

The joint pdf of the row with the largest norm, which is selected from the $N \times M$ channel matrix $\mathbf{H}$, is

$$f_{\mathbf{R}}(\mathbf{r}) = N\left(1 - e^{-\|\mathbf{r}\|^2}\sum_{m=0}^{M-1}\frac{\|\mathbf{r}\|^{2m}}{m!}\right)^{(N-1)}\frac{1}{\pi^M}e^{-\|\mathbf{r}\|^2}, \tag{43}$$

where $\mathbf{r} = [c_1, c_2, \ldots, c_M]$ containing M complex channel coefficients. With the definition $v_i = |c_i|^2$, $1 \le i \le M$ and $w = v_1 + v_2 + \ldots + v_M$, the pdf can be written as

$$\begin{aligned} &f_{\hat{C}_1,\hat{C}_2,\ldots,\hat{C}_M}(c_1, c_2, \ldots, c_M) \\ &= N\left(1 - e^{-w}\sum_{m=0}^{M-1}\frac{w^m}{m!}\right)^{(N-1)}\frac{1}{\pi^M}e^{-w}. \end{aligned} \tag{44}$$

After finding the pdf for the selected row which corresponds to the channel coefficients for the selected receive antenna, we select L_T transmit antennas. For simplicity, we call the pdf for the largest L_T elements of the selected row $f_{\hat{C}_1,\hat{C}_2,\ldots,\hat{C}_{L_T}}(c_1, c_2, \ldots, c_{L_T})$ as f. Similar to set of equations in (31-39), f can be written as

$$\begin{aligned} f = &\left(\frac{N.M!}{(M-L_T)!L_T!}\right) \\ &\sum_{p=1}^{L_T}\int_0^{v_p}\cdots\int_0^{v_p}\left(1 - e^{-w}\sum_{m=0}^{M-1}\frac{w^m}{m!}\right)^{(N-1)} \\ &\frac{1}{\pi^{L_T}}e^{-w}I_{R_p}(v_1, \cdots, v_{L_T})dv_{L_T+1}\ldots dv_M, \end{aligned} \tag{45}$$

where $I_{R_p}(v_1, \cdots, v_{L_T})$ is the indicator function which is 1 if and only if v_p is the minimum of all v_i for $1 \le i \le L_T$, otherwise, $I_{R_p}(v_1, \cdots, v_{L_T}) = 0$. After using the result in (12), we obtain

$$\begin{aligned} f \le &\left(\frac{N.M!}{(M-L_T)!L_T!}\right)\sum_{p=1}^{L_T}\int_0^{v_p}\cdots\int_0^{v_p}\left(\frac{w^M}{M!}\right)^{(N-1)} \\ &\frac{1}{\pi^{L_T}}e^{-w}I_{R_p}(v_1, \cdots, v_{L_T})dv_{L_T+1}\ldots dv_M. \end{aligned} \tag{46}$$

We note that

$$\begin{aligned} (v_1 + \cdots + v_M)^{M(N-1)} &= \left(\sum_{i=1}^{M} v_i\right)^{M(N-1)}, \\ &= \sum_{i_1=1}^{M}\cdots\sum_{i_M=1}^{M} v_{i_1}\cdots v_{i_{M(N-1)}}, \end{aligned} \tag{47}$$

where the indexes i_k in v_{i_k}, $k \in \{1, \cdots, M(N-1)\}$, take values from the set $\mathcal{J} = \{1, \cdots, M\}$. Assume the subscript index j appears l_j times among the subscripts of the term $v_{i_1}\cdots v_{i_M}$. Then,

$$\begin{aligned} v_{i_1}\cdots v_{i_M} &= \prod_{k=1}^{M} v_{i_k}, \\ &= \prod_{j=1}^{M}(v_j)^{l_j}, \end{aligned} \tag{48}$$

such that $\sum_{j=1}^{M} l_j = M(N-1)$. Therefore, we can use

$$(v_1 + \cdots + v_M)^{M(N-1)} = \sum_{i_1=1}^{M}\cdots\sum_{i_M=1}^{M}\prod_{j=1}^{M}(v_j)^{l_j}, \tag{49}$$

to obtain

$$\begin{aligned} f \le \beta &\sum_{i_1=1}^{M}\cdots\sum_{i_{M(N-1)}=1}^{M}\sum_{p=1}^{L_T}\int_0^{v_p}\cdots\int_0^{v_p} \\ &\prod_{j=1}^{M}(v_j)^{l_j}e^{-v_j}I_{R_p}(v_1, \cdots, v_{L_T})dv_{L_T+1}\ldots dv_M, \end{aligned} \tag{50}$$

where $\beta = \left(\frac{\frac{N.M!}{(M-L_T)!L_T!}}{(M!)(N-1)}\right)$. Each integral can be written as follows

$$\begin{aligned} \int_0^{v_p} v_j^{l_j}e^{-v_j}dv_j &= l_j!\left[-e^{-v_p}\sum_{k=0}^{l_j}\frac{v_p^{(l_j-k)}}{(l_j-k)!}\right] \\ &= l_j!\left[-e^{-v_p}\sum_{m=0}^{l_j}\frac{v_p^m}{m!}\right] \le l_j!\frac{v_p^{(l_j+1)}}{(l_j+1)!}, \end{aligned} \tag{51}$$

where we used $m = l_j - k$ for a simpler expression, and the result in (12) for the last inequality. Then the pdf becomes

$$\begin{aligned} f \le \beta &\sum_{i_1=1}^{M}\cdots\sum_{i_{M(N-1)}=1}^{M}\sum_{p=1}^{L_T}\prod_{k=1}^{L_T}e^{-v_k}(v_k)^{l_k} \\ &\prod_{j=L_T+1}^{M}\frac{v_p^{(l_j+1)}}{(l_j+1)}I_{R_p}(v_1, \cdots, v_{L_T}). \end{aligned} \tag{52}$$

In order to write the PEP bound, we integrate over $0 \le v_i \le \infty$ region instead of integrating in the region defined by I_{R_p} which further loosens the upper bound, resulting in

$$\begin{aligned} P(\mathbf{S} \to \hat{\mathbf{S}}) \le &\int_0^{\infty}\cdots\int_0^{\infty}e^{-\gamma(v_1+\ldots+v_{L_T})}\beta\sum_{i_1=1}^{M}\cdots\sum_{i_{M(N-1)}=1}^{M} \\ &\sum_{p=1}^{L_T}\prod_{j=L_T+1}^{M}\frac{v_p^{(l_j+1)}}{(l_j+1)}\prod_{k=1}^{L_T}e^{-v_k}(v_k)^{l_k}\prod_{k=1}^{L_T}dv_k, \end{aligned} \tag{53}$$

where $\gamma = \frac{\rho}{4\times L_T}\hat{\lambda}$ and the integration with respect to angle cancels out the π^{L_T} term. After interchanging the summation and integration, then, taking the integrals for the two cases $k = p$ and $k \ne p$ separately with the help of (18), we finally obtain

$$\begin{aligned} P(\mathbf{S} \to \hat{\mathbf{S}}) \le &\sum_{i_1=1}^{M}\cdots\sum_{i_M=1}^{M}\left(\beta\prod_{j=L_T+1}^{M}\frac{1}{(l_j+1)}\right) \\ &\frac{\left(l_p + (\sum_{j=L_T+1}^{M}(l_j+1))\right)!}{(\gamma+1)^{\left(l_p+1+(\sum_{j=L_T+1}^{M}(l_j+1))\right)}}\prod_{k=1,k\ne p}^{L_T}\frac{l_k!}{(\gamma+1)^{(l_k+1)}}. \end{aligned} \tag{54}$$

We observe that the exponent of γ, and thus the exponent of the SNR term ρ, will be $\sum_{j=1}^{M}(l_j+1) = MN$ which shows that full diversity is achieved even though only L_T transmit antennas and only a single receive antenna are used. This PEP expression can be useful in designing new STCs with joint transmit and receive antenna selection.

D. Selection of Arbitrary Number of Transmit and Receive Antennas

Due to the complexity in deriving the joint pdf of the selected channel coefficients for joint transmit and receive antenna selection scheme for arbitrary number of antennas $L_T > 1$ and $L_R > 1$, we do not deal with the PEP bound for

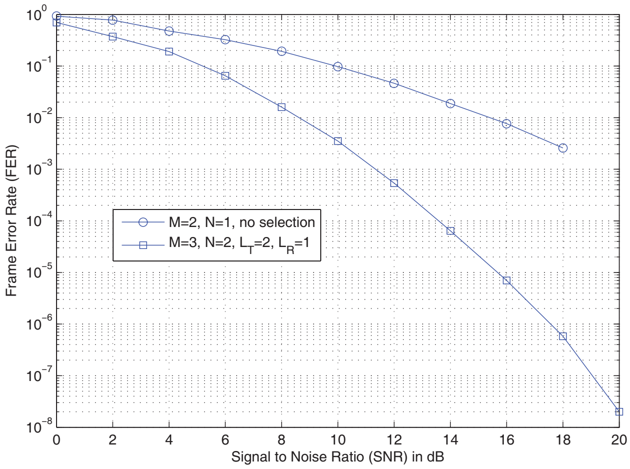

Fig. 5. FER for a full rank STC using $(5,7)_{octal}$ convolutional code with joint transmit and receive antenna selection.

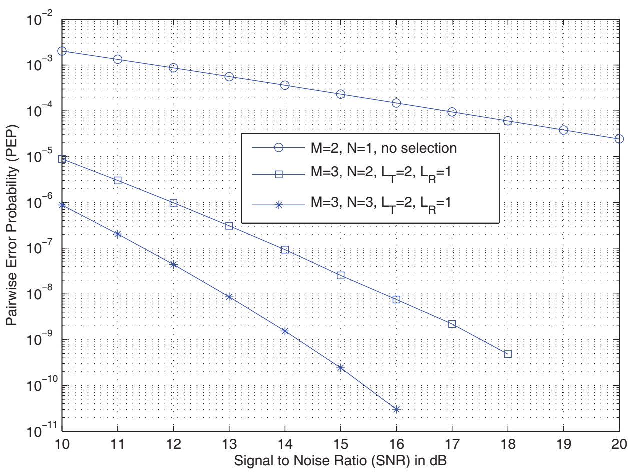

Fig. 6. PEP (from (4)) for a full rank code with joint transmit/receive antenna selection.

this case. Clearly, since we have shown that full diversity is achievable with $L_R = 1$, selecting multiple receive antennas will also achieve full spatial diversity if a full-rank STC is used.

Let us present an example. Figure 5 shows the FER plots for the system with M transmit and N receive antennas. We transmit a full rank STC, using $(5,7)_{octal}$ convolutional code, over 2 antennas. For $M = 2$ and $N = 1$, i.e. no antenna selection, this STC achieves full space diversity order of 2. With joint transmit and receive antenna selection, $L_T = 2$ and $L_R = 1$, the diversity order increases to 6 for a system with $M = 3$, $N = 2$. Figure 6 shows the PEP plots for the system with joint transmit and receive antenna selection with $L_T = 2$ and $L_R = 1$. We use the same codeword pair as provided in the previous section. With no antenna selection, the full rank code achieves a diversity order of 2 for the $M = 2$, $N = 1$ system. When $M = 3, N = 2$, the diversity order increases to $MN = 6$ and when $M = 3$, $N = 3$ the diversity order becomes 9 as expected. Obviously, the system with joint transmit and receive antenna selection results in much lower error rates although the same number of RF chains are used.

V. Antenna Selection with Rank-Deficient Space-Time Codes

Until now, we considered full-rank STCs and observed that they will achieve a spatial diversity of MN. To complete the picture, we consider the performance of rank deficient STCs with antenna selection. We consider two cases, i.e., transmit antenna selection and joint transmit/receive antenna selection, separately.

A. Transmit Antenna Selection

For rank-deficient space-time codes, when $L_T > 1$ transmit antennas are selected, the derivation of the PEP bound will follow the same lines as that of the full-rank codes, i.e., expressions (6)-(19). When a space-time code is used with rank $q = rank(B) < L_T < M$, then $(L_T - q)$ many of the λ_i terms will be zero. Therefore, Equation (15)

$$\mathcal{I}_l^{(1)} = \kappa \left(\frac{1}{\prod_{i=1, i\neq l}^{L_T} 1 + \frac{\rho\hat{\lambda}}{4L_T}} \right)^N,$$

and Equation (19),

$$\mathcal{I}_l^{(2)} = \left(\frac{1}{N!}\right)^{M-L_T} \sum_{n_1=1}^{N} \cdots \sum_{n_{NM-NL_T}=1}^{N} \frac{l_1! \cdots l_N!}{(\frac{\rho\hat{\lambda}}{4L_T}+1)^{l_1+1} \cdots (\frac{\rho\hat{\lambda}}{4L_T}+1)^{l_N+1}},$$

will be computed for only nonzero eigenvalues, where $\hat{\lambda}$ is the minimum of the nonzero eigenvalues. From the summation of the SNR exponents for $\mathcal{I}_l^{(1)}$ and $\mathcal{I}_l^{(2)}$, we see that the exponent for rank-deficient codes will be at least Nq. With a similar argument as in [1], we claim that this is the true diversity order as opposed to MN for full-rank codes. The coding gain depends on the eigenvalues of the codeword difference matrix. We also note that the derivation of lower bound on PEP will result in the same diversity order Nq (as in [1]). Therefore, we claim that the diversity order will be Nq for rank deficient STCs with transmit antenna selection.

After providing the theoretical diversity order, we now present several examples to verify our expectations. Figure 7 shows the FER for the system with a rank deficient STC using $(3,3)_{octal}$ convolutional code. With no antenna selection, for an $M = 2$ and $N = 1$ system, this STC achieves no spatial diversity, as the rank of this code is only $q = 1$. With transmit antenna selection $L_T = 2$, when $M = 5$, the diversity order still remains as 1. When $M = 3$, and $N = 2$ the diversity order becomes $qN = 2$ which corroborates our theoretical expectation. Figure 8 shows the PEP plots from the expression in (4) averaged over channel fading for the system with transmit antenna selection with $L_T = 2$. Two codeword matrices with QPSK symbols

$$\mathbf{S} = \begin{pmatrix} j & j & j & j \\ j & j & j & j \end{pmatrix} \qquad \widehat{\mathbf{S}} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix},$$

where $j = \sqrt{-1}$ and rank $q = 1$, are used in the simulations. We observe that the diversity order remains same $qN = 1$ for

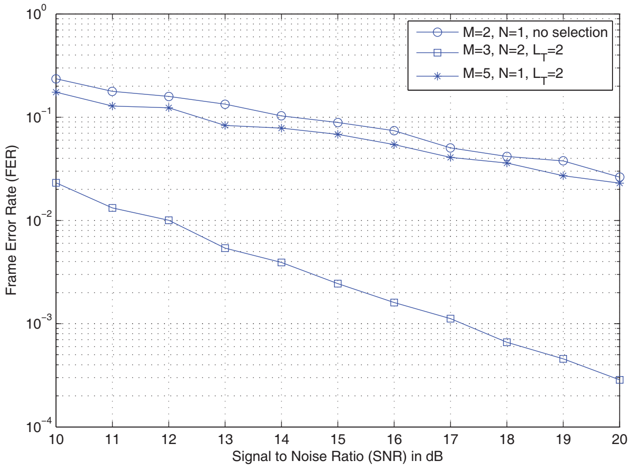

Fig. 7. FER for a rank deficient STC using $(3,3)_{octal}$ convolutional code with transmit antenna selection.

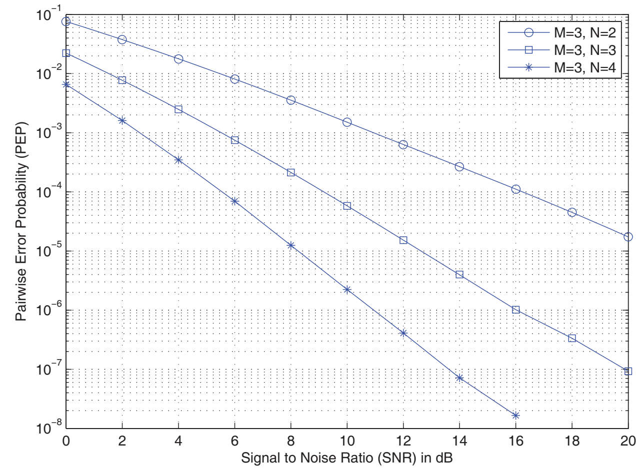

Fig. 9. PEP from (5) for a rank-deficient code with transmit antenna selection, $L_T = 2$ (for different number of receive antennas).

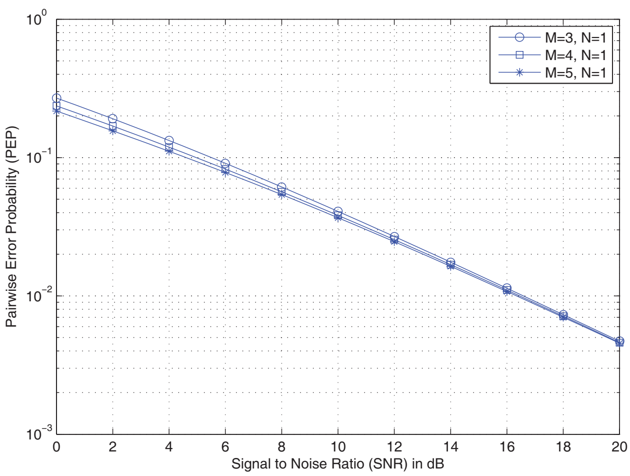

Fig. 8. PEP from (5) for a rank-deficient code with transmit antenna selection, $L_T = 2$ (for a fixed number of receive antennas, $N = 1$).

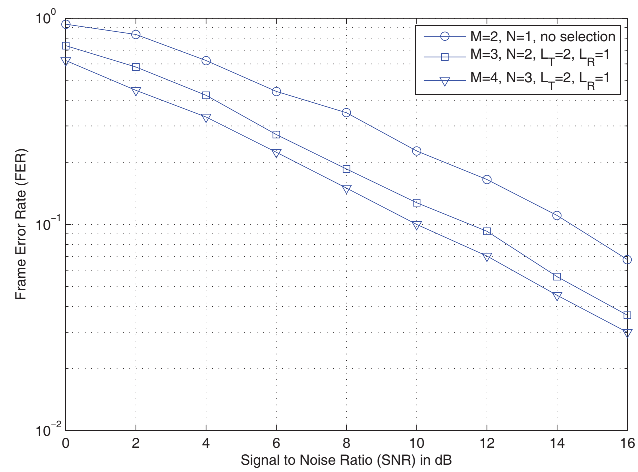

Fig. 10. FER for a rank deficient STC with joint transmit and receive antenna selection.

$N = 1$ and several different values of $M \in \{3, 4, 5\}$. Figure 9 shows the PEP plots with different number of receive antennas where only $L_T = 2$ out of $M = 3$ transmit antennas are used in actual transmission. The achieved diversity orders are 2, 3 and 4 when N is 2, 3 and 4, respectively.

B. Joint Transmit and Receive Antenna Selection

For the PEP expression in (54) which is derived for $L_R = 1$, we note that if the underlying space-time code was a rank-deficient code with rank $q < L_T$, then the diversity order would be q since $N - q$ SNR terms would disappear due to zero eigenvalues. Although we do not have a PEP expression for arbitrary $L_T > 1$ and $L_R > 1$, based on empirical results, we claim that the diversity order for rank-deficient STCs in general case of selecting $L_T \times L_R$ from $M \times N$ system is qL_R.

Figure 10 shows the FER plots for a rank-deficient STC with joint transmit and receive antenna selection where the $(3,3)_{octal}$ convolutional coded sequence is transmitted from 2 antennas. With no antenna selection, when $M = 2$ and $N = 1$, this STC achieves no diversity, since the rank of this code is only $q = 1$. With antenna selection, the diversity order still remains as $qL_R = 1$ for larger systems with $M = 3$ and $N = 2$, or with $M = 4$ and $N = 3$, as theoretically expected. Although the diversity orders are the same, obviously, when selection is done among more antennas, smaller error rates can be obtained. Figure 11 shows the PEP plots for the system with several M, N, L_Rs. We use the same rank-deficient codeword pair as in the transmit selection example. With no antenna selection, when $M = 2$ and $N = 1$, this STC achieves no diversity ($q = 1$), although the available spatial diversity is 2. When $M = 3, N = 2$, or $M = 3, N = 3$ the diversity order remains as $qL_R = 1$. For $M = 3, N = 4$ system, if $L_R = 2$ receive antennas are used, then the diversity order becomes $qL_R = 2$. Similarly, if $L_R = 3$ receive antennas are used, then the diversity order becomes $qL_R = 3$.

VI. Conclusions

In this paper, we have considered transmit and joint transmit/receive antenna selection for space-time coded MIMO systems over flat fading channels. We assumed that the antenna selection is based on maximum received SNRs. We derived PEP bounds and demonstrated that by employing antenna selection full spatial diversity can be achieved provided that

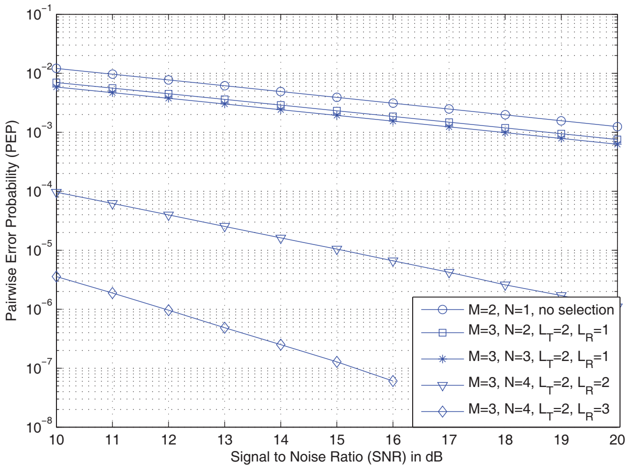

Fig. 11. PEP from (4) for a rank-deficient code with joint transmit/receive antenna selection.

the underlying STC is full-rank. Otherwise, the diversity achieved will depend on the rank of the codeword difference matrix and the selected number of antennas. These results mainly extend the results of [1] to a more complicated scenario of transmit/receive selection.

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