

Navigation of Autonomous Mobile Robots in Connected Groups

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Abstract—The navigation of autonomous mobile robots as a group is considered in this paper. Definitions adopted from the graph theory are given to characterize the robot group. A local steering strategy is proposed such that when each robot in the group applies this steering scheme, the overall result is that the whole group is displaced without losing its connectivity. This is achieved using only limited-range position sensors and without any communication between the robots.

I. INTRODUCTION

There are some tasks that are either impossible or infeasible to be accomplished by a single robot. A group of robots, rather than a single robot, are intuitively expected to be successful in performing such tasks. Indeed, one can see several species in nature which exhibit very beautiful examples of collective working. Schooling fish, flocking birds, ant and bee colonies are quite well-known for their collective working behaviors [1]-[3].

Several works are available in the literature which utilize the cooperation of autonomous agents. One of the first efforts to model species which work collectively was given for flocking birds in 1987 [4]. In that work, it is asserted that large group behaviors arise as a result of simple principles about the motion of each member of the group. An important application of this idea in discrete-time was given in 1995 [5]. Since then, the concept of cooperative motion has greatly evolved. The formation of robot groups [6]-[8] as well as the utilization of potential functions and artificial forces to accomplish a group behavior [3],[9],[10] has been widely studied. Some methodologies rely on limited communication between robots for desired group task [11],[12]. A more general discussion of the initial studies conducted in this area and concept development can be found in [13] and the references therein.

In this paper, we develop a methodology for the navigation of autonomous robots in connected groups, using concepts from graph theory. The connectivity of robot groups is first defined similarly, and a local steering strategy is built, which guarantees the connectivity of the group for the whole time of interest. We assume that the robots have no communication capability and their position sensors are of limited range. These assumptions make our work more realistic, as we form an analogy between the mobile robots and aggregating organisms in nature. There are other works in the literature too,

which make use of graph theory, such as [14]-[17]. However, as far as we know, no work is available yet that guarantees the connectivity of the robot group without using communication. In [15] and [16], it is assumed that a state vector consisting of position and velocity is measurable and every robot can sense any other robot in the group without any restrictions. These works and also the study in [18] assume group connectivity during the motion as a pre-requisite for the success of their methods. However, in the current study, a methodology is presented that results in the navigation of a robot group having dynamic topology using only limited-range position sensors with guaranteed connectivity. Therefore, our work aims to fill an important gap in this field.

This paper is organized as follows: In the following section we present the formulation of the problem based on elementary graph theory. In Section III, we propose a local steering strategy for the autonomous motion of the robots. The computational cost of the steering strategy is analyzed in Section IV and a method for the simplification of computations is presented. The proposed methodology is tested for several groups of robots in Section V. Concluding remarks and a discussion are provided in Section VI.

II. PROBLEM FORMULATION

The robots in this study are assumed to be identical in physical properties. No communication between the robots is considered. Each robot, represented by R , can move in all directions and is equipped with limited-range position sensors. The position sensors can accurately sense other robots within range. This establishes a *link* between the sensing robot and each robot in its sensing range. We assume that the sensing capability is continuous and available equally in all directions. Sensing other robots means acquiring the position information of the robots in the neighborhood. Note that this does not imply recognizing a specific robot. The robots have no ID's. Moreover, each robot does not even know its own label.

A number of such mobile robots constitute a group. The problem studied in this work is the navigation of an autonomous mobile robot group. We use a graph-theoretic approach to formulate the problem. The following definitions are adopted from graph theory. The interested reader is referred to standard textbooks such as [19] or [20] for details.

Definition 1: A **group** \mathcal{G} is a set of autonomous mobile robots $\{R_i, i = 1, \dots, N\}$, that can be connected by links.

Definition 2: The group \mathcal{G} is **connected** if there is a path from any robot to any other robot in the group through links. A group which has at least one pair of robots having no path inbetween is **disconnected**.

The robots' position sensors are assumed to provide continuous and exact position information in all directions in a range denoted by d_{max} . That is, each robot can continuously scan its surrounding space (a sphere of radius d_{max}). Since we assume that this sensing range is limited and the total number of robots in the group can be large, a robot may not sense all other robots in the group. Based on this, we define subgroups as follows.

Definition 3: A **subgroup** \mathcal{S}_i is a group of robots sensed by the robot R_i .

Since there are N robots in the group, by definition, there are N subgroups, i.e. one for each robot. \mathcal{S}_i has a spherical shape with R_i lying at its center. If R_i cannot sense any other robots in the group, then \mathcal{S}_i is an empty set. Obviously, a necessary condition for the connectivity of a group is that none of the subgroups is an empty set. On the other hand, letting the largest distance between the robots in \mathcal{G} be denoted as D_{max} , \mathcal{G} will be connected according to Definition 2, if $d_{max} \geq D_{max}$. However, one faces nontrivial and more interesting cases whenever $d_{max} \ll D_{max}$, which corresponds to robot groups of a relatively large number of individuals having limited sensing ranges.

Fig. 1 depicts a group consisting of three robots. It is seen that $R_2 \in \mathcal{S}_1$, and $R_1 \in \mathcal{S}_2$. This establishes the links C_{12} and C_{21} , respectively. The links between R_2 and R_3 are formed likewise. Note that the robot R_2 has the position information of both R_1 and R_3 , but R_1 and R_3 cannot sense each other, as the distance between R_2 and R_3 is larger than d_{max} .

The group is required to navigate from a starting location to a target location. We assume that the location of the target is unknown to all group members except one robot. This robot is assigned as the *leader* of the group and denoted as R_L . The leader has the same physical properties and capabilities as the other robots. The only difference is that the direction of the target is given to R_L so that it can determine its movement using this target information. However, in this study, the leadership is *hidden*. None of the robots recognize the leader as a distinguished group member. In other words, if R_L is sensed by a robot R_j , i.e. $R_L \in \mathcal{S}_j$, R_j can only see it as one of its neighbors and the leadership of R_L does not affect the local steering strategy of R_j . In the remaining part of the paper, we will consider the group of N robots as one leader, R_L , and $N - 1$ as *followers*, R_j , where $j = 1, \dots, N - 1$.

Using these definitions and assuming that a set of robots initially form a connected group; our objective now is to develop a decentralized steering methodology that yields efficient navigation of the group while preserving its connectivity in the sense of Definition 2.

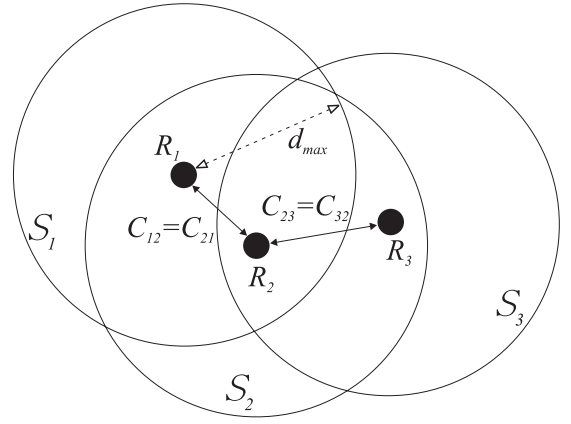


Fig. 1. Illustration of a robot group with three robots and subgroups

III. AUTONOMOUS MOTION

At any time t , let $t + \Delta t$ be the next sampling time, where $\Delta t > 0$ is the interval at which a robot senses the positions of other robots in its range. We will denote the position of a robot R_i at time t , as $X_i(t)$, $i = 1, \dots, N$. Since all robots in the group steer autonomously, we will set up local moving rules for each robot. Here, we propose a local steering strategy that is inspired by the preliminary study of Reynolds in [4]. At each time t , while the leader R_L aims in a given global target direction, each follower robot R_j , $j = 1, \dots, N - 1$, acquires the positions of other robots in its sensor range, i.e. in the subgroup \mathcal{S}_j , and determines a local target location for itself. This is most conveniently described in terms of the local coordinates of R_j , with R_j being at the origin. Let us denote the position vector in local coordinates as $x(t)$. We will use a notation such that the superscripts in x relate the coordinate frame to a robot, and the subscripts in x indicate which robot's position it is. For example, x_k^j represents the position vector of R_k in the coordinate frame of R_j . By definition, the local position vector x_j^j of R_j in its own coordinate frame is zero. For the robots in \mathcal{S}_i , $i = 1, \dots, N$, we have

$$x_k^i(t) \stackrel{def}{=} X_k(t) - X_i(t), \quad k = 1, \dots, M \quad (1)$$

where M is the number of robots in \mathcal{S}_i . Then, we propose the following local steering strategy.

Local Steering Strategy: At each sampling time t , each robot in the group computes a target location for the time $t + \Delta t$ and moves towards that location, such that,

- 1) for each follower robot R_j , $j = 1, \dots, N - 1$, the target location $x_j^j(t + \Delta t)$ minimizes a cost function, $J(x_j^j(t + \Delta t))$, derived from the positions of the robots in \mathcal{S}_j ,
- 2) for the leader R_L , the target location is always towards the given global target,

and for both R_L and the follower robots R_j , the movement is subject to

$$\|x_i^i(t + \Delta t)\| \leq \frac{1}{2}(d_{max} - \max_k \|x_k^i(t)\|) \quad (2)$$

where $i = 1, \dots, N$ and $x_k^i(t)$ is as defined in (1).

Note that both $x_k^i(t)$ and $x_k^i(t + \Delta t)$ are positions in coordinates of the local reference frame attached to R_i at time t . The position that R_i is aiming at, i.e. $x_k^i(t + \Delta t)$, is restricted by an upper bound in (2). This upper bound is incorporated to guarantee the connectivity of the group.

Several types of cost functions can be used in implementing the local steering strategy. One example of these cost functions may be given for each follower robot R_j as

$$J(x_j^j(t + \Delta t)) = \max_k \|x_j^j(t + \Delta t) - x_k^j(t)\|,$$

which makes each robot try to decrease the distance to the farthest robot that it senses. Another possible approach could be to force the robots to keep their distances with the robots in their subgroups as close to a desired distance as possible. Let us denote the desired distance as d_0 ($d_0 \leq d_{max}$). Then we can define a suitable cost function for each follower robot R_j , $j = 1, \dots, N - 1$, as

$$J(x_j^j(t + \Delta t)) = \sum_{k=1}^M \left(\|x_j^j(t + \Delta t) - x_k^j(t)\| - d_0 \right)^2. \quad (3)$$

Regarding the connectivity of the group we can state the following theorem.

Theorem 1: Consider a group \mathcal{G} of N autonomous mobile robots which are connected at $t = 0$ as defined in Definition 2. If the robots in the group move according to the Local Steering Strategy defined above, the group preserves its connectivity for $t > 0$.

Proof: Let R_a and R_b be any two robots within their mutual sensing range, that is, $R_a \in \mathcal{S}_b$ and $R_b \in \mathcal{S}_a$ at time t . Let M_a and M_b be the number of robots in \mathcal{S}_a and \mathcal{S}_b , respectively. From (2), we have

$$2\|x_a^a(t + \Delta t)\| + \max_k \|x_k^a(t)\| \leq d_{max}, \quad k = 1, \dots, M_a \quad (4)$$

and

$$2\|x_b^b(t + \Delta t)\| + \max_l \|x_l^b(t)\| \leq d_{max}, \quad l = 1, \dots, M_b. \quad (5)$$

Noting that $\max_k \|x_k^a(t)\| \geq \|x_b^a(t)\|$, $\max_l \|x_l^b(t)\| \geq \|x_a^b(t)\|$, and $\|x_b^a(t)\| = \|x_a^b(t)\|$, it follows from (4) and (5),

$$\|x_a^a(t + \Delta t)\| + \|x_b^b(t + \Delta t)\| + \|x_b^a(t)\| \leq d_{max}. \quad (6)$$

Further, by triangular inequality, we get

$$\|x_a^a(t + \Delta t) - (x_b^a(t) + x_b^b(t + \Delta t))\| \leq d_{max}. \quad (7)$$

Note that the term $x_b^a(t) + x_b^b(t + \Delta t)$ is the position of R_b at time $t + \Delta t$ as expressed in the local coordinate frame attached to R_a at time t . Therefore, (7) shows that the distance between the robots R_a and R_b will not be larger than d_{max} . This holds for any robots R_a and R_b providing $R_a \in \mathcal{S}_b$ and $R_b \in \mathcal{S}_a$. Hence, any two robots sensing each other at time t will still be linked when they moved to their locations at $t + \Delta t$; that is, they will *stay connected*. Since group connectivity is formed through those robots which sense each other, and the group is connected at $t = 0$, it will also be connected for $t > 0$. ■

Remark 1: According to Theorem 1, the robots in the group can move *simultaneously*. However, another possibility is to assign certain time slots to each robot so that only one robot can move in the group at a time. This *sequential* movement can be achieved without communication, using synchronized clocks. If this is applied, then the constraint on the motion given in (2) can be relaxed so that

$$\|x_i^i(t + \Delta t)\| \leq d_{max} - \max_k \|x_k^i(t)\| \quad (8)$$

For such a sequential motion, Theorem 1 still holds with the bound in (8).

Remark 2: For both types of coordinated (sequential or simultaneous) motion of the robots, each subgroup \mathcal{S}_i will keep the robots that are initially in \mathcal{S}_i . So, the number of robots in \mathcal{S}_i will not decrease; nevertheless, it may grow as new robots appear in the sensing range of R_i and hence, become a member of \mathcal{S}_i .

IV. COMPUTATIONAL REMARKS

The robots in this study are supposed to be quite simple and limited devices, especially from the computational point of view. Our purpose is to provide a decentralized control methodology which can be applied to such simple robots, yet also lead to a satisfactorily good group navigation. Below, we propose an iterative method to reduce the computational burden in the implementation of the Local Steering Strategy.

The minimum points of the cost function given in (3) are the locations which each follower robot R_j aims to reach at each sampling time. The minimization of (3) requires higher computational power as the number of robots in \mathcal{S}_j increases.

When there is only one robot, say R_m , in a subgroup \mathcal{S}_j , the solution is a circle in \mathbb{R}^2 or a sphere in \mathbb{R}^3 . In this case, the solution set has infinite number of points which minimize J , but the robot R_j selects the nearest point in the solution set. This leads to the movement of R_j in the line which connects R_j to R_m . The movement direction is either towards R_m if the distance between R_j and R_m is larger than d_0 , or away from R_m if the distance is smaller than d_0 . Obviously, no movement is required if R_j is already on an optimal point at that time.

When there are only two robots, say R_m and R_n , in \mathcal{S}_j , one may consider different cases as far as the number of the optimal points is concerned. If the distance between R_m and R_n is larger than or equal to $2d_0$, the optimal point is unique both in \mathbb{R}^2 and \mathbb{R}^3 , and is at the center of the line segment which connects R_m and R_n . Otherwise, in \mathbb{R}^2 , there are two optimal points which lie symmetrically at each side of the line connecting R_m and R_n . The robot R_j selects the nearest point as the local target. In \mathbb{R}^3 , the number of the optimal points are infinite and they lie in a circle whose center is the center of the line segment connecting R_m to R_n . This line segment is also normal to the circle of solution points. Again, the robot R_j aims at the point on this circle which is nearest to itself.

It is clear that, the computation task is relatively easy when only one or two robots are sensed by each follower robot at a time. However, whenever three or more robots are in

a subgroup \mathcal{S}_j , the minimization of J given by (3) requires the solution of a set of nonlinear equations. At the extremum points, we have

$$\frac{\partial J}{\partial x_j^j} = \sum_{k=1}^M \frac{\partial}{\partial x_j^j} \left(\|x_j^j(t + \Delta t) - x_k^j(t)\| - d_0 \right)^2 = 0 \quad (9)$$

which results in a nonlinear set of equations. The solution of the system in (9) may not be unique. After solving this system, whether or not the solution points are a minimum must then be tested. If more than one minimum is present, we find the global minimum by evaluating the cost function given in (3) at these points.

It should be noted that the optimal points are computed for each follower robot R_j at every sampling time. The location of the optimal points depends on the positions of the robots in the subgroup \mathcal{S}_j . Since the sensed robots in \mathcal{S}_j also move autonomously, the location of the computed optimal points will change at each time. This change will very likely happen before they are reached by R_j . Hence, the solution will provide only a direction to the optimal points, because the solution will change before R_j gets to that location.

This fact gives us the chance to decrease the computational burden in the Local Steering Strategy. Rather than solving for the minimum points of the cost function, each robot R_j can move in the direction of the negative gradient of the cost function, evaluated at the position of R_j for each time. That is,

$$x_j^j(t + \Delta t) = x_j^j(t) - \gamma \left. \frac{\partial J(x_j^j(t + \Delta t))}{\partial x_j^j(t + \Delta t)} \right|_{x_j^j(t + \Delta t) = x_j^j(t)} \quad (10)$$

where $\gamma > 0$ is a positive gain, and x_j^j is the position vector of R_j in its local coordinates. From (3) it follows that

$$\begin{aligned} \frac{\partial J(x_j^j(t + \Delta t))}{\partial x_j^j(t + \Delta t)} &= 2 \sum_{k=1}^M \left(\|x_j^j(t + \Delta t) - x_k^j(t)\| - d_0 \right) \\ &\quad \times \frac{x_j^j(t + \Delta t) - x_k^j(t)}{\|x_j^j(t + \Delta t) - x_k^j(t)\|}. \end{aligned} \quad (11)$$

Further, since $\dot{x}_j^j(t) = 0$, from (10) and (11) we obtain

$$x_j^j(t + \Delta t) = 2\gamma \sum_{k=1}^M \left(\|x_k^j(t)\| - d_0 \right) \frac{x_k^j(t)}{\|x_k^j(t)\|}. \quad (12)$$

The gain γ in (12) is adjusted to assure the constraint given by (2). The application of (12) is much simpler than solving the system in (9). It gives the direction of the next movement and the movement in this direction is applied only if it satisfies the inequality in (2). In the next section, we will test the proposed methodology with the local steering of robots in the direction of the negative gradient, as given in (10).

V. SIMULATION RESULTS

We verify the theorems and theoretical results of previous sections by computer simulations. The simulations are performed in MATLAB and the working space is a section of

xy -plane in \mathbb{R}^2 . The sensor range (d_{max}) is 8 units, and desired distance (d_0) is 5 units. The following scenario is applied: A group of robots start navigation under the Local Steering Strategy. The local steering is achieved by the use of (12) with the gain $\gamma = 0.01$. Total simulation length is 2600 iterations. The trajectory to be followed by the leader is piecewise linear having turns at the 400, 800 and 2200th-iterations. The trajectory of the leader is seen by the dashed lines on the figures.

In the first part of the simulations, four robots are present: One leader and three follower robots. The result is seen in Fig. 2. Initially, the leader is sensed only by the robot at $(-14, -6)$, and the robot at $(-22, 4)$ senses only the robot at $(-19, 0)$. The connectivity of the group is preserved throughout the navigation. In Fig. 3, the same group navigates sequentially with synchronized clocks. Towards the end of the simulation, all robots sense at least two robots; this increases the connectivity of the group.

In the second part of the simulations, we include 17 robots. This is important to show the efficiency of our methodology for large robot groups. The sensor range (d_{max}) and desired distance (d_0) are taken as 12 and 6 units, respectively, in this case. The navigation of the robots is seen in Fig. 4. Since the number of robots is high, we analyze this navigation in three parts. Fig. 4a reflects the first 800 iterations of the simulation. Although the robots are initially distributed over a rather large area, the algorithm gathers them in the very beginning of the simulation. However, the leader looks to be a little bit apart from the group. When the sensor range is large, the number of sensed robots increases. If the leader and the rest of the group are on separate sides of a fictitious plane, the robots just behind the leader are affected by the other robots in their subgroup so that the negative gradient of the cost function is not towards the leader but towards the rest of the group. For that reason, the leader adjusts its motion according to the group that it leads. When the group lags behind, the leader waits for the group to preserve the connectivity. Fig. 4b depicts the view at about the 1200th-iteration and Fig. 4c shows the positions of all robots at the end of the simulation. Only the trajectory of the leader is provided in these figures, and the trajectories of the follower robots are omitted in order not to make the figures too complex.

Fig. 5 displays the connectivity of the group of 17 robots during the navigation. Initially, the group starts navigation with a connectivity formed of 59 links. The number of links increases gradually as the navigation continues. After the sharp turn in the trajectory of the leader at the 800th-iteration, the number of links reaches a value of 136. It is clearly seen that the connectivity is increased during the navigation with the proposed scheme. Also note that for the case of 17 robots, the maximum possible number of links in the group is $17 \cdot (17 - 1) / 2 = 136$. Hence we can say that in this simulation the methodology has provided a navigation with maximum connectivity. However note that it cannot always be possible to reach the maximum possible number of links in the group. Larger number of the robots and desired distances inbetween

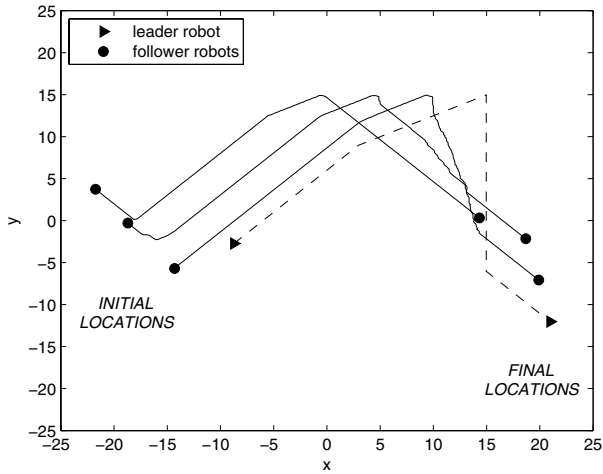
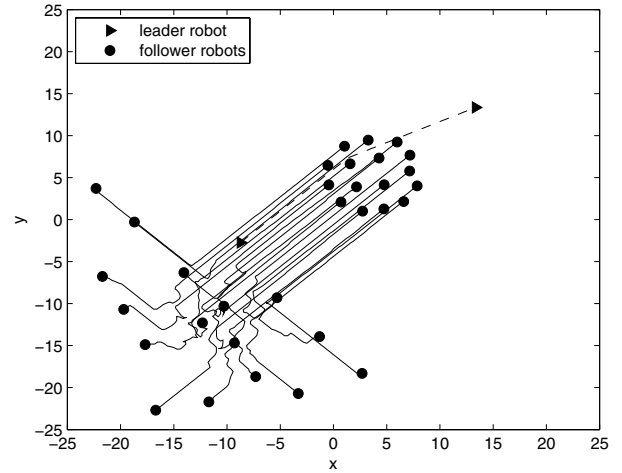


Fig. 2. Navigation of 4 robots



(a)

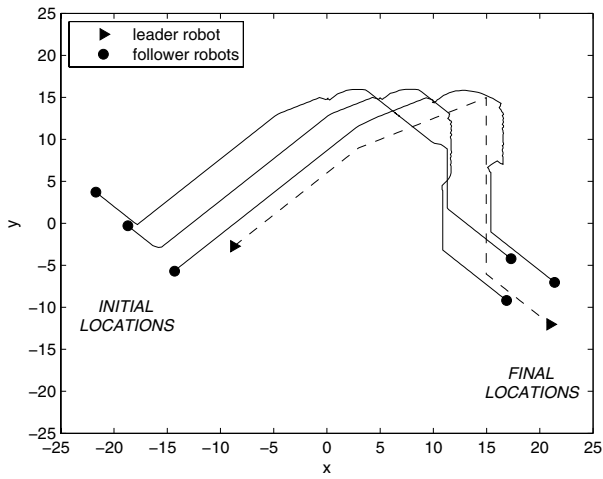
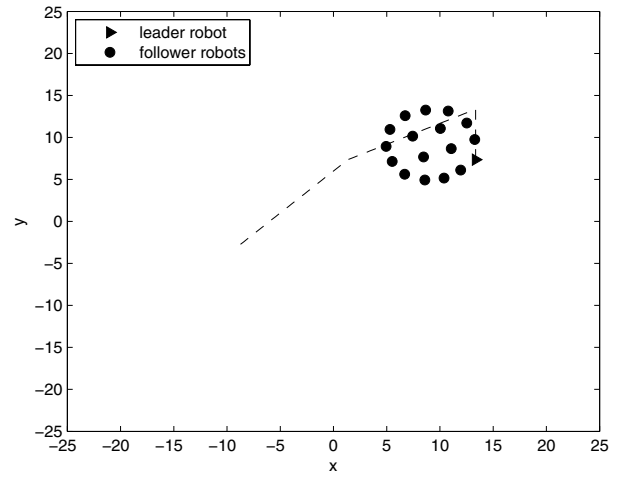


Fig. 3. Navigation of 4 robots (sequential movement, see Remark 1)



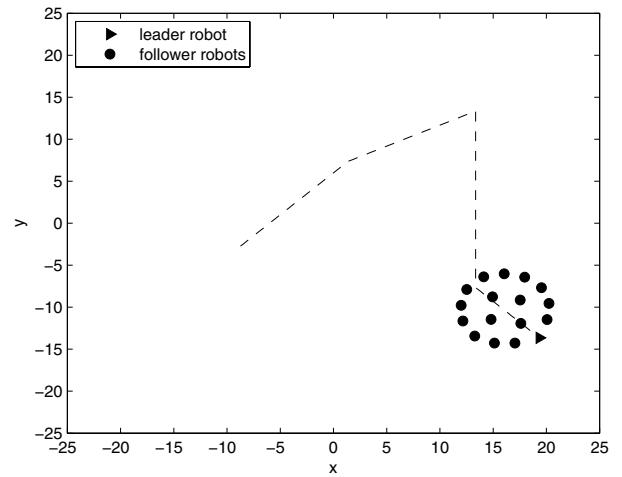
(b)

them and smaller sensor ranges may result in a connectivity with a number of links less than the maximum possible value.

VI. CONCLUSIONS

The connectivity of mobile robot groups has been studied in this work. Whenever there is neither a central control mechanism nor communication between the robots, the autonomous motion of mobile robots can break up group connectivity. We have proposed a local steering strategy to preserve group connectivity throughout navigation that has been inspired by some animal groups such as schooling fish and flocking birds. In the proposed methodology, there is a certain optimal distance that each robot tries to maintain between itself and the other robots it senses. The movement of the robots is restricted by fixed bounds so that the connectivity of the group is guaranteed.

The simulations have verified the success of the methodology. The groups have not only preserved but also increased



(c)

Fig. 4. Navigation of 17 robots

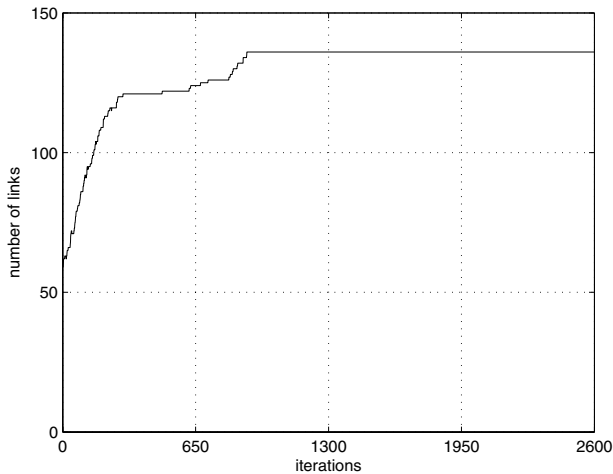


Fig. 5. Connectivity of the group of 17 robots

connectivity during navigation, for both the simultaneous motion and sequential motion cases.

The relation of the desired distance d_0 to d_{max} is of interest. When $d_{max} \gg d_0$, the number of sensed robots in each subgroup becomes larger and this increases the tendency of each follower robot to track the rest of the follower robots rather than the leader. Our steering strategy gives its best results when $0.5 d_{max} \leq d_0 \leq 0.9 d_{max}$.

The upper bound for the motion given in (2) may cause a special case in which all the robots in the group go into *deadlock*. This pathological case occurs especially if the initial distances between all the robots are as large as d_{max} . However, Theorem 1 still holds in this case. To avoid this deadlock case, one may relax the magnitude constraint in (2) for the robots that are at the *corners* of the subgroups.

The proposed methodology does not address collision avoidance, as it arises naturally as a result of the Local Steering Strategy. As long as $d_0 > 0$, that is, the desired distance is nonzero, the strategy provides each robot with local targets that are neither too far nor too close to neighbor robots.

This study has provided a simple yet reliable methodology for the navigation of mobile robots in connected groups. The fact that no communication or hierarchy among the robots is required allows new members to be accepted to the group very easily. Similarly, the departure of some members from the group does not cause any problems for the rest of the group as long as they do not break the overall connectivity. Further studies may focus on loosening the restrictions imposed in the local steering strategy without breaking the connectivity and may consider obstacles in the paths of the robots. The methodology can also be combined with a formation control mechanism.

REFERENCES

[1] A. Okubo, "Dynamical aspects of animal grouping: Swarms, schools, flocks, and herds", *Advanced Biophysics*, vol. 22, pp. 1-94, 1986.

[2] K. Warburton and J. Lazarus, "Tendency-distance models of social cohesion in animal groups", *J. Theoretical Biology*, vol. 150, pp. 473-488, 1991.

[3] V. Gazi and K. M. Passino, "Stability analysis of swarms", *IEEE Trans. Automatic Control*, vol. 48, pp. 692-697, 2003.

[4] C. Reynolds, "Flocks, birds, and schools: A distributed behavioral model", *Computer Graphics*, vol. 21, pp. 25-34, 1987.

[5] T. Vicsek, A. Czirok, E. Ben-Jacob, I. Cohen and O. Shochet, "Novel type of phase transition in a system of self-driven particles", *Phys. Rev. Letters*, vol. 75, pp. 1226-1229, 1995.

[6] T. Balch and R. C. Arkin, "Behavior-based formation control for multirobot teams", *IEEE Trans. Robotics and Automation*, vol. 14, pp. 926-939, 1998.

[7] H. Yamaguchi, "A cooperative hunting behavior by mobile-robot troops", *Int. J. Robotics Research*, vol. 18, pp. 931-940, 1999.

[8] M. Egerstedt and X. Hu, "Formation constrained multi-agent control", *IEEE Trans. Robotics and Automation*, vol. 17, pp. 947-951, 2001.

[9] J. H. Reif and H. Wang, "Social potential fields: A distributed behavioral control for autonomous robots", *Robotics and Autonomous Systems*, vol. 27, pp. 171-194, 1999.

[10] N. E. Leonard and E. Fiorelli, "Virtual leaders, artificial potentials and coordinated control of groups", in *Proc. Conf. Decision and Control*, Orlando, FL, Dec. 2001, pp. 2968-2973.

[11] D. J. Stilwell and B. E. Bishop, "Platoons of underwater vehicles", *IEEE Control Systems Magazine*, vol. 20, pp. 45-52, 2000.

[12] A. V. Savkin, "The problem of coordination and consensus achievement in groups of autonomous mobile robots with limited communication", *Nonlinear Analysis*, vol. 65, pp. 1094-1102, 2006.

[13] L. Bayındır ve E. Şahin, "A review of studies in swarm robotics", *TÜBİTAK Turkish J. Electrical Eng. & Computer Sciences*, vol. 15, pp. 115-147, 2007.

[14] J. P. Desai, J. Ostrowski and V. Kumar, "Modeling and control of formations of nonholonomic mobile robots", *IEEE Trans. Robotics and Automation*, vol. 17, pp. 905-908, 2001.

[15] H. G. Tanner, A. Jadbabaie ve G. J. Pappas, "Stable flocking of mobile agents, Part I: Fixed Topology", in *Proc. Conf. Decision and Control*, Maui, Hawaii USA, Dec. 2003, pp. 2010-2015.

[16] H. G. Tanner, A. Jadbabaie ve G. J. Pappas, "Stable flocking of mobile agents, Part II: Dynamic Topology", in *Proc. Conf. Decision and Control*, Maui, Hawaii USA, Dec. 2003, pp. 2016-2021.

[17] Z. Lin, M. Broucke ve B. Francis, "Local control strategies for groups of mobile autonomous agents", *IEEE Trans. Automatic Control*, vol. 49, pp. 622-629, 2004.

[18] A. Jadbabaie, J. Lin ve A. S. Morse, "Coordination of groups of mobile autonomous agents using nearest neighbor rules", *IEEE Trans. Automatic Control*, vol. 48, pp. 988-1001, 2003.

[19] W. Mayeda, *Graph Theory*, New-York: John Wiley & Sons, 1972.

[20] W. Kocay ve D. L. Kreher, *Graphs, Algorithms and Optimization*, Boca Raton: Chapman & Hall/CRC Press, 2005.